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Title: NON-DATA AIDED PARAMETRIC BASED CARRIER FREQUENCY ESTIMATORS FOR BURSTY GMSK COMMUNICATION SYSTEMS

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Estimating the carrier frequency from a modulated waveform is one of the most important functions of a coherent signal receiver. Good performance and low bit error rates are obtained by coherent demodulation. Therefore, exact knowledge of the received signal carrier frequency is critical for communication systems. Also due to the spectral crowding, a high probability of channel interference can be observed. Under moderate carrier frequency offsets, data-aided estimators have been developed which have a high accuracy of estimation. However, for high frequency offsets the frequency estimator does not have the information of data or timing. In this thesis we propose a parametric based carrier frequency estimation of GMSK, which has improved performance over ad-hoc methods (delay and multiply) and has high resolution capability.

In this thesis three methods are implemented over GMSK data to improve the performance and their results compared with the standard delay and multiply method. Two of these methods are parametric based estimators and one is a fast frequency estimator. Parametric based estimators were chosen partly due to their high resolution capabilities and mainly for their proven performance. Parametric based estimators were seen to have high computational load, and hence an alternate fast frequency estimator was implemented. The tradeoffs involved with respect to computational load and performance were shown.

The contributions of this thesis include the verification of the validity of applying a parametric based approach on GMSK data, and compare the performances of parametric methods and fast frequency estimator. It is shown that such an approach has a better performance compared to non-data aided ad-hoc delay and multiply methods. A closed loop configuration of the open loop parametric methods is suggested in the end.

NON-DATA AIDED PARAMETRIC BASED CARRIER FREQUENCY ESTIMATORS FOR BURSTY GMSK COMMUNICATION SYSTEMS

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Ajay Kandukuri, Author

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TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

Page

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Estimating the carrier frequency plays an important role in communication receivers. This research investigates the Carrier Frequency measurement for burst mode applications, which have short data bursts. The concentration is on the continuous phase modulation (CPM) which is a popular modulation format and has widespread usage in communications. This is due to its attractive spectral efficiency and constant modulus property. The data available to estimate the frequency is limited under burst mode assumption, and improved frequency estimators are necessary for faster lock. The digital frequency estimators are broadly classified as data aided and non-data aided estimators. Data aided estimators make use of the signal timing or data information to achieve low variance estimates. Non-data aided techniques on the other hand do not use any advanced information on the timing and data information. These estimators on the other hand exhibit poorer performance with respect to data aided methods. The case in study focuses on the estimation of large carrier frequency offsets. At large frequency offsets of the order larger than 0.1 percent of the symbol period, extracting the timing information or data information is extremely difficult. Under these circumstances non-data aided techniques are preferable. A refinement approach is proposed as a means to improve the performance of a non-data aided delay and multiply method most often used in ad-hoc estimation [12].

Parametric frequency estimation problems have been studied extensively before [2] [14] and their superior frequency resolution for short data records is well established. These estimators due to their high resolution have also the added advantage of estimating the dominant frequency while rejecting any interference. In this thesis we improve the performance by using eigenvalue based frequency estimation over a delay and multiply method, and compare the performance of signal subspace methods and noise subspace methods, along with their advantages over delay and multiply methods. The aim of this work is to implement an improved estimator for CPM formats with improved performance compared to ad-hoc estimators. The case in study would be a bursty transmission of GSM data. GSM stands for global system for mobile Communications, is a popular standard used extensively today. GMSK (Gaussian Minimum Shift keying) is a popular modulation type, which is used in the GSM standard. The algorithm developed can be also implemented on other continuous phase modulation formats as very few restrictions are placed on the format while deriving our structure. The restrictions placed have been verified to hold for MSK (Minimum Shift Keying) also, but other CPM modulations have not been checked. The research focuses on a digital algorithm as such algorithms can be implemented on a DSP chip. MATLAB and simulink are used for the simulations that verify the analytic results.

1.2 MOTIVATION

The goal of this research is to come up with a digital algorithm for carrier recovery in burst transmissions. The ability to measure the carrier when the local carrier offset varies over a wide range compared to the symbol period is important for the case of not so good oscillator stabilities. Added to this problem is the non availability of data symbols or timing information in the case of large carrier frequency offsets. In such instances the frequency recovery has to be done without the aid of timing and data symbols. Such techniques are broadly classified as non-data aided methods. An ad-hoc delay and multiply method was proposed [12] under the above constraints for carrier estimation. As discussed above we will try to improve the performance of the delay and multiply algorithm for the case of continuous phase modulations under the above assumptions.

1.3 GSM OVERVIEW

GSM is the most popular standard in the world. It uses a binary GMSK modulation with a BT=0.3, where B is the 3db bandwidth of the Gaussian filter and T is the symbol period. GSM uses a combination of time division and frequency division multiplexing. The FDMA part involves the division by frequency of the total 25 MHz bandwidth into 124 carrier frequencies of 200 kHz bandwidth each. One or more carrier frequencies are assigned to each base station. Each of these carrier frequencies is then divided in time, using a TDMA scheme, into eight time slots.

Each time slot has a duration of 4.615 ms. The unmodulated carrier is transmitted in 1 time slot of the frame every 10 frames.

1.4 RESEARCH OVERVIEW

Normally a phase locked loop is used to recover a modulated signal carrier frequency. Phased lock loops have very slow acquisition capabilities and need some kind of an aid to acquire a signal faster. A frequency ramp provided to the VCO central frequency could speed up the process, but would require longer data intervals before it can lock. A high rate of frequency ramp would help in reducing estimation times and there by lock on with limited data. But a very large rate as required for large frequency offsets and short data records would make it lose lock and is not feasible. When considering burst transmissions, the window of data to acquire lock is very small. Hence, alternate methods have to be looked into for this purpose. Digital algorithms provide a good means of estimation in such cases. With the advent of high speed DSPs, it has become possible to work with large frequency offsets in the discrete time domain. Apart from the above constraints the over crowding of the frequency spectrum introduces a high probability of co-channel interference (CCI). Hence an estimator must also be good at rejecting such interference and avoiding false lock.

In GSM as we have seen above, an unmodulated carrier is transmitted every 10 frames in 1 time slot. If the carrier is tracked while the data is being transmitted, the estimate could be used during the unmodulated carrier slot to make a quick and accurate measurement. This would reduce the acquisition time during that slot as the carrier is being constantly tracked while the data is being transmitted and would be in close proximity of the actual value when the unmodulated carrier is sent.

CHAPTER 2

GMSK BURSTY TRANSMISSION

2.1 INTRODUCTION

Modulation is a means by which information is encoded into signals. Carrier modulation is often used for wireless communications due to many reasons. It involves translating the baseband message signal to a higher frequency by either multiplying it with a sinusoid at that frequency or by inserting it into either the phase or the frequency of a sinusoidal carrier. These carrier modulations can be classified into two types, namely linear and non-linear modulations. Modulation schemes that have amplitude modulated (AM) components like BPSK and QPSK fall under the linear modulations category. Constant envelope modulations such as MSK and GMSK fall under the non-linear modulation category. Linear modulations have good spectral efficiencies but they need highly linear RF power amplifiers before transmission. As linear amplifiers are expensive and inefficient, an alternative is to use highly efficient non-linear amplifiers. Non-linear modulations which have constant envelope are less sensitive to the amplifier non-linearities.

2.2 CONTINUOUS PHASE MODULATION (CPM)

Continuous phase modulation is a non-linear modulation where the information of the signal is encoded into the phase or frequency of the carrier. The advantages of

CPM can be demonstrated with the help of a simple example. A QPSK modulated signal would undergo phase transitions of 180° for some bit periods depending on the QPSK mapping. An OQPSK signal on the other hand undergoes phase transitions of 90° for every bit period. This results in a reduction in out of band interference. This suggests that the out of band interference can be further suppressed by reducing the discontinuous phase transitions in the signal. Since the data information is incorporated into the phase of the signal, CPM signals have a constant envelope nature. As discussed before, this is very useful when dealing with non-linear amplifiers. These advantages led to the many CPM formats being designed and deployed in the many communications systems around the world today. The only draw back of the CPM formats is increased inter symbol interference (ISI).

The complex envelope of a CPM signal has the form [5]

$$s(t,\alpha) = e^{j\psi(t,\alpha)}$$

where

$$\psi(t,\alpha) = 2\pi h \sum_i \alpha_i q(t - iT)\,. \tag{2.2.1}$$

In the above equation h is the modulation index, T is the symbol interval and α_i are the encoded data belonging to the binary alphabet. $q(t)$ is the phase response of the system given by the relationship[4]

$$q(t) = \int_{-\infty}^{t} f(\tau)d\tau\,. \tag{2.2.2}$$

The pulse $f(\tau)$ is time limited to the interval (0, LT) and satisfies the conditions

$$f(t) = f(LT - t), \tag{2.2.3}$$

$$\int_0^{LT} f(\tau)d\tau = q(LT) = \frac{1}{2}. \tag{2.2.4}$$

The signal formats above, for which L=1 are called full response formats and for L>1 are called partial response formats.

2.3 GMSK

MSK (Minimum Shift Keying) is a popular CPM modulation format because of its spectral efficiency, constant envelope and ease of implementation. MSK uses a modulation index $h=0.5$ and $L=1$. GMSK which is used in GSM is one of the many CPM formats. GMSK is in essence an extension of MSK, where increase in suppression capabilities of out of band interference is traded off with the bit error rates. A GMSK also has a modulation index $h=0.5$ but L is infinity. But, for all practical purposes L could be considered to be equal to 4 [15]. The out of band suppression is achieved by using a pre-modulation linear filter. When the filter is a Gaussian filter, the output waveform becomes a GMSK wave. This wave is obtained by feeding the NRZ (non return to zero) data signal to a Gaussian filter whose impulse response is given by [1]

$$h(t) = \sqrt{\frac{2\pi}{\ln 2}} B \exp\left(-\frac{2\pi^2 B^2 t^2}{\ln 2} \right) \tag{2.3.1}$$

It should be noted that the filter will introduce considerable ISI (inter symbol interference) [8] as this makes the adjacent symbol pulses overlap one another. The tail of one pulse extends into adjacent pulse intervals interfering in the detection of that symbol. For GSM a BT of 0.3 is used as trade off between ISI and BER (Bit Error Rate) performance, where B is the 3db bandwidth of the Gaussian filter and T is the symbol time period. The presence of the pre-modulation filter spreads the pulse over a time greater than T (pulse interval) that is L>1. Hence GMSK is a partial response signal. The real part of the complex baseband GMSK signal for BT=0.3 is given in the figure below.

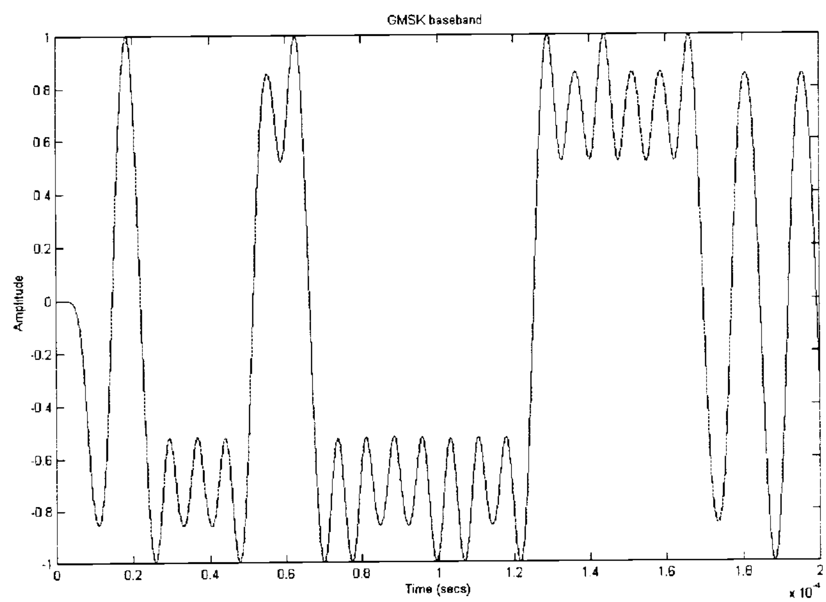

Figure 2-1 GMSK baseband signal

2.4 GSM CHANNEL STRUCTURE

As stated before, GSM is a popular global standard in wireless communications. It uses a combination of time and frequency division multiple access. GSM uses two frequency bands of 25 MHz. One is a forward link and the other is a reverse link. Each 25 MHz band is frequency division multiplexed into 124 carrier frequencies of 200 kHz bandwidth. Each base station is allotted one more carrier frequencies. Each of the carrier frequencies is divided into 8 time slots using TDMA. One slot is used for the transmission and one for the reception. This is done so that the mobile station would not transmit and receive at the same time. The structure of the time slot is shown in the figure below

3	57	1	26	1	57	3	8.25

Tail Data Bits Training Data bits Tail

Guard

Bits sequence bits bits

Figure 2-2 A single GSM TDMA slot

The overall bit rate is 270.833 kbps. The training sequence is used for equalization and the guard bits are a buffer for propagation delay. A group of 8 time slots is one TDMA frame. One frame is sent every 4.615 ms. There are two types of multiframe, containing 26 or 51 TDMA frames. The 26-frame multiframe contains 24 traffic channels (TCH) and two slow associated control channels (SACCH) which supervise each call in progress. The SACCH in frame 12 contains eight channels, one for each of the eight connections carried by the TCHs. In addition to the above channels, GSM uses other control channels for different purposes. Some of the control channels are :

2.4.1 Broadcast Control Channel (BCCH): Continually broadcasts, on the downlink, information including base station identity, frequency allocations, and frequency-hopping sequences.

2.4.2 Standalone Dedicated Control Channel (SDCCH): Used for registration, authentication, call setup, and location updating. Implemented on a time slot, together with its SACCH selected by the system operator.

GSM also uses Slow Frequency hopping to overcome problems like multipath fading. GSM traditionally uses coherent demodulation mainly because of the improvement in performance over non-coherent methods.

2.5 SUMMARY

Chapter 2 gives us a brief overview of GSM and the GSMK modulation format it uses. It gives us an insight into popular burst mode applications, GSM in our case. It outlines the advantages of the GMSK modulation format and CPM modulations in general.

CHAPTER 3

PARAMETRIC ESTIMATION

3.1 INTRODUCTION

Spectrum estimation of data has received extensive attention by researchers over the years due to its numerous applications in diverse fields. Spectrum estimation could be broadly classified into two classes. One is the non-parametric estimation and other is the parametric estimation. The non-parametric classes of estimators rely on the idea of estimating the auto correlation sequence of a random process from the measured data, and taking the fourier transform to obtain a power spectrum of it. One of the disadvantages of the non-parametric methods are they do not incorporate any information about the data known apriori. Apart from that, the assumption of unobserved data outside the measurement window being zero results in poor resolution and smearing of the spectrum for finite and short data records.

Parametric methods on the other hand are based on fitting an appropriate model for the process to estimate the power spectrum. The advantage of a model based approach is inherent in the ability to predict the nature of the measured process outside the measurement interval. This overcomes the problem of using a window function in our estimation, and consequently the distortion caused by such an assumption. The model that we are interested in this thesis is one in which the data is comprised of a complex exponential embedded in noise. We are more interested in the estimation of the frequency than the spectrum of the data itself. Though these frequencies could be estimated from the peaks of the spectrum, this method will not fully utilize the parametric form of the above process. Therefore, we will discuss methods based on the eigen decomposition of the auto correlation matrix into signal and noise subspace. Once these are determined, the frequencies are obtained with the aid of a frequency estimation function. A brief description of the eigen decomposition and an explanation of the MUSIC algorithm (noise subspace method), a forward backward linear predictor (Tufts-Kumaresan method) [2] which is a signal subspace method are given in this chapter. These two methods are applied in our algorithm and compared in performance with respect to a delay and multiply method.

3.2 EIGEN DECOMPOSITION

Before going into the description of a MUSIC algorithm, a brief review of eigen decomposition is given.

Consider the first order process

$$x(n) = Ae^{jnw_1} + w(n) \qquad n=0,1,\ldots N \;, \tag{3.2.1}$$

that consists of a single complex exponential in noise, where $w(n)$ is white noise with variance σ^2 and N is the length of the signal. The auto correlation of the above sequence would be

$$r_x(k) = P_1 e^{jkw_1} + \sigma^2\delta(k) \qquad k=0,1,\ldots N-1, \tag{3.2.2}$$

where $P_1 = |A|^2$, the power in the exponential and k is the lag. If only the first M elements of the auto correlation are chosen, then (3.2.2) can be represented in matrix form as

$$R_x = R_s + R_n, \tag{3.2.3}$$

where

$$R_s = P_1 \begin{bmatrix} 1 & e^{-jw_1} & . & . & e^{-j(M-1)w_1} \\ e^{jw_1} & 1 & . & . & e^{-j(M-2)w_1} \\ . & . & . & . & . \\ . & . & . & . & . \\ e^{j(M-1)w_1} & e^{j(M-2)w_1} & . & . & 1 \end{bmatrix}, \tag{3.2.4}$$

is the signal autocorrelation matrix of size MxM. It can be easily verified that it has a rank of one. The noise autocorrelation matrix is diagonal, i.e., $R_n = \sigma^2 I$. I is the identity matrix of dimension M. Now, if we define $e_1 = [1, e^{jw_1}, e^{j2w_1}, \ldots\ldots, e^{j(M-1)w_1}]^T$, then

$$R_s = P_1 e_1 e_1^H \tag{3.2.5}$$

where e_1^H is the hermitian matrix of e_1. Since rank of $R_s = 1$, it has only one non-zero eigenvalue. Therefore,

$$R_s e_1 = P_1 (e_1 e_1^H) e_1 = M P_1 e_1, \tag{3.2.6}$$

and the non-zero eigenvalue is equal to MP_1 and that e_1 is the corresponding eigenvector. In addition since R_s is hermitian the remaining eigenvectors $v_2, v_3, v_4, .. v_M$ will be orthogonal to e_1. That is

$$e_1^H v_i = 0 : i = 2,3,4,.M. \tag{3.2.7}$$

If λ_i^s are the eigenvalues of R_s, then the eigenvalues of R_x are

$$\lambda_i = \lambda_i^s + \sigma^2. \tag{3.2.8}$$

For a single exponential process, λ_i^s for $i=1,2...M$, would have only one non zero eigenvalue and it would be equal to MP_1. Thus, the largest eigenvalue of R_x is $MP_1 + \sigma^2$, and the remaining eigenvalues are equal to σ^2. The eigenvector corresponding to the largest eigenvalue is called the signal eigenvector, and the set of eigenvectors corresponding to the rest of the eigenvalues are called the noise eigenvectors. The same argument can be applied to two (or more) complex exponentials embedded in noise. In which case, the eigenvectors corresponding to the two largest eigenvalues will be the signal eigenvectors, and the rest the noise eigenvectors. Thus the eigenvalues and eigenvectors of R_x can be divided into two groups. The first group with the eigenvectors corresponding to the two largest eigenvalues are referred as signal eigenvectors, and span a two dimensional subspace called the signal subspace. The second group with the eigenvectors corresponding to the eigenvalues equal to σ^2 are referred to as noise eigenvectors, and span a (M-2) dimensional subspace called the noise subspace.

Under the assumption that the exact autocorrelation is known, the power of a single exponential process and the frequency can be determined from the following:

$$P_1 = \frac{1}{M}(\lambda_{\max} - \lambda_{\min}), \tag{3.2.9}$$

where λ_{max} is the largest eigenvalue and λ_{min} is the smallest eigenvalue. The frequency can be obtained from the signal eigen vector from,

$$w_i = \arg\{v_{max}(1)\}. \tag{3.2.10}$$

In reality, however, the above method is not useful as the exact autocorrelation is usually not available. Instead, an estimated autocorrelation is used in place of the exact one. But this results in the eigenvalues and eigenvectors being only approximately equal to the true values. The MUSIC [14] and the Tufts-Kumaresan (TK) [2] methods include improvements that will mitigate the problems due to errors in estimation.

3.3 MUSIC FREQUENCY ESTIMATION

Multiple Signal Classification Method (MUSIC) was first developed by Schmidt [14]. MUSIC algorithm is a parametric method, which means it takes into the account the known properties of the signal in noise and exploits that information to get a better result. MUSIC is also a noise subspace method. Let us assume that x(n) is a random process consisting of p complex exponentials in white noise with a variance σ^2 and let R_x is the autocorrelation of x(n). Taking the order of R_x as M where M > p+1. Take the eigenvalues of R_x and their corresponding eigenvectors, and arrange them in descending order of eigenvalues. These eigenvectors can be divided into two groups. The p eigenvectors corresponding to the p largest eigenvalues, and the M-p eigenvectors corresponding to the M-p noise eigenvalues.

These noise eigenvalues should be ideally equal to σ^2. Since we do not have the exact correlation values, we only get approximate results of σ^2.

Let the z transform of the noise eigenvector be referred to as an eigen filter. This can be represented as [6]

$$V_i(z) = \sum_{k=0}^{M-1} v_i(k) z^{-k} \; ; i = p+1, \ldots M, \tag{3.3.1}$$

where v_i are the noise eigenvectors. $V_i(z)$ is referred to as the noise subspace eigen filter. For the signal auto correlation R_x of dimension M, the eigenvectors will have a length of M. Since the eigenvectors of R_x are of length M, each of the noise subspace eigenfilters will have M-1 roots. Ideally p of these roots will lie on the unit circle at the frequencies of the complex exponentials. This is because of the orthogonality of noise eigenvector with the signal eigenvector. Consider a noise eigenvector v_i of the autocorrelation matrix. Computing the discrete time fourier transform of the coefficients in v_i [6]

$$V_i(e^{jw}) = \sum_{k=0}^{M-1} v_i(k) e^{-jkw} = e^H v_i \,. \tag{3.3.2}$$

But, the orthogonality condition implies that the above is zero at $w = w_1$. Hence taking the Z transform of v_i, and rooting the polynomial will result in p roots lying on the unit circle. This can be easily seen as transform $z = e^{-jkw}$ corresponds to the z transform evaluated on the unit circle. The eigen spectrum will be given by

$$|V_i(e^{jw})|^2 = \frac{1}{|\sum_{k=0}^{M-1} v_i(k)e^{-jkw}|^2}, \tag{3.3.3}$$

where v_i are the noise eigenvectors. This spectrum will exhibit sharp peaks at the frequencies of the complex exponentials. Ideally they would be infinite, but due to the errors in the estimation they would be very large. This would still result in spurious peaks if only a single noise eigenvector is considered mainly due to the uncertainties of using estimated values. The noise eigenvectors are formed by the noise component in the signal, and they are highly prone to errors. Since the auto correlations are not exact, the effects of the spurious peaks due to them is mitigated by averaging over all the available noise eigenvectors, using

$$P(e^{jw}) = \frac{1}{\sum_{i=p+1}^{M} |e^H v_i|^2}, \tag{3.3.4}$$

the frequencies are taken as the p largest peaks. For a signal with a single complex exponential, the value of p is 1. But instead of searching for the largest peaks, we will use a method called root MUSIC, which involves finding the roots of the polynomial

$$D(z) = \sum_{i=p+1}^{M} V_i(z)V_i^*(1/z^*), \tag{3.3.5}$$

where V_i^* is the complex conjugate of V_i .

The p roots that are closest to the unit circle are used to estimate the frequency. It was shown by Rao and Hari [9] that a root MUSIC is preferred over spectral MUSIC because of the better resolution when compared to the spectral MUSIC. This is well explained by the effect of an error Δz_i in the signal zero z_i, on the signal frequency. If the error in the root is radial, then there is no corresponding effect in the estimate of the frequency. This is not the case for spectral MUSIC where radial errors affect the peaks of the spectrum. Hence, root MUSIC has better resolution capability.

3.4 KUMARESAN AND TUFTS METHOD

Tufts and Kumaresan suggested a principal component or signal subspace method [2], which was an improvement over a modified covariance method of frequency estimation. The modified covariance method which is also known as the forward backward linear predictor was suggested by Nuttall/Ulrych [16]. The modified covariance method is an auto regressive model based approach, which involves finding the coefficients of a linear predictor and then rooting the polynomial formed by these coefficients. Consider a case of estimating the frequencies of the M complex exponentials embedded in noise where

$$x(n) = A_1 e^{jnw_1} + A_2 e^{jnw_2} + \ldots + A_M e^{jnw_M} + w(n) \quad n=0,1,\ldots N. \tag{3.4.1}$$

The equation for finding the coefficients of the prediction filter, in matrix form for $x(n)$, and an AR model of order p is [3]

$$\begin{bmatrix} c_{xx}[1,1] & c_{xx}[1,2] & \bullet & c_{xx}[1,p] \\ c_{xx}[2,1] & c_{xx}[2,2] & \bullet & c_{xx}[2,p] \\ \bullet & \bullet & \bullet & \bullet \\ c_{xx}[p,1] & c_{xx}[p,2] & \bullet & c_{xx}[p,p] \end{bmatrix} \begin{bmatrix} a[1] \\ a[2] \\ \bullet \\ a[p] \end{bmatrix} = - \begin{bmatrix} c_{xx}[1,0] \\ c_{xx}[2,0] \\ \bullet \\ c_{xx}[p,0] \end{bmatrix}, \tag{3.4.2}$$

where $c_{xx}[j,k] = \dfrac{1}{2(N-p)} \left(\sum_{n=p}^{N-1} x^*[n-j]x[n-k] + \sum_{n=0}^{N-1-p} x^*[n+k]x[n+j] \right)$

are the elements of the modified covariance matrix. The above equation can be rewritten as $\mathbf{a} = R^{-1}r$,

where $\mathbf{a} = [a[1], a[2], \cdots a[p]]^T$,

$$R = \begin{bmatrix} c_{xx}[1,1] & c_{xx}[1,2] & \bullet & c_{xx}[1,p] \\ c_{xx}[2,1] & c_{xx}[2,2] & \bullet & c_{xx}[2,p] \\ \bullet & \bullet & \bullet & \bullet \\ c_{xx}[p,1] & c_{xx}[p,2] & \bullet & c_{xx}[p,p] \end{bmatrix}, \text{ and } r = - \begin{bmatrix} c_{xx}[1,0] \\ c_{xx}[2,0] \\ \bullet \\ c_{xx}[p,0] \end{bmatrix}$$

In a linear prediction notation, the prediction filter is given by the column vector $\mathbf{a} = [a[1], a[2], \cdots a[p]]^T$, and a prediction error filter with its impulse response given by

$$\hat{\mathbf{a}} = [1, a[1], a[2], \cdots a[p]]^T, \tag{3.4.3}$$

where T stands for transpose. The transfer function of a prediction error filter

$$H(z) = 1 + \sum_{k=1}^{p} a[k] z^{-k} . \tag{3.4.4}$$

It was shown by Kumaresan [2] that this prediction error filter has M zeros on $|z| = 1$, in the complex z plane, at angles corresponding to the M sinusoidal frequencies. The rest of the p-M extraneous zeros fall inside the unit circle. This helps in identifying the M signal zeros. The modified covariance matrix equation given above can be represented in terms of an eigen decomposition as

$$\mathbf{a} = \sum_{i=1}^{W} \left(\frac{u_i^* r}{\lambda_i} \right) u_i \tag{3.4.5}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$ are the eigenvalues of R and u_i are their corresponding eigenvectors. W is the rank of the data matrix.

The noise eigenvectors effect the values of **a** significantly. Since the noise eigenvalues are usually small, these can amplify the results as they are in the denominator. Hence the noise eigenvectors increase the probability of spurious peaks. Kumaresan and Tufts [2] have suggested an improvement over this method by considering only the signal components in estimating the coefficients, that is,

$$\bar{\mathbf{a}} = \sum_{i=1}^{M} \left(\frac{u_i^* r}{\lambda_i} \right) u_i \tag{3.4.6}$$

where $\bar{\mathbf{a}}$ forms the coefficients of the prediction filter. This principal component approximation of a modified covariance matrix can be seen as filtering out a part of the noise, and in the process improving the estimates. It can be seen that only M principal components are used in estimating the coefficients. This allows increasing the model order without increasing the spurious peaks due to noise eigenvectors.

3.5 AUTO CORRELATION METHOD

The main disadvantage with eigen decomposition methods is their high computational load while calculating the eigenvectors and eigenvalues of a high order covariance matrix. To overcome this problem, an auto correlation method is proposed which has much lower computational load compared to eigenvalue based methods. It will be shown in the next chapter that the trade off involved in an autocorrelation method is a loss in performance, but this would still have improved performance over a delay and multiply method.

This method involves taking the auto correlation of the sequence to estimate the frequency in a sinusoid embedded in noise. This method was first proposed by Fitz [17] to estimate the frequency of a complex exponential embedded in noise with reduced computational load. This was later improved by Luise and Regiannini [10].

Let

$$z(k) = e^{j2\pi fkT+\theta} + n(kT), \tag{3.5.1}$$

be the corrupted signal, where $n(kT)$ is white noise. The correlation of the above signal is given by

$$R(m) = e^{j2\pi m\nu T} + n^{'}(m), \tag{3.5.2}$$

where $1<m<L_0-1$. Here L_0 is the number of samples. Ideally $n^{'}(m)$ would be zero, as the autocorrelation of white noise is an impulse at zero lag and zero for all other lags. Since we only have estimates of the autocorrelation, the noise $n^{'}(m)$ would be a zero mean process with small magnitudes.

Now, summing the auto correlation over the range $1 \leq m \leq N$ and dividing by N,

$$\frac{1}{N}\sum_{m=1}^{N} e(m) = \frac{1}{N}\sum_{N}^{1} \arg\{R(m)\} - \pi(N+1)vT, \tag{3.5.3}$$

where

$$e(m) = \arg\{R(m)\} - 2\pi m v T \tag{3.5.4}$$

v is the frequency to be estimated. It can be noted that the values of the above equation are restricted to the interval [$-\pi, \pi$]. The estimation ranges are restricted to $|v| \leq \frac{1}{NT}$. Minimizing the above error gives

$$\bar{v} = \frac{1}{\pi N(N+1)T}\sum_{m=1}^{N} \arg\{R(m)\}. \tag{3.5.5}$$

This method is also known as the Fitz [17] method, and has better error variance and mean square errors at low SNRs compared to a delay and multiply method.

CHAPTER 4

IMPLEMENTATION OF EIGEN DECOMPOSITION BASED FREQUENCY ESTIMATORS TO ESTIMATE THE CARRIER FREQUENCY OF BURSTY TRANSMISSION

4.1 INTRODUCTION

Synchronization is considered as a parameter estimation problem and the techniques of estimation theory could be used to realize synchronization structures. Synchronization circuits are also devised on an ad hoc basis and the usefulness of the approach could be proved through simulation. Coherent demodulation is used with passband digital modulations when better error performance is required. Coherent demodulation implies that the carrier in the signal is removed by multiplying a local reference which has the same carrier frequency and phase. Circuits that estimate these parameters are referred to as carrier synchronizers. With the advances in microprocessor chips, digital methods of estimation have become very popular in communication technology. This is because of less stringent tolerances, low power consumption and lower costs of these chips added with increasing speeds with time.

The objective would be to measure the frequency from short data records which can be used to recalibrate the base station. The measurement need not be real time, and therefore the only constraint in our case would be short data records. The performance of a frequency estimator is optimum when it is able to remove the modulation from the signal which results in a pure sinusoid tone in noise, which is much easier to estimate. Extensive research has been done in this field, and very good estimators with good performance at low SNR (signal to noise ratio) have been devised for the sinusoidal case. Many of the optimum frequency estimators remove the modulation of the signal by using information about the timing and the data. It was shown that under moderate frequency errors, accurate timing information can be extracted from the received signal, and used to remove the modulation. But for large frequency offsets, this is not possible and we will have to rely on estimators which do not use any information about the timing or the data. An ad-hoc blind approach followed under the above constraints is a delay and multiply estimator. An advantage of the delay and multiply estimator is its robustness to different modulation types, unlike other estimators which use some sort of match filtering to improve performance. These optimum estimators require information about the modulation type, and the signaling pulse to be able to effectively match filter the signal. But on the other hand ad-hoc estimators are sub optimal and tend to have performance parameters much below the optimal estimators.

Extensive work has been done on delay and multiply estimators by Mengali [12], and the performance parameters analyzed. Parametric estimators on the other hand have been shown to offer good performance in estimating frequencies in exponentials especially for short data records. This has been the motivation in implementing a parametric estimation based schemes to improve the performance of a delay and multiply method. We will show in this chapter that the performance can be improved by these estimators with a slight increase in computational load. Since the estimated frequency is a measurement, it is not done in real time and computational complexity is not an issue in this study.

4.2 PASS BAND SYSTEMS

Pass band systems are obtained by modulating a baseband signal onto a sinusoidal carrier. The mathematical model for a modulated carrier signal is given by

$$S(t) = \mathrm{Re}\left\{S_{ce}(t)e^{j2\pi f_c t}\right\}, \tag{4.2.1}$$

where f_c is the carrier frequency and $S_{ce}(t)$ is the signal complex envelope relative to f_c. The expression $S_{ce}(t)$ depends on the modulation format used. Generally to obtain the transmitted information, the pass band received signal is down converted to an intermediate frequency (IF). This is accomplished by multiplying the pass band received signal with local reference oscillators. Demodulation can be performed either at this IF frequency or at baseband after another stage of down conversion and filtering. In general, the local frequency term is not exactly the same as f_c and this results in a frequency difference. The process is shown in a block diagram below, where it has been assumed that the noisy received signal is available at an intermediate frequency.

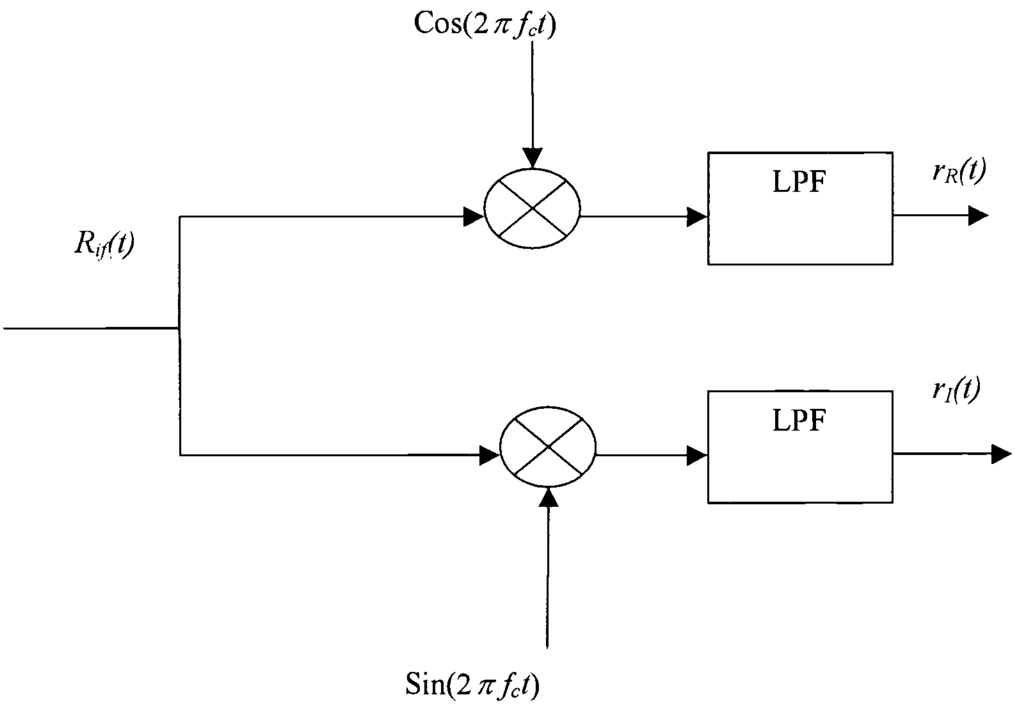

Figure 4-1 Down conversion

It can be shown that the filter outputs $r_R(t)$ and $r_I(t)$ may be represented as a single complex waveform $r(t) = r_R(t) + j\, r_I(t)$, where $r(t) = S(t) + n(t)$,

$$S(t) = e^{j(2\pi v t + \theta)} S_{ce}(t), \qquad (4.2.2)$$

and $n(t)$ is the noise term. It can be seen that the base band signal has the following unknown parameters. v is the frequency offset and θ is the phase offset. To be able to estimate the exact carrier frequency, an exact estimate of v is needed which can be fed back to the local oscillator for it to make the necessary adjustment. Hence carrier estimation is essentially the estimation of the frequency offset.

4.3 PERFORMANCE BOUNDS

In verifying the accuracy of any estimator, a bound in the accuracy that could be achieved is important. That will give an idea on how much the estimator could be improved in the case of the estimates being far away from the bound. It provides a benchmark in the accuracy that could be set and help compare the estimator. This bound is given by the modified Cramer Rao bound (MCRB) [11] which is a variant of the Cramer Rao bound(CRB). MCRB gives us the lowest error variance for a given SNR that could be achieved by an estimator.

The MCRB was introduced by Mengali [11] to over come the difficulties of calculating the CRB. Without going into the details of the derivations of the bounds, we will confine here to just the MCRB of frequency estimation. It was shown that a MCRB for frequency estimation is given by the following formula

$$T^2 MCRB(v) = \frac{3}{2\pi^2 L_0^{\,3}(E_s / N_0)} \,. \qquad (4.3.1)$$

where L_0 is the number of samples available in the observation interval, T is the sampling period and $\frac{E_s}{N_0}$ is the SNR.

4.4 DELAY AND MULTIPLY ESTIMATOR

At large frequency offsets, the signal timing cannot be extracted from the signal and phase information is not available. Hence the ability of using information about the signal symbols and timing information to improve performance is lost. Hence a method under the above limitations operates in a non-data aided and non-clock aided fashion.

An ad hoc open-loop delay and multiply method was suggested by Mengali [12] either in a closed-loop or open-loop architecture for faster frequency estimation. The continuous phase signal after down converting is given by

$$x(kT_s) = s(kT_s) + n(kT_s),\ k=0,1,2\ldots NL_0 \tag{4.4.1}$$

where

$$s(kT_s) = e^{j(2\pi v kT_s+\theta)}\sqrt{\frac{2E_s}{T}}e^{j\psi(kT_s-\tau,\alpha)}, \tag{4.4.2}$$

$n(kT_s)$ is zero mean filtered white noise, v is the frequency offset after down conversion, τ is the time delay introduced by the channel, T_s is the sampling interval, α are the data symbols, and $\psi(t,\alpha)$ is the information bearing phase. L_0 is the observation length in symbol periods, N is the oversampling ratio.

The carrier frequency offset for a delay and multiply method is estimated through the formula [12]

$$\bar{v} = \frac{1}{2\pi DT_s}\arg\{\sum_{k=0}^{NLo-1} z(kT_s)\}, \tag{4.4.3}$$

where L_0 is the observation interval and $z(kT_s)$ is given by

$$z(kT_s) = x(kT_s)x^*[(k-D)T_s]. \tag{4.4.4}$$

where D is the delay in samples and $x^*[.]$ is the complex conjugate of $x[.]$. The validity of the above equation is verified below as given by Mengali [12]. Substituting $x(kT_s)$ given by (4.4.1) and (4.4.2) into (4.4.4), we get

$$z(kT_s) = \frac{2E_s}{T} e^{j2\pi\nu DT_s} e^{j\{\psi(kT_s-\tau,\alpha)-\psi[(k-D)T_s-\tau,\alpha)]\}} + N(kT_s), \tag{4.4.5}$$

where $N(kT_s)$ is a zero mean noise term. Taking the expectation of $z(kT_s)$ gives

$$\mathrm{E}\{z(kT_s)\} = \frac{2E_s}{T} A(kT_s-\tau)e^{j2\pi\nu DT_s}, \tag{4.4.6}$$

where

$$\mathrm{A(t)} = E\{e^{j\{\psi(t,\alpha)-\psi(t-DT_s,\alpha)\}}\}$$

can be expressed as,

$$\mathrm{A(t)} = \prod_{i=-\infty}^{\infty}\{\frac{1}{M}\frac{\sin[2\pi hMp(t-iT,DT_s)]}{\sin[2\pi hp(t-iT,DT_s)]}\} \tag{4.4.7}$$

with $p(t,DT_s) = q(t) - q(t-DT_s)$,

where $q(t)$ is the phase response of the modulation. The proof of the equation (4.4.7) is given in appendix A. Coming back to the sum given in (4.4.3) between $0 \le k \le NL_0 - 1$ gives

$$\mathrm{E}\{\sum_{k=0}^{NLo-1} z(kT_s)\} = \frac{2E_s}{T} NL_0\overline{A}\, e^{j2\pi\nu DT_s}, \tag{4.4.8}$$

where $\overline{A} = \frac{1}{N}\sum A(kT_s-\tau)$ and N is the over sampling rate. Therefore NT_s=T, where T is the symbol period.

In all practical cases of CPM modulations, $\overline{A}$ is positive. Taking the argument of (4.4.8) and solving for ν gives

$$\nu = \frac{1}{2\pi DT_s} \arg\{\mathrm{E}\{\sum_{k=0}^{NLo-1} z(kT_s)\}\}. \tag{4.4.9}$$

Since we do not have the expectation of the sum of $z(kT_s)$, we instead just take the sum for our estimation. As the sum instead of the expectation of sum was taken, the frequency estimate ν cannot be exactly calculated. Instead what we get is a close noisy estimate, whose error variance depends on the signal-to-noise ratio. The delay chosen depends on the modulation format and the oversampling ratio. Hence the optimum delay to get the best estimate varies depending on the above factors. We could use a delay of one as default, but it would not be the best choice. A simple block diagram for a feed forward delay and multiply method is shown in Figure 4.2.

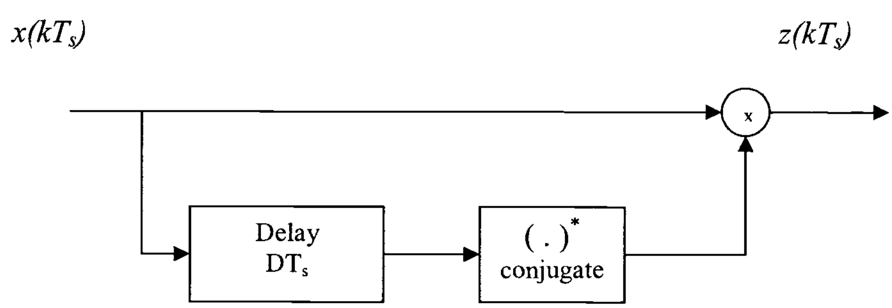

Figure 4-2 Block diagram of delay and multiply method

4.5 REFINING DELAY AND MULTIPLY METHOD

The delay and multiply method is not optimal because the noise suppression capability is poor in this algorithm. Added to that we have discussed, that the delay has to be changed depending on the modulation format and the sampling frequency. This can only be ascertained by simulating for various values of delays and verifying the estimator with the best variance over about 500 trails. This is not possible for an estimator. As seen above, ideally the noise term is removed by the sum, the noise process being zero mean. The estimated values could be improved, by either reducing the noise from the signal $x(kT_s)$, or by removing the noise term from the estimates. That would require some sort of filtering, which might remove or greatly attenuate the offset frequency component too. An alternative to filtering would be the implementation of parametric methods on the data which have proven performance parameters in estimating the frequency components embedded in noise. Simulations have shown that such an approach indeed improves performance, but at the expense of increased computational load.

A parametric estimation done on the GMSK data was shown to improve performance due to the nature of the autocorrelation of GMSK. Simulations have shown that this method has a better normalized error variance than the delay and multiply method. This way, finding the best delay for the estimator would not be necessary. The focus would be on the characteristics of the auto correlation of the data, and fitting our data to the model of the parametric estimation. As seen from (4.4.8),

$$\mathrm{E}\{\sum_{k=0}^{NLo-1} z(kT_s)\} = \frac{2E_s}{T} NLo\overline{A}\, e^{j2\pi vDT_s}. \tag{4.5.1}$$

Since the actual expectation is not available, we substitute it with the sum $\sum_{k=0}^{NLo-1} z(kT_s)$. This results in

$$\sum_{k=0}^{NLo-1} z(kT_s) = \frac{2E_s}{T} NLo\overline{A}\, e^{j2\pi vDT_s} + \overline{N}(kT_s)\ , \tag{4.5.2}$$

where $\overline{N}(kT_s)$ is a zero mean noise term. By varying the delay from 1 to N, and using

(4.4.8), we get

$$\mathrm{y} = G_n\, e^{j2\pi vnT_s} + \overline{N}(nT_s), \tag{4.5.3}$$

for n=1:N and G_n is some arbitrary positive parameter which varies with n. The term $\overline{N}(nT_s)$ will ideally be an impulse function, because the above method of taking the outputs of a delay and multiply method for various delays is equivalent to finding the estimated auto correlation of the GSM data. The noise corrupting the signal is zero mean white noise, and hence its auto correlation will be an impulse at lag of zero, and whose amplitude will be given by the variance of the noise. This problem hence boils down to a frequency exponential embedded in noise, whose autocorrelation is given by the above equation (4.5.3).

The structure of the auto correlation as given in (4.5.3) is an exponential with varying amplitudes. This structure cannot be used in an eigen based estimators unless the varying amplitudes G_n are positive. This is because the frequency information contained in the phase of the exponential is altered for negative amplitude exponentials. In the parametric methods discussed in chapter 3, we found the frequencies are obtained from the roots closest to the unit circle. The same cannot be applied to this auto correlation as G_n is not a constant. But thesis research has shown that the same methods can be used in this case also. This is due to the nature of the parameter G_n. It can be seen that the auto correlation will have the form,

$$\mathrm{E}=A(k)e^{jwk}\ , k=1,2\ldots\mathrm{N}\ , \qquad (4.5.4)$$

where w is the frequency to be estimated and the signal eigen vector would be an element by element product of A(k) and $[1,e^{jw},e^{j2w},e^{j3w},\ldots]$, j is an imaginary number. If the parameter is trivial, that is A(k) would be of the form b*[1,1,1,...], b is some arbitrary constant. This would result in a regular parametric estimation where the roots closest to the unit circle could be identified as the signal roots. But for changing A(k), the spectrum will be given by the convolution of the varying constant and the signal eigen vector. Convolution results in spectral leakage of energy and as long as the constant A(k) has a small bandwidth, the spectrum of E would still retain the spectral properties of complex exponential data. As was showed in (4.5.3) the parameter G_n is obtained for increasing delay values from 1 to N. It can be seen from (4.5.2) and (4.5.3) that G_n is dependent on $\overline{A}$. Under these circumstances, the parameter G_n varies slowly for increasing delays and hence it has a very small bandwidth. The pattern of $\overline{A}$ for delays ranging from 1 to 50 is shown in the figure below.

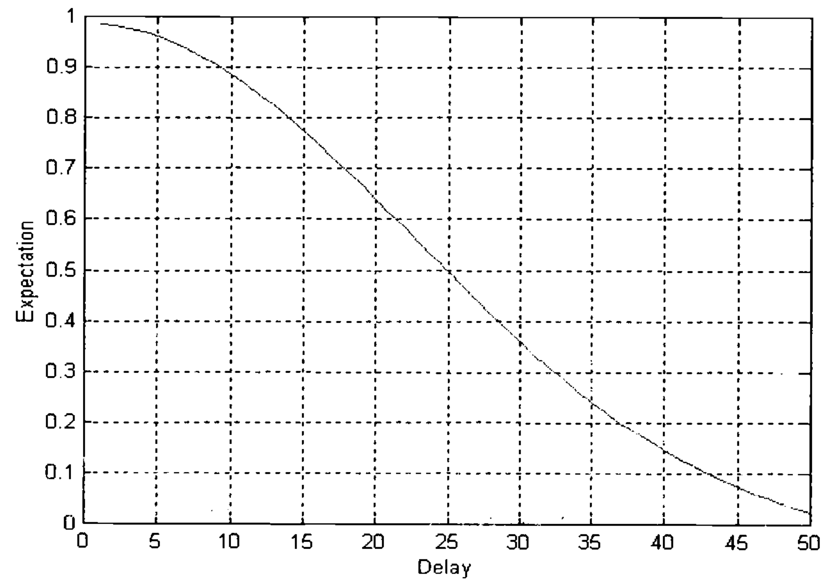

Figure 4-3 Expectation A(t) for delays 1 to 50

Though the expectation $\overline{A}$ varies slowly, the plot shows that the expectation is close to zero for delays greater than 40. It was observed that it goes to negative values beyond delay of 100. Results have shown that the expectation is always positive for delay upto the oversampling rate which is 25 in this case and generally about twice the oversampling rate. This would decide the order of the autocorrelation matrix that could be taken to achieve the best performance in a parametric based estimation.

Therefore a MUSIC or other eigen based method would still be able to identify the peaks through the roots of the polynomials as long as the order of the autocorrelation is within the bounds discussed above. This can be verified by observing the behavior of the roots of the polynomials for MUSIC and Tufts-Kumaresan methods. These roots are obtained by the eigen decomposition of the GMSK data. Z plane plot for the MUSIC method is given in the figure below.

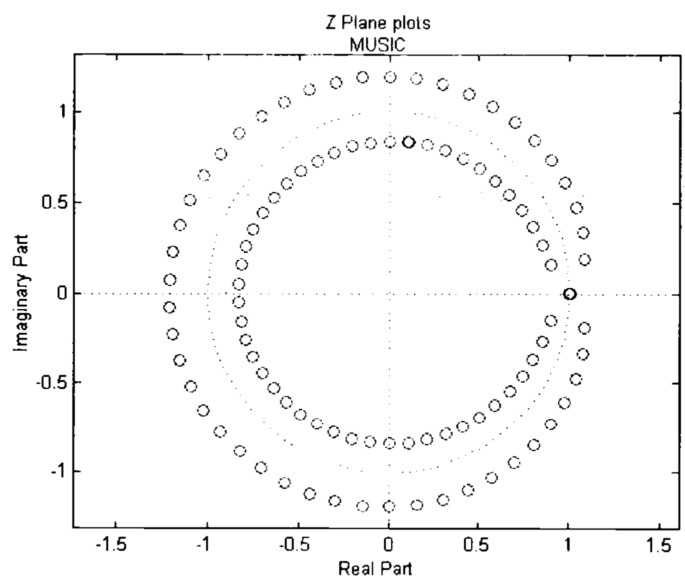

Figure 4-4 roots of the root MUSIC method for an autocorrelation order N=25

The above simulation was done for an SNR of 15 db on GMSK data, and run 30 times. It can be seen from the Z plane plots of the complex roots that the roots have a uniform structure and the roots corresponding to the frequency are closest to the unit circle. The spurious roots fall well below the unit circle. Hence for our case, the roots closest to the unit circle can be taken as the frequency estimates.

Under these circumstances, the orthogonality of the noise eigenvectors of the above auto correlation to the signal eigenvectors can be used to identify the peaks. These peaks give the frequency in the exponential. This is true if the exact auto correlation of the sequence is known. But since there is only an estimated autocorrelation, the frequency estimate will only be an estimate to the actual value, the accuracy of which depends on the SNR of the signal. The Z plane plot for the Tufts-Kumaresan method is given below.

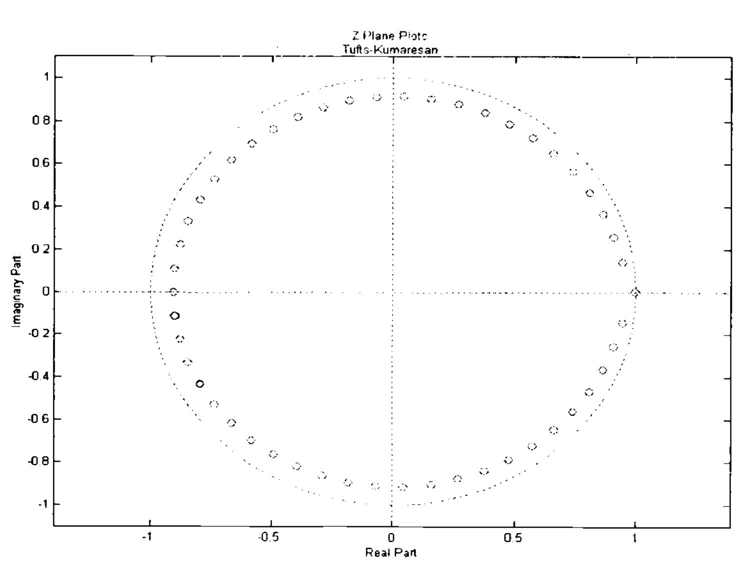

Figure 4-5 roots of the Tufts-Kumaresan method for an autocorrelation order of N=25

It can be seen that even in the Tufts-Kumaresan case the roots for the GMSK data follow the behavior of the roots for the case of an exponential in noise. It is also observed that the number of roots in the TK method are half of the Root Music method. This is because, though both methods use an eigen decomposition of the covariance matrix, they start from different directions. TK follows the path of linear prediction to form a polynomial representation of the signal in consideration. This is achieved by representing the signal covariance matrix in a eigen decomposition and rejecting the non-principal components. Then extracting the roots of the polynomial gives the frequencies where they are estimated from the roots closest to the unit circle. The rest of the spurious roots fall well inside the unit circle. Whereas MUSIC follows the path of representing the signal covariance in an eigen decomposition, and using the property of orthogonality of signal eigenvectors and noise eigenvectors to estimate the frequencies. So, though both use the principal and non-principal components of the eigen decomposition for frequency estimation, the philosophies behind the approaches are different.

By using a MUSIC and TK algorithm in Matlab, the estimate of the frequency offset ν was shown to improve by a reduction in the error variance. Simulation results have shown that the normalized frequency offset error variance is lower than the delay and multiply method. This method is computationally more intensive than a straight forward delay and multiply method. But the computational complexity could be controlled by keeping the order of the autocorrelation matrix in the eigen based methods small. That would result in degradation in performance.

Simulations have shown that the increase in performance is related to the order of the auto correlation matrix but hits a threshold at a certain value. Any attempts to increase the order of the autocorrelation matrix results in spurious peaks, thereby degrading the error variance of the estimator.

4.6 SIMULATION

For simulation purposes, a baseband GMSK signal with BT=0.3 and a known carrier offset is used. The data is over sampled by a factor of 25 i.e., N=25. The simulation can be represented by the block diagram as given below.

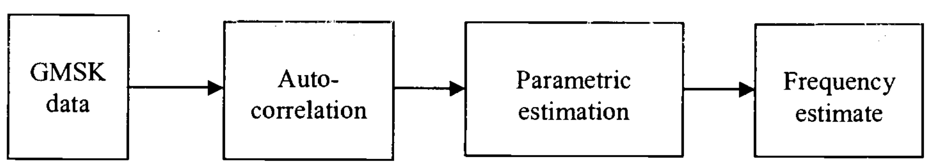

Figure 4-6 Open loop frequency estimation

The performance of an estimator can be measured by the MSE (Mean Square Error) and Error Variance. The MSE is given as

$$MSE\{\hat{\nu}\} = E\{|\nu - \hat{\nu}|^2\}, \qquad (4.6.1)$$

where $\hat{\nu}$ is the estimate of the frequency ν. The following is the normalized error variance plot for delay and multiply method, the MUSIC, and TK algorithms. This was obtained by running the estimator 1000 times. These plots are for a frequency offset of 1 Khz, and covariance order of 25.

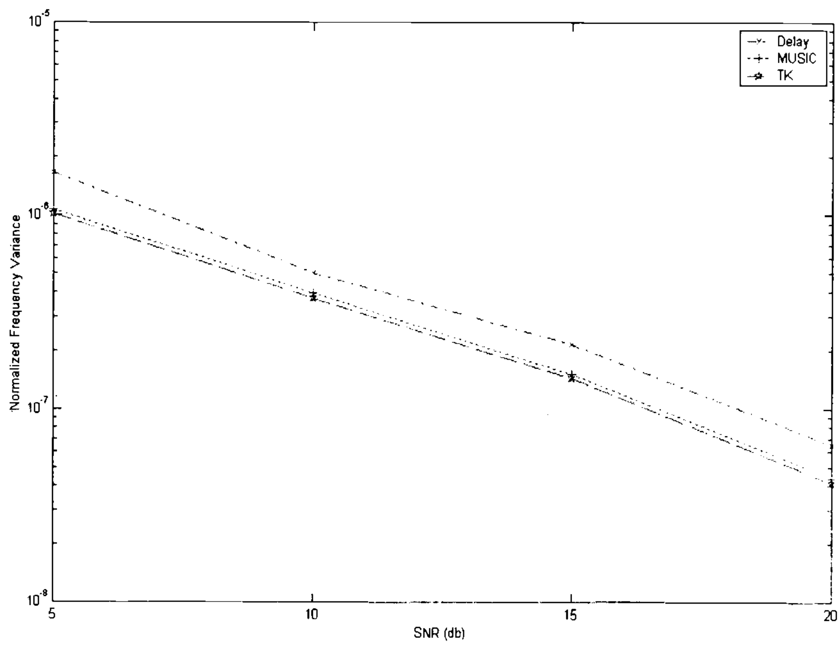

Figure 4-7 Normalized error variance with respect to symbol period 'T' Vs 'SNR' for the delay and multiply method, TK, and MUSIC

The above plot shows the improvements in performance for parametric estimation based algorithms compared to the optimal delay and multiply method. The 'dashed' line indicates the error variance for TK , the 'dashed period' line indicates the variance for the delay and multiply method for the optimal delay (delay of 15) and the 'dotted' line for the MUSIC method. The optimal delay was found to be 15 for this case through simulations. This delay is mainly dependent on the modulation, bit period T and the sampling interval T_s. Hence the best delay for another CPM modulation format might not be 15, and it has to be found out through simulations. The MUSIC and TK estimation has the advantage of improvement in performance with respect to the delay method without having to have apriori knowledge of the simulation parameters and optimum delay. It can also be seen that the TK method has a slightly better performance when compared to MUSIC. This can be attributed to the components of the eigen decomposition each method uses. It was shown earlier in chapter 3 that the MUSIC uses the noise eigenvectors while the TK method uses the signal eigenvectors. Coming back to the case of a single exponential in noise, for a P order signal covariance matrix, we will have one signal eigenvector and P-1 noise eigenvectors. This is because we are considering the case of only one frequency embedded in noise. This gives the advantage to TK method of increasing the order P of the covariance matrix without a corresponding increase in the spurious peaks due to noise eigenvectors as was shown in chapter 3.

The Cramer Rao bound for frequency estimation that was given before shows that the performance improves as a third power of the length of the data N and linearly with SNR. Therefore it can be seen that the order P should be as large as possible to have a large aperture for the prediction filter. But at the same time, increasing it to a very large value results in too many extraneous zeros. As can be seen, the number of roots are equal to the order of the covariance matrix for the TK method and twice the number for root Music. Hence there is a good chance that these redundant roots might fall close to the unit circle, resulting in spurious peaks. The

Mean square errors for the above methods are shown below. It can be seen that the TK method has a slightly better performance compared to Root Music, and both perform much better than a delay and multiply method.

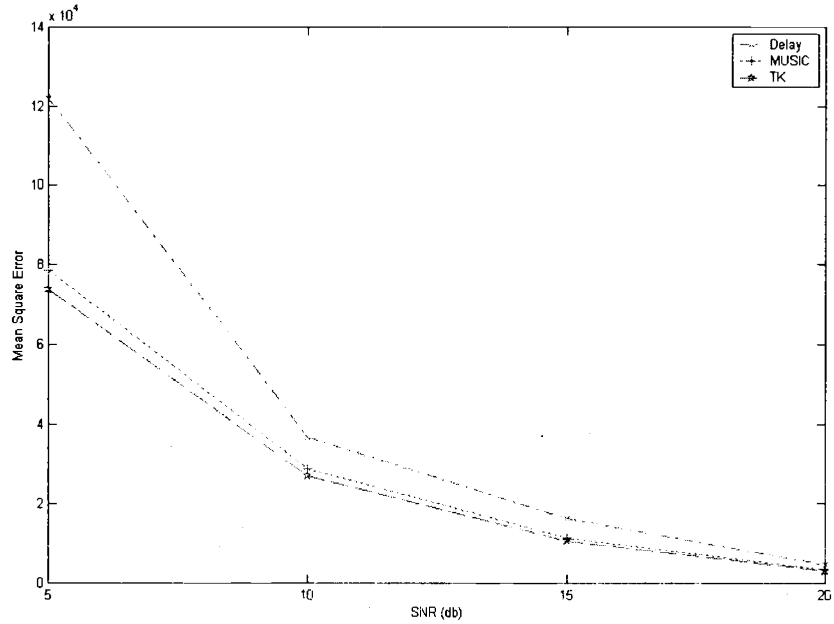

Figure 4-8 Mean square errors Vs SNR

4.7 AUTO CORRELATION METHOD

We now apply the auto correlation method instead of the parametric methods to the available data. The details of this method can be found in Chapter 3 and the cited references.

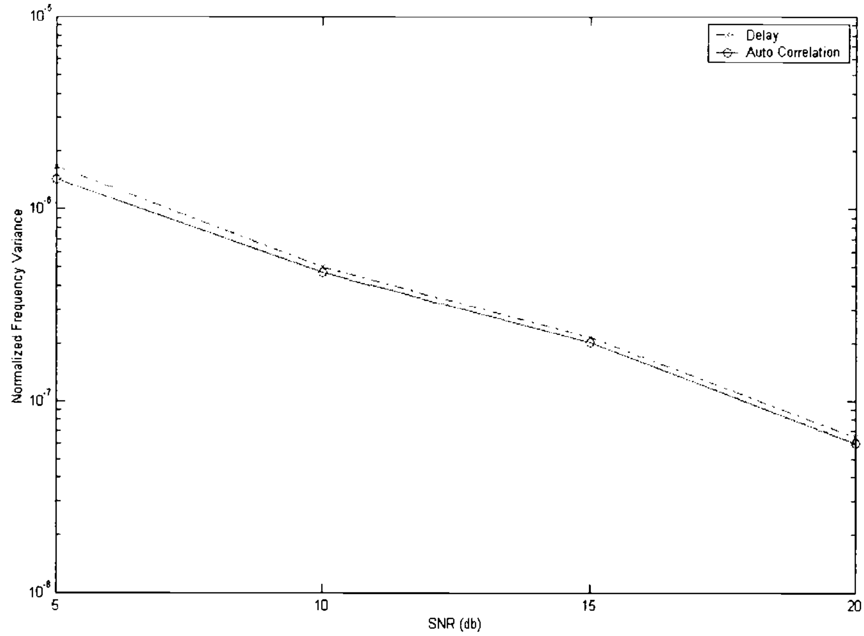

Figure 4-9 Normalized frequency variance Vs SNR.

This plot shows the improved estimation capabilities of the Auto correlation method, compared to a delay and multiply method. The increase in performance is slightly better than the delay and multiply method. This is being compared to the optimal delay of 15 in our case. The difference is greater at low SNRs, but as the SNR increases the accuracy of both the estimators seem to merge.

Now we will look at the plot of the mean square error between the delay and multiply method and the auto correlation method.

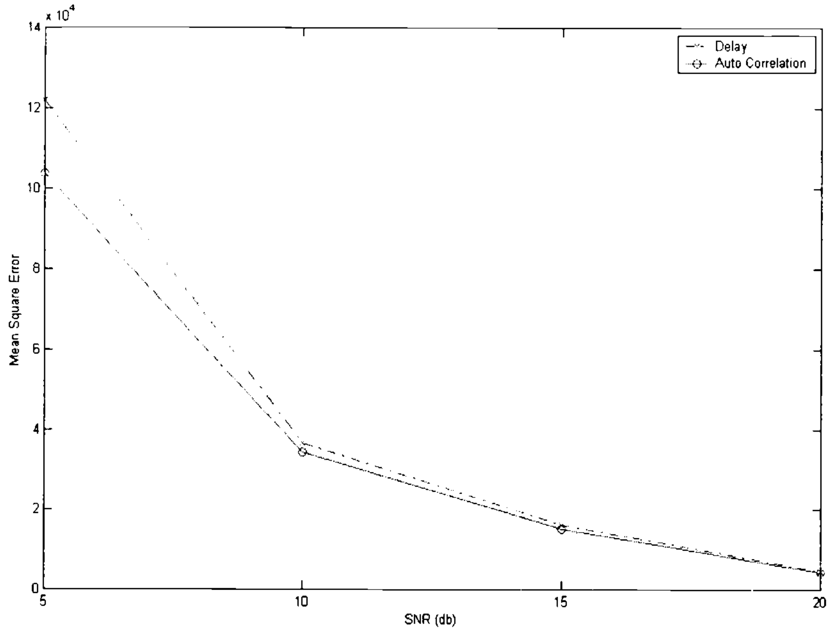

Figure 4-10 Mean square error Vs SNR

This plot shows the average mean square errors, for SNR= [5 10 15 20] db. It can be seen that at low SNRs, the auto correlation method has better performance compared to the delay and multiply method, but the errors seem to converge at higher SNRs. So in all, the above methods show us three ways of improving the performance, especially at low SNRs.

4.8 SIMULATION OF CLOSED LOOP STRUCTURE FOR DELAY AND MULTIPLY METHOD

The above methods have open loop architecture, as the frequency estimates are obtained from one single block of data. They can also be modified into a closed loop configuration for frequency estimation by feeding the frequency estimates to the signal and thereby steering the frequency estimates to the actual frequency. Consider a closed loop block diagram below

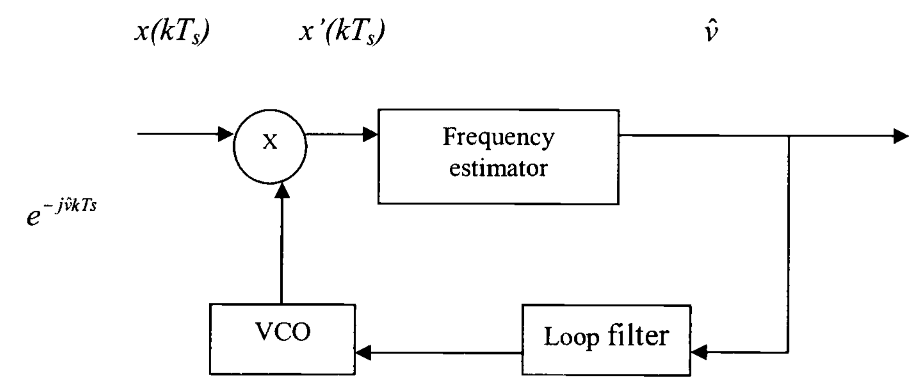

Figure 4-11 Block diagram of a closed loop estimator

The frequency estimator block implementing any of the three methods discussed before could be used to give an estimate of the frequency over a block of data and that frequency could be fed back to update the frequency in the signal $x(kT_s)$. Let $\hat{v}$ be the estimated frequency out of the vco block. Then

$$x'(kT_s) = x(kT_s)e^{-j2\pi\hat{v}kT_s} \tag{4.8.1}$$

It can be seen from (4.8.1) that $x(kT_s)$ and $x'(kT_s)$ have the same signal structure, but now the frequency offset is v- $\hat{v}$. This can be used as an error signal to guide $\hat{v}$ closer to v. The VCO frequency is updated as

$$\hat{v}[(n+1)T] = \hat{v}(nT) + D\ e(nT)\ , \tag{4.8.2}$$

where e(nT) is the frequency error estimate out of the frequency estimator block at time nT and D is the error gain. For this simulation a GMSK signal is used, which is sampled every T_s equal to $1.4769*10^{-7}$ seconds and the signal is corrupted by white Gaussian noise. This is passed onto a low pass filter and a buffer is used, which accumulates the data. This is fed to a frequency estimator block which gives out an estimate of the frequency offset. Any of the frequency estimators discussed in sections 4.5 and 4.7 can be used to implement a frequency estimator. The buffer size can be varied and is a design variable L. This estimate is used to improve on the lock by feeding it back to $X(kT_s)$. Now the next estimate will be the difference of the initial offset and the current estimate. This method has a draw back, that is, under low SNR conditions the estimates can be off by a large margin and will make the loop oscillate at large frequency offsets. This can be overcome to a certain extent by having a discrete time integrator in the feed back path. An integrator acts like a low pass filter, thereby reducing the effect of noise and also does an averaging operation on the loop estimates. This would reduce the errors which are caused by the noise. The integrator is followed by a gain block which controls the acquisition time and the mean square errors. The higher the gain the faster the acquisition speed, but this will be at the expense of larger frequency oscillations. The frequency counter is updated every T_s seconds, where it accumulates the previous outputs of the integrator and updates the frequency value every T_s secs. The simulink schematic is provided below

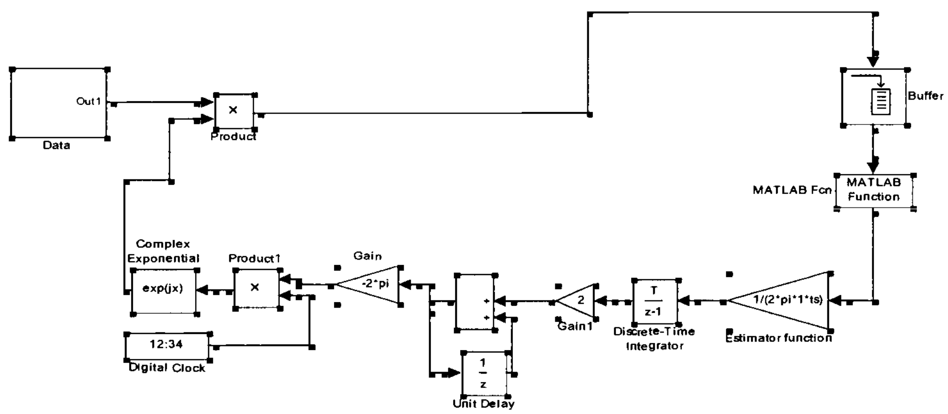

Figure 4-12 Simulink schematic for closed loop carrier lock.

The carrier lock achieved, the estimate of which is close to 1Khz offset is shown below for 20 db SNR with a MUSIC frequency estimator.

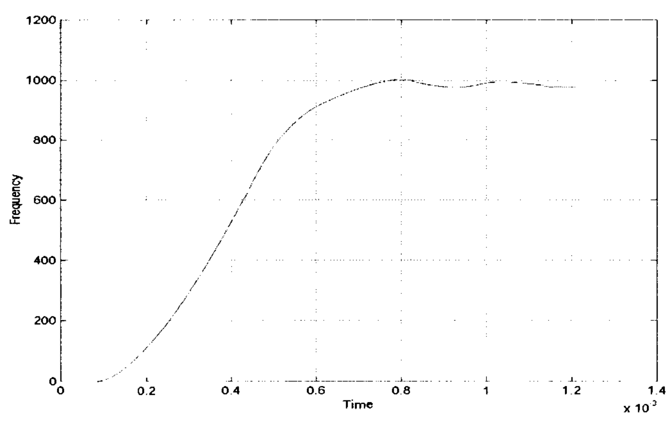

Figure 4-13 Carrier lock for 1 KHz offset

It can be seen that the above architecture gives a close enough estimate of the carrier offset of 1 KHz. Results have shown that by varying the gain, a quicker lock could be obtained. But it will be at the expense of larger estimation errors. It is a design criterion to decide the amount of offset that can be tolerated so that it is within the lock-in range of a phase lock loop or a data aided frequency estimator. For a 10db SNR value, lock differences have been in the range of 200-300Hz.

This estimate could be used by a phase locked loop with aided acquisition to lock on to the carrier faster when the unmodulated carrier is transmitted. The carrier lock for a 2 KHz offset is given in figure 4-14.

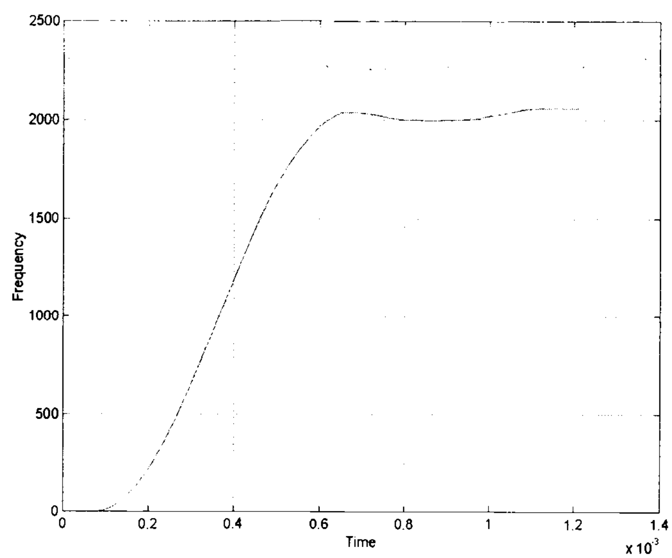

Figure 4-14 Carrier lock for a 2 KHz offset

One way of overcoming the frequency oscillations in high noise environment would be to use the output of the discrete time integrator as a lock detector. The output of the integrator going below zero or a small positive threshold value could be used as an indicator of frequency lock. This happens, due to the fact that, as the estimation gets closer to the frequency offset, the estimates of the buffer often become negative. That results in the change in the slope of the output of the integrator resulting in a decrease in the amplitude after an initial increase. This could be used as an indicator of lock depending on the threshold decided. As soon as this condition is met, the loop is disabled and the frequency estimate noted. This way, we can track the frequency of the carrier to a close value during the transmission of the data, and this result could be used in the fast lock of a carrier offset when the unmodulated carrier is transmitted. The output of the discrete time integrator for the above simulation is shown below.

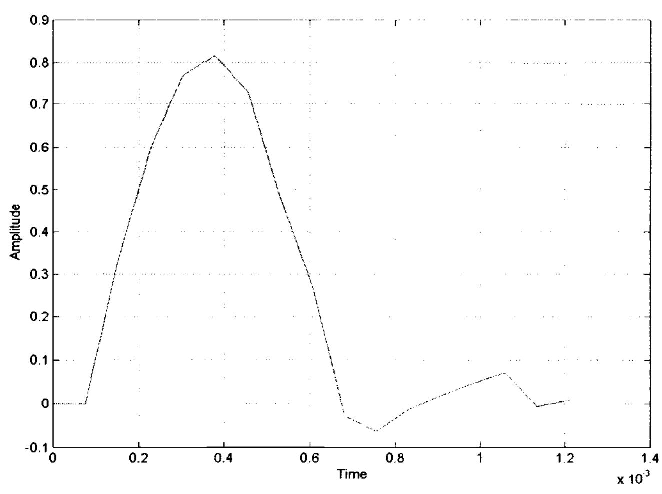

Figure 4-15 Output of the integrator. Threshold lock detector for 2Khz offset

4.9 OBSERVATIONS

It was seen in this chapter that an open loop parametric based estimation of frequency for GMSK signal improves performance over an optimum delay and multiply method by about 3 db. This improvement is at the cost of increased computational complexity. It was also observed that the performance of the estimator increases with the order of the autocorrelation matrix. The maximum allowable order of the matrix would be about 2N, where N is the over sampling rate. It was shown by Kumaresan [2] experimentally that an order of 3L/4 is optimal where L is the length of the data. The same principle cannot be applied for our case due to the reasons given in section 4.5.

The methods proposed being a non-data aided one, the need to extract timing and symbol information is not required. A data aided method would have better performance but the ability to have data aided synchronization is not practical in many cases. A data aided method would also have a relatively narrow lock in range. However, the non-data aided acquisition could be used to achieve an initial frequency offset estimate which will fall under the lock in range of a data aided method. A data aided method can then be implemented for higher performance. In most cases the symbol timing information could be extracted at low frequency offsets. In such a scenario, the non-data aided methods could be used to achieve low frequency offsets whereby symbol timing could be extracted to implement a clock aided carrier recovery. When the computational complexity has to be reduced, an auto correlation method discussed in chapter 3 was shown to have a slight improvement in performance over the delay and multiply method. Hence it would be a trade off between computational complexity and performance. Though the estimators perform well compared to a delay and multiply method, the normalized error variance of these estimators still falls short of the MCRB by a big margin.

A parametric based estimation would have a performance close to the data aided methods when an unmodulated carrier is sent. This is because most of the data aided methods use the symbol and timing information to remove the modulation from the data thereby resulting in an exponential at the frequency offset. Once this is achieved the problem is changed to the estimation of frequency in a complex exponential. In such a case many estimators achieve the Cramer Rao theoretical bound. A parametric based estimation would attain the CRB for all SNRs over 7 db for TK method, and for all SNRs over 15 db in the case of MUSIC [3]. Hence while estimating the unmodulated carrier, non-data aided parametric estimation techniques have similar performance as data aided methods.

The added advantage of using a parametric based estimation is its high resolution capabilities compared to other methods. This reduces the probability of false lock due to interference from other carriers. As was noted in chapter 2, GSM sends an unmodulated carrier once in every 8 time slots and transmits 124 carriers in a 25MHz band. To use bandwidth effectively, the channel separation is kept low. This results in the congestion of the frequency spectrum with different carriers being close to each other. With the congestion of the frequency spectrum, it would result in a good probability of interference from other carriers. Using a parametric based estimation, the dominant frequency could be found rejecting the interference from other carriers.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 CONCLUSIONS

This thesis presents an investigation into the model based, parametric frequency estimation for estimating the carrier frequency offset in a non-data aided fashion. Chapter 2 gives a brief description of the CPM signals with an emphasis on GMSK. In chapter 3 the parametric estimation methods are given in detail with mainly eigen decomposition based methods. Chapter 4 uses various frequency estimators discussed in chapter 3 by fitting the auto correlation of a GMSK data to the models in discussion in chapter 3. Different methods are compared to the more conventional ad-hoc approach of delay and multiply methods for non-data aided estimation. The results show a clear reduction in error variances of these estimators compared to a delay and multiply methods. An eigen decomposition based parametric estimation, which uses the information about the structure of the signal autocorrelation to improve performance has been proposed and the performance characteristics of signal subspace methods and noise sub space methods are compared. These two are the main contributions in this thesis.

5.2 SUGGESTIONS FOR FUTURE RESEARCH

One of the main constraints faced in communications is channel interference (CI). It is possible that during the estimation of the carrier frequency, frequency components from the other channel could fall in the current channel, and thereby result in the phase locked loops or other estimation algorithms locking onto the interference channel frequency. As we have mentioned earlier already, the methods introduced in chapter 3 have very high resolution for short data records. Therefore, these methods might be useful in rejecting the unwanted frequencies and only identifying the main frequency component. The true carrier offset of the channel would have higher energy compared to the interference components, and thereby decreasing the chances of false lock. More research is needed to verify the interference rejection capabilities of such a process. Apart from the above, the mathematical bounds in the frequency offset beyond which the methods fail could be calculated. The results of our methods for different CPM modulations (other than MSK, GMSK) could be verified, and the effect of the parameters of the CPM on the performance could be ascertained.

Also, more research needs to be done on the performance characteristics of a closed loop estimator. A statistical analysis of the threshold detector mentioned in the closed loop estimator can be undertaken. The usefulness of using the zero crossings, and the probability of zero crossings for different SNRs can be verified.

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APPENDIX

EXPECTATION

The result was shown by Mengali in [12]

We will calculate the expectation of $\mathrm{E}_{\tilde{\alpha}}\{e^{j[\psi(t_2,\tilde{\alpha})-\psi(t_1,\tilde{\alpha})]}\}$, where

$$\psi(t,\tilde{\alpha}) = 2\pi h \sum_{i=-\infty}^{\infty} \tilde{\alpha}_i q(t-iT) \tag{A.1.1}$$

where h is the modulation index , q(t) is the frequency response of the CPM format, and T is the bit period. $\psi(t,\tilde{\alpha})$ is the information bearing phase function. Since $p(t,\Delta T) \equiv q(t) - q(t-\Delta T)$,

$$\mathrm{E}_{\tilde{\alpha}_i}\{e^{j[\psi(t_2,\tilde{\alpha})-\psi(t_1,\tilde{\alpha})]}\} = \mathrm{E}_{\tilde{\alpha}_i}\{\exp[j2\pi h \sum_{i=-\infty}^{\infty} \tilde{\alpha}_i p(t_2 - iT, t_2 - t_1)]\}$$

$$= \mathrm{E}_{\tilde{\alpha}_i}\{\prod_{i=-\infty}^{\infty} \exp[j2\pi h\, \tilde{\alpha}_i p(t_2 - iT, t_2 - t_1)]\} \tag{A.1.2}$$

As the symbols $\tilde{\alpha}_i$ are independent the above equation can be averaged separated as given below.

$$\mathrm{E}_{\tilde{\alpha}_i}\{e^{j[\psi(t_2,\tilde{\alpha})-\psi(t_1,\tilde{\alpha})]}\} = \prod_{i=-\infty}^{\infty} \mathrm{E}_{\tilde{\alpha}_i}\{\exp[j2\pi h\, \tilde{\alpha}_i p(t_2 - iT, t_2 - t_1)]\} \tag{A.1.3}$$

As $\tilde{\alpha}_i$ takes values belonging to the set $\{\pm 1, \pm 3 \ldots. (M-1)\}$, the right side of the equation A.1.3 can be separated as

$$= \frac{1}{M} \sum_{m=1,3}^{M-1} e^{j2\pi hmp(t_2 - iT, t_2 - t_1)} + \frac{1}{M} \sum_{m=1,3}^{M-1} e^{-j2\pi hmp(t_2 - iT, t_2 - t_1)} \tag{A.1.4}$$

Also,

$$\frac{1}{M}\sum_{m=1,3}^{M-1} e^{j2\pi hmp(t_2-iT,t_2-t_1)} = \sum_{n=o}^{M/2-1} e^{j2\pi h(2n+1)p(t_2-iT,t_2-t_1)}$$

$$= \frac{1}{M}\frac{\sin[2\pi hMp(t_2-iT,t_2-t_1)]}{\sin[2\pi hp(t_2-iT,t_2-t_1)]} \tag{A.1.5}$$

Substituting in the equation (A.1.3)

$$\mathrm{E}_{\tilde{\alpha}_i}\{e^{j[\psi(t_2,\tilde{\alpha})-\psi(t_1,\tilde{\alpha})]}\} = \prod_{i=-\infty}^{\infty}\frac{1}{M}\frac{\sin[2\pi hMp(t_2-iT,t_2-t_1)]}{\sin[2\pi hp(t_2-iT,t_2-t_1)]} \tag{A.1.6}$$