AN ABSTRACT OF THE THESIS OF

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Title: CALCULATION OF TREE VOLUME AND SURFACE AREA BY THE HEIGHT-ACCUMULATION METHOD

Abstract approved: David P. Paine
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This study was initiated to determine the accuracy and limits of the height-accumulation method of measuring tree volume and surface area for Pacific Northwest second-growth trees. Thirty trees were selected from various thinning treatments in a 55 year old Douglas-fir forest. Each tree was measured with the highly accurate Barr and Stroud optical dendrometer and height, surface area, and cubic-foot volume were calculated on a Control Data Corporation 3300 electronic computer.

A Spiegel-Relaskop was used to measure taper steps for height-accumulation. These results were then compared with the dendrometer results to determine the accuracy of this technique.

The Relaskop was found to be inadequate when used alone, but when a 4-power telescope was mounted, it greatly enhanced the usefulness of the instrument.

The telescopic Spiegel-Relaskop, using the height-accumulation method, was found to be capable of measuring volume with comparable accuracy to the optical dendrometer. The study revealed that volumes determined using height-accumulation, measured with a telescopic Spiegel-Relaskop, in conjunction with point sampling should estimate volume of a stand as accurately as any other cruising technique. In addition this method, like other recently developed methods, eliminates the use of volume tables and their inherent biases.

Calculation of Tree Volume and Surface Area by the Height-Accumulation Method

by

James Douglas Arney

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INTRODUCTION

Foresters have come to realize that trees are a manageable forest crop. By advanced silvicultural practices the form of tree stems can be manipulated to meet more demanding and varied management objectives. At the same time foresters try to explain the factors which control the growth form of the forest crop. The spatial element required to grow even a few trees involves a multitude of independent and interdependent factors; very few of which can be controlled or explained. Since the area is necessarily so large, it is impractical to measure every tree. Thus foresters are involved with sampling designs in order to measure the forest crop. These designs are always limited by that part of the environment which cannot be explained in the model (Grosenbaugh, 1948).

To determine when to harvest it is necessary to obtain an estimate of the volume of the standing trees. The size and inaccessibility of measuring trees before harvest led to felled tree measurements. But felling the trees prior to the planned harvest to estimate the volume destroys any possibility of future management plans for those trees. Therefore, trees outside the management unit must be utilized for felled tree volume. Volume tables where constructed with volume over one independent variable. The one variable usually being diameter breast high. The limitations are readily apparent. Foresters have strived to improve this estimate by adding more variables, and complex interactions of variables (i.e., species height, form class, form quotient, site, tariff access, et cetera).

The Problem

Felled tree volume usually does not possess the distinct characteristics inherent in standing trees. Volumes obtained in this manner contain bias that usually cannot be removed, even when corrected by a few rigorously valid sample trees from the management unit. The forester must eventually get back to making direct non-destructive measurements on the standing trees in the management unit.

The best estimate of the true volume of a stand is a good unbiased sample of precisely measured volumes of individual trees. There are a number of valid unbiased sampling schemes appropriate to any forest stand. But no objective measure of standing tree volume has been discovered. The only alternative at present is to measure some function of volume. The function must be objectively measured, directly correlated to volume, and a sensitive predictor of variation in volume between individual trees. A series of diameter and height measurements comes theclosest to fulfilling these requirements. Height is easily obtained for any point on the tree. Diameter, except near the ground, has been difficult, if not impossible, to obtain. The most accurate instrument designed to measure upper stem diameter is the Barr and Stroud optical dendrometer. A recent study showed that it can measure diameter at any visible point on the tree to the nearest one-tenth inch (Bell and Groman, 1968). But, since the instrument does not allow for direct field interpretation of readings of diameter or height, spacing of measurements is purely arbitrary. Thus, the interpreter has little, if any, objective measure of the distinct form of the sample tree. To sample the stand effectively the measurements must be precise and sensitive to the stem form of each sampled tree. David Bruce of the Pacific Northwest Forest and Range Experiment Station made a slide rule for field interpretation of diameters and height using the Barr and Stroud dendrometer. It is not precise and is slow to use.

Purpose of Study

In 1954, Dr. Lewis R. Grosenbaugh produced a paper which introduced a new tree measurement concept. It not only does away with volume tables, but also is sensitive to individual tree stem form. He presented extensive mathematical proofs on the concept and called it height-accumulation.

His basic argument in proving the theory is that the integral function,

$$\sum \frac{\pi}{4} \int_{0}^{L} X^2 \, dY,$$

explains the relationship of diameter, X, and height, Y, to volume of a series of frustra of solids of revolution. This function holds true for all frustra of solids of revolution of the general form $X^2 = BY^{K-1}$; where B is an arbitrary constant and K equals two for parabolic frustra, three for conic frustra, and four for neoloidic frustra. The constant K is often arbitrarily selected as two for ease of calculation with height as the independent variable and diameter as the dependent variable at various heights on the tree.

Grosenbaugh (1967) pointed out that extrapolation of these basic forms must not be carried out where frustra are too long or where upper and lower diameters differ by greater than 20 percent. He also points out that least squares, minimax principles, and maximum likelihood do not provide a valid guide for testing or evaluating the merits of any arbitrary projections.

It has been proven that the integral function

$$\sum \frac{\pi}{2} \int_{B}^{o} XY \, dX,$$

is equally valid where diameter and height form an upward progressive total of heights times a diminishing arithmetic progression of diameters. Height is now the dependent variable and diameter is the independent variable. This method has none of the defects which make conventional felled tree methods impractical on standing trees. Height at any point on the tree is easily measured, whereas diameter is not.

Hans Enghardt and H. J. Derr of the Alexandria Research Center, in central Louisiana, have used the height-accumulation method since 1955. It has proven rapid in the field and in office computations and has yielded estimates close to those by other methods. They found it well suited for thinning studies because the task of constructing local volume tables at each thinning interval was time consuming.

Their experimental stands of long leaf (*Pinus palustris* Mill.), slash (*Pinus elliottii* Engelm.) and loblolly pine (*Pinus taeda* L.) ranged from 10 to 30 years old. Over 1900 sample trees were measured with the Speigel-Relaskop using height-accumulation. In a comparison with cubic volume from Mesavage's form class volume tables, average total cubic-foot volumes, inside bark, differed by less than 0.1 percent, with a maximum difference of 1.8 percent on a single plot. In another study they found less than three percent difference in volumes obtained from plot sampling with height-accumulation compared to measuring all trees. No individual tree comparisons were made.

In a study in southern New Hampshire by James P. Barrett, 26 white pines were felled and measured with 1-inch taper steps.

The trees ranged from 5.1 to 28.0 inches Diameter Breast High and from 31 to 103 feet in total height. Volumes were computed by the graphical method and height-accumulation formula to one inch taper steps. There was less than a 1-in-20 chance that individual tree volume would vary from graphical estimates by more than 1.25 cubic feet using one inch taper steps (Barrett, 1964).

The only other work that is being done with height-accumulation started in 1963 in Shelton, Washington. Simpson Timber Company is investigating application of height-accumulation to determine regression equations for volume by tree diameter under their Continuous Forest Inventory. They plan to complete their study in 1968 (Petzold, 1966).

Objectives

The present study was designed to:

1) Determine the accuracy and limits of the height-accumulation method of measuring tree volume and surface area for Pacific Northwest second-growth forests.

2) Determine the possibility of applying this method with a Spiegel-Relaskop and variable plot cruising. The Spiegel-Relaskop is in common use in the Pacific Northwest for variable plot cruising; and it is one of the few instruments which measures upper stem diameters over 30 inches.

3) Provide an easily applicable and understandable computer program for quick and easy programming of height-accumulation prediction methods developed.

Minor objectives of this study were to determine the time involved to make the necessary measurements and to determine the effect of field conditions on the accuracy of the measurements.

MATERIALS AND METHODS

Methods

Basic Concepts of Volume Formulas

The most commonly used formulas in forestry for determining the cubic-foot content by logs in standing trees are:

Smalian formula: $$V = \frac{(B_1 + B_2)}{2} L$$

Huber formula: $$V = B_{\frac{1}{2}} L$$

Newton formula: $$V = \frac{(B_1 + 4B_{\frac{1}{2}} + B_2)}{6} L$$

Where

V = cubic-foot volume of a log

B_1 = basal area, in square feet, of large end of log

$B_{\frac{1}{2}}$ = basal area, in square feet, of the middle of the log

B_2 = basal area, in square feet, of small end of the log

L = length in feet of the log (Chapman and Meyer, 1949)

Both the Smalian and Huber formulas calculate the volume of a perfect parabolic frustum. Where the log assumes any other form than a paraboloid, these formulas are only an approximation of the true volume. Therefore, their use should be restricted to the upper portions of the tree. Providing the form is symmetrical, the Newton formula will approximate a parabolic, conic, or neoloidic frustum.

The height-accumulation method involves making the diameter, D, on standing trees the independent variable arbitrarily selected in an arithmetic progression up the tree, and measuring the associated height, H. Thus the measurement of the bole of each tree is directly related to the growth form. The volume of a tree can be considered as the sum of the volumes of several frusta of solids of revolution. If each section of the bole is chosen such that the difference in upper and lower diameters is consistent, then the difference can be considered a constant. This will, in effect, remove diameter as a necessary variable to compute cubic volume. The constant difference in diameter can be chosen before any measurements are taken. This difference is commonly referred to as a taper step.

The height-accumulation formula will calculate volume of frusta of solids of revolution as cones, neoloids, or paraboloids simply by adjustment of a single constant, K. The general form being

$$V = \frac{\pi T^2}{2(144)} \left[\sum_{i=1}^{j} H_i + \frac{\sum_{i=1}^{j} L_i}{2K} \right]$$

where

V = volume, in cubic feet, of the entire stem

π = 3.1415927

T = taper step, in inches

$\sum_{i=1}^{j} H_i$ = sum of the upward progressive totals of height to each diameter increment, in feet

$\sum_{i=1}^{j} L_i$ = sum of the lengths, in feet, between each diameter measurement.

j = number of taper steps measured on the tree

K = shape divisor, 2 for paraboloid, 3 for conoid, and 4 for neoloid

The Concept of Basal Area Sampling

One of the newest and most efficient sampling schemes is Bitterlich point sampling. It is objective and easily applied to forest sampling. All trees are counted which subtend an angle greater than some preset horizontal angle; the number of trees counted multiplied by the basal area factor equals the estimate of basal area per acre. Trees of the stand are sampled by unequal probability sampling proportional to basal area. Equal probability sampling is proportional to frequency which frequently samples heaviest in size classes of least interest. Bitterlich point sampling is the best unbiased estimator of stand basal area. A precise measure of volume for each tree of the Bitterlich sample will determine a volume-basal area ratio with which to estimate stand volume. The form of the basal area estimator is

$$\hat{Ba} = \bar{n}FA$$

where

$\hat{BA}$ = estimated basal area of the stand, in square feet

$\bar{n}$ = the average number of trees per point in the sample

$F = \dfrac{43560}{Csc^2(\frac{\theta}{2})}$, the basal area factor

A = the number of acres in the stand

θ = a horizontal angle of a specified size used to determine if a tree is to be sampled

The estimator, $\hat{T}_y$, of the volume of the stand is

$$\hat{T}_y = \frac{FA}{m} \sum_{j=1}^{m} n_j \left[\sum_{j=1}^{k} \sum_{i=1}^{n_j} \frac{y_{ij}}{x_{ij}} \right] \Bigg/ \sum_{j=1}^{k} n_j$$ [1]

where

m = the number of points sampled (count points plus measured points)

nj = the number of trees sampled on the jth point

k = the number of points measured for volume

y_{ij} = the volume measured by height-accumulation on the ith tree on the jth point

x_{ij} = the basal area of the ith tree on the jth point

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[1] See Appendix II for a more complete discussion of the sampling theory.

The value of the volume estimator, $\hat{T}_y$, depends on the height-accumulation method to predict true stem volume for a given basal area. On each sample point the observer counts the number of point sampled trees. A subsample of the points is measured with height-accumulation to estimate the volume/basal area ratio. Volume trees are measured using height-accumulation with one, two, or four-inch taper steps. Each point sample is a complete unbiased estimate of stand basal area. The average of all the point samples for volume/basal area ratio times the average number of trees per point and the appropriate constants yields an estimate of the stand volume per acre.

Design of the Study

To estimate the volume of a stand the estimator must be unbiased, consistent, and efficient. The Bitterlich point sample overlayed with a small sample of precisely measured trees will fulfill these requirements. The height-accumulation formula is exact for volume,[2] but it is not known if the method of measuring trees for height-accumulation can be efficiently applied to the tall dense stands of the Pacific Northwest. If height-accumulation with a Spiegel-Relaskop can be applied, the interpreter should have an

[2]See Appendix I for a more complete discussion of height-accumulation theory and estimating procedures.

estimating scheme that is inexpensive, unbiased, consistent, and efficient.

A large number of trees have previously been measured with the highly accurate Barr and Stroud optical dendrometer to determine their true cubic-foot content. These measurements were made for another study but appeared to be ideally suited for use as controls to gauge the performance of height-accumulation.

Initially two electronic computer programs were designed by this author. The foundation for the first was taken from a program written for the Forest Research Laboratory, Corvallis, Oregon. It was designed to calculate the total volume, surface area and length of individual trees measured with the optical dendrometer. The second program reads the output from the first program plus the observations on the same trees using the telescopic Spiegel-Relaskop. The latter program computes volume, surface area and length by the height-accumulation method and compares paired results by statistical methods. The program also computes the totals of all trees measured, their mean difference, standard deviation of the difference, coefficient of variation for mean difference and for control mean, standard error of mean difference, and results of a paired t test. This analysis was made for volume, surface area and length individually. Results from these analyses should demonstrate the efficiency of height-accumulation, its unbiasness, and limitations in various stand conditions.

Area of Study

The sampling for this study was done on the Black Rock Experimental Forest. It is located approximately 24 miles northwest of Corvallis, Oregon. The stands sampled were 55 year old Douglas-fir ranging in diameter from 8.0 to 26.0 inches and in height from 80 to 140 feet. The stands are part of a long term study on thinnings.

Materials

The following discussion presents a pair of electronic computer programs and a new look at the adaptability of the Spiegel-Relaskop. The flow charts and program listings are covered in Appendix V. The Relaskop with optics was tested in the field before adoption into this project. A more complete description of the Spiegel-Relaskop with optics is found in Appendix III.

Formation of a Computer Program

In order to calculate and compare volumes of a number of measured trees quickly and accurately, an electronic computer program (HTACCUM), was written. It compares individual volume estimates against true dendrometered volumes and displays trends or biases in the estimating procedure. A second program, II, was written to include the Bitterlich point sample theory. Program II yields individual tree volume, surface, and length; tree and stand identification; stand volume, surface, and length; and tree and stand variances and errors of estimates.

Explanation of Program I (HTACCUM)

1. Program I reads one control card, which sets the upper diameter limits for volume computations.
2. The program reads a set of cards which represents one tree. These cards contain data of the form:
   1) the true volume, surface, and length of the ith tree in the stand from the Forest Research Lab revised program
   2) an observation card for the ith tree which contains the plot number, tree number, DBH, distance of instrument from tree, first even diameter measured, taper step, number of measurements on tree, and a list of measured heights on the tree.
3. The program subdivides the tree into three basic forms for determination of volume. The lowest segment is automatically loaded into the neoloid summation. Each following segment is loaded dependent on its length when compared to the previous segment. Thus, if

$L_{i+1} > L_i$ load L_{i+1} in neoloid summation

$L_{i+1} = L_i$ load L_{i+1} in conoid summation

$L_{i+1} <$ Li load L_{i+1} in paraboloid summation

where

Li = the ith segment length, in feet

L_i+1 = the segment immediately above L_i

The last segment is always loaded into the conoid summation.

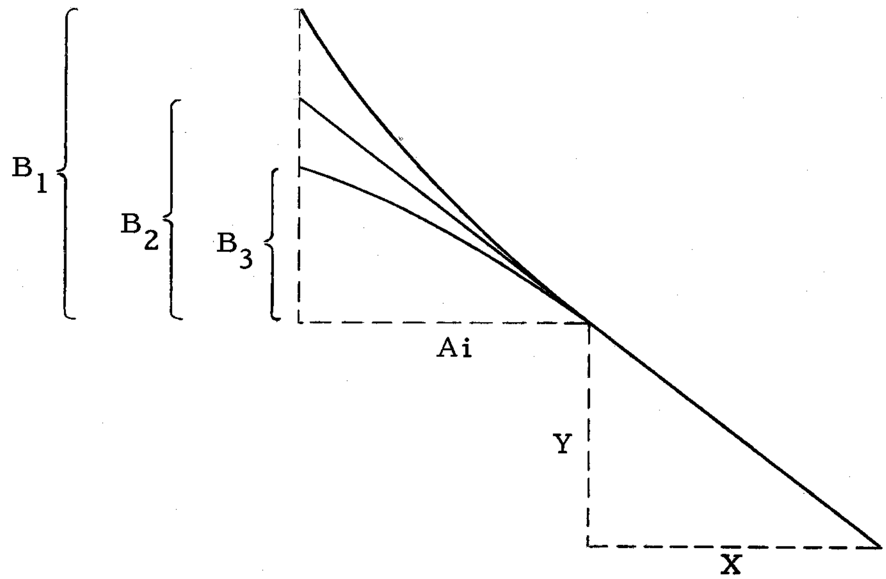
4. After summation by form is complete the program sums all segments and computes volume, surface, and length for the tree.

5. The computed volume, surface, and length are compared to the true volume, surface, and length. The differences and percent differences are recorded.

6. The program repeats steps 2 through 5 until all tress have been summed and recorded.

7. All trees are included in final volume, surface, and length calculations. Differences, percent differences, and error variances are computed.

8. All data are printed out and labeled for analysis.



When $\frac{Y}{X} > \frac{B_3}{Ai}$ use parabolic formula

$\frac{Y}{X} = \frac{B_2}{Ai}$ use conic formula

$\frac{Y}{X} < \frac{B_1}{Ai}$ use neoloidic formula

Where $L_i = Y$ and $L_{i+1} = B_i$, and Ai is constant and equal to X for height-accumulation theory.

Figure 1. Theory of formula selection for stem form.

Explanation of Program II

1. Program II reads one control card, which specifies the data, top merchantable limit, basal area factor, number of acres in stand, unit identification, and species identification.
2. The program then reads cards, which specify the number of variable point samples taken and lists the number of sampled trees at each point.
3. Cards are then read containing plot number, tree number, DBH, distance of instrument from tree, first even diameter measured, taper step used, number of measurements on tree, and a list of measured heights on the tree.
4. Program II does the same as parts three and four of Program I.
5. The final calculations of the program determine the volume-basal area ratio and then the volume per acre.
6. Final volume, surface, and length for the stand are computed and printed out for analysis.

The Spiegel-Relaskop

The wide scale Spiegel-Relaskop was chosen for this study because it is more suitable for measuring diameters of large trees at various heights and for tree counts with small and large basal area factors. The Relaskop was fitted with 4X magnification in order to reduce the scale and improve visibility of upper stem diameters.

This is possible because each Relaskop unit is 1/50 of the horizontal distance to the tree and each quarter unit is 1/200. Magnification of the stem but not the scale by four makes each quarter unit equivalent is 1/800 of the horizontal distance. The Relaskop gains the capacity to read upper stem diameters to the nearest one-quarter inch at approximately one chain horizontal distance.

Collection of the Data

To test the accuracy of measuring individual tree volumes, a sample of 30 trees was taken on the Black Rock Forest. The trees selected were a sample of approximately 200 trees measured with the optical dendrometer. The dendrometer measurements were made within three months of the height-accumulation measurements. All measurements were made after the growing season. Sixteen of the trees came from a heavily thinned plot on a high site. They were 13.0 to 26.0 inches DBH and visibly well tapered. The thinnings wereten years old. The remaining 14 trees came from adjoining stands with little or no thinning, high site, and cylindrically appearing stems. They ranged from 10.0 to 18.0 inches DBH.

The measurements made with the optical dendrometer were from ground level to tree tip, with diameter measurements arbitrarily chosen over the visible stem. The trees were numbered and marked to define the point of DBH measurement and the aspect from which the upper stem diameters were taken. A pole was used to find the top of the first 16-foot log . All upper stem height readings were based on the 16-foot point on the pole.

The Spiegel-Relaskop was then set up on the same aspect as was the optical dendrometer. It was positioned 66 feet horizontal distance from the center of the tree. This distance was necessary to read one-inch taper steps along the stem. The first reading was taken at stump height. Only the elevation was recorded. The next reading was the first taper step record. It is the first upward even inch of taper from the stump. The elevation to the point was recorded and the diameter was recorded as read from the Relaskop. At each successive taper step upward an elevation was recorded. If the taper steps became obscured a final total height reading was taken to interpolate the form to the top.

Difficulties arising in the field were recorded as were field conditions and time requirements.

PRESENTATION OF RESULTS

The data analysis from the computer programs showed that the height-accumulation method in conjunction with the telescopic Spiegel-Relaskop can predict individual tree volume. However, on the first trial a paired t test disclosed a highly significant difference in volume. This difference is more than should be encountered under normal conditions. The dendrometer measurements were plotted and a profile of each tree was made. Similar graphs of the height-accumulation measurements were overlaid on the dendrometer measurements. Any trends that may occur could then be evaluated for possible reasons for bias. Figure 2 shows the profile of one of the typical tree measurement trends. Through more field measurements, stem taper was found to be so gradual over the majority of the stem that the observer has the tendency to call the taper step too low. The statistical analysis of this trial is presented in Table 1.

The height-accumulation data were recorded for the same trees in the field a second time. In this trial the Relaskop was moved up the stem until the diameter point was clearly passed over. Then the instrument's scale image was backed down and tapped gently to assure proper settling of the gravity activated scales. This method produced much better results without an appreciable addition of time. During the last few critical feet, finding the taper step, the movement

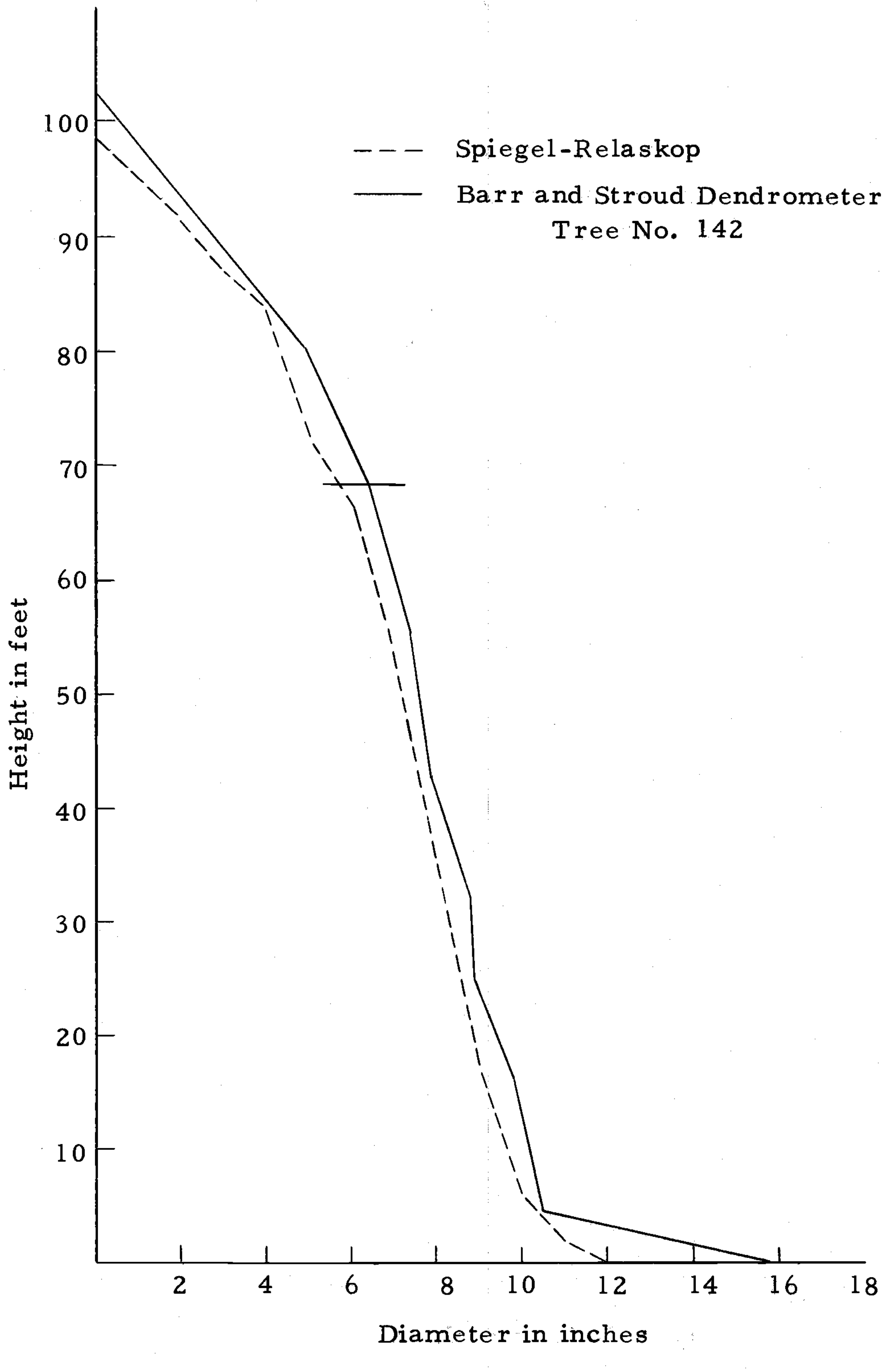


Figure 2. Stem profile.

Table 1. Computer printout analysis for Barr and Stroud dendrometer using the Smalian formula versus the Spiegel-Relaskop using the height-accumulation formula.

1 INCH TAPER STEPS — ARITHMETIC MEAN DIA. = 10.90

TOP DIAMETER 0 INCHES — QUADRATIC MEAN DIA. = 11.89

SOURCE	LENGTH	SURFACE	VOLUME	MEAN	DIFF	S.D.*	C.V.*	C.V.**	S.E.	T
HEIGHT-ACCUMULATION	3263.5			108.8	2.51	1.61	64.2		.294	4.974
COMPARISON	3338.7			111.3				1.45		
HEIGHT ACCUMULATION		8723.3		290.8	9.37	7.59	80.9		1.386	5.761
COMPARISON		9004.5		300.1				2.53		
HEIGHT-ACCUMULATION			2310.7	77.0	3.27	2.76	84.6		.504	5.300
COMPARISON			2408.7	80.3				3.44		

AVERAGE LENGTH PER INCH OF TAPER 7.20

TOTAL TREE COUNT 30

* of the mean difference

** as percent of the COMPARISON mean.

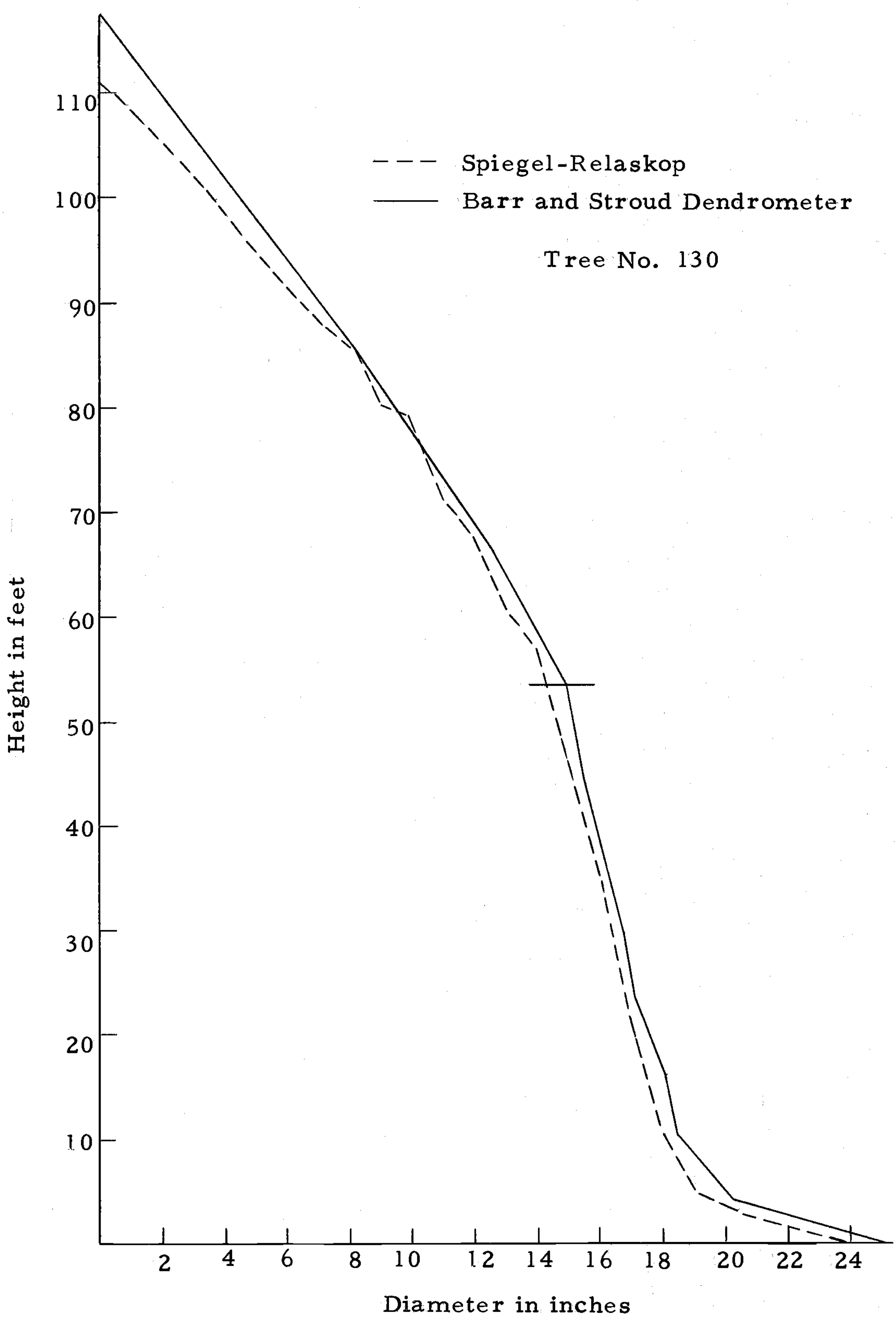


Figure 3. Stem profile.

was so slight, the scale had the tendency to stick and not respond. A slight tap on the case alleviates the problem.

The t test still expressed a significant difference in height. Measurements with the optical dendrometer were checked in an attempt to explain the bias. Dendrometer volumes were consistently higher, as were surface and height. The stump and tip of the trees were definite images for most trees generally discounting operator error in defining them. Error in height must be from technique rather than interpretation.

A reference point for positioning the 16-foot pole for the optical dendrometer is a paint mark at 4.5 feet from the high side of the stump at ground level. This mark was not used with the Relaskop since height is dependent on diameter. Upon investigation the paint marks were found to be generally biased. They averaged 0.3 to 0.5 feet low.

The measurement of total height for the optical dendrometer is done with a clineometer. Since it has no magnification and was hand held, it is inferior to the Relaskop on a tripod with four power magnification. A transit would be the optimum instrument for measuring height. The Relaskop in use was determined to be as good an estimator of height as could be brought to the field.

It is of interest to determine the optimum distance for measuring the height of any tree. The average height of the sample trees is 110 feet. Angles necessary to detect a two-foot difference at 110 feet height for various distances are plotted in Figure 4. Contrary to popular belief, "the farther away from the tree, the better the height estimate" is a fallacy. The most sensitive point to measure trees 110 feet in height is 110 feet horizontally distant from the base of the tree. In other words, tree height is measured with most precision where the angle to the point of interest is 45 degrees. Any other angle is less optically sensitive for interpretation of true height.

All the trees in the sample were then measured for total height at approximately optimum distance with the telescopic Relaskop on a tripod. These heights were substituted for the optical dendrometer for comparison to height-accumulation. A significant difference is still the result of comparison of height. Height-accumulation is slightly biased by 1.1 percent. The bias probably occurred because height-accumulation measurements were made too close to the tree to be optimum for total height. This was accepted since height readings had to be made over the entire length of the stem. The mean difference of height between Relaskop and optical dendrometer was 1.26 feet. To read heights more precisely at 110 feet of height requires the observer to distingusih subdivisions smaller than one-third of one degree.

The form of the measurements were graphed again. Instead of

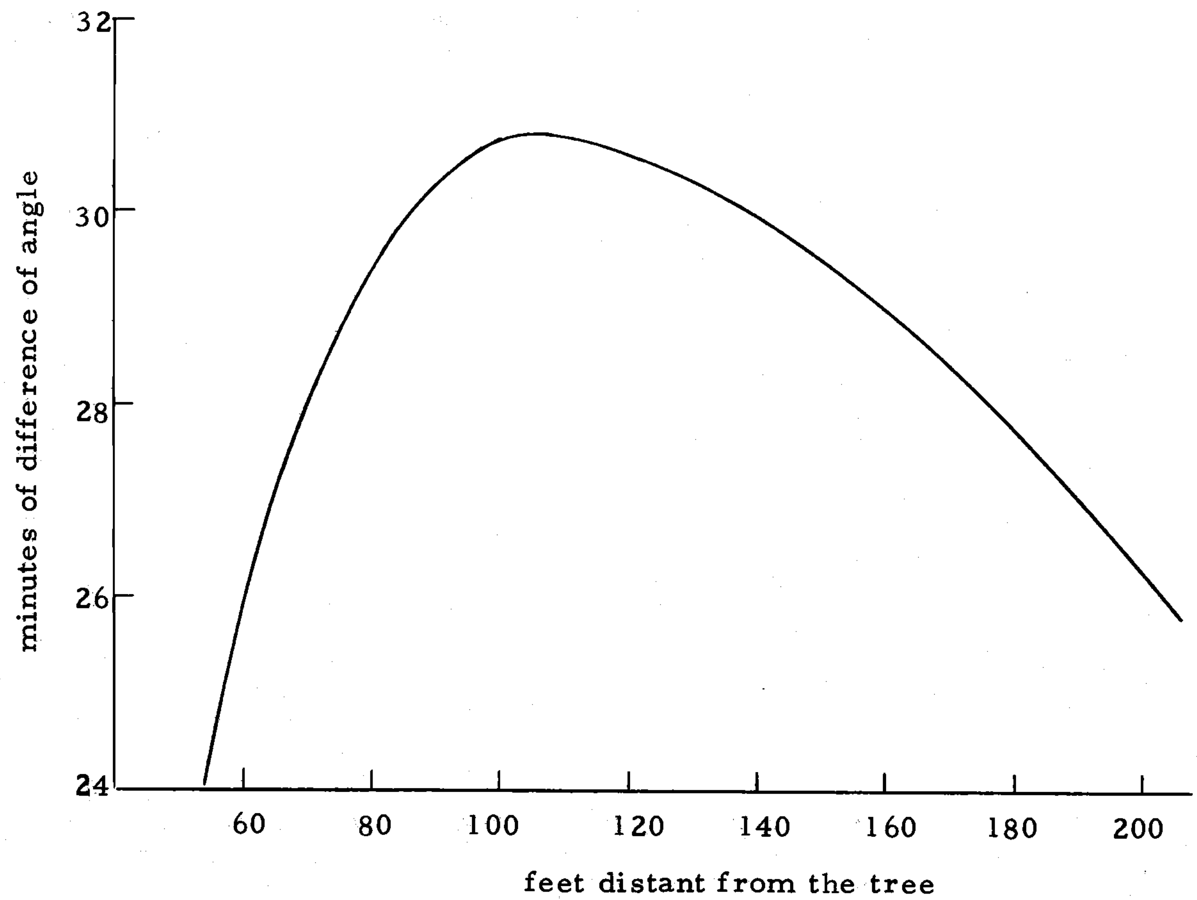


Figure 4. Difference in angle for 110 and 112 feet of height.

connecting points by straight lines, the actual formula curves were graphed to see the effect by each method. These comparisons may be observed in Figure 5.

Tree measurements graphed with the predicted form gave the most insight into why volumes were different. Both methods were predicting volume as a summation of parabolic segments. The optical dendrometer was more precise for each diameter but only a few measurements were taken. The formula filled in these voids along a parabolic curve. This curve misrepresents the tree at the stump and top. The result was to overestimate the volume of the tree, a situation recognized before sampling but not expected to be so critical to the comparison of volume estimation methods.[3]

The height-accumulation method had the same problem plus the restriction of a lean sensitive estimator of diameter. The volume determined by this method was much more reliable, however, because there were many more measurements taken. The abundance of measurements clustered heaviest at the problem spots, the stump and top. This had the effect of curbing the inflation brought on by extrapolation to the parabolic formula. To make the comparison more equitable an addition was made to the computer programs for both methods.

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[3] See example in Appendix IV of Smalian volume overestimating true volume.

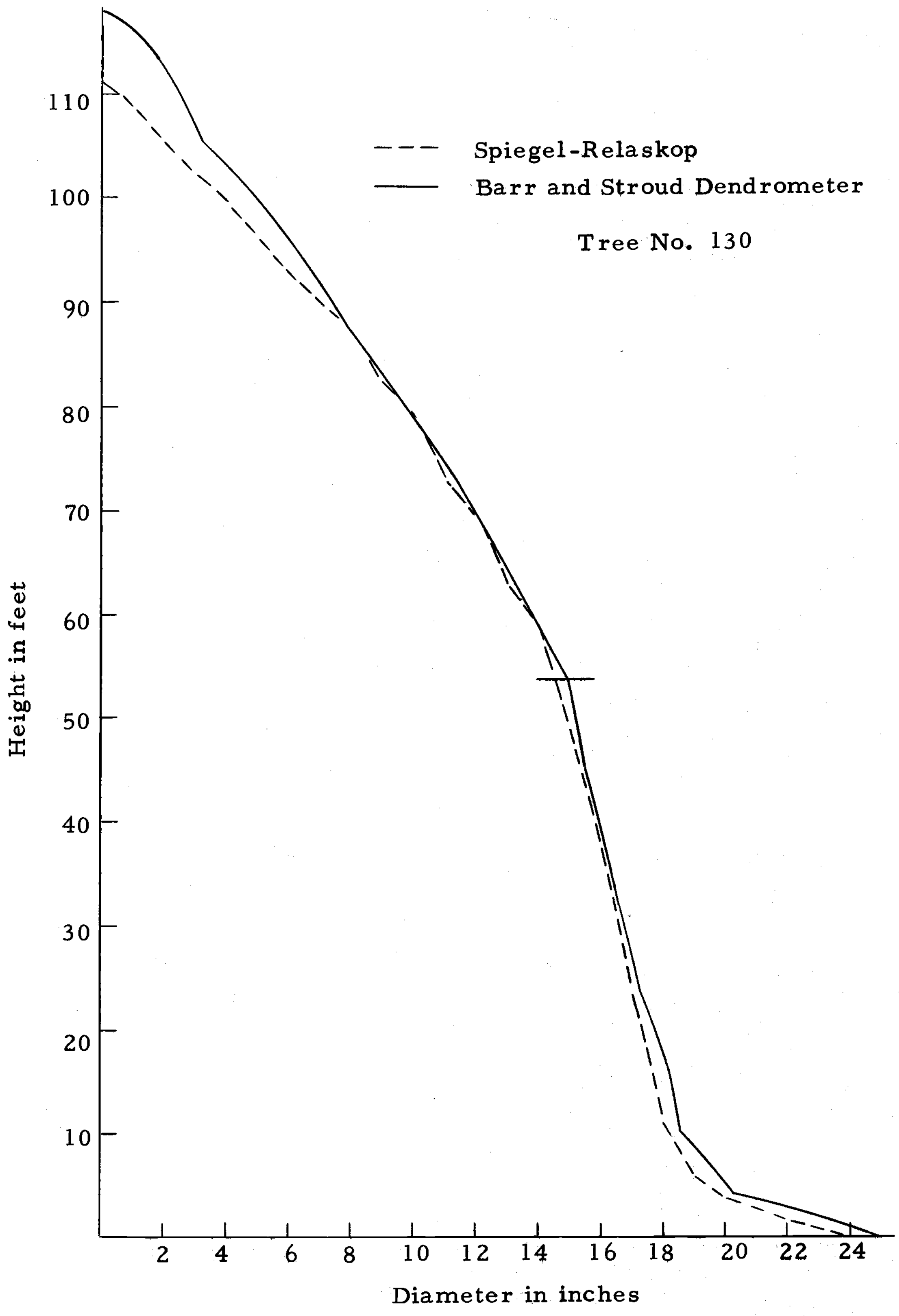

Figure 5. Stem profile as predicted by formula.

Table 2. Computer printout analysis for volumbe by trio (neoloid, conoid, paraboloid) and height by optimum distance method. Measured by Barr and Stroud dendrometer versus the telescopic Spiegel-Relaskop.

1 INCH TAPER STEPS — ARITHMETIC MEAN DIA. = 10.90

TOP DIAMETER 0 INCHES — QUADRATIC MEAN DIA. = 11.88

SOURCE	LENGTH	SURFACE	VOLUME	MEAN	DIFF	S.D.	C.V.*	C.V.**	S.E.	T
HEIGHT-ACCUMULATION	3263.5			108.8	1.25	1.11	88.3		.203	4.887
COMPARISON	3301.2			110.0				1.01		
HEIGHT-ACCUMULATION		8649.9		288.3	9.72	6.85	70.4		1.250	7.253
COMPARISON		8941.6		298.1				2.30		
HEIGHT-ACCUMULATION			2298.2	76.6	2.11	2.20	103.9		.401	4.313
COMPARISON			2361.6	78.7				2.79		
AVERAGE LENGTH PER INCH OF TAPER				7.20						
TOTAL TREE COUNT				30						

* percent of mean difference

** percent of COMPARISON mean

For the bottom segment both programs were rewritten to calculate the volume as a neoloid. Each successive segment was tested for diameter and length to determine the form in respect to the previous (lower) segment. For comparable diameter changes, length greater than the previous segment called for the neoloid formula. Lengths equal called the conic formula; and lengths shorter called the parabolic formula. After investigating the trend of all the top segments the programs were designed to calculate only conic bole using height-accumulation and the telescopic Relaskop. The t test for paired difference was less for all three variables: height, surface and volume. The variation, however, was greater than previously. This was probably due to the imaginary top of eight inches as interpreted by the two instruments. The analysis shows that there is a 1-in-50 chance that height-accumulation surface areas will differ from those of the Barr and Stroud dendrometer by more than 7.4 square feet (3.0 percent). There is less than an 8-in-10 chance that the volumes will differ by more than 0.14 cubic feet (0.19 percent). The difference in height was determined to be due in part to the interpolation technique used for the optical dendrometer for finding an eight inch top. The Barr and Stroud dendrometer cannot find predetermined diameters and must rely on biased interpolation schemes. The results of the analysis to an eight inch diameter top are shown in Table 3.

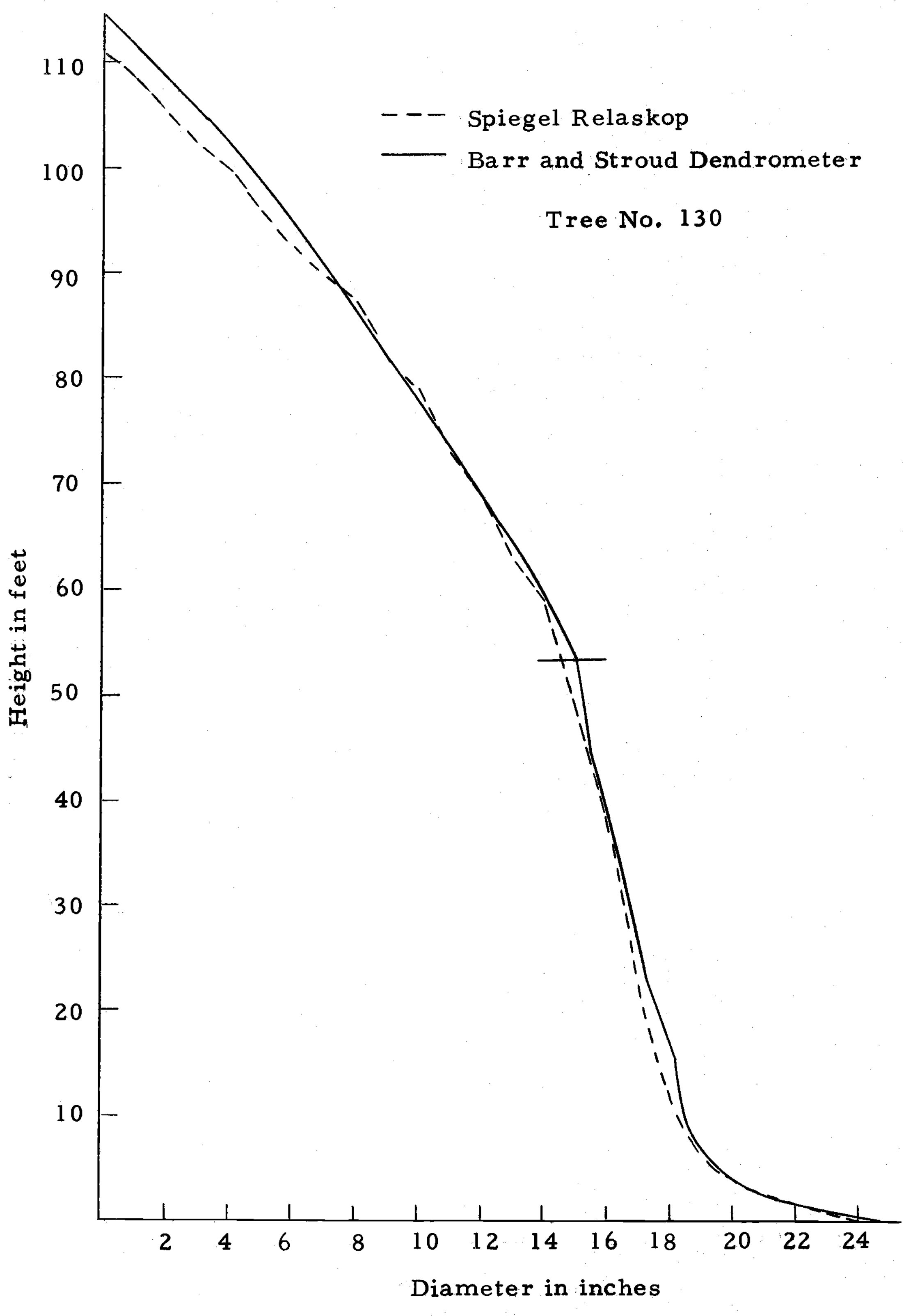

Figure 6. Stem profile as predicted by trio of formulae.

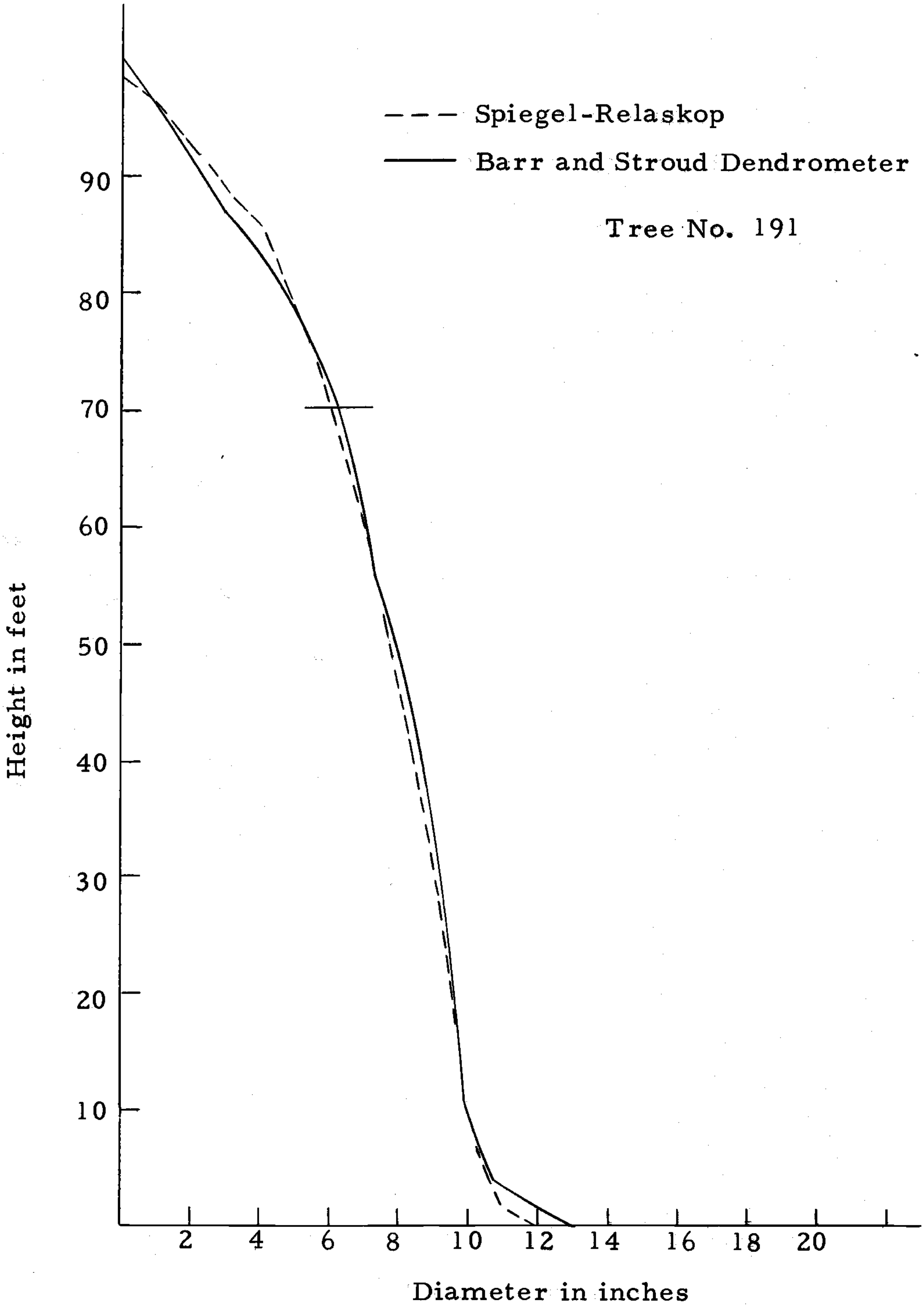

Figure 7. Stem profile as predicted by trio of formulae.

Table 3. Computer printout analysis for volume by trio(neoloid, conoid, paraboloid) measured by Barr and Stroud dendrometer versus telescopic Spiegel-Relaskop to an 8-inch top diameter.

1 INCH TAPER STEPS — ARITHMETIC MEAN DIA. = 12.90

TOP DIAMETER 0 INCHES — QUADRATIC MEAN DIA. = 13.39

SOURCE	LENGTH	SURFACE	VOLUME	MEAN	DIFF	S.D.	C.V.*	C.V.**	S.E.	T
HEIGHT-ACCUMULATION	2217.8			73.9	-4.632	7.10	153.0	10.25	1.296	-3.274
COMPARISON	2078.8			69.3						
HEIGHT-ACCUMULATION		7488.4		249.6	-7.402	16.25	220.0	6.71	2.967	-2.123
COMPARISON		7266.3		242.2						
HEIGHT-ACCUMULATION			2167.2	72.2	-0.141	3.18	2220.0	4.42	.581	-0.178
COMPARISON			2163.0	72.1						
AVERAGE LENGTH PER INCH OF TAPER				7.40						
TOTAL TREE COUNT				30						

* percent of mean difference

** percent of COMPARISON mean

Based on measurements for this study the height-accumulation method with the telescopic Spiegel-Relaskop took approximately nine minutes to execute per tree (from discovering the tree to completion of data collection). With an assistant to take DBH, measure the base line, and record the average time was five minutes. At this rate with an assistant, the interpreter could expect to measure approximately 90 trees per day. Without assistance, he would expect to measure about 50 trees per day under comparable conditions. Fifteen minutes per tree is the best time with the Barr and Stroud dendrometer. The interpreter would expect to measure approximately 30 trees per day using this instrument with an assistant.(Bell and Groman, 1968).

SUMMARY AND CONCLUSIONS

Height-accumulation with the telescopic Spiegel-Relaskop is one of the most sensitive measures of standing tree volume and form yet discovered. The ability of the method is limited only by the instrument used. The technique of using a trio of formulae (paraboloid, conoid, and neoloid) made a small but significant gain in accuracy. This technique is most useful for mixed stands, stands under intensive management, and stands of unusual characteristics (as might be found under irrigation and fertilization) which cannot be characterized by tariff tables or other similarly derived volume tables.

Some error and time would be saved under most field conditions to take the first reading at DBH and round to the nearest taper step interval. A fixed height to DBH could be worked into the computer program to add on the necessary length to the stump.

For trees greater than 120 feet in height it may be necessary to back farther away than 66 feet. It was found in this study that magnification changes with focus adjustment. The instrument could be used at 72 feet and still read one-inch taper steps by adjusting the focus and increasing the magnification. The telescopic Relaskop may also be used at 133'4" for two-inch taper steps. The decrease in accuracy is slight for two-inch taper steps but the time required to measure the trees is nearly half. The cruiser using height-accumulation may find two or four inch taper steps adequate for his purposes.

The measurements to a merchantable top of eight inches are more precise than to the top of the tree. This was apparent in the field. Above the eight to six-inch diameter the crown usually became an obstruction. Measurements in heavy crowns were difficult and sometimes impossible to take. If total stem volume is needed and the last few measurements cannot be seen, the computer program will interpolate the diameters to calculate the volume to the top. If, however, volume is required only to a certain lower diameter, the observer need not estimate diameters above that point. The computer program will calculate volume to the last measurement taken. This will be so regardless of smaller diameters requested. If the observer requests volume to a four-inch top and only measures to six inches, the program will only compute to a six-inch top.

It may be necessary with some Relaskops to tap the instrument lightly before taking each reading. Once the observer has noted that he has passed over the taper step he backs down to take the reading. The instrument may have a sticky scale which will not adjust readily to small movements without a slight tap. This can be a possible source of bias in estimating volume.

The telescopic Spiegel-Relaskop is sufficiently accurate to measure upper stem diameters. The smallest angle represents

1/800 the horizontal distance with four power magnification. The telescopic Relaskop measures upper stem diameters to the nearest two-tenths of an inch at even-inch intervals. On lower diameters of smooth barked trees the observer can read to one tenth of an inch at even-inch intervals. The scales in the Relaskop[4] allow trees to be measured in one-inch increments up to 48 inches in diameter. At 133 feet 4 inches, the observer can read diameters by two-inch increments up to 96 inches in diameter. There are other optical calipers and optical forks equally precise for small diameters, but none which will read diameters greater than 36 inches.

The telescopic Spiegel-Relaskop could be mounted with different magnification depending on the use and stand conditions. It is necessary under most conditions to have at least four-power to take one-inch taper steps from an optimum distance. As the magnification goes up the observer will find it increasingly difficult to steady the instrument. For most old-growth stands of high site, the four-power magnification with two-inch taper steps at 133 feet should give good results. Four-power with one-inch taper steps will measure second growth most efficiently where recurring thinnings and other treatments take place. In the Pacific Northwest these measurements will frequently be greater than 30 inches. The telescopic

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[4] The Wide Angle Spiegel-Relaskop by Dr. Walter Bitterlich.

Spiegel-Relaskop is the most feasible instrument devised for this area. It is lighter than the Barr and Stroud dendrometer, more economical and faster in the field. The observer takes one reading at each point on the tree and progresses the entire length without removing his eye from the instrument image. No other instrument has this capacity of efficient measurement. The Barr and Stroud dendrometer requires three readings with four eye adjustments at each point on the stem.

Taper steps with the Spiegel-Relaskop appears to be one way for the forest land manager to measure the reaction of the stand to his treatments. Though more study is needed, it appears that the growth form of individual trees may be measured using height-accumulation. Of the four stands used in this study, one was under intensive thinning treatments and three had little or no thinning. The average length per inch of taper measured was 6.32, 8.18, 8.27, and 8.20 feet, respectively. The stand under intensive management appears to be putting on more growth throughout the entire length of the stem than are the other three. The reason most obvious is that all the suppressed, intermediates, and codominants were removed. This leads to the possibility of using different lengths per inch of taper on individuals in a stand as a predictor of stand treatment and growth. Those trees of short length per inch of taper may be the vigorous trees in the stand. The land manager may be able to determine when and which stems to thin on the basis of measured length per inch of taper.

Bitterlich Sampling with Height-Accumulation

Trees selected for volume measurement from the point samples may be sampled on as many points as the observer sees necessary. They should be chosen systematically over the set of sampling points to cross any variation present. A further refinement would be an estimate of volume on all point sampled trees. A small number of height-accumulation measured trees would correct any bias in the estimations. This technique could be expanded to include grade and cull deductions.

Once the height-accumulation technique has been accepted, it is simple to expand to include board-foot volume by any rule. The user simply adds some coefficients to the cubic formulas to predict volume in board feet. A mill owner may wish to make his own coefficients for a particular yield of interest. He must determine the cubic content before processing and the net yield after processing for a particular run of wood. All future net yields could then be predicted given the gross volume, surface and length. This is the basic idea of volume, surface and length as brought forth by Lewis R. Grosenbaugh (1965).

BIBLIOGRAPHY

Barrett, James P. 1964. Testing accuracy of height-accumulation volumes. Journal of Forestry 62:824-825.

Beers, Thomas W. and Charles I. Miller. 1964. Point sampling: research results, theory, and applications. Lafayette, (Indiana Agricultural Experiment Station. Research Bulletin no. 786)

Bell, John F. and William A. Groman. 1968. Unpublished research on accuracy of the Barr and Stroud optical dendrometer model FP12. Corvallis, Oregon, Oregon State University, School of Forestry.

Bitterlich, Walter. 1962. Relaskop with wide scale. Allgemeine Forstzeitung (Vienna) 73:62-65. (Translated by Richard K. Hermann, Forest Research Laboratory, Oregon State University. Translation no. 13, 1963. 11 p.)

Bruce, David. 1967. New tools and methods in forest mensuration. Portland. 10 p. (Pacific Northwest Forest and Range Experiment Station. Research Note PNW-67)

Chapman, Herman H. and Walter H. Meyer. 1949. Forest mensuration. New York, McGraw-Hill. 522 p.

Enghardt, Hans and H. J. Derr. 1963. Height-accumulation for rapid estimates of cubic volume. Journal of Forestry 61:134-137.

Grosenbaugh, Lewis R. 1948. Forest parameters and their statistical estimation. Paper presented at Auburn Polytechnic Institute Conference on statistics applied to research. Auburn, Alabama. September 8.

Grosenbaugh, Lewis R. 1954. New tree measurement concepts: height-accumulation, giant tree, taper and shape. New Orleans. 32 p. (Southern Forest Experiment Station. Occasional Paper 134.)

______________ 1958. Point-sampling and line-sampling: probability theory, geometric implications, synthesis. New Orleans. 34 p. (Southern Forest Experiment Station. Occasional Paper 160.

Grosenbaugh, Lewis R. 1963. Optical dendrometers for out-of-reach diameters: a conspectus and some new theory. Washington, D.C. 47 p. (Society of American Foresters. Forest Science Monograph 4)

__________ 1965. Modern mensuration. Paper presented at Oregon State University, School of Forestry, Corvallis. November 19.

__________ 1967. Tree form: definition, interpolation extrapolation. In: Wood Measurement Conference Proceedings, ed. by F. Buckingham. Toronto. p. 1-16. (University of Toronto)

Johnson, Floyd A. 1961. Standard error of estimated average timber volume per acre under point sampling when trees are measured for volume on a subsample of all points. Portland. 5 p. (Pacific Northwest Forest and Range Experiment Station. Research Note 201)

Petzold, Albert O. 1966. Supervising Forester, Simpson Timber Company. Personal correspondence. Shelton, Wash. November 22.

Sutter. 1965. Diameter measurement of large trees with the wide-scale relaskop. Journal of Forestry 63:101-102.

APPENDICES

APPENDIX I

It is a simple procedure to transform formulae for cubic volume of frusta of solids of revolution to the equation used in estimating cubic volume by the height-accumulation method.

r_1 = upper radius of frusta of solids of revolution

r_2 = lower radius of frusta of solids of revolution

D = diameter at some point on a solid of revolution

T = taper step, defined as 1, 2, 3, or 4-inch change in diameter (D) between diameter measurements

L = length in feet or number of 4-foot sections between each diameter measurement

H = upward progressive totals of measurements of L

H' = upward progressive totals of heights (H)

K = shape divisor, 2 if parabolic, 3 if conic, and 4 if neoloidic.

Derivation of formula for parabolic frusta of solids of revolution:

$$V = \frac{\pi L}{2}\left[r_1^2 + r_2^2\right]$$

$$= \frac{\pi L}{2}\left[\frac{D^2}{4} + \frac{(D+T)^2}{4}\right]$$

$$= \pi\left[\frac{2D^2T^2L}{2(4T^2)} + \frac{2DT^2L}{2(4T)} + \frac{T^2L}{2(4)}\right]$$

$$= \frac{\pi T^2}{4}\left[L\left(\frac{D}{T}\right)^2 + L\left(\frac{D}{T}\right) + \frac{L}{K}\right];$$ where K = 2 and all measurements

are in the same units (i.e., inches, feet, meters)

Cubic volume for conic frusta of solids of revolution

$$= \frac{\pi L}{3} \left[r_1^2 + r_1 r_2 + r_2^2 \right]$$

$$= \frac{\pi L}{3} \left[\frac{D^2}{4} + \frac{D(D+T)}{4} + \frac{(D+T)^2}{4} \right]$$

$$= \frac{\pi L}{3} \left[\frac{3D^2}{4} + \frac{3DT}{4} + \frac{T^2}{4} \right]$$

$$= \pi \left[\frac{3D^2T^2L}{3(4T^2)} + \frac{3DT^2L}{3(4T)} + \frac{T^2L}{3(4)} \right]$$

$$= \frac{\pi T^2}{4} \left[L(\frac{D}{T})^2 + L(\frac{D}{T}) + \frac{L}{K} \right] \text{ ; where } K = 3.$$

Cubic volume for neoloidic frusta of solids of revolution

$$= \frac{\pi L}{4} \left[r_1^2 + 2r_1 r_2 + r_2^2 \right]$$

$$= \frac{\pi L}{4} \left[\frac{D^2}{4} + \frac{2D(D+T)}{4} + \frac{(D+T)^2}{4} \right]$$

$$= \pi \left[\frac{4D^2T^2L}{4(4T^2)} + \frac{4DT^2L}{4(4T)} + \frac{T^2L}{4(4)} \right]$$

$$= \frac{\pi T^2}{4} \left[L(\frac{D}{T})^2 + L(\frac{D}{T}) + \frac{L}{K} \right] \text{ ; where } K = 4.$$

This formula is readily adjustable for any particular part of the tree. The stump assumes a neoloidic frusta, the main bole assumes a conic frusta, and the upper bole a parabolic frusta. Then, as L. R. Grosenbaugh described in his 1954 paper, "New

Tree Measurement Concepts", the $L(\frac{D}{T})^2$ and $L(\frac{D}{T})$ can be logically substituted with the upward progressive totals of these lengths (L). The revised formula takes the form:

$$V = \frac{\pi T^2}{2} \left[\Sigma H' + \frac{\Sigma L}{2K} \right] \qquad (2)$$

An example might help to visualize this concept of substitution of arithmetic progressions. The tree in this example is 16 inches D.B.H. and 60 feet to an 8-inch merchantable top. All diameters smaller than the merchantable top have zero length and the same height as the merchantable top. Measurements are in 2-inch taper steps.

	dob.	X	L	XL	H	X^2L	H'
	2	1	0	0	60	0	352
	4	2	0	0	60	0	292
	6	3	0	0	60	0	232
merch. top	8	4	8	32	60	128	172
	10	5	12	60	52	300	112
	12	6	24	144	40	864	60
	14	7	12	84	16	588	20
D.B.H.	16	8	4	32	4	256	4
totals			60	352	352	2136	1244

To derive the revised form, substitute X for the $(\frac{D}{T})$. If the diameter breast high is chosen as a multiple of T, the X value will diminish in a decreasing arithmetic progression to one. Therefore, equation (2) equals

$$\frac{\pi T^2}{4}\left[\Sigma LX^2 + \Sigma LC + \frac{\Sigma L}{K}\right], \tag{3}$$

where $X = \frac{D}{T}$. The sums of the upward progressive totals of L equal ΣLX, which also equal ΣH. By the same rule, the sum of the upward progressive totals of H equal $\frac{1}{2}(\Sigma LX^2 + \Sigma LX)$. Equation (3) then becomes

$$\frac{\pi T^2}{2}\left[\frac{\Sigma LX^2}{2} + \frac{\Sigma H}{2} + \frac{\Sigma L}{2K}\right]$$

Which is logically substituted to the form:

$$V = \frac{\pi T^2}{2}\left[\Sigma H' + \frac{\Sigma L}{2K}\right] \qquad \text{equation (2).}$$

When applying this method to estimation of some product other than cubic feet, the surface integral (ΣH) must be maintained to permit proper evaluation. One of the advantages of this method is that it can be completely worked out on a standard adding machine. The lengths recorded for each diameter class for all trees can be added before doing any calculations. Then one grand total calculation can be made for all trees by multiplying π and a couple of constants times the upward progressive totals of lengths. No further calculations are needed for volume by any log rule to any merchantable top diameter.

APPENDIX II

Theory of Bitterlich Point Sampling Applied to Height-Accumulation

The Bitterlich point sample is commonly used in forestry and is recognized as an extremely useful sampling scheme for determining basal area of a stand. Since basal area is linearly related to volume for an even-aged stand, using the point sample in conjunction with a sample of volume by height-accumulation should produce a reliable estimate of stand volume. The volume sample will estimate volume only on the point sampled trees. The general scheme may be defined as follows:

$U = \{u_1, u_2, u_3, \ldots, u_N\}$, the set of N trees in the stand

$X = \{x_1, x_2, x_3, \ldots, x_N\}$, the set of N values where

x_i = the basal area of the ith tree in the stand

$Y = \{y_1, y_2, y_3, \ldots, y_N\}$, the set of N values where

y_i = the volume, in cubic feet, of the ith tree in the stand

Then the total basal area of the stand is

$$T_x = \sum_U x_i$$

and the total volume of the stand is

$$T_y = \sum_U y_i$$

Point sampling is a special case of unequal probability sampling where the inclusion probability is proportional to some variable, which in this case is basal area. Basal area of the stand may be expressed by the general unbiased estimator

$$T_x = \sum_U x_i / \pi_i$$

where

$\sum_S$ = the summation over S, the sample set

π_i = the inclusion probability of the $i\underline{th}$ tree

The estimator, $\hat{T}_x$, of the basal area of the stand is

$$\hat{T}_x = \sum_{S_{II}} x_i / \pi_i$$

which by substition yields

$$\hat{T}_x = \sum_{S_{II}} \frac{x_i 43560 A}{x_i \operatorname{Csc}^2(\frac{\theta}{2})}$$

which may be simplified to

$$\hat{T}_x = nFA$$

where

S_{II} = the set of sample trees

n = the number of angle-counted trees at the sample point

$F = \frac{43560}{Csc^2(\frac{\theta}{2})}$, the basal area factor

A = the number of acres in the stand

$$\pi_i = \frac{x_i CSC^2(\frac{\theta}{2})}{43560\ A}$$

θ = a horizontal angle of specified size used to select trees for the sample

For each random point, an estimate of the total basal area is obtained. Since there is a known relationship of volume to basal area in a given stand, a precise measure of volume on a single point sample should predict stand volume. The form is

$$\hat{T}_y = \hat{T}_x \frac{t_y}{t_x}$$

where t_y = the sum of the measured volume on the sampled trees on the sample point

t_x = the basal area of the trees, summed over the same sample

Then each possible sample point in the stand will estimate total volume. A sample, S_I, of m sample points may be taken of the possible set of sample points to yield a more precise estimate of total volume and an estimate of variance. Then the estimator, $\hat{T}_y$, of total volume from a sample of m points is

$$\hat{T}_y = \frac{1}{m} \sum_{S_I} T_{yj}$$

which may be simplified to

$$\hat{T}_y = \frac{FA}{m} \sum_{j=1}^{m} \left[\frac{\sum_{i=1}^{n} y_{ij}}{\sum_{i=1}^{n_j} x_{ij}} \right]$$

or

$$\hat{T}_y = \frac{FA}{m} \sum_{j=1}^{m} \left[\frac{t_{yj}}{t_{xj}} \right]$$

where

n_j = the number of trees sampled by the point sample on the jth point

m = the number of sample points taken

y_{ij} = the height-accumulation volume of the ith tree on the jth point

x_{ij} = the basal area of the ith tree on the jth point

The estimator of variance, $\hat{V}(\hat{T}_y)$, for a simple random sample of m points is

$$\hat{V}(\hat{T}_y) = \frac{(FA)^2}{m(m-1)} \left[\sum_{j=1}^{m} \frac{t_{yj}^{\ 2}}{t_{yj}} - \frac{\left[\sum_{j=1}^{m} \frac{t_{yj}}{t_{xj}} \right]^2}{m} \right]$$

Where not all sample points are measured for volume, the estimator changes such that

$$\hat{T}_y = \frac{FA}{m}\left[\sum_{j=1}^{m} nj\right] \frac{\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{y_{ij}}{x_{ij}}}{\sum_{j=1}^{k} n_j}$$

where

m = the number of sample points taken (count points plus measured points)

The new estimator of variance for a single random sample of m points and volume/basal area ratio over k points becomes

$$\hat{V}(\hat{T}_y) = (FA)^2 \left[\frac{\sum_{j=1}^{k}\left[\sum_{i=1}^{n}\frac{y_{ij}}{x_{ij}}\right]^2 - \frac{\left[\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{y_{ij}}{x_{ij}}\right]^2}{k}}{k-1} + \frac{\sum_{j=1}^{k}\left[\sum_{i=1}^{n}\frac{y_{ij}}{x_{ij}}\right]^2 - \frac{\left[\sum_{j=1}^{k}\hat{R}\, n_j\right]^2}{k}}{k-1} \left[\frac{m-k}{k}\right] \right]$$

where

$$\hat{R} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{y_{ij}}{x_{ij}}}{\sum_{j=1}^{k} n_j}$$, the volume/basal area ratio estimator

Then the standard error of the estimated average volume per acre equals

$$SE_{\bar{y}} = \sqrt{\frac{V(T_y)}{A^2 m}}$$

The standard error of the tree count in percent is

$$SE_{\bar{n}} = \sqrt{\frac{\sum_{j=1}^{m} n_j^2 - \frac{\left[\sum_{j=1}^{m} n_j\right]^2}{m}}{(m-1)\sum_{j=1}^{m} n_j}}$$

and the standard error of the average volume/basal area ratio in percent is

$$SE_{\hat{R}} = \sqrt{\frac{\sum_{j=1}^{k}\sum_{i=1}^{n}\left[\frac{y_{ij}}{x_{ij}}\right]^2 - \frac{\left[\sum_{j=1}^{k}\sum_{i=1}^{n}\frac{y_{ij}}{x_{ij}}\right]^2}{\sum_{j=1}^{k} nj}}{\left[\sum_{j=1}^{k} nj - 1\right]\sum_{j=1}^{k}\sum_{j=1}^{n}\frac{y_{ij}}{x_{ij}}}}$$

APPENDIX III

The Spiegel-Relaskop with Optics

The Relaskop is ideally suited for use with height-accumulation. It adjusts automatically as the interpreter sights up the tree. The taper steps are well defined by the instrument when viewing the tree. The only problem has been a lack of accuracy because the interpreter must be at least one chain distant to view the entire stem. Each Relaskop unit is equivalent to 16 inches at one chain.

To enable the interpreter to estimate one-inch taper steps some magnification was necessary. No magnification was built into the Relaskop for direct viewing of the object. The scales were magnified to bring each unit up to 1/50 of the horizontal distance to the object. Each quarter unit is equivalent to 1/200 of the horizontal distance. It was found that a telescope mounted in front of the Relaskop magnifies the object and maintains the scale dimensions within the instrument. A four power telescope would make each unit equivalent to 1/800 the horizontal distance. One Relaskop unit equals the following diameters on the object at the prescribed distances with four power magnification:

1 unit	$\frac{1}{4}$ unit	Distance
2"	$\frac{1}{2}$"	33'4" or 400"
4"	1"	66'8" or 800"
6"	1$\frac{1}{2}$"	100' or 1200"
8"	2"	133'4" or 1600"

A common rifle scope was found to have a sufficient focal length to mount in front of the Relaskop and maintain the desired field of vision. A 4 × 33 Western Field rifle scope was obtained. The model 60-1208 had an luminosity of 66 which was found to be the minimum for dark stands of timber. An aluminum mount was designed and constructed for the scope and Relaskop. The Relaskop was mounted without special work on its case. Some slippage occurred and two screws in the Relaskop were removed and reset through the aluminum mount. This prevented slippage but was not used as a major support. An aluminum strap and bolts held the Relaskop in position. The mount was equipped with a tripod shoe for quick mounting to a tripod.

Two tripods were tested. The best was the Safe-Lock Model "FL". The leg controls are at the top for quick set-up and adjustment. The Relaskop, mount, and scope were attached to the tripod and leveled with a surveyor's level. The Relaskop was slightly out of adjustment and corrected. The scope caused the light rays to bend, if not leveled, and produced false readings. Two leveling stations were set up approximately 80 feet apart to check the instrument.

In the day to day use of the instrument, it was checked to discover any slippage in the mount or scope. After three months, 1000 miles in a pickup truck, and some inclement weather no adjustment had to be made. The instrument is lighter than the Barr and Stroud Optical Dendrometer and easily carried in the woods. With the Relaskop the entire apparatus can be built for about $200. The mount was made of one piece of 4 × 14 inch aluminum, two detachable rifle mounts, four machine screws and nuts, and a 1 × 4 inch aluminum strap. A four-inch piece of spring steel was used to release the scales during readings.

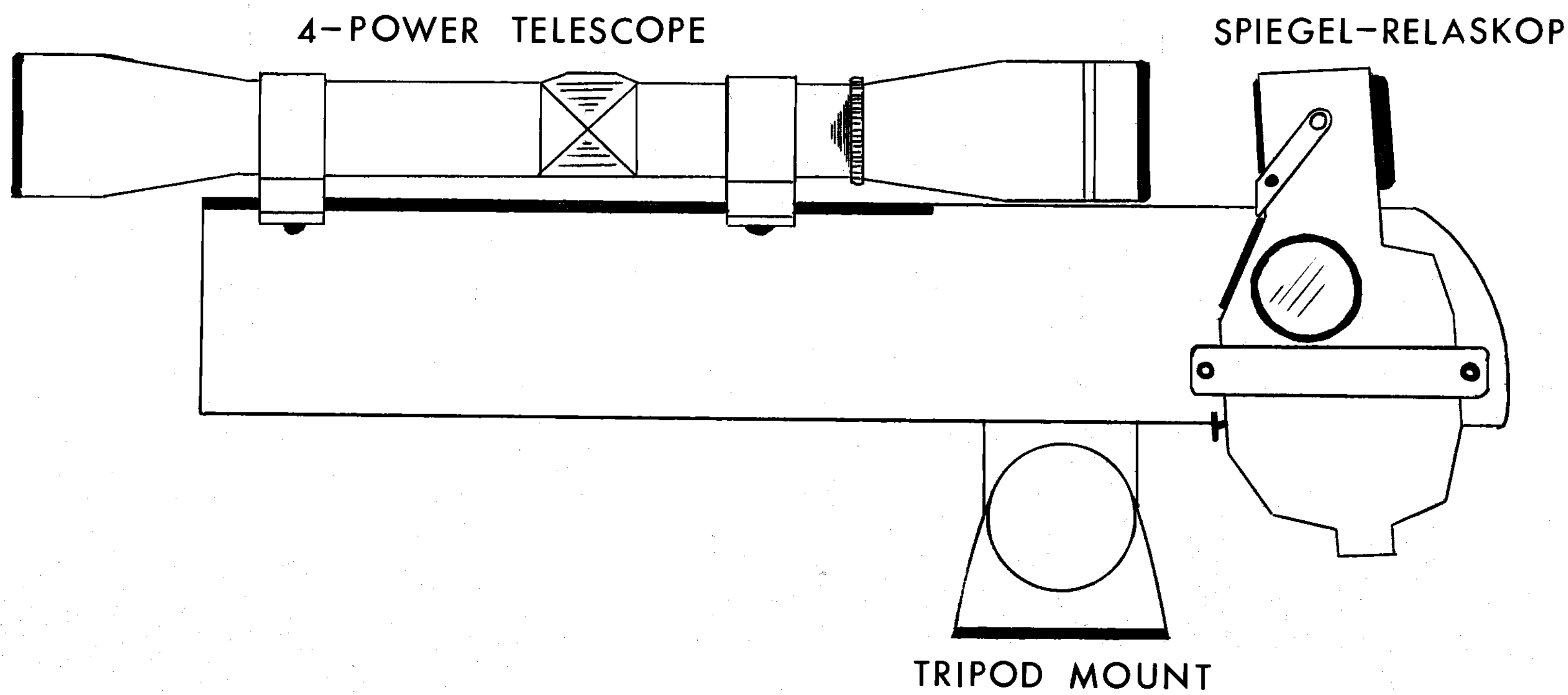


Diagram of telescopic Spiegel-Relaskop

APPENDIX IV

Example of error when using Smalian formula for butt logs. In this example the butt log is 16 feet long with a large end diameter of ten inches and a small end diameter of six inches. The Smalian formula is parabolic at a constant rate of taper. This sample log tapers at a rate of four inches per eight feet for the first four feet and one and one third inches per eight feet for the remaining 12 feet. Height-accumulation measures the log by one-inch taper steps. The computer program senses that the log is neoloidic. Volume by height-accumulation and the inaccurate Smalian formula are calculated below. The error is apparent when using formula inappropriate for the form of the tree.

d. o. b.	L	H	H'
2	0	16	52
4	0	16	36
6	12	16	20
8	4	4	4
	16	52	112

Height-accumulation volume =

$$\frac{4\pi}{(2)144}\left(112+\frac{16}{8}\right)$$

$$= \frac{228\pi}{144}$$

Smalian volume =

$$\frac{16\pi}{(2)144}\left(\frac{6^2+10^2}{4}\right)$$

$$= \frac{272\pi}{144}$$

Error in Smalian formula =

$$\frac{272 - 228}{228}(100) = +19.298 \text{ percent}$$

APPENDIX V

List and definitions of flow chart symbols

for computer program "II"

Data(A) = DM, DD, DA, UNIT, ACRES, SPEC, TOP, BAF, NVP, BTR

Data (B) = NTP, TCP_i (part 1)

= JP, TREE, DBH, DIST, DOB, TS, N, HT_i (part 2)

Data (C) = UNIT, ACRES, DM, DD, DA, SPEC, TOP, BTR, VBAR, VPA, SPA, SLPA, BAPA, BINT, BSCR, TVOL, TBINT, TBSCR, TS, SP, STC, BAF, NTP, SET, SEVB, CSE

DM = month

DD = day

DA = year

UNIT = name of unit

ACRES = number of acres in unit

SPEC = species name

BAF = basal area factor

NVP = number of volume count points in sample

BTR = bark thickness ratio, inside bark/outside bark

NTP = number of sample points for basal area

TCP_i = number of trees on the ith point

JP = sample point number

TREE = tree number

DBH = diameter breast high

DIST = instrument distance from tree

DOB = first diameter measured with the instrument

TS = taper step

N = number of height measurements on tree

HT_i = angle to the ith point on the tree

SL = upward progressive total of lengths on the tree

SH = upward progressive total of SL

SHP = upward progressive total of SH

SAA_i = calculated value where i = 1 is volume, 2 is surface, and 3 is length

VBAR = volume/basal area ratio

VPA = volume per acre in cubic feet

SPA = surface area per acre in square feet

SLPA = length per acre in linear feet

BAPA = basal area per acre

BINT = International board feet per acre

BSCR = Scribner board feet per acre

TVOL = total cubic volume on unit

TBINT = total international board foot volume on unit

TBSCR = total scribner board foot volume on unit

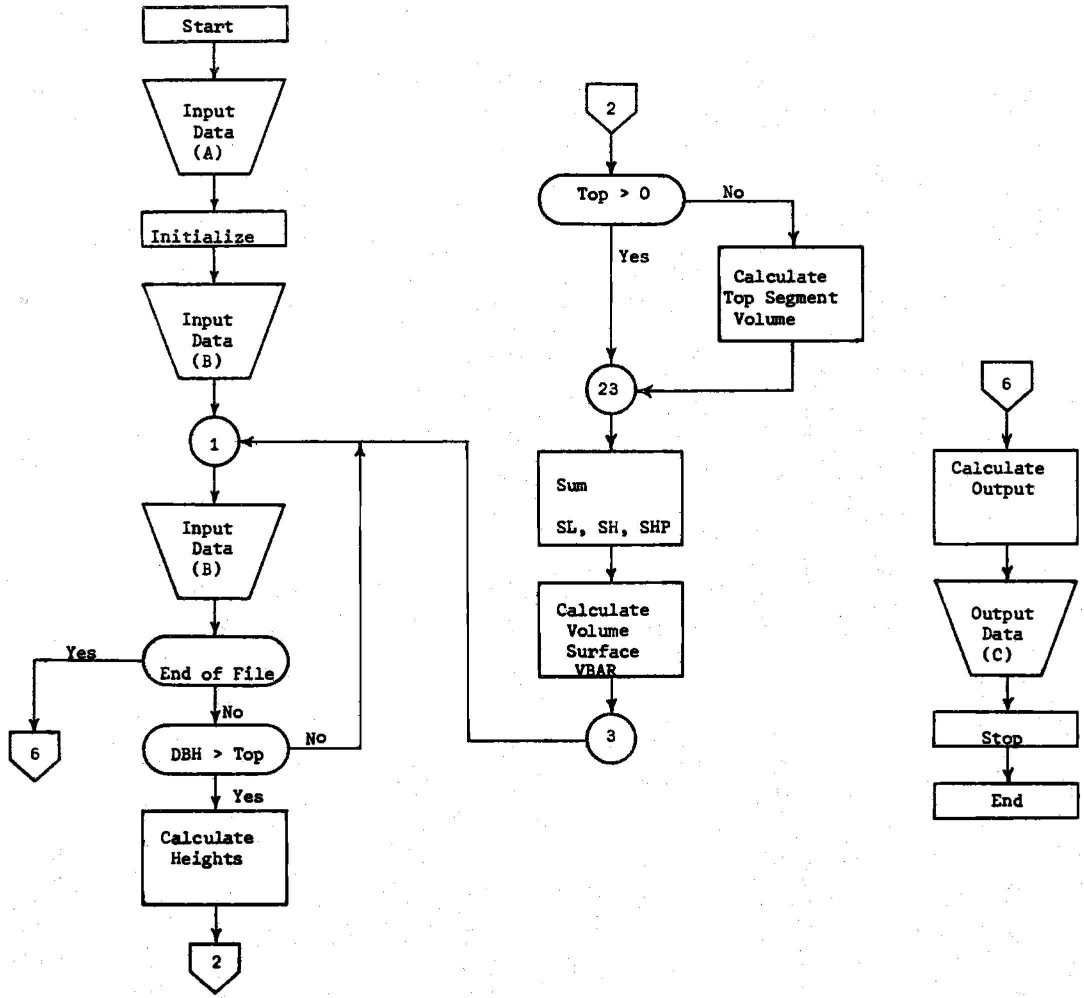
SP = number of volume measured trees

STC = number of tree counts for sample

SET = standard error of tree count

SEVB = standard error of volume estimate

CSE = standard error of estimated volume per acre



A flow chart for the computer program "II" for this study

List and definitions of flow chart symbols

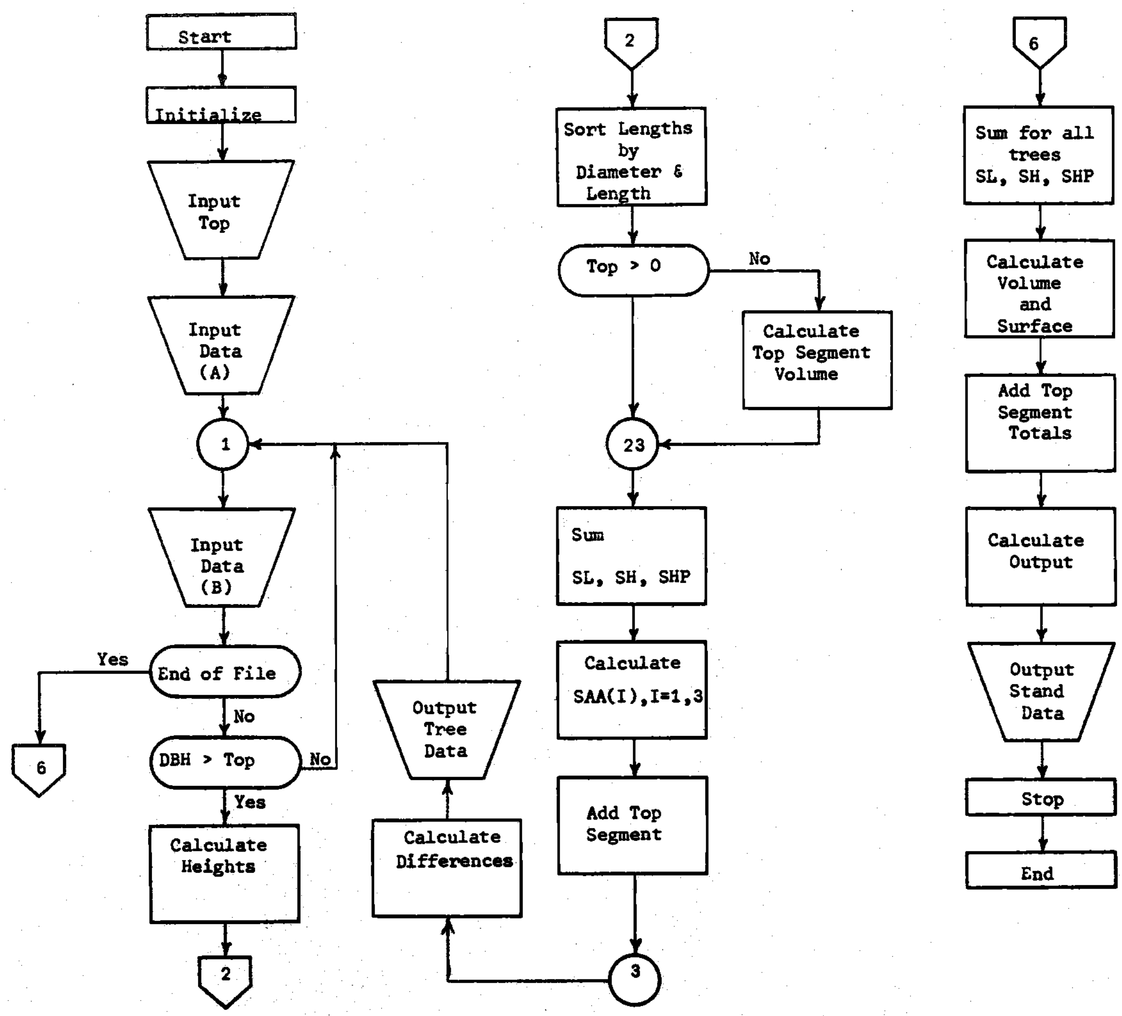
for computer program "HTACCM"

Data (A) = comparative volume, surface and length of each tree in the study measured with the Barr and Stroud dendrometer

Data (B) = JP, TREE, DBH, DIST, DOB, TS, N, HT_i (as previously defined)

Tree Data = volume, surface, and length as computed by both methods, percent differences, and true differences

Stand Data = total volume, surface, and length statistics, standard deviations, standard errors, coefficients of variation, and t tests of significant difference



A flow chart for the computer program "HTACCUM" for this study

The computer program "II" used on the CDC 3300 for this study

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      PROGRAM II
C**** A TIMBER CRUISE SUMMARY PROGRAM FOR CALCULATING VOLUME, SURFACE,
C     AND LENGTH BY THE HEIGHT ACCUMULATION METHOD IN CONJUNCTION WITH
C     DOUBLE SAMPLING OVER A BITTERLICH POINT SAMPLE DESIGN
C     PREPARED JANUARY 24, 1968 BY JAMES D. ARNEY SCHOOL OF FORESTRY
C     OREGON STATE UNIVERSITY
      DIMENSION HT(50),RL(50),TCP(200),SVBAR(50),SSBAR(50),SLBR(50),
     1VCP(50)
      READ(60,100) DM,DD,DA,UNIT,ACRES,SPEC,TOP,BAF,NVP,BTR
  100 FORMAT(3I2,2F8.1,F3.0,2F2.0,I3,F8.6)
  101 FORMAT(I3,F3.0,F4.1,F4.0,2F2.0,I2,(15F4.1))
      SRLT=0.0
      SSRF=0.0
      SVOL=0.0
      SHAPE=3.0
      IF(NVP.LE.50)16,17
   17 WRITE(61,126)
   16 READ (60,201)  NTP,(TCP(I),I=1,NTP)
  201 FORMAT (I5,(25F3.0))
      SQVB=0.0
      IF(NTP.LE.200)31,30
   30 WRITE(61,127)
   31 DO 50 I=1,50
      SVBAR(I)=0.0
      SSBAR(I)=0.0
      SLBR(I)=0.0
   50 VCP(I)=0.0
      STC=0.0
      SQTC=0.0
    3 READ (60,101)  JP,TREE,DBH,DIST,DOB,TS,N,(HT(I),I=1,N)
      GOTO(6,4) EOFCKF(60)
    4 IF(DBH.LE.TOP)3,9
    9 VCP(JP)=VCP(JP)+1.
      U=.017453293*HT(1)
      FTB=-DIST*SINF(U)/SQRTF(1.-SINF(U)*SINF(U))
      DO 14 I=2,N
      SU=SINF(.017453293*HT(I))
   14 HT(I)=DIST*SU/SQRTF(1.-SU*SU)+FTB
      HT(1)=0.0
      K=(DOB-TOP)/TS+1.
      IF(TOP.GT.0.)25,24
   24 N=N-1
   25 IF(K.LE.N)13,12
   12 K=N
   13 RLT=0.0
      SRF=0.0
      VOL=0.0
      J=DOB+TS
      NTS=TS
      DO 5 I=2,K
      J=J-NTS
      M=I-1
    5 RL(J)=HT(I)-HT(M)
      IF(TOP.LE.0.)22,23
   22 RLT=HT(N+1)-HT(K)
      FTJ=J
      FTJ=FTJ*BTR
      VOL=3.1415927*RLT*FTJ*FTJ/1728.
      SRF=3.1415927*FTJ*SQRTF(FTJ*FTJ/576.+RLT*RLT)/24.
   23 NDOB=DOB
      SL=0
```

```
      SH=0
      SHP=0
      L=DOB/TS
      K=K+1
      DO 11 I=K,L
      J=J-NTS
   11 RL(J)=0
      DO 8 JJ=1,L
      J=NDOB-NTS*JJ+NTS
      SL=SL+RL(J)
      SH=SH+SL
    8 SHP=SHP+SH
      CUFT=0.010908*TS*TS*BTR*BTR*(SHP+SL/(SHAPE*2.))+VOL
      SUR=0.261799*TS*BTR*(SH+SL/2.)+SRF
      SL=SL+RLT
      BA=DBH*DBH*.005454154
      VBAR=CUFT/BA
      SVBAR(JP)=SVBAR(JP)+VBAR
      SSBAR(JP)=SSBAR(JP)+SUR/BA
      SLBR(JP)=SLBR(JP)+SL/BA
      SQVB=SQVB+VBAR*VBAR
      SRLT=SRLT+RLT
      SSRF=SSRF+SRF
      SVOL=SVOL+VOL
      GO TO 3
    6 CONTINUE
      TP=NVP+NTP
      VBAR=0.0
      SBAR=0.0
      SLBAR=0.0
      VBS=0.0
      SPS=SP=0.0
      SWX=0.0
      DO 52 I=1,50
      SP=SP+VCP(I)
      VBAR=VBAR+SVBAR(I)
      SBAR=SBAR+SSBAR(I)
      SLBAR=SLBAR+SLBR(I)
      SPS=SPS+VCP(I)*VCP(I)
      SWX=SWX+VCP(I)*SVBAR(I)
   52 VBS=VBS+SVBAR(I)*SVBAR(I)
      VP=NVP
      DEVA=(VBS-VBAR*VBAR/VP)/(TP*(VP-1.))
      RHAT=VBAR/SP
      DEVB=(VBS+RHAT*RHAT*SPS-2.*RHAT*SWX)/(VP*(VP-1.))
      DEVB=DEVB*(TP-VP)/TP
      CSE=BAF*SQRTF(DEVA+DEVB)
      DO 41 I=1,NTP
      STC=STC+TCP(I)
   41 SQTC=SQTC+TCP(I)*TCP(I)
      SUMP=NTP
      SMTC=STC/SUMP
      BAPA=BAF*SMTC
      VPA=VBAR*BAPA/SP
      SPA=SBAR*BAPA/SP
      SLPA=SLBAR*BAPA/SP
      BINT=9.1236*VPA-0.70846*SPA+0.042222*SLPA
      BSCR=9.057*VPA-0.852*SPA-0.112*SLPA
      SDT=SQRTF((SQTC-STC*STC/SUMP)/(SUMP-1.))
      SDVB=SQRTF((SQVB-VBAR*VBAR/SP)/(SP-1.))
      SET=SDT*100./(SQRTF(SUMP)*SMTC)
```

```
      SEVB=SDVB*100./(SQRTF(SP)*VBAR/VP)
      TVOL=VPA*ACRES
      TBINT=BINT*ACRES
      TBSCR=BSCR*ACRES
      WRITE(61,120) UNIT,ACRES,DM,DD,DA,SPEC,TOP,BTR
      WRITE(61,121) RHAT,BAPA,VPA,SPA,SLPA
      WRITE(61,124) BINT,BSCR
      WRITE(61,125) TVOL,TBINT,TBSCR
      WRITE(61,122) TS,SP,NVP,STC,BAF,NTP
      WRITE(61,123) SET,SEVB,CSE
  120 FORMAT(1H1,9X,6HUNIT  ,F8.0,12X,F8.1,25H ACRES                DATE ,
     12(I2,1H/),I2,//10X,10HSPECIES   ,F4.0,10X,12HTOP DIAMETER,F3.0,
     27H INCHES,//10X,20HBARK THICKNESS RATIO,F14.6)
  121 FORMAT (////15X,24HVOLUME/BASAL AREA RATIO ,F14.1,11H CUBIC FEET,
     3//15X,24HBASAL AREA PER ACRE     ,F14.1,12H SQUARE FEET,
     5//15X,24HVOLUME PER ACRE         ,F14.I,11H CUBIC FEET,
     1//15X,24HSURFACE AREA PER ACRE   ,F14.1,12H SQUARE FEET,
     2//15X,15HLENGTH PER ACRE,9X,F14.1,12H LINEAR FEET)
  122 FORMAT ( ///10X,17HSUMMARY BASED ON,,
     1 //15X,45HHEIGHT ACCUMULATION VOLUME MEASUREMENT METHOD,
     2//15X,F3.0,21H INCH TAPER STEPS ON ,F3.0,11H TREES OVER,I5,
     37H POINTS,//15X,15HPOINT SAMPLE OF,F5.0,12H TREES USING,F4.0,
     44H BAF,//15X,2HON,I4,14H SAMPLE POINTS)
  123 FORMAT (///10X,20HSTATISTICAL SUMMARY,,
     1 //15X,32HSTANDARD ERROR OF TREE COUNT     ,F8.2,8H PERCENT,
     2//15X,32HSTANDARD ERROR OF VOL/BASAL AREA,F8.2,8H PERCENT,
     3///15X,51HSTANDARD ERROR OF ESTIMATED AVERAGE VOLUME PER ACRE,
     4//15X,18HBY JOHNSONS METHOD,8X,F14.2,11H CUBIC FEET)
  124 FORMAT (
     1 /15X,29HINTERNATIONAL VOLUME PER ACRE,F11.1,8H BD. FT.,
     2//15X,24HSCRIBNER VOLUME PER ACRE,2X,F14.1,8H BD. FT.)
  125 FORMAT(///10X,20HTOTAL VOLUME ON UNIT,
     1//15X,32HCUBIC FOOT VOLUME                ,F14.1,
     2//15X,32HBOARD FOOT VOLUME INTERNATIONAL ,F14.1,
     3//15X,32HBOARD FOOT VOLUME SCRIBNER      ,F14.1)
  126 FORMAT(24H*** JOB REJECTED ***    ,
     137HNUMBER OF VOLUME PTS. GREATER THAN 50)
  127 FORMAT(24H*** JOB REJECTED ***    ,
     137HNO. OF SAMPLE POINTS GREATER THAN 200)
      STOP
      END
```