AN ABSTRACT OF THE THESIS OF

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Title: SEPARABILITY TESTS ON WHEAT PRODUCTION FUNCTIONS IN OREGON

Redacted for Privacy

Abstract approved: John A. Edwards

Many natural resources such as water and forests have become more intensively used in recent years. Often, this has made it necessary to reallocate these resources from less to more efficient productive usage. The knowledge of the existing tradeoffs between alternative uses are necessary to make reallocative decisions. However, these resources also have strong public property character and are not usually amendable to demand analysis to determine willingness to pay. When the "price" is institutionally set, the productivity measurement often must be based on direct production function estimation. For sectoral reallocation of resources, some aggregate productivity measures are required. Such measurêments are feasible when aggregate production functions are estimated.

The aggregate production functions are however beset with a host of difficulties arising from their aggregate nature. The resulting aggregation bias must be eliminated if any aggregate productivity measure is to be the basis of policy recommendation. The improvement of

the results of aggregate productivity analysis hinges on the methods which reduce aggregation bias.

There are two major conditions under which the aggregation bias is minimized or eliminated altogether. These conditions are a) relative prices are fixed amongst factors that are aggregated and/or b) the aggregated factors are weakly separable from others in an economic sense. The first condition relates to Hicks' Aggregation Theorem and the second to Leontief Separability. The latter condition appears to be directly relevant from the practical standpoint, since relative prices are seldom fixed amongst all aggregated factors. Thus the existence of valid aggregate input indices in an aggregate production scheme can be assured only when there exists separability between these inputs in each aggregate input indices. The present study attempts to test for separability amongst inputs going into wheat production using county level Oregon Census of Agriculture data.

There is strong empirical evidence of weak separability amongst the biological process inputs such as fertilizer and the husbandry process inputs such as capital and irrigation service. And furthermore, a weather variable, rain, is found to be separable from both biological and husbandry process inputs. The tests were conducted utilizing the TRANSLOG type second order Taylor approximation to a general functional form. The separabilities imply various linear and nonlinear restrictions on the estimated coefficients of the translog function; and these restrictions were tested in conjunction with the usual F-distributed statistics of linear and non-linear restrictions on quadratic expressions. The results from linear restrictions were however ambiguous between inseparabilities among fertilizer and irrigation and among capital and irrigation. Similarly, for nonlinear restrictions, ambigueity resulted between inseparability amongst capital and fertilizer and capital and irrigation. However, in both cases inseparability amongst capital and irrigation does marginally better in terms of the sum of the squared error terms.

A logarithmic cubic approximation function was used to test for Sadan's perfect process complimentarity between biological and husbandry processes. This test using quadratic approximation models was rejected. However, the strongest evidence from the cubic approximation model was that the model with inseparability amongst capital and irrigation is the only one consistent with Sadan type high process complimentarity. Thus the implication is that, at the micro-level biological and husbandry process functions are valid because of Sadan's perfect process complimentarity, and, at the macro-level, wheat production function can be defined for aggregate inputs of capital and irrigation service, and fertilizer, precipitation, etc. In conclusion, the results appear to be supportive of the lay notion that wheat production consists of biological and the husbandry processes which are highly complementary to each other.

Separability Tests on Wheat Production Function
in Oregon

by

Binayak Prasad Bhadra

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

June, 1982

APPROVED:

Redacted for Privacy

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Date thesis is presented June 2, 1981

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ACKNOWLEDGMENT

I wish to express gratitude to Dr. John A. Edwards for his guidance and constant encouragement throughout the course of this study; to Dr. Roger G. Kraynick for his helpful suggestions and encouragement; to Dr. William G. Brown for his helpful suggestions and encouragement; to Dr. H.H. Stoevner, for his general guidance in the early part of this effort. I also thank Dr. David Faulkenberry and Dr. Ray Northam for their helpful criticism.

I express my heartfelt gratitude to Centre for Economic Development and Administration, Nepal and Agricultural Development Council, for their continued support, without which the present research would not be possible.

Sincere thanks are due to Mary Ann (Sadie) Airth for typing the thesis.

I express my appreciation of moral support from my wife and daughter.

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I. INTRODUCTION

The use of many natural resources such as water and forests have expanded rapidly in recent years. This has made it necessary to improve their use efficiency in individual applications and also has raised issues about reallocation of these natural resources from less efficient use to more efficient ones. Many such reallocative decisions have to be made both by the society and private entrepreneurs so that efficient use of the limited natural resource is assured. Such allocative decisions can only be made on the basis of the tradeoffs that exist between alternative uses of these resources. Thus measurement of the productivities of such natural resources as water and forests in various uses have become important from the policy formulation point of view. The increasing conflicts of interest has made it necessary to formulate these policies of resource use for the attainment of economic efficiency.

In case of many natural resources, traditional demand study involving price-quantity relationship have failed. This is caused by the public property nature of these natural resources. For example, water is allocated on the basis of traditional prior appropriation doctrine in the Northwest U.S.A. The "price" of the water is institutionally set and does not reflect the willingness to pay on the part of the users. Thus without a market determined price, the demand relation cannot be estimated to infer the marginal water productivity in agriculture. The resource allocation is socially inefficient in such instances and the policy formulation usually must fall back on measurement of resource productivities through direct estimation of the production function. This is one reason for the recent emphasis on the use of duality approach in applied production theory. This approach may be well suited to both the development and econometric application of the production theory.

In production function studies, the multitude of activities contained within a sector often requires considerable simplification for manageability of data gathering and subsequent analysis. The majority of the studies, at least in agriculture, appear to be either micro response functions based upon experimental data or the aggregate macro level sectoral (farm income) production functions based on aggregate state or county level data. This situation is expected since majority of data is available at these micro and macro levels. Though for general policy formulation the macro level production functions are relevant, these studies may incur extensive aggregation bias. The aggregation bias can result from aggregation of inputs, as well as from the aggregation of outputs, when there are multiple outputs present. Where all the outputs are produced individually in an independent manner, the issues of aggregation bias can be reduced to the issues of the bias of aggregating the inputs within a single output production scheme.

The Problem Statement

Conceptually, a complex production scheme can be broken down into simpler sets of activities, which are considerably fewer in number than the list of all the activity inputs. Thus, aggregation bias may be thought of as a result of misspecifying activities in terms of the wrong inputs. In this case the solution to the problem of eliminating the aggregation bias is to identify a set of inputs that truly belong to each given activity or process.

This subdivision of the production scheme into processes requires intimate knolwedge of the production activities, and therefore of the technology. Further, such knowledge should be empirically verifiable so that the existence of such processes and activities can be demonstrated. The economic significance of separate activities and processes lie in the fact that inputs into one activity can be altered without affecting the other activities. This is called the economic separability of the inputs in one group from those in the other. The simplification of a complex input-output relation can only take place through the notion of processes and sub-processes. If this conceptual breakdown of the production scheme is not feasible, then one cannot expect to be able to aggregate inputs without bias, and all the benefits of such simplified descriptions of the technology are lost.

Thus we can broadly define the problem posed in this study. Given a set of disaggregate input data and single output data, the problem consists of finding the best way to aggregate the input data to minimize the aggregation bias in aggregate production function estimates.

Conceptually, this problem can be posed for any level of production relation. The production relation can relate to the macro-level as well as the micro-level. For example, if the data is available for farm activities, the aggregation bias issues may be raised about the aggregate inputs explaining the farm level gross-output. Alternatively, if the data is available in disaggregate activity levels for the inudstry, the best method to aggregate these input data to explain the industry gross-output may be sought.

In the present context, the specific problem is posed as the following question: What is the method of aggregating the wheat production inputs which generates no aggregation bias in the wheat production function. If such a method is available, it could pave the way towards an unbiased farm production function.

The Objectives of the Study

In view of the previous problem statement, the following objectives have been set for the present study.

(1) To test the separability of inputs going into the wheat production function in Oregon, with the view to establish the bias free method of aggregation of inputs into aggregate inputs.

(2) To infer the validity of the notions of the processes within the wheat crop-growing activities through the use of the notion of economic separabilities of the process inputs.

(3) To test for complimentarities and substitutabilities between the processes (if they are defined), within the wheat growing activity

in Oregon. In particular, tests for complementarities between biological process of plant growth and non-biological husbandry processes are to be devised, if these processes are found to be separable.

(4) To infer the productivities of the factor inputs from the estimated production function, and the aggregate input functions.

It may be emphasized that, the purpose of the present study is merely to test the validity of aggregation, as is usually performed in aggregate production function studies taking the wheat production function as an example. The present study is not intended to carry on with the extension of separability tests for other crops and agricultural activities with a view to "construct" aggregate farm-output production function without aggregation bias at the macro-level. Though such an attempt would reward one with better estimates of sector level factor productivities, which could be of immediate policy relevance, it is beyond the scope of the present study.

Background

The theory of production, which deals with the decision making process of a producer unit (maximizing profit for a given level of resource endowment) treats products and factors as well defined entities. However, when we look at the empirical application of the theory, we find that the 'products' and 'factors' are actually aggregates of distinct goods and services. We may thus question whether or not the theory has any empirical relevance. Otherwise, there must exist conditions under which use of aggregate factors and products are justified in production theory. Further, these conditions must be testable. It is often noted that the variables used in production and derived demand studies are invariably some kind of aggregates. Since aggregation is ever present, it is sometimes argued that there cannot be an empirically meaningful way of dealing with aggregation bias. In what follows, this will be shown to be false.

The study of factor demand and factor productivity has been traditionally conducted using aggregate and disaggregate (experimental) production functions [Ruttan (1956), Heady (1957), Hock (1962), Holloway (1972), Thomas 1974), Lynne (1978), and Mittelhammer et al. (1980)]. Factor demands are estimated using aggregate demand [Cromarty (1959), Griliches (1959), Heady and Yeh (1959), Kako (1978)].

The latter approach can not be used for inputs without an observable market price, such as water. Lynn (1979) has indicated that non-market and the public property nature of water is primarily responsible for this. This indicates that, for many common property type resources, derived demand estimation approach fails. Thus resource use policies must be based on direct production function estimation and direct factor productivity measurements in these cases. Thus the issue of aggregation bias is a pertinent one in these aggregate production function studies.

Various Approaches to Aggregation

There have been three distinct approahces to deal with the issue of aggregation in production and factor demand studies. The usual approach has been to assume a true functional form for a production function and then to proceed from there to determine the best way to aggregate data. The main concern of course is the extent of bias that is generated due to aggregation.

In contrast to this functional form approach, there are approaches where the knowledge of the true functional form is not presumed. The functional form in these approaches is empirically decided. The pure statistical approach and the Process Function Approach fall into this category. These will be discussed separately below.

(i) Functional Form Approach

For a log-linear form of production function, Klein [1946] was the first to show that, the best way to aggregate independent variables was to take their geometric means rather than the arithmetic ones. He also indicated that if the arithmetic means are used instead of the proper geometric means, a downward bias occurs in the estimated coefficients of the log-linear form. The reason is that for small variations the geometric mean G can be shown to be approximately equal to $\overline{X}\,[1 - \frac{1}{2}\,(\frac{\sigma^2}{\overline{X}^2})] \leq \overline{X}$, where, $\overline{X}$ and σ^2 are the arithmetic mean and the variance of X, respectively.

Nataf (1950) showed that for a sensible aggregation of the micro function to a macro one, the assumption of additive (in factors) production function is necessary. Thus, because Cobb-Douglas production function is additive in log-linear form, use of geometric means mostly eliminates aggregation bias in estimation of the aggregate production function. For a log-linear form, a simple average of the logarithms of the variables turns out to be the logarithm of the geometric mean.

The geometric mean, $G = \sqrt[n]{(X_1 \cdot X_2 \cdots X_n)}$ can also be expressed as, $e^{(\frac{1}{n} \Sigma \ln X_i)} = e^{\overline{\ln X}}$. Thus, $\ln G = \frac{1}{n} \Sigma \ln X_i$ and for the log-linear model, the relevant average is, $\overline{\ln X} = \frac{1}{n} \Sigma \ln X_i$. Thus the appropriate method of aggregation of the log-linear form of micro-level function to a log-linear form of the macro-level function is to use the geometric averages instead of the arithmetic ones. Similar methods of aggregation of variables into "means" could also be derived for other functional forms (e.g., the Constant Elasticity of Substitution form) which can be changed into linear functions through transformation of the variables.

(ii) Statistical Approach

Another approach that has been indicated to be feasible is the statistical approach. Theil (1954) poses the question as to the possibility of fitting a macro-relation to the aggregate values when the micro-relations are given along with the form of aggregation [Walters (1963), p. 10]. For example, let the micro relations be Cobb-Douglas production functions in their logarithmic form, for each i^{th} firm,

$$X_i = \alpha_i l_i + \beta_i l_i k_i + a_i \qquad i = 1, \ldots, n.$$

where, X_i, l_i, k_i represent output, labor and capital, respectively in logarithms. Then we may attempt to estimate a macro form,

$$X = \alpha\ 1 + \beta\ k + a + \varepsilon,$$

using the aggregate values of a time series.

The individual firm demand for inputs could be estimated by the following regressions,

$$1i = \beta_{1i}\ 1 + C_{\ell i}k + D_{\ell i} + U_{\ell i}$$

$$k_i = B_{ki}\ 1 + C_{ki}k + D_{ki} + U_{ki},$$

utilizing the aggregate variables 1 and k. The regression coefficients D, B and C represent the connection between the macro and micro variables. They represent how micro quantities have to move when the macro variables move. Thus, knowing D, B and C's we can estimate a macro function, $X = \alpha 1 + \beta k + a + \varepsilon$, by substituting for 1i and ki in the firm production functions: $X_i = \alpha_i 1_i + \beta_i k_i + a$: for all i's. The macro function parameters [Walters (1963)] are,

$$\alpha = \overline{\alpha} + n \ \ \text{Cov}\ (\beta_i B_{1i}) + \text{Cov}\ (\beta_i B_{1i})$$
$$\beta = \overline{\beta} + n \ \ \text{Cov}\ (\beta_i C_{1i}) + \text{Cov}\ (\beta_i C_{ki})$$
$$a = n \left\{ \overline{a} + \text{Cov}\ (\overline{\alpha}_i D_{1i}) + \text{Cov}\ (\beta_i D_{ki}) \right\}$$

One advantage of this approach is that it gives the covariance terms above as the <u>aggregation bias</u> in the macro parameters, α, β, a. However, not knowing α_i, β_i, a_i, it is hard to estimate the size of these covariance terms a priori. This appears to be a major drawback of Theil's approach from the empirical standpoint.

Houthakker (1955) has also shown that if the input/output ratios of the firm level linear production functions have Pareto type distribution the aggregate production function for the industry as a whole is in the form of a Cobb-Douglas. Though this represents an adequate approach to cross-sectional data, there still remain questions when the input/output ratios are non-Pareto distributed. Thus in practical terms, this approach is also inadequate.

(iii) Process Function Approach

Another approach to aggregation is based on aggregating the process functions. These functions are defined according to the analytical conveniance of the scientist or the engineer [Chenery (1949); Ferguson (1953); Heady and Dillon (1962)]. They have been estimated on the basis of agronomic or engineering data [Moore (1959); Markowitch (1953); Hirsch (1952 and 1956)]. Attempts have been made to integrate process functions to 'factory' or 'plant' functions or even 'firm' level production functions [Smith (1961)]. Problems with the process functions have been that they normally exclude indirect inputs such as management machinary, land, etc. from consideration [Smith (1961)]. Process and 'plant' production functions are useful in deriving the 'plant' cost curves, though they are not useful for asking questions about the returns to scale at the farm or 'plant' levels [Walters (1963)].

When process functions are used for the farm as a whole, specification bias results from ignoring the indirect inputs (management) into the farming process [Grileches (1957); Mundalak (1961)].

Comparative Statics and Aggregation Bias

The use of aggregate variables in production and derived demand studies can be justified in three fundamentally different ways. The simplest way is to use the notion of a linear technology, such that, the input-output ratios within aggregated inputs are fixed. This kind of perfect complementarity within a subset of inputs is unrealistic. Therefore the assumption of linear technology is not a generally acceptable rationale for the use of aggregate inputs in empirical studies.

A better justification, popularly given, is based on the important theorem of comparative statics, known as the Hicks' Aggregation Theorem. Unfortunately, the theorem holds only for the unlikely situation where the relative prices of the aggregated inputs do not change at all. The final justification for aggregation is provided by the existence of a restricted type of technology, characterised as weak separability of aggregated inputs.

Though all the three justifications above are testable empirically, this has generally not been done. So, as will be argued below, there is always a possibility of excessive aggregation bias in studies which utilize aggregate input and output data. More importantly, however, the three justifications can also furnish some insights into the style of aggregation which will result in minimum aggregation bias. This is an important consideration wherever there exist disaggregate data requiring aggregation for manageability and ease of analysis. In the next sections we will deal with the Hicks' Aggregation Theorem and Leontief Separability Theorem, respectively.

Hicks' Aggregation Theorem

In Value and Capital (1946, pp. 311-313) Hicks demonstrated the following very important aggregation theorem. The theorem states that if the relative prices within a group of commodities are fixed, the value aggregate of such commodities behaves exactly as if it were a separate intrinsic commodity. This is the basis of using Hicks-Allen money in popular two dimensional geometry of demand analysis.

If the prices $p_1, \ldots, p_r$ of the commodities $X_1, \ldots, X_r$ move in exact proportion, we can define the composite commodity, as, $\overline{X}_1 = \sum_{i=j}^{r} p_i X_i$, with the usual properties of a single commodity. Samuelson warns that this definition merely results in the change of frame of reference and not in any change in the dimensionality of the problem from n to (n-r+1) (see pp. 142, The Foundation of Economic Analysis).

Samuelson has originally shown (The Foundations of Economic Analysis, pp. 129-143, 1947) that the comparative static results remain invariant under some kinds of transformation of variables, provided these transformations preserve value magnitudes. Let the price and quantity vectors p,X, respectively be transformed to new coordinates $\overline{p},\overline{X}$ such that $p'X = \overline{p}'\overline{X}$. This has been called a contragradient linear transformation of p,X into $\overline{p},\overline{X}$ by Samuelson. He shows that we can indicate the transformation through a non-singular matrix c, such that, $\overline{X} = c^{-1}X$, $\overline{p} = c'p$ (Note p' and $\overline{p}'$ are transposed p and $\overline{p}$ respectively).

Consider the production problem, where output y=f(X) is to be maximized under an expenditure constraint $M=p'X$. The Lagrangian associated with this problem is, $\max_{\lambda,X} L=f(X) + \lambda\ (M-p'X)$, where, λ = Lagragian multiplier. Under the contragradient linear transformations, the Lagrangian is, $L=f(c\overline{X}) + \lambda\ (M-\overline{p}'\overline{X})$, the form of which has not changed. Thus we can re-state the problem by the Lagrangian.

$$\max_{\lambda,\overline{X}} \overline{L} = \overline{f}(\overline{X}) + \lambda\ (M-\overline{p}'\overline{X}) \text{ where } \overline{f}(\overline{X}) = f(c\overline{X})$$

The necessary conditions for the maximum of $L(X,\lambda)$ are,

$$\frac{\partial L}{\partial X_i} = 0 \quad \text{or} \quad f_i - \lambda p_i = 0 \quad i=1,\ldots,n$$

$$\frac{\partial L}{\partial \lambda} = 0 \quad \text{or} \quad M - \sum_i^n p_i X_i = 0 \quad i=1,\ldots,n$$

and the sufficient condition is that the bordered Hessian,

$$T = \left[\begin{array}{c|c} \dfrac{\partial^2 L}{\partial X_i\ \partial X_j} & \dfrac{\partial^2 L}{\partial X_i\ \partial \lambda} \\ \hline \dfrac{\partial^2 L}{\partial \lambda \partial X} & \dfrac{\partial^2 L}{\partial \lambda^2} \end{array}\right] = \left[\begin{array}{c|c} \left(\dfrac{\partial^2 f}{\partial X_i X_j}\right) & -p \\ \hline -p' & 0 \end{array}\right]$$

is a negative definite matrix of size (n+1) x (n+1)

Analogously the necessary and the sufficient conditions for the maximum of $\overline{L}\ (\overline{X},\lambda)$, in the new coordinates $\overline{X}$, are

$$\frac{\partial \overline{L}}{\partial \overline{X}_i} = 0 \quad \text{or } \overline{f}_i - \lambda\ \overline{p}_i = 0 \quad i=1,\ldots,n$$

$$\frac{\partial \overline{L}}{\partial \lambda} = 0 \quad \text{or } M = \sum_i^n \overline{p}_i \overline{X}_i = 0 \ i=1,\ldots,n$$

and

$$T = \left[\begin{array}{c|c} \dfrac{\partial^2 \overline{L}}{\partial \overline{X}_i\ \partial \overline{X}_j} & \dfrac{\partial^2 \overline{L}}{\partial \overline{X}_i\ \partial \lambda} \\ \hline \partial^2 \overline{L}/_{\partial \lambda} \partial \overline{X} & \dfrac{\partial^2 \overline{L}}{\partial \lambda 2} \end{array}\right] = \left[\begin{array}{c|c} \dfrac{\partial^2 \overline{f}}{\partial \overline{X}_i\ \partial \overline{X}_j} & -\overline{p} \\ \hline -\overline{p}' & 0 \end{array}\right]$$

is negative definite. But it should be noted that, the negative definiteness of T implies that $\bar{T}$ is also negative definite, and therefore the optimization of $\bar{L}$ using transformed inputs $\bar{X}$, is possible and valid.

Further, explicit differentiation of, $\begin{cases} f_i = \lambda p_i, & i=1,\ldots,n \\ M = \Sigma p_i X_i \end{cases}$ gives the following matrix equation,

$$\left[\frac{\partial x}{\partial P}\right] = \begin{bmatrix} \frac{\partial X}{\partial p} & \frac{\partial X}{\partial M} \\ \frac{\partial \lambda}{\partial p} & \frac{\partial \lambda}{\partial M} \end{bmatrix} = \lambda \begin{bmatrix} \left[\frac{\partial^2 f}{\partial X_i \, \partial X_j}\right] & -p \\ -p' & 0 \end{bmatrix}^{-1} = \lambda T^{-1}$$

where, $x = \begin{bmatrix} X \\ \lambda \end{bmatrix}$ and $P = \begin{bmatrix} p \\ M \end{bmatrix}$. Define, $\bar{x} = \begin{bmatrix} \bar{X} \\ \lambda \end{bmatrix}$ and $P = \begin{bmatrix} \bar{p} \\ M \end{bmatrix}$ so that

$$\bar{x} = \begin{bmatrix} \bar{X} \\ \lambda \end{bmatrix} = \begin{bmatrix} c^{-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ \lambda \end{bmatrix} \text{ and } \bar{p} = \begin{bmatrix} \bar{p} \\ M \end{bmatrix} = \begin{bmatrix} c' & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ M \end{bmatrix}$$

Thus, $\left[\frac{\partial \bar{x}}{\partial \bar{P}}\right] = \left[\frac{\partial \bar{x}}{\partial X}\right] \left[\frac{\partial X}{\partial P}\right] \left[\frac{\partial P}{\partial \bar{P}}\right]$

or $$\left[\frac{\partial \bar{x}}{\partial \bar{P}}\right] = \begin{bmatrix} c^{-1} & 0 \\ 0 & 1 \end{bmatrix} \left[\lambda T^{-1}\right] \begin{bmatrix} c^{-1} & 0 \\ 0 & 1 \end{bmatrix}' = B\,[\lambda T^{-1}]\,B'$$

Because multiplication by $B = \begin{bmatrix} c^{-1} & 0 \\ 0 & 1 \end{bmatrix}$ and B' represents a congruent transformation, it preserves the <u>definitness</u> of T^{-1}. Therefore, $\left[\frac{\partial \bar{x}}{\partial \bar{P}}\right]$ is negative definite. But, $\left[\frac{\partial \bar{x}}{\partial \bar{P}}\right] = \lambda\, \bar{T}^{-1}$, we therefore conclude that, $\bar{T} = \begin{bmatrix} \left[\frac{\partial^2 \bar{f}}{\partial \bar{X}_i \, \partial \bar{X}_j}\right] & -\bar{p} \\ -\bar{p}' & 0 \end{bmatrix}$ is also a negative definite matrix. Therefore, $\left[\frac{\partial \bar{x}}{\partial \bar{P}}\right]$ matrix is embedded with the same qualities as $\left[\frac{\partial x}{\partial P}\right]$ matrix.

Therefore, the following relations hold for the transformed input variables, $\bar{X}$, analogous to the untransformed X.

The Slutsky equation,

$$\left(\frac{\partial \overline{X}_i}{\partial \overline{p}_j}\right) = \left.\frac{\partial \overline{X}_i}{\partial \overline{p}_j}\right|_{y=\text{constant}} + \overline{X}_j\left(\frac{\partial \overline{X}_i}{\partial M}\right)$$

where, $\left.\frac{\partial \overline{X}_i}{\partial \overline{p}_j}\right|_{y=\text{constant}} = \lambda \frac{|\overline{T}_{ij}|}{|\overline{T}|} = \overline{k}_{ij}$, is explained as the, residual variability of the i^{th} transformed input variable for a compensated change in the j^{th} price. It is the pure substitution effect for i^{th} transformed input variable, $\overline{X}_i$, due to change in price of $\overline{X}_j$. Similarly, $\overline{X}_j$ $(\frac{\partial \overline{X}_i}{\partial M})$ represents the expenditure or budget effect of j^{th} price change.

The negative definite $\overline{T}$ implies the inequalities characteristic of conditional factor demand, $\overline{X} = \overline{X}$ $(\overline{p},y)$, e.g.

e.g. $\frac{\partial \overline{X}_i}{\partial \overline{p}_j} = \overline{k}_{ij} < 0$, and $\overline{k}_{11} \cdot \overline{k}_{22} - \overline{k}_{12}^2 > 0$, etc.

so that the principle minors of $\overline{k}$ alternate in sign. Thus the demand functions, $\overline{X} = \overline{X}(\overline{p},y)$ satisfy the same inequalities satisfied by un-transformed variable demand functions, $X = X$ (p,y).

Now let us define the contragradient transformation on X so that the first element of $\overline{X}$, $\overline{X}_1 = \sum_{i=1}^{r} p_i x_i$, is the composite input $\overline{X}$

$$\overline{X} = \begin{bmatrix} \overline{X}_1 \\ \vdots \\ \overline{X}_n \end{bmatrix} = \left[\begin{array}{c|c} p_1 & p_2 \cdots p_r 0 \cdots 0 \\ \hline \vdots & \\ 0 & I_{n-1} \end{array}\right] \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix} = c^{-1} \cdot X$$

$$\overline{p} = \begin{bmatrix} \overline{p}_1 \\ \vdots \\ \vdots \\ \overline{p}_n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ p_{r+1} \\ \vdots \\ p_n \end{bmatrix} = \left[\begin{array}{c|c} 1/p_1 & 0 \\ \hline -p_2/p_1 & \\ \vdots & \\ -p_r/p_1 & I_{n-1} \\ 0 & \\ \vdots & \\ 0 & \end{array}\right] \cdot \begin{bmatrix} p_1 \\ \vdots \\ \vdots \\ p_n \end{bmatrix} = c' p.$$

Because, $p'X = \bar{p}'\bar{X}$, the value magnitudes are preserved as required for this particular, c. It should be noted that when the prices $p_1,\ldots,p_r$ change in the same proportion with respect to $p_{r+1},\ldots,p_n$, all held constant, only the first element of $\bar{p}$ changes from 1.0 to θ. The other (r-1) elements of $\bar{p}$, $\bar{p}_2,\ldots,\bar{p}_r$ all remain fixed at zero, and $\bar{p}_{r+1},\ldots,\bar{p}_n$ all remain fixed at $p_{r+1},\ldots,p_n$, respectively.

Thus under the new coordinate transformation, the proportional change of first r prices $p_1,\ldots,p_r$ is transformed to a single change in $\bar{p}_1$, price of the composite input commodity, $\bar{X}_1$. However, we have already shown that the composite input variables, $\bar{X}_1$, $\bar{X}_2,\ldots,\bar{X}_n$ behave exactly as the original variables $X_1,\ldots,X_n$ under the assumption of optimizing behavior. The same necessary and sufficient conditions were derived for contragradient transformed variables, $\bar{X}$ as is true for X. Because negative definite T implies negative definite $\bar{T}$, we can show that there are qualitatively the same properties for the factor demand functions, $X = X(p,y)$ and $\bar{X} = \bar{X}(\bar{p},y)$. It should also be noted that the Lagrangian multiplier is independent of the transformation, so that, $\lambda = \lambda(p,y) = \bar{\lambda}(\bar{p},y)$.

Thus we conclude that, under the condition when the relative prices of $X_1,\ldots,X_r$ are fixed, contragradient transformation on $X_1,\ldots,X_r$ may be performed to define a composite commodity, $\bar{X}_1 = \sum_i^r p_i X_i$ which has all the economic attributes of a single commodity. This is Hicks' Aggregation Theorem in its simple form for a single group. It can be extended to more groups.

In conclusion, as long as the relative prices do not change within a group, the production function, $y = \bar{f}(\bar{X})$, is a valid production function in terms of the composite input variables, $\bar{X}$. The production function, $\bar{f}(\bar{X})$, under prices, $\bar{p}$, constraint, $M=\bar{p}'\bar{X}$, and the maximizing behavior, generates, the usual conditional factor demand function, $\bar{X}_1 = \bar{X}_1(\bar{p}_1,\ldots,\bar{p}_n,Y)$. Note, $\bar{X}_1 = \bar{X}_1(\bar{p}_1,0,\ldots,0,\bar{p}_{r+1},\ldots,\bar{p}_n,Y) = \bar{\bar{X}}_1$ $(\bar{p}_1,p_{r+1},\ldots,p_n,Y)$. Thus, factor demand can be analyzed with (n-r+1) variables using the $\bar{\bar{X}}_1(.)$ relations.

This aggregation theorem, though proven briefly by Hicks and Samuelson, is proven precisely by Gorman (1953). Though the result of the theorem is strictly valid only for fixed relative prices within the aggregate inputs, Diewert (1978) has shown that even if the individual prices do not change in exact proportions, Hicks' Aggregation Theorem holds approximately. Though this allows for small changes in relative prices, it is difficult to judge the quality of approximation at hand.

More importantly however, the relative prices can be stable only under specialized situations over a long period of time. One possibility is that the inputs are close substitutes, so that if the price of one changes the price of the other also changes in the market. On the other hand, we can also have the extreme opposite, i.e. perfect complementarity. The latter would also result in close relative price movement. The importance of the theorem is undermined because both these requirements are often not met in reality.

Thus if the price changes in the aggregated group are not in the same proportion, the use of aggregate variables in production function estimation results in an aggregation bias. It is expected that the bias would increase with the increase in differences in relative prices. In reality relative prices don't usually stay the same for a long time and aggregation bias would result if the Hicksian theorem alone provides the basis of aggregation. If the aggregation bias could be foretold a priori, the changing relative prices would not be detrimental to aggregation; compensations could be provided to correct the bias. Unfortunately this is not so. So we now turn to Leontief Separability Theorem.

Leontief Separability Theorem

The final justification of aggregation is provided by the property of weak separability of the overall production function. Though this notion of separability is mathematical in character, its origin lies in economics. Leontief originated the concept of separability in the context of a production function. The 1947 article in Econometrica, "Introduction to a Theory of the Internal Structure of Functional Relationships", starts out with the problem of simplifying an over-all production function $Y = F(X_1,\ldots,X_n)$ of variables $X_1,\ldots,X_n$ into a function of a few intermediate variables $f_1,\ldots,f_r$, where $r < n$. These intermediate variables, f_1's are themselves functions of subsets of the variables $X_1,\ldots,X_n$, such that the subsets are mutually exclusive. So the following identity would hold,

$$F(X_1,\ldots X_n) \equiv \phi\,(f_1(X_1{}^1,\ldots X^1{}_{n_1}),\ldots,f_r(X_2^r\ ,\ldots X_{n_r}^r)).$$

According to Leontief, a complex production scheme can be represented through a set of intermediate production functions, $f_1(X_1^1,\ldots,X_{n_1}^1)$, (See pp. 362, Econometrica, 1947) provided there exist appropriate technical informations on the intermediate steps of the overall production process. Though the knowledge of the intermediate production functions $f_1(\cdot\cdot)$ can be easily combined to construct the overall function, $\phi(f_1, \ldots, f_r)$, the reverse process of determining $f_1, \ldots, f_r$ given the properties of $F(X_1, \ldots, X_n)$ is not easy.

Leontief thus raised the question about the conditions or nature of $F(\cdot\cdot)$ under which the over-all production function, $F(\cdot\cdot)$, can be functionally separated into intermediate production functions, as, $F(\cdot\cdot) \equiv \phi(f_1, \ldots, f_r)$. This pertains directly to the justification of aggregation in production and demand functions. If $F(\cdot\cdot)$ can be expressed as $\phi(f_1, \ldots, f_r)$, we are justified in aggregating inputs $X_1, \ldots, X_n$ into r mutually exclusive categories, each represented as, $X_1^1, \ldots, X_{n1}^1$.

In another 1947 article, A note on the Interrelation of Subsets of Independent variables of a Continuous function with Continuous First Derivaties (The Bulletin of the American Mathematical Society), Leontief demonstrates that the necessary and sufficient condition for weak separability of input set X into the mutually exclusive complimentary subsets S and $\overline{S}$ is that the "rate of technical substitution" between two inputs X_i, X_j in S is independent of input X_k in $\overline{S}$ i.e.

$$\frac{\partial}{\partial X_k}\left(\frac{\partial F/\partial X_i}{\partial F/\partial X_j}\right) = 0,$$

where $F(X) = \phi(f(S), g(\overline{S}))$.

The necessary part of the above result can be easily demonstrated as follows:

$$\frac{\partial F}{\partial X_\ell} = \frac{\partial \phi}{\partial X_\ell} = \frac{\partial \phi}{\partial f} \cdot \frac{\partial f}{\partial X_\ell}, \quad \ell = i,j$$

and therefore, $\frac{\partial}{\partial X_k}\left(\frac{\partial F/\partial X_i}{\partial F/\partial X_j}\right) = \frac{\partial}{\partial X_i}\left(\frac{\partial f/\partial X_i}{\partial f/\partial X_j}\right) = 0.$

This condition, $\frac{\partial}{\partial X_k}[\frac{\partial F/\partial X_i}{\partial F/\partial X_j}] = 0$, is also sufficient for, $F(\cdot\cdot) \equiv \phi(f,g)$. This can be shown as follows: Define, $\Psi(S) = F(S,\overline{S}_0)$, where $\overline{S}_0$ represents the set of all X_k fixed at the value, X_k^o. Now, the subset S consisting of $X_1,\ldots,X_v$ is locally functionally separable in X if the function F(X) can be shown to be expressible in $\Psi(S)$ and $\overline{S}$, i.e. $F(X) = \phi(\Psi(S),\overline{S})$. Let us now consider X_k's in $\overline{S}$ as parameters of F(X), then we can construct the following matrix of derivatives of the function F(X) and $\Psi(S)$ with respect to $X_1,\ldots,X_v$.

$$\begin{vmatrix} \partial F/\partial X_1 & \ldots & \partial F/\partial X_v \\ \partial \Psi/\partial X_1 & \ldots & \partial \Psi/\partial X_v \end{vmatrix}$$

The first row of the matrix above is proportional to the second row if, we have,

$$\frac{\partial \Psi/\partial X_i}{\partial \Psi/\partial X_j} = \frac{\partial F/\partial X_i}{\partial F/\partial X_j} \text{ for } i,j \leq v.$$

This is true when, $X_k = X_k^o$ for all $k > v$.

Because, $\frac{\partial}{\partial X_k}(\frac{\partial F/\partial X_i}{\partial F/\partial X_j}) = 0$, for $i,j \leq v$ and $k > v$, we must have,

$$\frac{\partial F/\partial X_i}{\partial F/\partial X_j} = \frac{\partial F(S,\overline{S}_0)/\partial X_i}{\partial F(S,\overline{S}_0)/\partial X_j} = \frac{\partial \Psi/\partial X_i}{\partial \Psi/\partial X_j}$$

as the ratio is independent of all X_k's. Therefore we see that the rows of the matrix are linearly dependent on each other. However if we make the usual assumption that, $\frac{\partial \Psi}{\partial X_v} > 0$, then according to the general theorem on functional dependence we can express, $F(X) = \phi(F(S,\overline{S}_0),\overline{S}) = \phi(\Psi(S),\overline{S})$. The same arguments could be repeated by taking the first set $\overline{S}$ and defining $F(\overline{S},S^o) = \overline{\Psi}(\overline{S})$ etc. Then we would conclude that, $F(X) = \phi(\overline{\Psi}(\overline{S}),S)$.

Thus, in general, $F(X) = \phi(\Psi(S), \overline{\Psi}(\overline{S}))$ which we recognize as, $F(X) = \phi(f,g)$.

We recognize that Separability is a very restrictive assumption. Berndt and Christensen (1973) have shown that Leontief's Separability Conditions imply that partial elasticities of substitutions between input X_i^S in S and input $X_k^{\overline{S}}$ in $\overline{S}$ must remain the same locally for all $i \leq v$ and $k > v$. This result will be demonstrated later when we deal with Separability Theory.

The implication of this result for the usual aggregate production function in agriculture is devastating. Consider, the input category expenditure in the usual Cobb-Douglas farm production function. Expenditure is an aggregate of fuel, fertilizer chemicals, pesticides, etc. Now, because Cobb-Douglas is log-linear, it is also weakly separable in all inputs, capital, labor, etc., including expenditure. The Berndt-Christensen results thus imply that, fuel and capital have the same elasticity of substitution as fertilizer chemicals and capital. Realistically, fuel is complementary to capital and a substitute to fertilizer. Thus the usual pattern of aggregation of fuel, fertilizer chemicals into expenditure group appears highly tenuous.

At this stage, it serves well to recognize that the Hicksian Aggregation Theorem and the Leontief Separability Theorem provide the "necessary" basis for aggregation. These considerations though paramount in importance, do not exhaust all. Separability restricts the functional form and so the aggregation issues are also related to the choice of functional form to be employed empirically. The functional form, most appropriate at a given level of analysis (i.e. firm level, sector level, process level, etc.), is necessarily consistent with forms at higher and lower levels of analysis.

For example, the micro level process function may be presumed to have increasing returns to scale, operate under limited entrepreneural ability, and often work under rather elastic factor prices. For the macro-functions, constant or decreasing returns to scale, with variable entrepreneural ability but faced with inelastic factor supplies becomes a more realistic assumption. So exactly the same parametric specification of macro and micro-production function would not be desirable. But when micro level functional forms are known, the criterion of choice of the macro function is that, "the model give rise to a sensible aggregate relationship, which, in some sense, corresponds to the micro-relations" [Walters (1963)].

The present analysis has so far preceded without considering the issues of aggregation over time. It can be demonstrated that provided the inputs are labeled separately for each time period, the Leontief Separability Theorem can be easily extended to deal with aggregation over time. In contrast to the comparative statics case however, the presence of dynamic growth complicates the matter considerably under the dynamics. So presently we will consider aggregation over time by extending the static production function to a dynamic one, the production functionals.

Dynamics and Aggregation over time

Under dynamics, the micro-level production relation should describe the process of growth and change at the micro level. For agricultural processes, the process description of biological growth requires Functionals rather than Functions [Georgescu-Roegen (1971), The Analytical Representation of Process and the Economics of Production, Chapter 9, pp. 234-238]. The 'inputs' to plants and animals are time functions (fund-flow rates in Georgescu-Roegen's definition) and 'outputs' are resultant growths of bio-mass, again time functions. This picture is immediately relevant to the case of crop and livestock activities.

The dynamic nature of crop production has been well recognized. For example, a few studies have asserted the need to explicitly consider water application regimes rather than the aggregate water input in crop production functions [Edwards (1963), Minhas, Parikh and Srinivasan (1974), Stegman (1980)]. However, Edwards explicitly considers the growth of plants in determining their yields. In his study, temperature-moisture regimes during various growth stages and the previous growth is used to explain the final yields.

Though it appears plausible in many instances, that biological growth achieved finally becomes a simple function of time aggregates of inputs, this need not generally be true. This puts a serious limitation on the time aggregated production function as an approximation to the dynamic functional. Yet when the growth produces a final cumulative result (say the production of a seed after plant matures), the effect of all input flows may be assumed to be accumulated at the end. If the inputs at any stage of growth were independent of the inputs required at a different stage, then this time wise separability would allow, time-aggregated inputs in specifying an aggregate yield function. This independence of dated inputs however is quite contrary to the known laws of biological growth. The nutritional (and basic) requirements of plants and animals depend upon the present ambiance and on the previously attained growth.

Further, even assuming the stage wise separability of inputs, one has to consider that inputs required for controls will always be independent of controlled inputs at all stages of growth. This may not be a realistic assumption in general, and time aggregation will not be generally valid. If, however, these separability assumptions hold, then aggregate yield functions with time aggregate inputs will be valid, especially when the input applications cannot be reversed, and therefore become asymmetrically independent over time.

Separability Theory

In his 'Theory of Internal Structure of Functional Forms' in Econometrica (1947), Leontief demonstrated that weak separability was a necessary condition for the existence of a subaggregate index for a group of inputs. That is, a production funtion, $Y = F(X_1,\ldots,X_n)$ can be written as,

$$y = \phi\left\{ f^1(X_1^1,\ldots,X_{N_1}^1),\ldots,f^r(X_1^r,\ldots,X^r_{N_r}) \right\}$$

when the input set $X_1,\ldots,X_n$ can be divided into mutually exclusive groups or subsets $X^i = X_1^i, \ldots, X_{Ni}^i$ for i = 1,2,...r, such that the subsets are weakly separable from each other. The condition of weak separability was shown to be

$$\frac{\partial}{\partial X_k}\left(\frac{\partial F}{\partial X_i} \Big/ \frac{\partial F}{\partial X_j} \right) = 0$$

when $X_i, X_j \varepsilon X^r$ and $X_k \varepsilon X^s$, where, $s \neq r$. Alternatively, $F_iF_{jk} - F_jF_{ik} = 0$, also indicates weak separability, where $F_{ik} = \frac{\partial^2 F}{\partial X_i \partial X_k}$ and $F_i = \frac{\partial F}{\partial X_i}$, etc.

This last expression indicates that the marginal rate of substitution $(\partial F/\partial X_i)/(\partial F/\partial X_j)$, between inputs X_i and X_j in group r is independent of inputs outside this group, X_k. Solow (1955) has called $f^r(X_1^r, \ldots, X_{Nr}^r)$ a consistent aggregate index of inputs $X_1^r, \ldots, X_{Nr}^r$. So a consistent aggregate index of a subset of inputs exists, if and only if, the subset is weakly separable from all other inputs [Green (1964)].

Berndt and Christensen (1973) showed that for homothetic production functions, the separability restrictions imply certain equality restrictions on the Allen-partial-elasticity-of-substitutions (AES). The AES is defined between two inputs X_i and X_k, as,

$$\sigma_{ik} = \left[\sum_{\ell} F_\ell X_\ell \, |\overline{H}_{ik}| \,/\, X_i X_k\right]$$

where, $\overline{H}_{ik}$ is the i^{th} row and k^{th} column element of the inverted bordered Hessian of $F(\cdot)$. This AES_{ik} measures the response of derived demand of input X_i to a change in price of input X_k, holding output and other prices fixed. Under the assumption of efficient production and perfect supply elasticity of inputs, the AES_{ik} has the following property,

$\sigma_{ik} = \frac{\eta_{ik}}{W_k}$ where, $\eta_{ik} = \frac{\partial X_i}{\partial P_k} \cdot \frac{P_k}{X_i}$, price of elasticity of demand, and $W_j = P_j X_j / \sum_i P_i X_i$, cost-share of X_j and P_i is the price of factor X_i.

For homothetic production functions, Shephard (1953) showed that there exists a dual cost function of the form,

$$C\,(Y, P_1, \ldots, P_N) = \Theta\,(Y) \cdot \Psi(P_1, \ldots, P_N)$$

where, θ and Ψ are some functions such that the cost is an increasing function in Y and P's. The above implies that, if the production function is weakly (strongly) separable in X's then the cost function would also be weakly (strongly) separable in the P's. This was shown by Shephard (1970) for separable homothetic production functions, though Lau (1969) had shown a similar result of homothetic separability between direct utility function U $(X_1, \ldots, X_N)$ and its dual indirect utility function, θ (I) $\cdot$ Ψ $(P_1, \ldots, P_N)$ [obtained by replacing the optimal values of X_i's in U $(X_1, \ldots, X_N)$ with their demand expressions, I is the budget constraint]. This result allows an important conclusion. Under homothetic separability, a consistent sub-aggregate price index, $\Psi^r(P_1^r, \ldots, P_{Nr}^r)$ exists, if and only if, the corresponding consistent sub-aggregate quantity index $f^r(X_1^r, \ldots, X_{Nr}^r)$, exists [Berndt and Christensen (1973, pp. 405)].

Under efficient production, and perfect factor supply elasticies, Samuelson (1948, pp. 61-69) has shown the comparative statics result,

$$\frac{\partial X_i}{\partial P_k} = \frac{\bar{H}_{ik}}{\lambda} \text{ for all } k,\ i = 1,2,\ \ldots,\ N,$$

and also $F_i = \frac{P_i}{\lambda}$, where λ is the Lagrangian multiplier in the constrained optimization problem.

Substituting these into the expression for AES we obtain,

$$\sigma_{ik} = \left|\frac{\partial X_i}{\partial X_k}\right| \cdot \frac{E}{X_i X_k} \quad , \sigma_{jk} = \left|\frac{\partial X_j}{\partial X_k}\right| \cdot \frac{E}{X_j X_k}$$

for all i,j = 1, ..., N and $E = \sum_i P_i X_i$

At this stage Hotelling's Lemma (1934) can be invoked. This Lemma states that, under the maximization hypothesis, the optimal factor levels are expressible as,

$$X_i^* = \left(\frac{\partial C(Y,P)}{\partial P_1}\right) = \theta(Y)\left(\frac{\partial \Psi}{\partial P_i}\right) = \theta(Y)\cdot\Psi_i \qquad i = 1,\ldots,N$$

where, C(Y,P) represents the cost function dual to the production function. So that, we have,

$$X_j^* = \theta(Y) \cdot \Psi_j, \; X_k^* = \theta(Y) \cdot \Psi_k$$

and $\quad \partial X_i^* / \partial P_k = \theta(Y) \cdot \Psi_{ik}$ etc.

Also, $E = \sum_i P_i X_i^* = C^*(Y,P) = \Psi^*(P_1,\ldots,P_N)$,

where, * represents the optimal value of the variable. By substituting we may show that

$$AES_{ik} = \sigma_{ik} = \frac{\Psi\Psi_{ik}}{\Psi_i\Psi_k} \text{ and } AES_{jk} = \frac{\Psi\Psi_{jk}}{\Psi_j\Psi_k}.$$

So that, $AES_{ik} = AES_{jk}$ is true, if and only if, $\Psi_i\Psi_{jk} - \Psi_j\Psi_{ik} = 0$.

Thus we conclude that the cost function $C(Y,P) = \theta(Y) \cdot \Psi(P_1, \ldots, P_N)$ is weakly separable in prices P_i, P_j from P_k, if and only if, $AES_{ik} = AES_{jk}$. Therefore, weak separability of prices in the cost function has been shown possible, if and only if, the homothetic production function is also weakly separable in the corresponding inputs quantities. Now we can conclude the Berndt and Christensen result, that equality of AES_{ik} to AES_{jk} implies that the production function is weakly separable into the factor groups $(X_1^r, \ldots, X_{Nr}^r)$ and $X_1^s, \ldots, X_{Ns}^s)$ and vice versa.

The above result is derived for homothetic production functions. Using the Shepard-Lau duality result (that homothetic separability on the production side gives rise to a smiliar separability on the cost side, in terms of corresponding prices of factors and vice-versa). This result is valid for the local point under consideration.

These results have been extended by Russell (1975) to non-homothetic production functions. It should however be noted that, for Russell's general results to hold, the aggregator or separable parts of the function, $f^r (X_1^r, \ldots, X_{Nr}^r)$, themselves need to be homothetic. Russell's results also hold globally rather than locally, provided some regularity conditions hold on the separable production function. The regularity conditions are that both the aggregate function and the separable parts of that function be continuous and at least twice differentiable.

Implications of Separability

Separability puts severe restrictions on the nature of technology and the form of the production function. Since separability is equivalent to equality of AES restriction, the following implications are summarized:

(1) An aggregate net output or value added functions can be defined under separability.

(2) There exist aggregate input indices or separable parts to the production function, (these will be referred to as process functions later on), and,

(3) There exists the possibility of having decentralization and/or multistage optimization procedures.

For example, Bruno (1978) has shown that if the intermediate inputs are separable from direct inputs, the net output (or value added) production function can be defined in terms of the primary inputs. This is true even if the underlying production function with all inputs is not homothetic or linearly homogeneous. Denny and May (1978) have tested the validity of separability in Canadian Manufacturing, and found negative results. This implies that unless the Hicksian aggregation conditions hold, no real value added aggregate function can be defined in this case. Thus separability test may be used to establish the validity of the empirically determined net value added aggregate production function.

Berndt and Christensen (1973) and Denny and Fuss (1977) tested for the existence of consistent aggregates for labor and capital (using U.S. manufacturing data) using the separability test. There was no evidence for labor separability, but they found support for capital aggregation provided labor aggregation is presumed. This is an example of success in using separability to test for validity of aggregate indices and functions.

Under separability, production efficiency can be achieved by sequential optimization. In consumer theory Strotz (1959), Gorman (1959) and Green (1964) have indicated that strong separability allows a budgeting procedure consistent with two stage maximization. Blackorby et al. (1970) showed that this budgeting procedure is also consistent with two-stage optimization, if and only if, weak separability holds.

When separability can be ascertained, the derived demand by factors can be simplified. Pollak (1969 and 1971) has shown that the factor demand functions are a function of the factor prices within the separable group and the cost allotted to that group. This does not mean that the quantities of factors demanded in one separable group are independent of factor prices in other groups or of total cost of inputs. All this means is that these variables enter into the demand functions only through their effect upon the cost allotment to that group. The cost allocated to a group of factor inputs are frequently known, so that the factor prices outside of the separable group can be ignored altogether. This results in a considerably simpler demand equation to empirically estimate.

When factor demands are so simplified, the dual cost function will also be weakly separable in corresponding factor prices [Shephard (1970)]. Separability also opens up the possibility of multistage estimation of macro production functions using consistent aggregates in later stages. For example, Fuss (1977) has used a two-stage procedure to estimate demand for energy in Canadian manufacturing. The net output or value added function can be legitimately estimated without considering the intermediate inputs, and Bruno (1978) shows that the marginal productivity of the primary inputs will be estimated accurately without bias.

Agricultural value added or farm income 'aggregate' model would only be legitimate under separability. The confirmation of the separability assumption in empirical terms is necessary prior to deriving any policy recommendations based upon possibly biased, aggregate value added functions.

Under separability, there exist aggregate input indices or separable parts to the production function. These indices or separable parts of the production function will be called process-functions later on. The reason for calling these separable parts "process-functions" is as follows. All the inputs that belong to a separable part are independent of inputs in other separable parts. Thus, these inputs within a separable part directly interact only with one another. Therefore, these direct interactions may be summarised as a "process." In the following chapter, it will be argued that various lay notions of "processes" or "activities" may coincide with the separable parts of the production function. Thus, the test of separability may be used to provide empirical support for the existence of process functions. If separability assumption holds, then it provides a natural basis for aggregation of inputs in a production function without introducing any aggregation bias. This approach of aggregation along the process functions will be dealt with in the following chapter.

II. PROPOSED APPROACH

Proposed Approach: Aggregation Along the Processes

In agriculture, there exists substantial heterogeneity across farms at the aggregate level. The farms differ from one another in terms of the types of activities pursued and also in terms of their intensities. It is conceivable that a large portion of the observed heterogeneity results from the variation in the mix of similar farming activities or processes.

The underlying biological processes within each activity is however homogeneous. The same is true of most husbandry processes which involve tillage, seedbed preparation and harvesting (especially for similar crops). Thus a complex farm production scheme can be divided into simpler homogenous subprocesses. The availability of process-wise data provides the basis for testing this homogeneity within mechanically and biologically similar processes.

This inherent integrity of the underlying biological processes, and to some extent, the mechanical processes, also implies that these farming subactivities are independent of one another, and therefore, economically separable. It is noted that a farm may exploit complimentarities between these otherwise independent processes. This separability between inputs going to different activities indicates the route through which aggregation bias may be minimized in aggregate production function estimation. The underlying heterogeneity of farms themselves indicates that farm wise aggregation of inputs and outputs is undesirable.

The separability of various activity inputs validate an aggregate production function of aggregate inputs, where factors are aggregated along the activities and not across farms.

Reduction of Aggregation Bias

The aggregation bias would be minimized if aggregation occurs along homogeneous processes rather than along heterogenous farms. If we limit ourselves to a cropping activity, and if all the mechanical and biological processes are spatially replicable (i.e. linear homogeneity) and similar factor ratios prevail, the aggregation bias will not increase in increasing the level of aggregation from one farm to a group of farms. Though factor ratios vary within the same process, they may vary very little. This indicates the possibility of utilizing the county level data available from the Census of Agriculture in the U.S.A.

For a single crop growing activity, the subactivities of tillage, seedbed preparation, fertilization, irrigation and harvesting are quite homogeneous within and between farms. The activities of extension agents and the standardization of machinery also contribute towards increasing this homogeneity. However, in spite of the highly plausible nature of process homogeneity and separability, it is at the end, an empirical issue. Thus for the validity of an aggregate crop function, the separability of these subprocesses must be demonstrated. Separability of inputs into these subprocesses must be tested before attempting an aggregate crop production function.

For example, for wheat crop, the husbandry process of tilling and havesting requires fuel, capital and labor. On the other hand, the biological process of plant growth does not require these inputs after seedbeds are prepared. Plant growth does, however, require moisture, sunshine, fertilizer, pesticides and herbicides, etc. Thus, based on the sequential nature of the husbandry and biological processes, we may deduce that the fuel, capital, labor inputs are independent and weakly separable from fertilizer, moisture, pesticide and herbicide inputs.

The Process Functions and Separability

When a production function is separable in inputs, and the inputs in each group are related to a specific subprocess, the separable part of the production function can be termed the process function. When the output of the process can be determined, the process function can be estimated empirically. An example of a biological process function would be a response function. These have traditionally been estimated to determine factor productivity and substitutability under a given experimental set up [Moore (1961), Heady et al. (1956), and Berringer (1961)]. Some studies seek out the effects of random weather variables [Edwards (1963), Hillel and Guron (1973)] and others that of changing input application rates [Heady et al. (1956), Knetsch (1959), Miller and Boersma (1966)]. Rarely, water and fertilizer application regimes are also studied for their effect on yield [Minhas, Parikh and Srinivasan (1974), Stegman (1980)].

These response functions are highly specific to the experimental conditions and are often criticized for leaving out other necessary husbandry inputs such as fuel, capital, labor and management. Since outputs of husbandry process are not directly observable such a process function cannot be directly estimated. But in leaving them out, the resulting specification error bias often reduces the applicability of these functions for policy analysis. There are some exceptions to this which will be dealt with later.

Reduction of Multicollinearity

On the other hand, most aggregate production functions utilizing more general data, have their own problems. Aggregation bias has been already discussed. Another problem, somewhat aggrevated by aggregation is the high degree of multicollinearity among the aggregate factors [Ruttan (1965)]. This produces inflated variances of the estimated coefficients; and the variable deletion is recognized as leading to specification bias [Hoch (1967), Brown (1973)]. To overcome this problem, ridge-regression has been successfully used [Brown (1973), Brown and Beattie (1975), Hoerl and Kennard (1970)]. Mixed estimators are also popularly employed [Theil and Goldberger (1961), Holloway (1972), Mittelhammer and Price (1978), Mittelhammer and Baritelle (1977)]. However, to the extent possible, multicollinearity should be reduced by performing aggregation along homogeneous processes i.e. along separable inputs. This will reduce the risk of serious multicollinearity, though the risk will not be eliminated entirely.

Process Mix Optima and Sadan's Partial Production Functions

The crop growing activity may be broadly separated into the biological and husbandry processes. Inputs such as water, fertilizer, pesticides and herbicides, go into the biological process, while fuel and services of capital, labor and irrigation go into the husbandry process. The final yield, Y, is therefore a function of the biological process output and the husbandry process output. Symbolically, we have, $Y = F(X_1,\ldots,X_N)$ and $(X_1 \ldots X_N)$ is the input set, such that $(X_1,\ldots,X_N)$ can be separated and renamed into two groups $(X_1,\ldots,X_n) = \bar{X}$ and $(Z_1,\ldots,Z_m) = \bar{Z}$. The independence of the two processes implies that we can write,

$$Y = F(X_1,\ldots,X_N) = \phi\,[f(X_1,\ldots,X_n),\ g(Z_1,\ldots,Z_m)]$$

where, $f(X_1,\ldots,X_n)$ and $g(Z_1,\ldots,Z_m)$ represent the process outputs from biological and husbandry processes separately.

It may be noted that specifying, $\phi[f,g]$, allows for the processes to be either complements or substitutes to one another. Thus separability in this case allows us to treat the yield Y as a function of two process outputs f and g. Consequently graphical analysis of production efficiency can be applied to this case with some modification. The ϕ-function may be represented with a set of convex isoquants in the f-g space as shown in Figure 1(a).

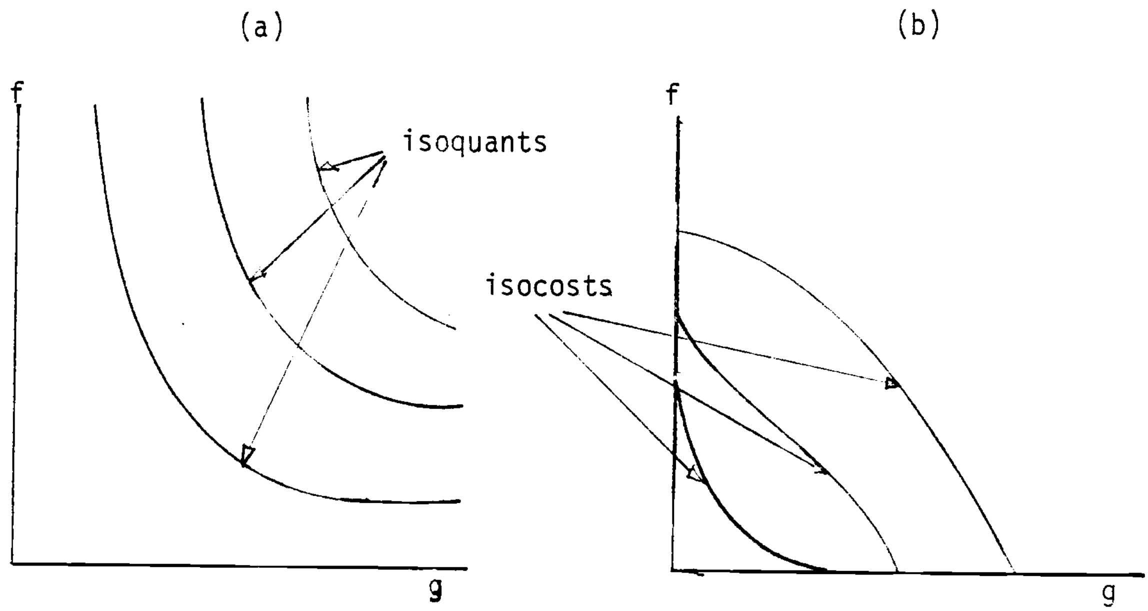


Figure-1: (a) Iso-quants (b) Iso-costs

In Figure 1 (b) above, the isocost curves are arrived at by considering the apportionment of a given cost dollars into biological and husbandry process and considering the individual process-cost functions. Since separability allows for two-stage optimization, we can define the process-cost functions under the following optimization rule. For a given process output f, say, the minimum cost may be derived for a given set of factor prices. Thus each process-function has a dual process-cost function. The four quadrant analysis on the left below indicates the basis of deriving the isocost curves in f-g space (Figure 2, a). The iso-cost curves near the origin are convex (to the origin) because

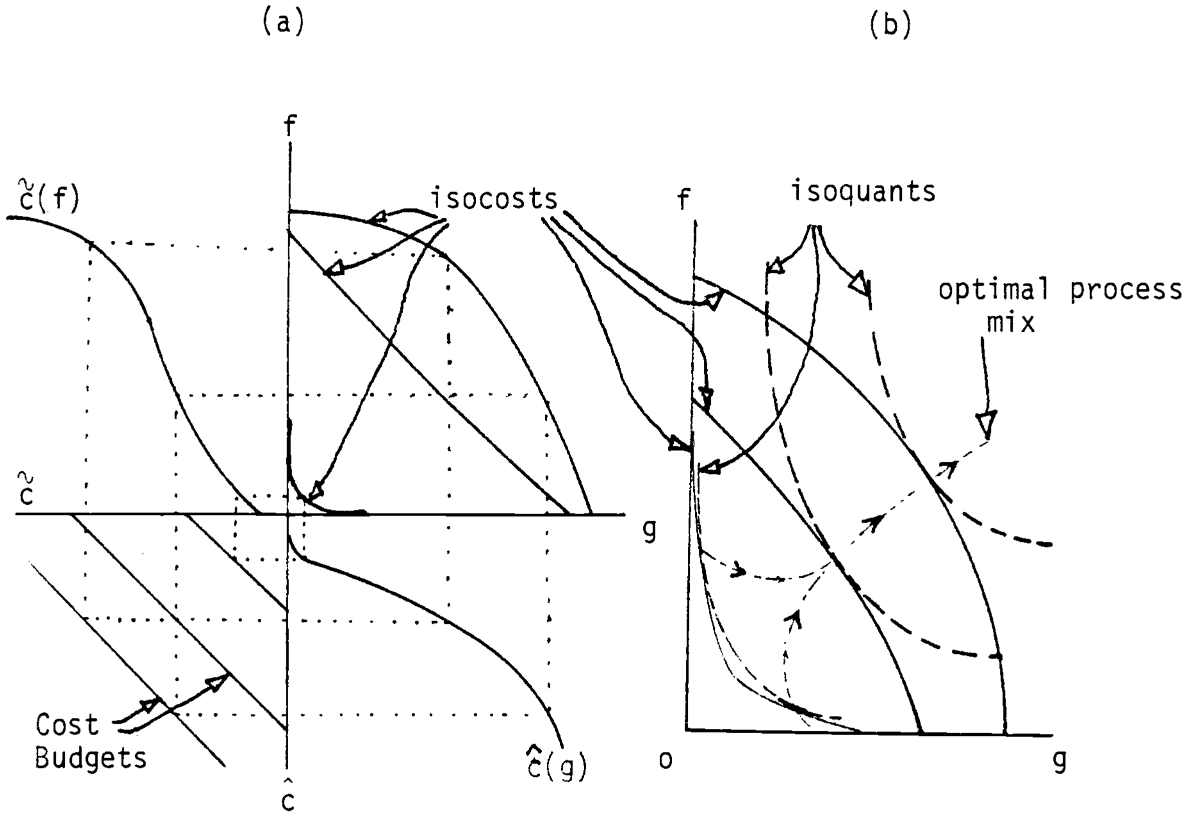


Figure-2: (a) Four quadrant analysis of iso-cost lines
(b) Optimal Process mix

each of the individual process-cost functions $\tilde{C}(f)$ and $\hat{C}(g)$ have concave sections near the origin. The important conclusion is that at lower levels of output, there is a likelihood of multiple process-mix optima, even when the factor prices are fixed. This is shown on the right above (Figure 2, b). Note that, with the same technology and factor prices there can be many optimum process-mixes. This indicates the extent of diversity that can be generated under separability and process substitution.

There is however one exception to be noted here. If for some reason, the processes under consideration are perfect complements to one another, then the optimum process-mix is unique for all levels of final yield. This is shown below (Figure 3).

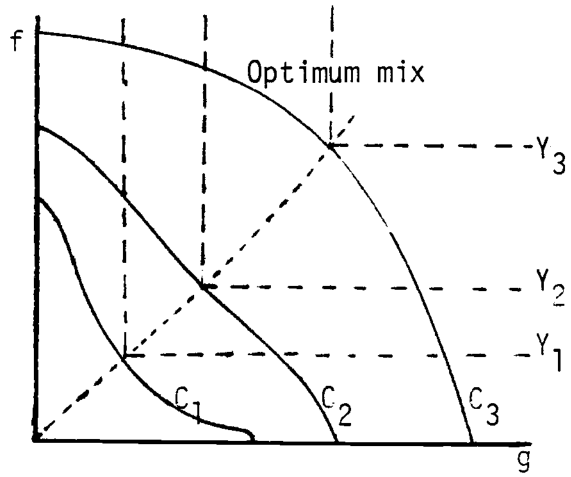


Figure-3: Sadan Model: Perfect Process Complementarity

This situation has already been recognized in the literature by Sadan (1970). The process-functions in this situation have been termed Partial Production Functions by Sadan. If we replace the general ϕ-function described above by a Leontief-technology in f-g-space i.e.

$$Y = \phi[f(\bar{X}), g(\bar{Z})] = \min[f(\bar{X}), g(\bar{Z})]$$

then, we see that we obtain the Sadan-type partial production functions. Thus, Sadan type separability puts a further restriction on the general separability specified by the Φ-function The restriction is that the biological and husbandry processes are perfect complements. As will be shown later, this type of separability may be tested for along with the general separability of the ϕ-function.

One important implication of this type of separability under efficient production is that the specification error bias incurred in estimating the biological process function without consideration of husbandry inputs would be zero. So estimated response functions are unbiased if efficient production occurs under the Sadan type technological structure, and they do accurately represent the biological process functions. Therefore, we may conclude that the biological process functions can be estimated using biological inputs only, when there exist perfect complementarity between biological and husbandry processes, and production is efficient.

The General and Specific Hypotheses

The hypothesis of weak separability appears plausible in agriculture. There is almost always a biological process in operation in conjunction with a non-biological husbandry process. If we take wheat as an example, the direct inputs such as water, fertilizer, pesticide and herbicide enter the biological process of growth. Growth is also influenced by the environment: rain, sunshine, and ambient temperature, etc. In conjunction with this process, there is the husbandry process utilizing indirect inputs; these inputs are generally services from machinery capital, irrigation networks, labor and management.

These hypotheses can be summarized mathematically in terms of a separable production function, $Y = \phi[f(X), g(Z)]$, where X, Z are biological and husbandry inputs, respectively. The presumption of this weak separability may be made more explicit by stating the, Leontief condition: $\frac{\partial}{\partial Z_k}\left(\frac{\partial\phi/\ X_i}{\partial\phi/\ X_j}\right) = 0$, and the Hicksian type of process complementarity:

$$\left(\frac{\partial^2\phi}{\partial f\ \cdot\ \partial g}\right) > 0.$$

A more specific thesis may be the assumption that, the biological and husbandry processes are perfect complements (i.e. no substitutability between the processes). Under such an extreme assumption, the partial production functions are equivalent to the response functions. This hypothesis may be mathematically stated as, Sadan complementarity: $\left[\frac{\partial^2\phi}{\partial f\ \cdot\ \partial g}\right] = \infty.$

The above hypotheses appear very plausible, though, they have not been tested so far in agriculture. Sadan's own analysis of Partial Production Functions were carried out under this perfect complementarity assumption, without testing it. Thus it becomes necessary to test separability along these "identifiable" sub-processes of a crop growing activity. Sadan (1970) was very likely aware of this need (See pp. 64).

If the separability hypotheses are empirically ascertained to be correct, only then it becomes valid to estimate the response and the production functions for various farming activities. These functions for crops, dairy and fodder activities can be estimated using consistent aggregates obtained by aggregating along the separate processes. A value added farm production function could then be efficiently estimated without aggregation bias.

III. METHODOLOGICAL BASIS

The basis of performing the separability test lies in the properties of the functional form. However, when the functional forms are themselves arbitrary, the test of separability also becomes arbitrary. There appear to be two ways to resolve this problem. One approach would entail going further into the analysis of the production functions at the micro level. That is, one can perhaps determine the growth-functions for plants and animals, based on the biological laws and begin to build up the macro-level relations from them. The other approach would entail approximating a general form locally, using Taylor expansions. The advantage here is that locally a very general form is assumed; the disadvantage is that the results obtained are mostly local.

Theory of the Test of Separability

Under the optimization hypothesis, there exists duality between the cost functions and the production functions [Shephard (1970)]. This has allowed inferences on the production parameters from the cost parameters. The test on weak separability can also be attempted using this duality. If the production function is homothetic and the cost-function dual to that production is assumed to have a

TRANSLOG form, the cost-shares are linear functions of the factor prices (in logarithms)and the output (in logarithms) [Binswanger (1973)].

Such linear cost-share functions and direct production functions have been popular in weak separability test [Berndt and Christensen (1973), Blackorby et al. (1977), Denny and Fuss (1977), and Applebaum (1977)]. Besides, the homotheticity assumption, the cost-share or derived-demand approach requires the use of factor prices. Generally, natural resources are characterized as 'public property' and they are not traded in the market but are allocated on an institutional basis, they do not have a market price. Thus these approaches are less useful in measuring the factor productivities of natural resources such as water.

The direct production function approach appears more suitable in productivity measurements of 'public property' type inputs. In agriculture, the major 'public property' type of input is irrigation water, and it often lacks a market price (since no open market exists for it). Thus the direct production function approach may dominate other approaches to measuring factor productivity in agriculture.

Since the Leontief condition for weak separability may also be simplified as,

$$\phi_i \phi_{jk} - \phi_j \phi_{ik} = 0,$$

we see that, weak separability is related to the properties of the second order cross-partials, ϕ_{ik}, vis a vis those of ϕ_i, that is, ϕ_{ik}/ϕ_i= constant, for all i = 1,2,...,n and k = 1,2,...,m. Thus a choice of functional form has important implication concerning separability. A choice of Cobb-Douglas (CD), Constant Elasticity of Substitution (CES) or Log-linear (LL) functional form would automatically predispose us to Strong Separability (SS) as a maintained hypothesis. These functions are unsuitable for testing Weak Separability (WS) assumptions, since SS implies WS.

Flexible Functional Forms and TRANSLOG

Thus for an empirical test for the WS assumption a more general form than Log-linear or CES is necessary. This requires the use of flexible functional form functions, such as, Quadratic (QUAD), Transcendental Logarithmic (TRANSLOG), Generalized Leontief Linear (GLL) and higher polymonials (POLY) to test the WS hypothesis. All these functions are linear in parameters and OLS procedures can be employed in their estimation.

Other examples of linear in parameters functions suitable for testing the WS assumption would be Diewert's (1973) Generalized Cobb-Douglas (GCD) and McFadden's (1978) Generalized Concave (GCON) functions. The many non-linear in parameter forms are less useful in econometric estimation because of the nonlinear estimation procedure they require. Presently, attention will be placed on linear in parameter forms only.

Alternatively, nonlinear in parameter production functions can be employed to test for weak separability. The nonlinear in parameter forms are analyzed primarily in terms of the AES. Fuss, et al.,

(1978) discuss various such functional forms. They have also generalized variants of the Cobb-Douglas and CES functions [Fuss et al., (1978), p. 242].

The most frequent functional form used to test the WS assumption has been the TRANSLOG [Berndt and Christensen (1973 and 1974), Denny and Fuss (1977), Corbo and Meller (1979)]. The main advantage of the TRANSLOG form is that it can be used to approximate any general function at a point up to a second-order Taylor expansion. Thus the test of the WS assumption using a TRANSLOG approximation (TRANSLOGAP) is a test of WS at a point in the factor space (X,Z). For a global test of WS, exact specification of a production function becomes necessary. Unfortunately, such exact TRANSLOG specification (TRANSLOGEX) is not without its own problem.

Exact Translog and Translog Approximation

Denny and Fuss (1977) and Blackorby, Primont and Russell (1977) have shown that the TRANSLOGEX does not have enough flexibility to specify weak separability and non-linearity together. They have proved that (1) ϕ is Log-linear in f(X) and g(Z) when f(X) and g(Z) are Log-linear in X and Z, respectively. Thus as Denny and Fuss (1977) have argued, the global test of Berndt and Christensen (1973) using TRANSLOGEX is not merely a test of the WS assumption but also a test of inflexibility of functional form as indicated above. The TRANSLOGEX specification generates inflexibility, that either ϕ is LL or f and g are LL. (Proof in the Appendix-A).

The TRANSLOG production function is given as,

$$\ln Y = \alpha_0 + \sum_i^N \alpha_i \ln X_i + 1/2 \sum_i^N \sum_j^N \gamma_{ij} \ln X_i \cdot \ln X_j$$

where, $\gamma_{ij} = \gamma_{ji}$, $i,j = 1,\ldots,N$.

The use of the WS assumption, provides the following Leontief conditions for the TRANSLOG.

$$(\alpha_i\gamma_{jk} - \alpha_j\gamma_{ik}) + \sum_{m=1}^{N} (\gamma_{im}\gamma_{jk} - \gamma_{jm}\gamma_{ik}) \ln X_m = 0$$

where, $m = 1,\ldots,N$. This leads to a sufficient condition, called the <u>Linear Restriction</u> of Berndt and Christensen, as, $\gamma_{jk} = \gamma_{ik} = 0$ for $X_i, X_j \in X$ and $Z_k \in Z$. This linear separability restriction leads to a ϕ-function which is Cobb-Douglas in TRANSLOG f(X) and g(Z). (See Appendix A).

Alternatively, the necessary and sufficient condition for WS to hold is that,

$$\alpha_i\gamma_{jk} - \alpha_j\gamma_{ik} = 0, \text{ and } \gamma_{im}\gamma_{jk} - \gamma_{jm} \cdot \gamma_{ik} = 0$$

for $m = 1,2,\ldots,N$. These imply that,

$$\frac{\alpha_i}{\alpha_j} = \frac{\gamma_{ik}}{\gamma_{jk}} = \frac{\gamma_{im}}{\gamma_{jm}} \quad \text{for } m;k = 1,2,\ldots,N.$$

This <u>nonlinear-restriction</u>, according to Denny and Fuss (1977) and Blackorby et al. (1977), implies that the ϕ- function is TRANSLOG in Log-linear f(X) and g(Z). The nonlinear separability restriction implies that the elasticity of substitution within the process inputs X_i and X_j is always one. The linearity restriction on the other hand allows for non-unitary, variable elasticity of substitution between the process inputs X_i and X_j etc. at the cost of constraining the elasticity of process substitution between f(X) and g(Z) at unity. Therefore, rejection of WS with TRANSLOGEX could result from the assumption that (a) the elasticity of substitution between X_i, X_j i.e. $\sigma_{ij} = 1 \cdot 0$, or, (b) the elasticity of substitution between f and g, i.e. $\sigma_{fg} = 1 \cdot 0$. In order to avoid this problem, a TRANSLOGAP specification is needed, though this does not allow for a global test of WS.

Other Forms and Monotonic Transformation of Variables

The separability property holds for all monotonic transformations of variables, i.e., if $F(X_1,\ldots,X_N)$ is made weakly separable, then $y = G(x_1,\ldots,x_N)$ can also be shown to be weakly separable when x_i is a transformation of X_i, such that, $\frac{\partial x_i}{\partial X_i} > 0$ or $\frac{\partial x_i}{\partial X_i} < 0$. (Proof in the Appendix A, Theorem-1). One very important implication of this result is that, the results derived for TRANSLOGEX above holds analogously for GLL and Quadratic forms. For the Quadratic, linear WS restrictions imply that the elasticity of process-substitution σ_{fg} is infinity (See Appendix A, Theorem-3B) because $G(x_1,\ldots,x_n)$ is a linear function of two (or more) process-functions f(x) and g(z), which are quadratic in x's and z's. The elasticity of substitution between x_i and x_j within the process f however is variable but finite.

With non-linear WS restrictions, the Quadratic $G(x_1,...,x_N)$ becomes a quadratic function of linear f and g. The elasticity of process-substitution σ_{fg} is variable and finite but the elasticities within the process σ_{ij} become infinitely large. The same results can be extended to the GLL specification [Blackorby et al. (1977)]. This indicates that TRANSLOGEX is a somewhat more general specification of any exact quadratic, in the following sense: it allows finite elasticity of substitution between processes under the nonlinear WS restriction and unitary elasticity of substitution under linear WS restriction.

Econometric Estimation and Test of Hypotheses

Corbo and Meller (1979) have used the direct production function approach in estimating the manufacturing sector production functions. They also used the TRANSLOG flexible form to test for weak and strong separability. Their results indicate that in the majority of cases the Log-linear (strong separability) assumption could not be rejected; therefore, their results indirectly support the aggregate industry production function using aggregate inputs.

In the present study, The TRANSLOGAP specification is used because, (i) it is linear in parameters and it provides a second order Taylor approximation to a general function (at a point) and (ii) it is flexible enough to permit a local test of weak and strong separability. The global weak separability test could be performed using (i) higher order Taylor approximations and (ii) Nonlinear in parameters forms.

Both methods above have their own drawbacks. Cubic and forth order approximations become cumbersome because of the rapid proliferation of parameters. The nonlinear in parameters forms on the other hand are computationally cumbersome since they need the use of iterative estimation procedures. Futhermore, these nonlinear estimators lack the BLUE properties of OLS estimators of linear in parameter models.

Even local tests of weak separability require performing the tests on the validity of the linear and nonlinear restrictions imposed on the TRANSLOGAP parameter (See pp. 47). The linear restrictions are equivalent to block-diagonalization of γ-matrix i.e. $\gamma_{ik} = 0$, $\tilde{x}_i \in X^s$, $\hat{x}_k \in X^r$ where $X^s \cap X^r = 0$ and $x^s \cup X^r = X$, the input set. TRANSLOGAP can be written as,

$$y = \alpha_0 + \tilde{\alpha}'\tilde{x} + \hat{\alpha}'\hat{x} + \tilde{x}'\tilde{\gamma}\tilde{x} + \hat{x}'\hat{\gamma}\hat{x}$$

where, $\tilde{\alpha} = (\tilde{\alpha},,\ldots,\tilde{\alpha}_{N_s})$, $\tilde{x} = (\tilde{x}_1,\ldots,\tilde{x}_{N_s})$, and $\tilde{\gamma} = \begin{bmatrix} \tilde{\gamma}_{11} & \cdots & \tilde{\gamma}_{1N_s} \\ \vdots & & \vdots \\ \tilde{\gamma}_{N_s 1} & \cdots & \tilde{\gamma}_{N_s N_s} \end{bmatrix}$, etc.

or alternatively as,

$$y = \alpha_0 + (\tilde{\alpha}' \vdots \hat{\alpha}')\begin{pmatrix} \tilde{x} \\ \cdots \\ \hat{x} \end{pmatrix} + (\tilde{x}' \vdots \hat{x}')\begin{pmatrix} \tilde{\gamma} & \vdots & 0 \\ \cdots & & \cdots \\ 0 & \vdots & \hat{\gamma} \end{pmatrix}\begin{pmatrix} \tilde{x} \\ \cdots \\ \hat{x} \end{pmatrix}$$

under the linear weak separability restriction (LWSR). Thus we see that the linear separability test requires testing the null hypothesis

$$H_o: \quad \gamma = \begin{bmatrix} \tilde{\gamma} & \vdots & 0 \\ \cdots & & \cdots \\ 0 & \vdots & \hat{\gamma} \end{bmatrix}.$$

We can estimate the TRANSLOGAP as a linear model $Y = X\beta + \epsilon$, where X represents a TXK observation matrix on K independent variables and there are K β-coefficients corresponding to these. It should be noted that, $K = (1 + N) + 1/2\, N(N + 1)$, where N is the number of inputs in the TRANSLOGAP production function. This is so because there are N linear terms in TRANSLOG form along with $N(N + 1)/2$ cross-product terms (1/2 due to the assumed symmetry) and one constant term. ϵ is the usual error term with mean zero and finite variance, σ^2.

Test Statistics for Linear Restriction

The linear restrictions can therefore be tested by applying the following linear restriction on the linear model representing the TRANSLOG,

$$R\beta = 0 \text{ or } \begin{bmatrix} 0_{11} & \cdots & 0_{1N} & \vdots & 1 & 0 \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{q1} & \cdots & 0_{qN} & \vdots & 0 & 1 \cdots & 0 \end{bmatrix} \begin{bmatrix} \beta_o \\ \vdots \\ \beta_N \\ \beta_{N+1} \\ \vdots \\ \beta_k \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

where, R is a q x K restriction matrix with first N columns of zeros and the rest K - N columns of zeros and zeros and 1's. If the diagonalization requires putting q β's to zero, the remaining K - N columns can be written as a q x q unitary matrix followed by columns with zeros. The right hand side consists of a column of zeros with q x 1 dimension.

This is a standard linearly restricted model. The OLS estimator under the null hypothesis, H_0: $R\beta = 0$, is given by the following expression:

$$b^* = b + (X'X)^{-1} R'[R(X'X)R']^{-1} [r - Rb],$$

where r = (q x 1) column of zeros in this case; and

$b = (X'X)^{-1} X'Y$ is the unrestricted OLS estimator of the true TRANSLOG parameters, β.

Therefore, the test of null hypothesis H_0: $R\beta = 0$, can be performed by using the following F-statistics:

$$F_{(q\ ,\ T-K)} = \left(\frac{1}{q\ S^2}\right)[(b - b^*)'(X'X)\ (b - b^*)]$$

where S^2 is the unconstrained linear least squares estimator of the error variance, σ^2 [Theil (1971), p. 143]. This test statistic can be simplified to:

$$F_{(q,T-K)} = \left(\frac{T-K}{q}\right)\left[\left(\frac{e_*'e_*}{e'e}\right) - 1\right],$$

where, $e'e = \frac{S^2}{T-K}$, the sum of the squared errors of the non-restricted model, and $e_*'e_*$ is the sum of the squared errors of the model restricted under H_0: $R\beta = 0$. The statistic can be shown to be equivalent to the maximum likelihood ratio test statistic [Theil (1971) p. 143].

The nonlinear weak separability restriction (NWSR) also imply non-linear restrictions on the α_i and γ_{ij} coefficients or the β's of the linear model representing TRANSLOGAP. However, these nonlinear restrictions cannot be incorporated in the linear form $R\beta = 0$. The implication is that the previously derived test statistics cannot be directly applied. Fortunately however, the nonlinear restriction can be built into the linear model prior to estimation; and therefore the nonlinearly restricted model can be expressed as a simpler non-linear model. These explicit non-linear in parameter models can be directly estimated using nonlinear techniques.

Test Statistics for Non-Linear Restriction

The nonlinear weak separability restriction implies that the terms of the TRANSLOG form can be generated from the two portions $\tilde{\alpha}$ and $\hat{\alpha}$ of the vector, $\alpha = (\alpha_1,\ldots,\alpha_N) = [(\tilde{\alpha}_1,\ldots,\tilde{\alpha}_{N_s}) \vdots (\hat{\alpha}_1,\ldots,\hat{\alpha}_{N_r})]$. In Appendix-A, Theorem-2, it has been shown that, if the inputs are 'nonlinearly' separated into $\tilde{X}$ and $\hat{X}$, the original γ-matrix of the TRANSLOG production function can be generated as cross-product matrix, as follows

$$\gamma = \begin{pmatrix} k\tilde{\alpha} \\ \cdots \\ l\hat{\alpha} \end{pmatrix} (k'\tilde{\alpha}' \vdots l'\hat{\alpha}') = \left[\begin{array}{c|c} kk'\tilde{\alpha}\tilde{\alpha}' & kl'\tilde{\alpha}\hat{\alpha}' \\ \hline k'l\hat{\alpha}\tilde{\alpha}' & ll'\hat{\alpha}\hat{\alpha}' \end{array}\right]$$

This implies that the translog can be simplified as:

$$y = \alpha_0 + \tilde{\alpha}'\tilde{X} + \hat{\alpha}'\hat{X} + kk'(\tilde{\alpha}'\tilde{X})^2$$

$$+ ll'\ (\hat{\alpha}'\hat{x})^2 + (kl + l'k)\ (\tilde{\alpha}'\tilde{X})\ (\hat{\alpha}'\hat{x})$$

We note that kk', ll' and k'l + l'k are scalars: Therefore the above expression is a nonlinear in parameters equation which implicitly contains the nonlinear restriction due to NWSR. The scalars kk', ll', kl' + lk' have to be estimated along with $\tilde{\alpha}$ and $\hat{\alpha}$ using iterative nonlinear estimators (e.g. Gauss Method).

Let the nonlinear least squares model be represented by $y = f(X_1\beta) + \epsilon$, where, $\epsilon \sim N(0,\sigma^2)$, where, $f(X_1\beta)$ is the nonlinear (explicit) function of β and X. Now let the nonlinear least square estimates of the true coefficient β be β^*. Let $\overset{*}{\sigma}{}^2$ represent the nonlinear least squares estimate of error variance, σ^2. Under some 'regularity' conditions, both β^* and $\overset{*}{\sigma}{}^2$ are consistent maximum likelihood estimators. [Judge et al. (1980), pp. 727]. Further, these estimates are assymptotically normal with mean β and σ^2, and they are assymptotically efficient.

Thus under the normality assumption, the maximum likelihood estimators β_{ML} and σ^2_{ML} are equivalent to the nonlinear least squares estimators $\overset{*}{\beta}$ and $\overset{*}{\sigma}{}^2$, respectively. The variance covariance matrix of β_{ML} is however known as,

$$Var(\beta_{ML}) = \sigma^2_{ML}[Z(\beta_{ML})'\ Z(\beta_{ML})]^{-1}$$

where, $Z(\beta_{ML})$, is defined at true parameter value β, as,

$$\left.\frac{\partial f}{\partial \beta}\right/_{\beta} = Z(\beta_{ML}).$$ [See Judge et al. (1980) pp. 725-727.]

As long as $f(X,\beta)$ meets some regularity conditions, i.e., differentiability and invertability of $[Z(\beta)'Z(\beta)]$ matrix, β_{ML} will be consistent and also asymptotically normal. This allows us to define,

$$e_*'e_* = S^2_{ML}(\beta_{ML}) = (y - f(X,\beta_{ML}))' \; (y - f(X,\beta_{ML}))$$

as a sum of independent and identically distributed asymptotic normal error terms. This means that $e_*'e_*/\sigma^2$ is asymptotically χ^2-distributed with T-K+q degrees of freedom (where q is the number of nonlinear restrictions implicitly employed on K β's in the original linear model).

This allows us to show that

$$\sigma^2_{ML} = \frac{S^2_{ML}(\beta_{ML})}{T-K+q} = \frac{e_*'e_*}{T-K+q}$$

is a reasonable estimator of true σ^2. [Judge et al. (1980), pp. 725.]

Now, $(e_*' e_*' - e'e)/\sigma^2$, the difference of two χ^2-distributed variables, is also a χ^2-distributed variable. though this is true only asymptotically. The degrees of freedom of the combined variable is (T-K+q) - (T-K) = q. Thus we may again take the ratio,

$$F = \frac{T-K}{q}\left(\frac{e_*'e_* - e'e}{e'e}\right).$$ We note that the denominator is

χ^2-distributed with T-K degrees of freedom, and the numerator is also asymptotically χ^2-distributed with q degrees of freedom. Therefore, the ratio, F, above, is asympotically F-distributed with q and T-K degrees of freedom [Theil (1971), pp. 80-81].

Therefore, in conclusion, we see that nonlinear restrictions cannot be tested using the regular F-statistics derived under linear restriction. In contrast to the linear restriction model, where normality assumption about the disturbance term was unnecessary, the non-linear restriction model requires normality of the disturbance terms before a test statistic can be constructed. In view of this, nonlinear restrictions are not as easily tested as the linear ones. The results of the test also need to be interpreted as asymptotically valid, and only under the normality assumption.

Negative Random Error Model Under Sadan Complementarity

When perfect complementarity between husbandry and the biological process holds, even under a wide range of relative factor prices, the process-mix will remain stable. This is due to the Leontief (input-output type) linear technology isoquants in the f-g-space as shown below:

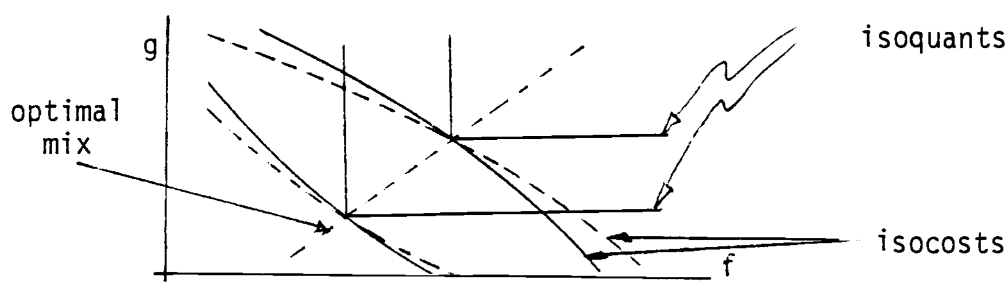


Figure-4: Sadan Model

This also implies that the regular econometric approach fails, because $y = \phi(f,g) = \text{Min}(f,g)$ is no longer a continuous function in f-g space. The function can be indicated in three dimension as the lower parts of two intersecting planes as shown in Figure 5.

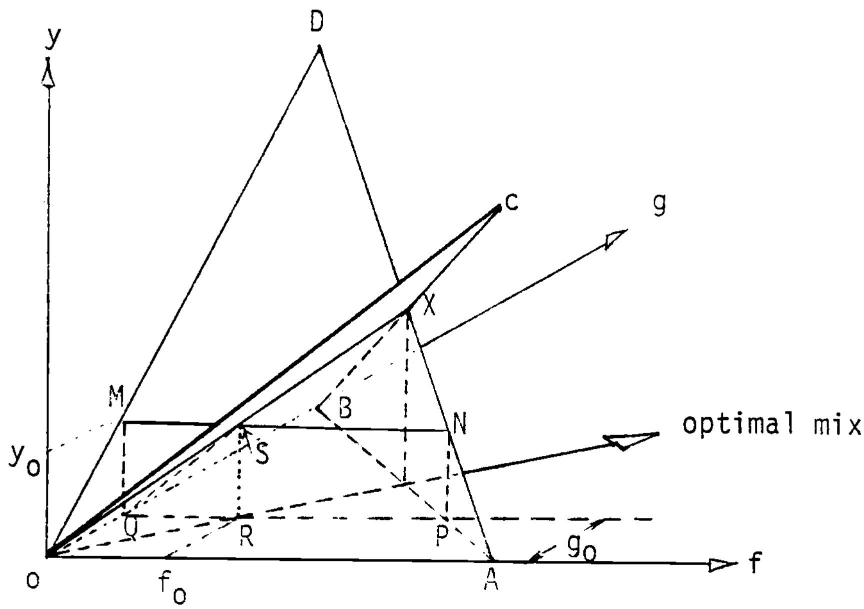


Figure-5: Perfect Process Complementarity and Discontinuous Yield Function

Plane OAD represents the potential of the husbandry process. For a given level of the process output, say at g_0, the maximum yield is y_0. The diagram clearly indicates y_0 to be independent of the biological process output f, along MN, as long as f is greater than f_0. When f falls below f_0, the yield does not follow MN line, but follows the line common to MNPQ and OBC planes. Thus, the feasible points are on or under the ridge represented by two planes OBX and OAX.

The points on the planes OXC and OXD are not feasible, except on the line OX. Under the assumption of efficient production, we expect the producer to regulate f and g such that the yield is always on OX or near it. Thus as far as, one single process output f and its relationship to yield y is considered, the possibility of y being equal to f depends upon whether g-output is constraining or not.

This may be summarized by saying that

$$y = f(X) + \epsilon$$

where, ϵ is always negative or zero, $\epsilon \leq 0$. Let us say that ϵ is a random variable with a negative expectation, $E(\epsilon) = \mu < 0$, and variance $V(\epsilon) = \sigma^2 < \infty$. Under these assumptions, we will find that the true model coefficients except the constant term may be estimated by the least squares method.

The coefficient estimates of the linear in-parameter form for f(X) will be seen to be unbiased except for the constant term, β_0. The estimate of β_0 will however be biased to a value, $\beta_0 + \mu$.

Let us define a new error term $\nu = \epsilon - \mu$, this random variable has a zero expectation, $E(\nu) = E(\epsilon - \mu) = E(\epsilon) - \mu = \mu - \mu = 0$. and the variance remains unchanged, $V(\nu) = V(\epsilon) + V(\mu) = V(\epsilon)$. Therefore the negative error model may be transformed into a general linear model,

$$y = X\beta + \epsilon = X\beta + \mu + \nu.$$

Note that the constant term β_0 in $\beta' = (\beta_0\ \beta_1 \ldots . \beta_k)$ can now be changed to represent a new coefficient vector, $b' = ((\beta_0 + \mu)\ \beta_1 \ldots . \beta_k)$. The difference between β' and b' is only in terms of the first term, $\beta_0 + \mu$, so that

$$y = Xb + \nu,\ E(\nu) = 0,\ V(\nu) = \sigma^2 < \infty$$

holds and we can use OLS to estimate, b, using the matrix equations,

$$\hat{b} = (X'X)^{-1}\ (X'Y).$$

We note that, all the coefficients $\beta_1,\ldots,\beta_k$ are estimated without bias by $\hat{b}_1,\ldots,\hat{b}_k$ respectively, because

$$E(\hat{b}) = E[(X'X)^{-1}\ (X'Xb + X'\nu)] = E(b) + 0$$

$$= \begin{bmatrix} \beta_0 + \mu \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}.$$

We also note that $E(\hat{b}_0) \neq \beta_0$ and therefore the constant term of the regression will be downward biased, by the unknown quantity $|\mu|$. The same arguments could also be repeated in relation to the g-function under Sadan complementarity. Thus under perfectly complimentary processes, the process functions can be estimated except for the constant terms. The estimates of the marginal productivities of the inputs used will however not be biased. The factor productivity coefficients will be asymptotically normal. The problem however remains as how best to test for perfect process complementarity.

The negative error models have already been used in the description of production frontiers [Judge et al. (1980) pp. 302, Aigner et al. (1976), Lee and Tyler (1978)]. Alternative specifications for the negative error terms are possible, but does not seem to provide any extra benefits beyond what has been already gained.

Schmidt (1976) has attempted maximum likelihood (ML) estimator by assuming half-normal distributions for ϵ,

$$f(\epsilon) = \frac{2}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\epsilon^2}{2\sigma^2}\right), \quad \epsilon \leq 0.$$

He indicates that one of the regularity conditions required for maximum likelihood estimation is violated under half-normal specification. Under this situation, the ML estimates are possible but their sampling properties are uncertain. Another possibility is to use two error terms, though this would imply two random processes at work, which is rather doubtful [Judge et al. (1980) pp. 302].

Sadan Model and a Simple Cubic Approximation

The previous discussion on the Sadan function, $y = \min(f,g)$, indicated that the function is represented as a slanted roof ridge over the f-g plane, shown below. Though this ridge is discontinuous

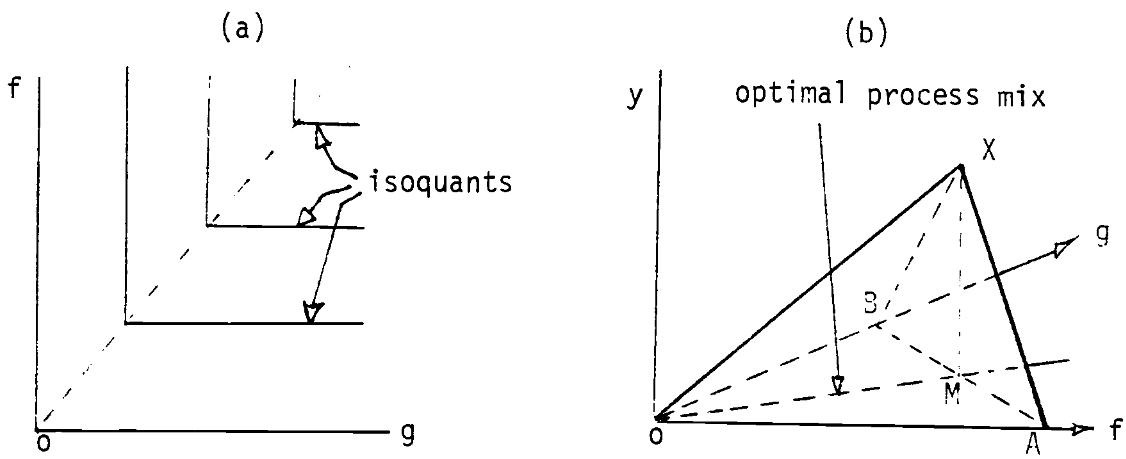


Figure-6: Discontinuous yield-surface in Y-f-g space along OX, a number of surfaces can be approximately fitted over this roof-ridge, particularly when most of the observation points are scattered along OX. One possibility is to approximate this ridge with the inverted inclined cylinder with a parabolic cross-section as shown below:

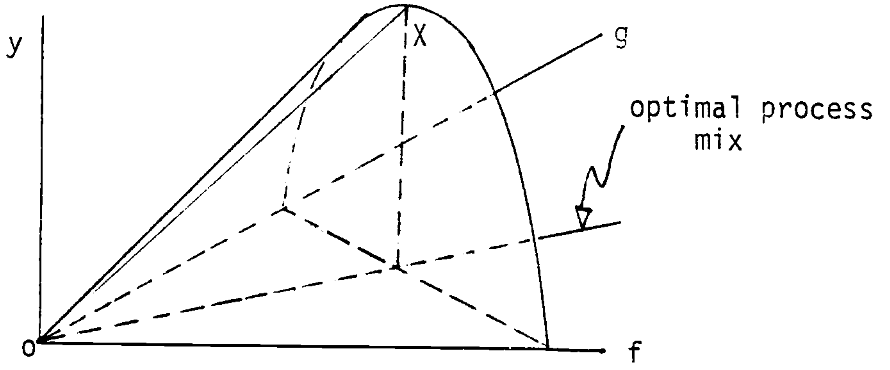


Figure 7: Inverted Inclined Cylinder

From the theory of conic sections it can be shown that such a surface can be represented in the positive f-g quadrant with the following quadratic in f and g:

$$y = (f + g) - \eta (f - g)^2$$

where, η is a positive constant. It is also known that increasing the power of the second right-hand term in the above equation sharpens the top part of the inverted parabolic cylinder (Figure 8).

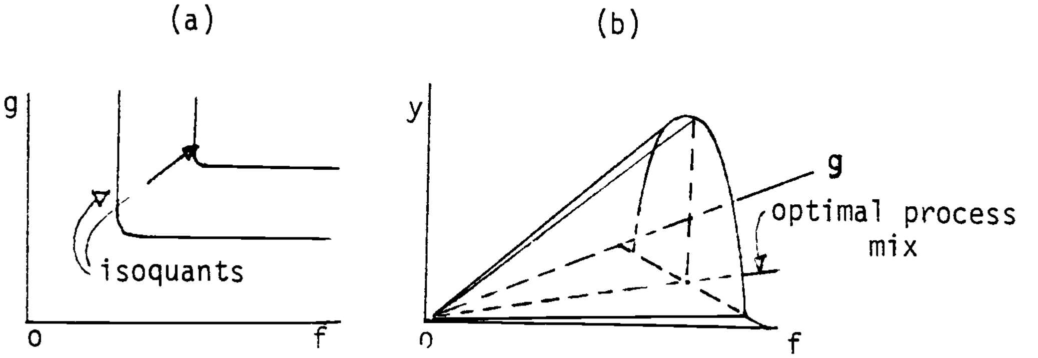


Figure-8: (a) Isoquants (b) Inverted Inclined Cylinder

On the other hand decreasing the power of the second term flattens the ridge of parabolic cylinder, see diagram below:

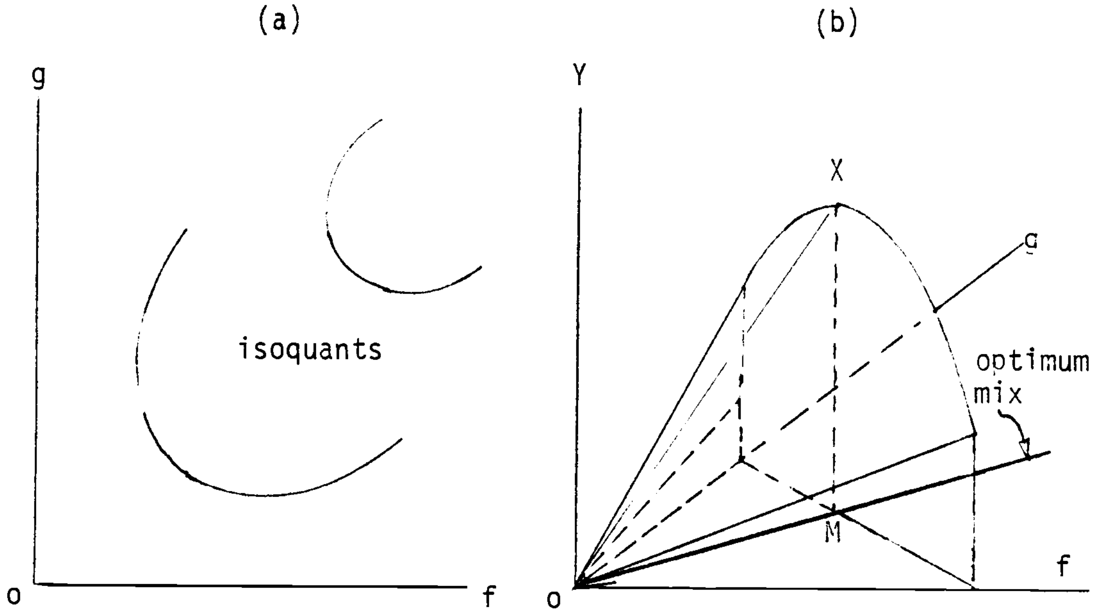


Figure-9: (a) Isoquants (b) Inverted Cylinder

Now, if f and g are quadratic functions of X and Z respectively, $Y = (f + g) - \eta|(f - g)|^{3/2}$ represents a cubic expression. Thus if Sadan complementarity exists, the sharp ridge of the function, $y = \min(f,g)$ may be approximated with, $y = (f + g) - \eta|(f - g)|^{3/2}$. The test of Sadan complementarity may be performed estimating the equation above and acertaining if η and other coefficients truly represent an inverted parabolic cylinder surface.
when the function,

$$y = (f + g) - \eta|(f - g)|^{3/2}$$

is estimated using a general quadratic for f(X) and g(Z), the coefficient η should be positive and statistically significant. The test would of course require a nonlinear function estimation and the use of asymptotic properties of these nonlinear estimators.

In contrast to the negative random error models, which can estimate either process function this simple cubic approximation allows the use of errors due to f and g simultaneously. From this standpoint, the simple cubic is better suited to test perfect complementarity. The negative random error model, on the other hand, is not capable of providing a test of perfect complementarity.

Nested Hypotheses Sequence

The hypothesis to be testedcan be group-wise linear and non-linear weak separability in the context of a translog function. More restrictively, we can have complete pair-wise strong separability and additive strong separability. Both of the presently discussed separabilities imply weak separability but not vice versa. Similarly, we may have further subdivision within each category, using linear homogeneity. This allows us to create and test a sequence of nested hypotheses.

For instance, if the strong separability hypothesis is not rejected statistically, it becomes unnecessary to test for weak separability. This is true when linear homogeneity and strong separability is not rejected. But even if strong separability is rejected there may still exist weak separability, so this needs to be tested. Linear homogeneity may be tested for at the first stage. (See Figure 10).

For the translog function, linear homogeneity is equivalent to the following linear restrictions on the α_i's and γ_{ij}'s:

$$\sum_{i=1}^{n} \alpha_i = 1 \cdot 0$$

$$\sum_{i}^{n} \gamma_{ij} = 0 \cdot 0 \text{ for all } j = 1,2,\ldots,n.$$

One may test for linear homogeneity using the linear restriction matrix, $R\beta = r$, as discussed above and applying the F-statistics to test the null hypothesis H_o: $R\beta = r$, representing the equations above [McFadden et al. (1978)].

Since monotonicity and quasi-concavity are not always ascertained for the translog, they need to be confirmed at some stage. If strong additive separability and linear homogeneity becomes the nonrejectable hypothesis, then the Cobb-Douglas form is implied and monotonicity and quasi-concavity is assured. Except in this case, the translog needs to be tested for both monotonicity and quasi-concavity over the relevant range of factor inputs. The test of monotonicity at a given factor combination point may be represented by the following null hypothesis,

$$H_o: \ (\alpha_i + \Sigma \gamma_{il} x_l) > 0.$$

(1) TEST LINEAR HOMOGENEITY

H_o: $\Sigma\alpha_i = 1$, $\sum_j \gamma_{ij} = 0$

(2) TEST COMPLETE

PAIR-WISE SEPARABILITY

H_o: $\gamma_{ij} = 0$, if $i \neq j$.

(3) TEST GROUP-WISE

LINEAR WS RESTRICTION

H_o: $\gamma_{ik} = 0$, if $X_i \in X^s$, $Z_k \in Z^r$.

(4) TEST GROUP-WISE NON-LINEAR

WS RESTRICTION

H_o: $\frac{\alpha i}{\alpha_i} = \frac{\gamma_{il}}{\gamma_{jl}} = e_j^i$ for all $l = 1, \ldots, N$.

and $\frac{\alpha p}{\alpha_{ql}} = \frac{\gamma_{pl}}{\gamma_{ql}} = e_q^p$ for all $l = 1, \ldots, N$.

( ) TEST MONOTONICITY

H_o: $\alpha_i + \Sigma_l \gamma_{il} x_l > 0$.

( ) TEST QUASICONVEXITY

H_o: $X'HX < 0$.

H is the bordered Hessian

of $\phi(f,g)$ at the point x.

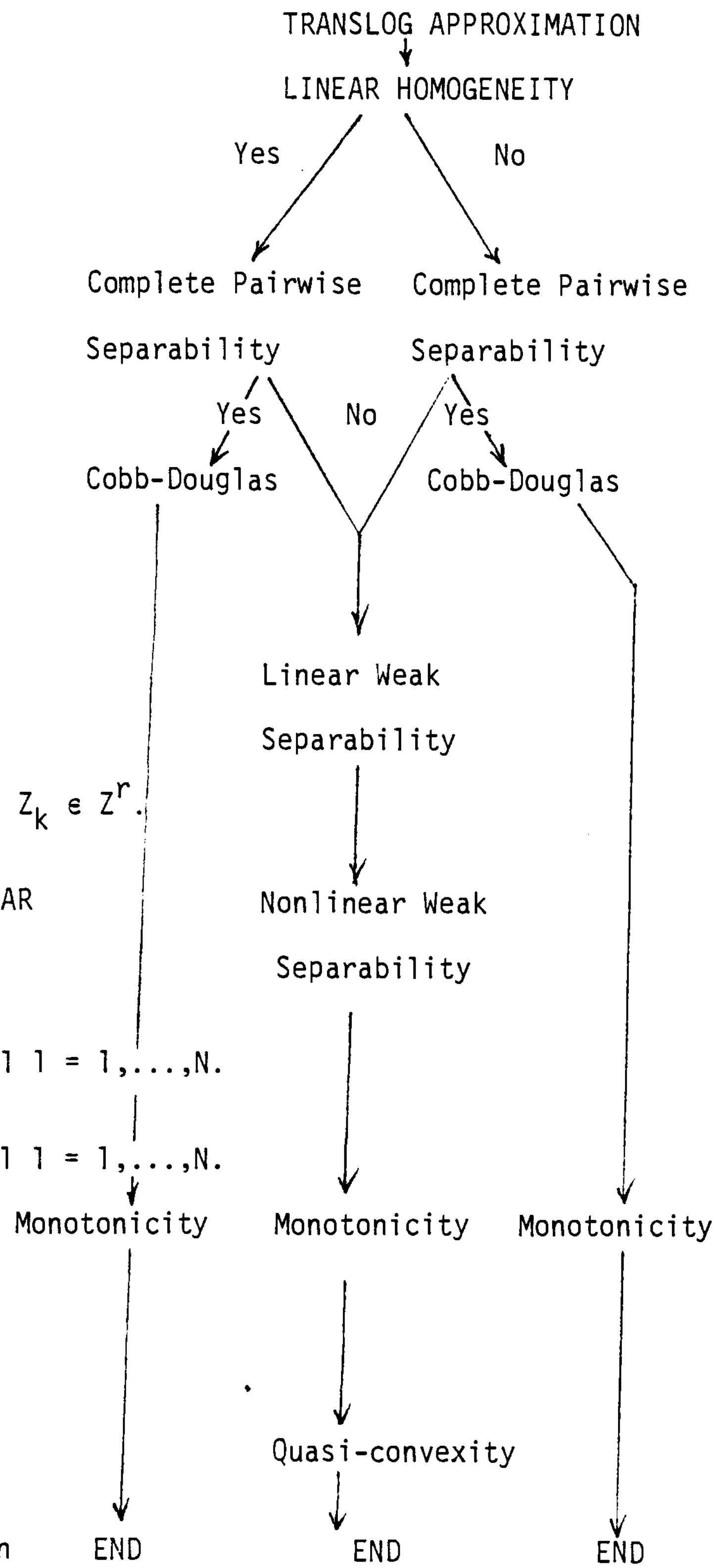


Figure-10: Nested Hypothesis Testing

The test of quasi-concavity requires the bordered Hessian of the translog production function to be negative definite at the given point. This requires that, $x'H\,x > 0$, where H = the bordered Hessian of the translog form [Corbo and Meller (1979)].

In Figure 10 above the sequence of tests on the nested hypothesis is indicated. If α% level of sequence of significance is allotted to each level of test, there being 4 levels of these tests, we have the overall significance of 4α%. So if $\alpha = 0{\cdot}025$, $4\alpha = 0.10$, and the overall significance is still 10%. The test would be a sequence of F-tests derived for the restriction for each level.

Single and Multiple Partitions Separability

Single and multiple partition separability is a generalization of the two extremes, complete pairwise separability and simple weak separability, into two groups. For example, let us consider a function of three variables, x_1, x_2 and x_3. We can stipulate the following partitions on these: three different single-partitions $[(x_1, x_2), x_3]$, $[x_1, (x_2, x_3)]$, $[(x_1, x_3), x_2]$, and one double-partition $[(x_1), (x_2), (x_3)]$. Let us further assume that x_1 and x_2 are, in fact, inseparable. If we test for all three single partitions and one double-partition, statistically, we expect three rejections and one non-rejection. The rejected restrictions would be $[x_1\ (x_2, x_3)]$, $[(x_1, x_3), x_2]$ and $[(x_1), (x_2), (x_3)]$. The non-rejected restriction would be $[(x_1, x_2), x_3]$.

This suggests that if the objective is merely to confirm separability of x_3 from (x_1, x_2), we may simply test this particular restriction. On the other hand if the objective is to test for any existing inseparabilities, we may have to perform all of the tests. The first test would appropriately be the most restrictive i.e. the double-partition (or global or pairwise separability), $[(x_1), (x_2), (x_3)]$. If this is not-rejected in a statistical test, further separability tests are unnecessary. At the other extreme, all the restrictions can be rejected, indicating no separability, and the non-existence of composite inputs or process-functions. In the case of three variables, if two single-partitions are not rejected, for the sake of logical consistency, we require that the double-partition be non-rejected also. But practically, spurious results can be expected where the choice between the non-rejects need to be made.

Extending these arguments to four-variables, we first test for complete pair-wise separability. If this test is rejected, then we may test for double-partitions; six of them: $[(x_1, x_2), x_3, x_4]$, $[(x_1, x_3), x_2, x_4]$, $[(x_1, x_4), x_2, x_3]$, $[(x_2, x_3), x_1, x_4]$, $[(x_2, x_4), x_1, x_3]$ and $[(x_3, x_4), x_1, x_2]$. If all of these six are also rejected, then we have total inseparability amongst $[(x_1, x_2, x_3, x_4)]$. If only one is not rejected out of six possibilities, this would be strong support for considering that there exists an non-separable pair amongst four x's. However, if more than one partition become not rejected then logical consistency would require that there is global pairwise separability. When direct test of global pairwise separability is rejected, but two or more of double-partitions are not rejected, a contradition arises which may not be resolved with the given data.

In such a situation (except when one (or more) of the non-rejections are marginal) further choice between the non-rejects (and therefore mutually competing notions of separability) is difficult on purely statistical grounds.

Thus in conclusion, we must note that though a complete rejection of separability is always possible, the reverse, where a specific type of separability alone is non-rejected, is possible but not always guaranteed. The data set may or may not be able to distinguish between two competing notions of separability. When such a case arises, the choice can be made only on the basis of some extraneous information outside the model and data. One possibility is to extend the order of approximations of a function, that is, to make a cubic out of the quadratic, and attempt to resolve the choice to a single model of separability. In models involving many variables, such a procedure, however, rapidly increases the number of parameters to be estimated. Another possibility is to use nonlinear in parameter functions with appropriate restrictions on AES to represent separability (along with Sadan type restrictions).

IV. MODEL SPECIFICATION AND HYPOTHESES

Model and Maintained Hypotheses

The validity of the biological and husbandry processes will be ascertained using separability tests on wheat production function. The model will be a general function locally approximated by the TRANSLOG function. The model is as follows:

$$Y = G(Q, B, F, P, T, I, N, K, M).$$

where, Y is per acre wheat yield, B is pesticide per acre, F is fertilizer per acre, I is percent of irrigated acres, P is precipitation in inches, T is temperature index, N is labor measured in dollar wage, K is capital service measured by fuel and oil expenditures. M represents the management input index and Q represents the land quality index. It may be noted here that there are no data available for Q, B, and M. They are listed here merely to indicate that they should ideally be included in a production function for wheat.

The model could also be specified in terms of total yield

$$Y = \overline{F}(Q, B, F, P, T, I, N, K, M, A)$$

where the variables B, F, N, K are corresponding total inputs, Q, P, T, M are corresponding indices, I is irrigated acres and A is total wheat acreage.

Since cropping activity is spatially replicable, we know the $\overline{F}(..)$ function will be linear homogenous in inputs F, I, N, K The function G(..) corresponds to the model described by $\overline{F}(..)$, in the sense that, G(..) be obtained from $\overline{F}(..)$, by dividing the total input and output variables by the total acreage A. The assumption that $\overline{F}(..)$ is linearly homogeneous will therefore be a maintained hypothesis in the present study.

This 'constant returns to scale' assumption is reasonable for a crop production function. The same assumption would not be valid if other activities were also present such as forage and cattle (or dairy). If there exist externalities between processes, the value added function may obtain increasing returns to scale. There is some evidence of increasing acreage size of the farms, indicating an existence of economies of scale.

This acreage growth of the farms may result from increasing returns to management inputs in 'management processes' and/or because of increased efficiency of capital use. Bigger and better machines may result in substantial reduction in labor and management inputs (for the same output) and allow for scale increases by relieving the slack created in management capability. When 'processes' are exact replicates, management inputs can show increasing returns to scale, because almost the same operations on the management side (e.g. procurement of inputs, keeping accounts and managing the inventory) result in large direct input utilization.

The issue of the growth of farms in recent years, though important, is not the focus here. The issue appears to require a more detailed study of the management process, which at present is lacking. The problem here arises from the fact that management inputs are diverse in quality, and there is no readily available index or quantified variable to represent it. This particular problem does not appear to have been satisfactorily dealt with so far. Another aspect of the problem consists of the fact that 'entrepreneural ability' is rather immobile between farms and is not marketed readily.

Proposed Hypotheses

It is hypothesized that any crop growing activity may be conceptually looked upon as consisting of the biological growth process and the husbandry process. The husbandry process creates a suitable culture medium by tillage, seedbed preparation, irrigation etc. so that the biological process may succeed. And after maturity of plants, the husbandry process continues again in terms of harvesting, drying etc. The sequential nature of these processes imply that the biological and the husbandry processes are economically separable. Thus it is hypothesized that,

$$Y = G(Q, F, P, B, T, I, M, K, N)$$

can be written in the following separable form

$$Y = \phi[f(Q, F, P, B, T), g(K, N, I, M)]$$

where, f is the biological process function and g is the husbandry function. If, the irrigation input were measured in terms of acre feet of water instead of percent of irrigated land, it would be an argument in the biological process function. However, when irrigation represents a service from irrigation capital (or infrastructure of pump sets and distribution pipe lines or the culverts and ditches), its logical place appears to be in the husbandry function.

Operationally, therefore, it is hypothesized that the per acre model will be weakly separable into at least two groups consisting of the weather/biological variables Q, F, P, B, T and the husbandry inputs K, N, I and M. It is possible that all of the inputs are strongly separable from one another; this would still be consistent with the notion of two process function f and g in ϕ-function. If, on the other hand, a pair of variables, one belonging to one group (say, biological process) and the other belonging to the other (say, the husbandry process) are found to be empirically inseparable, then this would mean an empirical refutation of the notion of those two subprocesses. Even that however does not exclude the possibility of having new processes redefined within the production scheme. There is a possibility of regrouping inputs into new separable groups, each group corresponding to a 'new' process. Such empirically constructed 'processes' may or may not have their theoretical justifications. Therefore, one should start with the a priori theoretical possibilities of subprocesses and weak separability, rather than reasoning from 'empirical separability' to 'processes'.

For example, in the case of cropping the knowledge of tillage, seedbed preparation, plant growth, harvesting as sequential activities leads one to assert input independence amongst these activities. So we hypothesize weak separability between fertilizer, water, pesticides and fuel, labor, capital,etc. When such intimate knowledge of the detail of the subprocesses is not available, it becomes necessary to allow the empirical identification of separability to lead the way to development of the theory.

Such may be the case of management inputs, if these inputs have satisfactory indices for them. Though management process output is not tangible, it is possible that certain attributes of the manager and the organization are good indicators of their effectiveness. Very often these attributes are vague, though each of them stand a better chance of being quantified rather than "management" itself. Their effectiveness is seen (often) clearly when they appear simultaneously.

In the absence of such indices, the major alternative appears to be as follows. If the management inputs (attributes) are assumed separable from other inputs, and there exists perfect complementarity between 'management process' and the rest of the 'processes', then, the partial production function of the remaining 'processes' may be estimated without specification bias, provided, efficient production is assumed.

Symbolically, if h(M) represents the management process output, then the yield is definable as,

$$Y \equiv \Psi[f(X),\ g(Z),\ h(M)]$$

under the weak separability assumption. Again, f(x) represents biological process function and g(Z) represents the husbandry process function. The perfect complimentarity between h, and, f and g, can be represented as,

$$Y \equiv \min[\phi(f(X),\ g(Z)),\ h(M)].$$

Under the assumption of efficient production, we have,

$$Y \equiv \phi(f(X),\ g(Z)) \equiv h(M).$$

Thus, only if we assume that weak separability with perfect process complimentarity between 'management' and other 'processes' exist, along with efficiency in production, it becomes valid to estimate the crop-function. Otherwise specifiction bias results. This is another maintained hypothesis in this model.

The husbandry process inputs, such as capital service, K, and labor, N, are assumed independent of the weather variables, precipitation, P, and the temperature index T. Furthermore, the biological input, fertilizer, F, is also initially assumed to be separable from the husbandry inputs, K and N. On the other hand, irrigation input may be dependent on P and T, because P can substitute for irrigation water. The later possibility has to be empirically tested.

Separability between F and K and N can, however, be tested afterwards if P and T are indeed separable from F, I, N, K. The separability structure then simplifies to the three distinct possible groups, weather, biological and husbandry inputs. On the other hand, weather inputs can also be regarded separable from economic inputs on the grounds that weather variables do not influence economic inputs because they are not known beforehand.

It is further hypothesized that in the case of a crop production process, the biological and husbandry processes are not only distinct and separable, but they are, in line with Sadan's thinking, perfect complements. The implication is that the ϕ-function is reduced to the case where,

$$Y = \text{Min}[f(X),\ g(Z)].$$

In other words, the substitutability between f(x), the biological and g(Z), the husbandry functions, is zero or there is perfect complementarity between them. This hypothesis is equivalent to weak separability and additionally, perfect complementarity between f and g, as has already been discussed. The isoquants in f-g-space are of Leontief-type, i.e., they are L-shaped. This discontinuity represents a problem in testing for this particular 'mix of hypotheses' in empirical terms, and approximations using cubic functions may be employed for testing purpose.

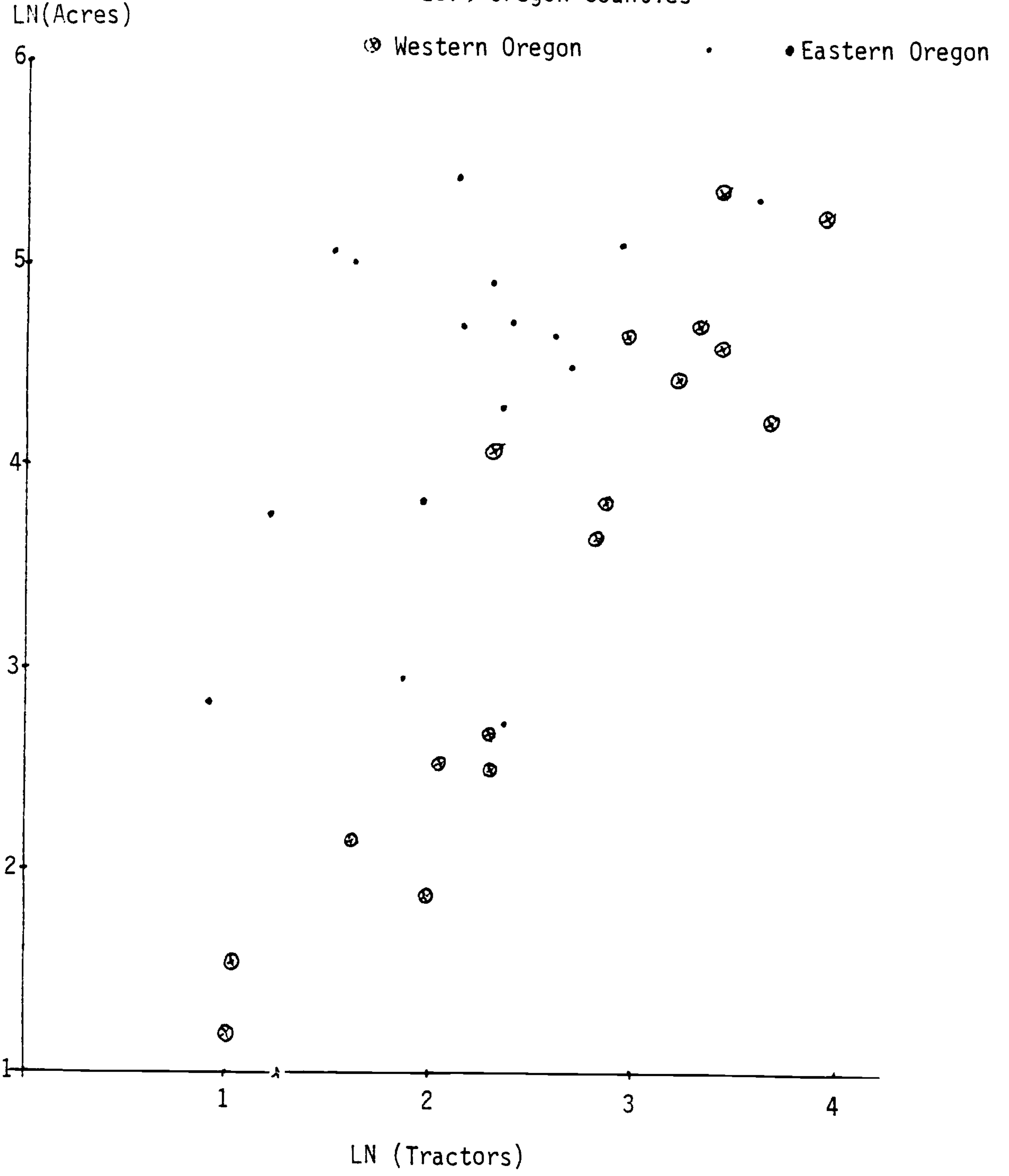
Regional Production Functions

Eastern Oregon is comparatively dry relative to western Oregon. This difference in precipitation influences the type of wheat grown in these two regions. The low annual precipitation in most of eastern Oregon is often unable to sustain annual wheat crops. Thus in eastern Oregon the common practice consists of winter wheat cultivated every alternate year. The moisture accumulated in the ground during a fallow year sustains the moisture requirement of the cropping year.

In western Oregon, on the other hand, spring wheat is the dominant class produced. The difference in variety of wheat grown and the incidence of precipitation makes it probable that there exists two distinct production functions--one for eastern Oregon and one for western Oregon. The high precipitation in western Oregon implies that there is little need to irrigate spring wheat. Irrigation in western Oregon is expected to affect yield to a lesser extent than in eastern Oregon.

Alternate year cropping in eastern Oregon also implies that the input of machinery services per unit of land cultivated there will be lower than that in western Oregon. The graph below indicates clearly that, as far as tractors, the major tillage machinery capital, is concerned, eastern Oregon employs fewer per acre than does western Oregon. This is the primary reason for believing that the number of tractors does not represent machinery service input into the production

GRAPH 1: Cropland Harvested and Tractors
LN (Acres) vs. LN (Tractors #)
1974 Oregon Counties



process. The 'stock' capital must be replaced with a 'fund' of services concept. Capital Service is thus better measured in terms of rate of utilization.

Summary of Hypotheses

There are these common maintained hypotheses for two types of models to be summarised later.

(i) 'management process' is perfectly complementary to other 'processes' (such as biological and husbandry);

(ii) production is efficient with respect to the 'management' and 'other' processes;

(iii) The original 'total' model, $\overline{F}(..)$, is linearly homogeneous in total inputs, land, fertilizer, etc.

The implication of (i) and (ii) is that the 'Other' process-function (or partial production function) can be estimated without specification error. The third assumption allows one to move from the 'total' model, $\overline{F}(..)$, to the 'per acre' model, G(..). There is a possibility that a) P and T, the weather variables in the biological inputs are separable from both the biological input F and the husbandry inputs K, I, N;

$$Y = \phi[P, T, G(F, K, I, N)].$$

or b) the husbandry inputs N, K are independent of all biological process inputs,

$$Y = \xi[W(P, T, F, I), V(K, N, I)].$$

Though a) and b) can both be tested, they are 'maintained' hypothesis in this study. Thus there are two models, model-A under (a) and model-B under (b).

The hypothesis to be tested using model-A are (i) Separability between husbandry and biological inputs,

$$Y = \phi[P, T, f(F), g(K, N, I)].$$

(ii) Perfect Complementarity between husbandry and biological processes,

$$Y = \min[f(P, T, (F)), g(P, T, (K, N, I))].$$

The hypothesis to be tested using model-B is that (P, T) is separable from (F, I), i.e.

$$Y = \xi[W((P, T), (F, I)), V(K, N, I)].$$

Sources of Data

Oregon county level data on inputs and outputs of wheat were obtained from the Census of Agriculture for the years 1954, 1959, 1964, 1969, and 1974. The data available are total wheat yield, acreage of wheat harvested, acreage of irrigated wheat, and the total amount of dry and liquid fertilizer applied. Unfortunately, the amount of water applied to wheat is not known by county. The husbandry inputs such as fuel (energy), labor and capital machinery services are not available specifically for wheat. The average per acre values in all uses may, however, be calculated from the data available. This measure is not entirely satisfactory since fuel and labor ratios may vary between crops to a considerable extent.

The source of climatological data for Oregon counties are the reports from the National Oceanic and Atmospheric Administration, U. S. Department of Commerce. [Annual Summary of Climatological Data for Oregon, Environmental Data Service, NOAA, USDC, 1954, 1959, 1964, 1969, 1974.] This source was particularly useful since it provides a yearly summary of precipitation and temperature. County level data were developed by choosing the 'representative' weather station in the county. Since annual averages are taken, the errors dur to geographic heterogeneity of precipitation and temperature will probably be small. Some error will, however, always remain. Appendix-C contains the list of weather stations chosen to represent the counties.

Data Description

Fuel and labor inputs vary from crop to crop. When county averages are used, error may be reduced somewhat provided that the averaging process was conducted with a single crop. The crops grown vary considerably from farm to farm. The crops grown vary less so from county to county. If the crop-mix between the counties are similar, fuel per acre (on the average) may be a reasonable measure of the total service of machinery capital.

Although capital stock could be employed as a variable, this too is heterogeneous in terms of horse-power, type of machine, age, and technical efficiency, and so this measurement of capital input becomes suspect. More importantly, 'stock' measures ignore utilization rates of these machines. Fuel on the other hand can measure the utilization rate accurately. However, when fuel and oil is aggregated over different mechanical equipment and operations, this accuracy might be lost.

Similar considerations on heterogeneity of labor service apply. However, it is remarkable to note that the ratio of deflated labor and duel (in logorithms) expenditures have remained relatively constant for 1954, 1959, 1964, 1969 and 1974, for the Oregon counties. [See Appendix-B, Graph (i)-(ii)]. The reason for this may be that fuel (and oil, etc.) and labor ratios vary little between different types of husbandry processes.

One important possibility is that the fuel consumption and labor required for the operation of a type of machinery (say a tractor) may be relatively fixed (for a given terrain and soil type, etc.). Further, if the machines required for the crops are similar (i.e., tractor, combine, trucks for most of the small grain crops) and require similar use rates, the above constancy of the fuel/labor ratio appears plausible. This collinearity between fuel and labor may lead to a multicollinearity problem in the estimation of the coefficients of inputs. The F-statistics used to test the weak separability hypothesis depend upon the sum of the squared errors in two models, multicollinearity may adversely affect this test. This is true particularly when collinearity is perfect and coefficient estimation becomes infeasible due to singularity of the (X'X)-matrix.

The land quality index variable ought to measure the intrinsic fertility of the soil. However, such an index of land quality is not available. Often the indices available are derived from some other "value of production" model, and their use in the present context would be 'circular'. Land quality indices for Oregon counties are available, however, in dissertations by Holloway (1972) and Thomas (1974).

The data on fertilizer input are also faced with the difficulty of heterogeneous summation. The census figures do not indicate whether the liquid and dry fertilizers have the same nutrient content (say for the primary component N_2). There appears to be some validity in assuming that the nitrogen concentrations in weight do not vary too much between dry and liquid forms. (This is based on telephone conversation with Peter White, Pendleton Grain Growers Cooperative, Pendleton, Oregon.) Accordingly, the measure of fertilizer input has been derived by adding the two types together.

Pesticide and herbicide inputs were not available in the Census of Agriculture; though their effects are important they have been left out in this study, resulting in some specification bias.

Precipitation was obtained for the chosen county weather station and was regarded to be representative of the county situation. The temperature variable employed was not the average annual temperature. A more relevant variable is believed to be the incidence (or lack) of extremely cold temperatures. A better index would consist of the length of the period during which the lowest tolerable temperature did occur. Thus the relevant information would consist of the number of days (in the growing season) when the temperature of the air and ground did not fall below 46°F (the minimum tolerance temperature for wheat. This information was not readily available, although an examination daily for each county for each year would have provided this information. The index chosen for temperature was, therefore, the 'Number of days between dates with 32°F or below'--a statistic which is reported by Environmental Data Service. It was felt that, though this variable was not the ideal, it may still be explanatory to some extent.

The irrigation variable used in this study represents a service derived from a stock of infrastructure. It does not measure the water applied in acre-feet. This in itself is, however, not a serious drawback. It is known in the case of irrigation that the water application regime is as important as the amount of water applied [Stegman (1980)]. The effect of a given water application varies greatly depending upon the temperature, sunshine and wind conditions and nature of the soil. Thus, the errors in "input of moisture" can vary greatly from one plot to the next, even under uniform application of water. The acreage of irrigated land (per acre of total harvested acreage) has therefore some advantage in that the measurement errors are small. The assumption is that the services from irrigation infrastructure definitely influence the 'biological process' itself, and so this variable captures some aspect of "irrigation input." The obvious disadvantage is that it ignores the 'heterogeneity' within 'irrigated acres' category resulting from water input differences.

The Units of the Variables

The units of the variables are as follows:

P, precipitation, in inches per unit area;

T, temperature index, number of days when minimum temperature is above 32°F;

F, Fertilizer per unit of land, tons/acre;

I, irrigation service, percent of land irrigated;

N, labor services, in thousands of 1954 dollars per acre, adjusted on the basis of farms expenditure index for the USA. [Prices Paid by Farmers: Index Numbers, Annual Average, U.S., 1950-1977 (1979)].

K, capital services, in terms of fuel and oil expenses, in thousands of 1974 dollars per acre, adjusted as above;

Q, land quality index, a pure number between a scale of 0-100 (or above);

Y, wheat yield per unit of land, bushels per acre.

The data set is shown in Appendix F.

V. RESULTS

Linear Computational Procedures

The translog model with linear restrictions was estimated using the Time Series Processor (1976) package (called T.S.P.) from the Computer Centre, The University of Western Ontario, Ontario, Canada. The linear least squares estimation procedure used in the T.S.P. package computes the estimates of the coefficients, their standard deviation, the t-statistics, R^2 and the Durbin-Watson statistics. It also gives the sum of the squared residuals, the standard error of regression, the sum of residuals, F-statistics of the regression and the estimate of the variance-covariance matrix of estimated coefficients (see Appendix E for a sample).

The T.S.P. has a plotting capability, which allows for presenting graphically observed and estimated dependent variable values, as well as the errors. The T.S.P. package also permits graphing the variables against one another. The T.S.P. package can be used for Farrar-Glauber multicollinearity test, auto-correlation detection and correction. It has the capability to compute simultaneous system of equations. The other main advantage of T.S.P. lies in the fact that the matrix manipulations required in testing the linear restrictions can be readily performed. The nonlinear equation estimation procedure is also available within T.S.P. as will be discussed later on.

Model-A Results

As indicated in Chapter IV, the Model-A hypothesis is that the weather variables and T are weakly separable from the other input variables; F,I,K,N. The weak separability assumed here is a simple type with linear restrictions $\gamma_{ij} = 0$, where we have

X_i = P,T and X_j = F,I,K,N.

The model has the following matrix form:

$$Y = \beta_o + \beta_p . P + \beta_r \cdot T + (\beta_F \ \beta_I \ \beta_K \ \beta_N) \begin{bmatrix} F \\ I \\ K \\ N \end{bmatrix} + (FIKN) \begin{vmatrix} \gamma_{FF} \gamma_{FI} \gamma_{FK} \gamma_{FN} \\ \gamma_{II} \gamma_{IK} \gamma_{IN} \\ \gamma_{KK} \gamma_{KN} \\ \text{sym.} \quad \gamma_{NN} \end{vmatrix} \begin{vmatrix} F \\ I \\ K \\ N \end{vmatrix}$$

The tests of separability, linear homogeneity and the Chow test for regionally different production functions can be conducted by applying various linear and nonlinear restrictions on the coefficients. As would be made clear consequently, the OLS procedure is adequate to handle all of these restrictions except that of nonlinear weak separability.

Regional Difference in Production Functions: Chow Test Results

The difference in precipitation between eastern and western Oregon results in major differences in the variety of wheat grown in these areas. Winter wheat is commonly grown in the east, spring wheat in the west. Therefore, it is expected that there are two distinct production functions for these regions.

If linear-in-parameter TRANSLOG form is used to approximate the production function, then the parameter vector, β, must be different in these two regions. Thus the test involves the refutation of the null hypothesis, $H_o : \beta_E = \beta_W$, where β_E and β_W represent the parameters for eastern and western Oregon, respectively. This null hypothesis can be put in the form of a linear restriction on the coefficients of a more general model in TRANSLOG written as a simple linear-in-parameter model, $Y = X\beta + \varepsilon$, where X now represents all the linear terms P,T,F,I,K,N, as well as the cross-products involving the last four.

Let the subscripts E and W represent eastern and western Oregon observations, respectively. The general model combining both the eastern and western Oregon data will be represented as follows:

$$\begin{bmatrix} Y_E \\ Y_W \end{bmatrix} = \begin{bmatrix} X_E & 0 \\ 0 & X_W \end{bmatrix} \begin{bmatrix} \beta_E \\ \beta_W \end{bmatrix} + \begin{bmatrix} \varepsilon_E \\ \varepsilon_W \end{bmatrix} ; \quad E \begin{bmatrix} \varepsilon_E \\ \varepsilon_W \end{bmatrix} = 0, \; V \begin{bmatrix} \varepsilon_E \\ \varepsilon_W \end{bmatrix} = \sigma^2 .$$

The null hypothesis, $H_o : \beta_E = \beta_W$, can be thought of as being a restriction on the above model, such as, $R\beta = 0$, where $R = [I : -I]$ and $\beta = \begin{vmatrix} \beta_E \\ \cdot \\ \beta_W \end{vmatrix}$, I is unit-matrix. The total sums of squares of the general models above and of the partial models, $Y_E = X_E\beta_E + \varepsilon_E$ and $Y_W = Y_W\beta_W + \varepsilon_E$ can be represented as $e'e$, $e_E{}'e_E$ and $e'_W e_W$, respectively. Then under the null hypothesis we can show that the ratio:

$$\frac{e'e - e'_E e_E - e'_W e_W}{(e'_E\, e_E + e'_W\, e_W)/(N_E + N_W - 2k)}$$

is the test statistic with F-distribution of $(k, N_E + N_W - 2k)$ degrees of freedom. Thus the null hypothesis can be tested using this F-statistic. The results presented in the table on the following page indicate

Table 1
Chow Test Results

VARIABLES	MODEL (1) $Y = X\beta + \varepsilon$ Oregon as a Whole Coeff.	(t-values)	MODEL (2) $Y_E = X_E\beta_E + \varepsilon_E$ Eastern Oregon Coeff.	(t-values)	MODEL (3) $Y_W = X_W\beta_W + \varepsilon_W$ Western Oregon Coeff.	(t-values)
P	0.072	(1.34)	0.169	(2.84)	0.175	(1.34)
T	-0.032	(-.66)	-.006	(-.17)	-.443	(-1.35)
F	0.162	(0.49)	-.029	(-.05)	-.748	(-.69)
I	0.160	(0.79)	0.856	(3.34)	-.198	(-.35)
N	0.452	(0.58)	-1.101	(-.88)	0.749	(0.37)
K	-.640	(-.32)	1.461	(0.37)	-.203	(-.04)
F^2	-.003	(-.18)	0.010	(0.39)	0.018	(0.35)
FI	0.001	(0.07)	0.011	(0.69)	-.212	(-.48)
FN	-.053	(-.80)	0.130	(1.29)	0.092	(0.63)
FK	0.044	(0.36)	-.177	(-.93)	-.298	(-1.12)
I^2	0.005	(1.11)	-.002	(-.64)	-.006	(-.23)
NI	-.042	(-1.26)	-.043	(-1.22)	-.042	(-.39)
KI	0.062	(0.87)	0.189	(2.38)	0.030	(0.14)
N^2	-.119	(-1.20)	0.154	(0.86)	-.338	(-1.65)
NK	0.382	(1.19)	-.556	(-1.04)	0.737	(1.21)
K^2	-.273	(-.82)	0.406	(0.73)	-.284	(-.39)
Constant	3.578	(0.99)	5.578	(0.76)	5.423	(0.58)
R^2	0.4116		0.7202		0.3790	
$e'e$	13.71650		2.85246		7.99868	
D.f.	142		71		54	
N	159		88		71	

$$\text{Computed } F = \frac{\Sigma e^2 - (\Sigma e^2_W + \Sigma e^2_E) \;/K}{(\Sigma e_W^2 + \Sigma e_E^2)/(N-2K)} = 1.94$$

Table value of $F_{(17,125)}$ at $\alpha = 0.05 = 1.65$ and at $(\alpha = .01) = 2.03$

CONCLUSION: Reject the Null Hypothesis, $H_o: \beta_E = \beta_W$ at $\alpha = .05$. (and at $\alpha = .01$, the test can be considered marginal).

that β_E and β_W are significantly different. One fairly important difference to be noted is that the coefficient of irrigation input in the western region is not significant, though it is significant for the eastern region at $\alpha = 0.01$ level. This perhaps reflects the fact that in western Oregon, irrigation has only a minor impact due to the high levels of precipitation. In the east where precipitation is rather low the precipitation coefficient is positive and significant (at $\alpha = 0.01$). However, there are no major differences in the significance of T between the two regions. Index variable for temperature does not explain variance in wheat yield very well in either eastern or western Oregon. The strong multicollinearity between the capital and labor services, K and N, may have caused the negative signs of N and K in eastern and western Oregon, respectively.

The Fararr-Glauber test (Table 2) confirms the presence of multicollinearity in both the eastern and western regional models. It may be noted that multicollinearity makes suspect the sign, magnitude, and variance of the coefficients. Although the Chow test is valid under multicollinearity, the power of the test would be reduced making it less likely that separability hypotheses would be rejected.

Table 2 indicates the result of regressing each of independent variable P,T,F,I,K,N with the others. The multicollinearity is severe in the case of K,N. It may be noted that the Fararr-Glauber test is a very cumbersome test when the number of variables are rather large. For that reason, the test has been performed only with the set, P,T,F,I,K,N rather than the set containing all of the square and cross-product terms.

Table 2

Farrar-Glauber Analysis of Multicollinearity

(I) Null Hypothesis: Perfect Multicollinearity

H_0: Det $[X'X] = 0$

Test-statistics

$\chi^2_{df=15} = 610.756$

Critical $\chi^2_{df=15} = 5.229$ at $\alpha = 0.01$

Conclusion: Reject H_0 at $\alpha = 0.01$.

(II) Regression of independent variables on others

H_0: X_i is not linear in other X_j's.

Dependent variable	R^2	F-statistic	Conclusion on H_0
P	0.4935	29.82	Reject
T	0.3044	13.39	Reject
F	0.3210	14.47	Reject
I	0.2512	10.26	Reject
N	0.9557	659.67	Reject
K	0.9547	644.76	Reject

Conclusion: The multicollinearity problem exists mainly between labor N and capital service K. The critical F with d.f. (15,153) = 3.14 at $\alpha = 0.01$ indicates that there exists a strong multicollinearity between N and K.

(III) See Simple Correlation Coefficient matrix in Appendix-D.

Test for Cobb-Douglas Structure

The most restrictive of all separability restrictions that can be applied to the TRANSLOG approximation is to reduce it from a log-quadratic to a log-linear function. The resulting Cobb-Douglas function is strongly separable, i.e., the technology allows for pair-wise separability of all inputs and so the elasticity of substitution is unity between inputs. The test involves deleting all second order terms (generated from F,I,N, and K) in the regression and computing the new sum of the squared errors $e_*'e_*$ to be used in F-statistics,

$$F = \frac{N-K}{q}\left(\frac{e_*'e_*}{e'e} - 1\right).$$

Table 3 presents the results of the analysis for both eastern and western Oregon.

Model (6) indicates that for western Oregon (at least) the Cobb-Douglas production function is a valid specification given the TRANSLOG approximation. The implication of this type of strong separability is that weak separability between all F,I,N and K inputs holds and therefore, one need not test other linear and nonlinear weak separability restrictions. This was the reason for proceeding along with the nested hypothesis sequence.

For the eastern Oregon, Model (5), however, the Cobb-Douglas functional form is rejected, so the remaining possibilities are in terms of weak separability amongst F,I,N,K or strong-pair-wise separability. The latter does not imply Cobb-Douglas but merely additive separability between F,I,N,K (i.e., all second order terms except the

Table 3
Test for Cobb-Douglas Structure

VARIABLES	MODEL (5) Cobb-Douglas Eastern Oregon Coeff. (t-value)	MODEL (6) Cobb-Douglas Western Oregon Coeff. (t-value)	MODEL (7) Additively Separable Eastern Oregon Coeff. (t-value)
P	0.086 (1.46)	0.152 (1.32)	0.161 (2.57)
T	0.002 (0.06)	-.322 (-1.11)	-.002 (-.06)
F	0.227 (7.06)	0.227 (4.19)	0.104 (0.74)
I	-.012 (-1.09)	-.058 (-1.51)	0.448 (1.54)
N	-.028 (-.42)	-.163 (-1.62)	-1.013 (-2.31)
K	0.093 (0.65)	0.197 (1.00)	5.406 (2.43)
F^2	-	-	-.014 (-.84)
I^2	-	-	0.005 (1.80)
N^2	-	-	-.100 (-2.25)
K^2	-	-	0.508 (2.41)
C	4.384 (8.78)	5.313	15.481 (3.21)
R^2	0.5985	0.3126	0.6569
N-K	81	64	77
$e_*'e_*$	4.09310	8.85271	3.49767
q	10	10	6
$e'e$	2.85246	7.99868	2.85246
$F = \frac{N-K}{q}\left(\frac{e'_*e_*}{e'e} - 1\right)$	3.09	0.58	2.90
Df (n_1, n_2)	(10,81)	(10,64)	(6,77)
Critical F	2.55 (α = .01)	2.61 (α = .01)	3.04 (α = 0.01)
d.f. (n_1, n_2)	(10,80)	(10,65)	(6,80)
Conclusion on Null Hypothesis	Rejected H_o	H_o Not Rejected	H_o Marginally Rejected at α = .01 Rejected at α = 0.05

cross-products are allowed). The next step in the analysis consists of performing these tests, in the order of additively restricted, linearly restricted and nonlinearly restricted separabilities.

Table 3, Model (7) indicates the result of additive separability for eastern Oregon. This type of additive separability was also rejected for eastern Oregon. The implication is that there are some interactions between F,I,N,K which are relevant in explaining the variation of yield.

Test of Linear Homogeneity of TRANSLOG in F,I,N and K

Linear homogeneity of the TRANSLOG implies that we have

$$\sum_{i}^{N} \alpha_i = 1.0$$

for the log-linear part of the function and

$$\sum_{i}^{N} \gamma_{ij} = 0 \qquad \text{for all } i = 1,\ldots,N$$

in the log quadratic part. The restriction equation, $R \quad = r$, can be specified in terms of the following:

$$\beta' = [\beta_P \beta_T \beta_F \beta_I \beta_N \beta_K \beta_{F2} \beta_{FI} \beta_{FN} \beta_{FK} \beta_{I2} \beta_{NI} \beta_{KI} \beta_{N2} \beta_{NK} \beta_{K2} C]$$

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & & 0 & \frac{1}{2} & 1 & 0 \end{bmatrix}$$

and $r' = [1 \quad 0 \quad 0 \quad 0 \quad 0]$.

Table 4

Test for Linear Homogeneity

MODEL (8)

$[R(X'X)^{-1}R']^{-1}$

$$= \begin{bmatrix} 147.057 & -1002.9 & -725.723 & -1423.17 & -1526.14 \\ & 7585.85 & 5494.90 & 9849.45 & 10546.6 \\ & & 6948.37 & 7299.51 & 7762.02 \\ \text{symmetrical matrix} & & & 14033.1 & 14856.8 \\ & & & & 15890.2 \end{bmatrix}$$

$$R\beta = \begin{bmatrix} 1.875 \\ -.008 \\ 0.077 \\ -.080 \\ 0.134 \end{bmatrix} \quad \text{and} \quad (r-R\beta) = \begin{bmatrix} 0.185 \\ 0.008 \\ -.077 \\ 0.080 \\ -.134 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F = \frac{(r-R\beta)'\,[R(X'X)^{-1}R']^{-1}(r-R\beta)}{q\cdot S^2} = \frac{210.69}{5} = 42.14$$

d.f. = (5,71) = (q,N-K).

F critical = 3.29 (at d.f. 5,70).
(α = .01)

Conclusion = Reject the null hypothesis $H_0: \beta R = r$ at α = 0.0

In this particular case, the most convenient way to compute the test statistics is to use the formula:

$$\frac{(r-Rb)' \, [R(X'X)^{-1}R']^{-1} \, (r-Rb)}{q \cdot S^2}$$

The T.S.P. package allows this computation. The results for the eastern Oregon model are presented in Table 4, Model (8) below. The matrix $[X'X]^{-1}/S^2$ was first computed using the OLS procedure of the T.S.P. package. This was premultiplied by R' and postmultiplied by R. The resulting matrix was then inverted to obtain

$$\left[\frac{R(X'X)^{-1}R'}{S^2}\right]^{-1}.$$

The coefficient vector b was also retrieved from the OLS procedure in the T.S.P. package. Then (r-Rb) was computed from b and R and r. The linear homogeneity test was performed using the F-statistic.

$$\frac{(r-Rb)' \, [R(X'X)^{-1} \, (r-Rb)}{q \cdot S^2}$$

The null hypothesis of linear homogeneity, or $H_0: R\,\beta = r$, was rejected at the 1% level of significance. The model, it should be noted, already presumes that total yield is a linear homogeneous function of total inputs such as land, capital, labor, fertilizers, etc. The present test of linear homogeneity (of the per unit land model) is a test for an additional restriction on the linear homogeneous technology already presumed for the 'totals' variable model.

Test of Linearly Restricted Weak Separability

Single Partitions

It was indicated in Chapter III, page 66, that hypotheses of linear weak separability can be tested with single and double partitions. There are seven single partitions possible with four variables F,I,N,K. There are three partitions with pairs, [(F,I), (N,K)], [(F,N), (I,K)] and [(F,K), (N,I)]; and four partitions with triplets, [F, (I,N,K)], [(F,I,N),K], [(F,I,K),N] and [(F,K,N),I]. These models can be simply estimated by deleting relevant cross-product terms from the quadratic, TRANSLOG.

For example, the [(F,I), (N,K)] partition can be represented by deleting the FN, FK, KI, NI cross-product terms from the general quadratic expression of TRANSLOG. Single partition separability can be tested using the F-statistic, $\frac{N-K}{q}\left(\frac{e_*'e_*}{e'e}-1\right)$, where $e_*'e_*$ represents the total sum of squares for the model without the relevant cross-product terms and e'e is that for the unrestricted TRANSLOG.

Table 5 and Table 6 below give the results of the restricted models with single pair-wise and single triplet partitions, respectively. The results indicate that amongst the single pair-wise partitions, [(K,I),(F,N)] can be rejected, whereas, [(F,I),(N,K)] and [(K,I),(F,N)] can not be rejected. This implies that one of the four interactions FI, KI, FN, or KN is necessary to explain the variance of yield in the model.

The results amongst single triplet partitions narrows the choice from the four cross-product terms considerably. Table 6 shows that

Table 5

Single Partitions (with pairs) on Eastern Oregon Model

Null Hypothesis	MODEL (13) H_0:[F,I),(N,K)]		MODEL (14) H_0:[(F,N),(K,I)]		MODEL (15) H_0:[(F,K),(N,I)]	
Variables	Coeff.	(t-value)	Coeff.	(t-value)	Coeff.	(t-value)
P	0.163	(2.67)	0.173	(2.91)	0.168	(2.69)
T	0.005	(0.16)	-.007	(-0.23)	-.0006	(-.02)
F	0.171	(1.16)	0.207	(0.66)	-.340	(-.52)
I	0.179	(3.14)	0.688	(3.56)	0.186	(2.09)
N	-1.325	(-1.24)	-.413	(-.65)	-1.297	(-2.06)
K	4.333	(1.66)	1.164	(0.40)	6.600	(1.94)
F^2	-0.015	(-.91)	-0.006	(-.39)	0.001	(0.04)
FI	0.036	(2.69)	-	-	-	-
FN	-	-	0.015	(0.26)	-	-
FK	-	-	-	-	-.106	(-.72)
I^2	0.003	(1.07)	-0.003	(-.94)	.002	(0.47)
NI	-	-	-	-	0.032	(1.64)
KI	-	-	0.130	(3.36)	-	-
N^2	0.033	(0.25)	-0.046	(-.90)	-.140	(-2.20)
NK	-.314	(-.76)	-	-	-	-
K^2	0.562	(1.48)	0.072	(0.26)	0.662	(1.82)
C	11.904	(2.38)	6.861	(1.19)	17.2651	(2.64)
R^2	0.6879		0.7019		0.6711	
$e_*'e_*$	3.18513		3.03936		3.35281	
d.f.	88-13		(88-13)		(88-13)	
F	2.0701		1.1488		3.1135	

Critical F (4,70) at $\alpha = 0.01$, (3.60), and at $\alpha = 0.05$ (2.5)

Conclusion on H_0	Can't Reject H_0	Can't Reject H_0	Can't Reject H_0 at .01 Reject H_0 at α = 0.05

Table 6

Single Partitions with Triplets on Eastern Oregon Model

	MODEL (9)		MODEL (10)		MODEL (11)		MODEL (12)	
Null Hypothesis	[F,(I,N,K)]		[(F,I,N,)K]		[(F,I,K),N]		[(F,K,N),I]	
Variables	Coeff.	(t-Value)	Coeff.	(t-Value)	Coeff.	(t-Value)	Coeff.	(t-Value)
P	0.170	(2.85)	0.166	(2.73)	0.169	(2.85)	0.162	(2.53)
T	-.009	(-.27)	0.006	(0.17)	-0.00004	(-.002)	0.00004	(0.001)
F	0.121	(0.86)	0.042	(0.13)	-0.035	(-.06)	-.437	(-.64)
I	0.836	(3.79)	0.267	(2.87)	0.617	(3.12)	0.057	(1.74)
N	-0.435	(-.39)	-.845	(-1.35)	-0.551	(-.87)	-1.88	(-1.48)
K	0.573	(0.19)	3.904	(1.44)	2.131	(0.60)	8.066	(2.19)
F^2	-.006	(-.35)	-.010	(-.64)	-0.0005	(-.02)	0.005	(0.19)
FI	-	-	0.032	(2.38)	0.019	(1.34)	-	-
FK	-	-	-	-	-0.050	(-.35)	-.191	(-.94)
FN	-	-	-.034	(-.57)	-	-	0.065	(0.64)
I^2	-.003	(-.87)	.00005	(.02)	-.003	(-.79)	0.005	(1.91)
NI	-.041	(-1.33)	.025	(1.30)	-	-	-	-
KI	0.195	(3.13)	-	-	0.101	(2.36)	-	-
N^2	-0.008	(-.06)	-.083	(-1.6)	-.056	(-0.87)	-0.36	(-.20)
NK	-0.040	(-.10)	-	-	-	-	-.327	(-.57)
K^2	0.016	(0.04)	0.376	(1.47)	0.194	(0.51)	0.98	(1.76)
C	5.281	(0.95)	11.89	(2.20)	8.36	(1.22)	19.286	(2.89)
R^2	0.7095		0.6930		0.7090		0.6614	
$e_*'e_*$	2.96114		3.12938		2.96661		3.45151	
d.f.	88-14		88-14		88-14		88-14	
$F_{3,71}$	0.9017		2.2976		0.9470		4.9488	
Conclusion on H_0	Can't reject		Reject		Can't reject		Reject	

[(F,N,K),I] and [(F,I,N)] are rejected. In both of these partitions, the KI cross-product term is deleted. On the other hand, [(I,N,K),F] and [(I,F,K),N] are not rejected indicating that deleting the FN and KN cross-product terms is permissible. Therefore, it appears that the cross-product term KI cannot be deleted. Though KI appears important, FI can also explain the variance of yield to some extent as indicated by non-rejection of [(F,I),(N,K)] partition. This generates some ambiguity as to whether the separation of K and I alone are responsible for the rejection of [(F,N,K),I], [(F,I,N),K], [(F,N,K),I] and [(I,F,N),K].

Thus double partitions were considered necessary to resolve this ambiguity. The ambiguity would be removed completely if partitions of the type [(F,N),K,I], [(F,K),N,I] and [(N,I),F,K] alone were to be rejected. Thus, if [(N,K),F,I] and [(K,I),F,N] were not rejected, this would imply that the FI cross-product term was not necessary to explain the variance in yield. In this case alone the ambiguity can be reduced in the importance of models with KI vis-a-vis those with FI.

Double Partitions

In case of four variables, F,I,N and K, there are altogether six double partitions. These always contain a pair, e.g., [(F,I),N,K], [(N,K),F,I], [(F,N),K,I], [(K,I),F,N], [(F,K),N,I] and [(N,I),F,K]. These models are more restrictive than single partitions, since more cross-product terms are deleted. The estimation procedure, however, remains unchange , and the F-statistics can be computed exactly as before.

Table 7

Double Partitions

Null Hypothesis	MODEL (16) H_o:[(F,I),N,K)		MODEL (17) H_o:[(N,K),F,I)]		MODEL (18) H_o:[(F,N)K,I)]	
Variables	Coeff.	(t-value)	Coeff.	(t-value)	Coeff.	(t-value)
P	0.159	(2.62)	0.163	(2.57)	0.162	(2.55)
T	0.007	(0.20)	-0.003	(-.08)	-0.002	(-.06)
F	0.202	(1.43)	0.088	(0.59)	0.054	(0.16)
I	0.167	(3.06)	0.048	(1.56)	0.045	(1.54)
N	-0.591	(-1.31)	-1.346	(-1.21)	-1.09-	(-1.70)
K	3.387	(1.49)	5.865	(2.22)	5.670	(2.06)
F^2	-0.011	(-0.70)	-0.015	(-.90)	-0.014	(-.84)
FI	0.035	(2.61)	-	-	-	-
FN	-	-	-	-	-0.010	(-.17)
FK	-	-	-	-	-	-
I^2	0.002	(0.92)	0.005	(1.80)	0.005	(1.79)
NI	-	-	-	-	-	-
KI	-	-	-	-	-	-
N^2	-0.602	(-1.32)	-0.060	(-.45)	-0.104	(-2.02)
NK	-	-	-0.139	(-.33)	-	-
K^2	0.325	(1.51)	0.617	(1.56)	0.533	(2.04)
C	11.297	(2.30)	15.839	(3.18)	15.890	(2.92)
R^2	0.6852		0.6574		0.6570	
e'e	3.20947		3.49281		3.49639	
d.f.	88-12		88-12		88-12	
F(d.f. = 5,71)	1.7653		3.1747		3.1925	

Critical F (α = 0.01) = 3.25 F (α = 0.05) = 2.33 at d.f. (5,70)

Conclusion	Can't Reject H_o	Reject H_o	Reject H_o

Table 7 (Cont.)

Double Partitions

Null Hypothesis	MODEL (19) H_0:[(K,I)F,N)		MODEL (20) H_0:[(F,K),N,I)		MODEL (21) H_0:[(N,I),F,K)	
Variables	Coeffi.	(t-value)	Coeff.	(t-value)	Coeff.	(t-value)
P	0.173	(2.93)	0.160	(2.54)	0.169	(2.72)
T	-.007	(-.22)	0.0005	(.01)	-.003	(-.09)
F	0.132	(0.99)	-0.375	(-.56)	0.121	(0.86)
I	0.682	(3.57)	0.048	(1.64)	0.183	(2.07)
N	-0.533	(-1.22)	-1.35	(-2.13)	-0.972	(-2.24)
K	1.595	(0.67)	7.30	(2.14)	4.771	(2.14)
F^2	-0.006	(-.41)	-.0005	(-.02)	-0.012	(-0.72)
FI	-	-	-	-	-	-
FN	-	-	-	-	-	-
FK	-	-	-0.111	(-0.74)	-	-
I^2	-0.003	(-.95)	0.005	(1.84)	0.001	(0.43)
NI	-	-	-	-	0.032	(1.67)
KI	0.128	(3.37)	-	-	-	-
N^2	0.053	(1.20)	-.134	(-2.09	-0.107	(-2.43)
NK	-	-	-	-	-	-
K^2	0.113	(0.49)	0.728	(2.00)	0.450	(2.12)
C	7.550	(1.48)	18.727	(2.86)	14.1328	(2.92)
R^2	0.7016		0.6593		0.6689	
e'e	3.04214		3.47284		3.37578	
d.f.	88-12		88-12		88-12	
F(5,71)	0.9329		3.0754		2.5926	

Critical F ($\alpha = 0.01$) = 3.25; F ($\alpha = 0.05$) = 2.33 at d.f. (5,70)

Conclusion	Can't Reject H_0	Reject H_0	Reject H_0 at $\alpha = 0.05$

The results are indicated in Table 7 below. These again support the results obtained with single partitions. The double partitions, [(N,K),F,I], [(F,N),K,I], [(F,K),N,I] and [(N,I),F,K] are again strongly rejected. However, [(K,I),F,N] cannot be rejected while [(F,I),N,K] is marginal. Thus the ambiguity between FI and KI cross-products is not entirely resolved. But because F-statistic for Model (19) with KI is much smaller than that for Model (16) with FI, Model (19) with Ki appears to be the preferred model.

It may, however, be noted that the significance of the FI term, given that the model already contains the KI term, is rather low. This type of significance can be tested by considering the models with the following partitions, [(I,F,F),N] and [(K,I),F,N]. Here the model with the partition [(I,F,K),N] is regarded as the unconstrained model and the [(K,I),F,N] partition represents the restricted model. The F-statistic was computed, using $F = \frac{N-K}{q}\left(\frac{e_*'e_*}{e'e}-1\right)$, to be 0.95 with (2,74) degrees of freedom. The critical value of $F_{2,74}$ is 4·90 at α0.01 and 3·12 at $\alpha = 0.05$ (Table 8). Consequently, the cross-product terms FI and FN can be deleted from the model with partition [(I,F,K),N], assuming this partition is correct.

The significance of the KI term given that the model already includes the FI term is, however, not very strong. It can be seen that with the partition [(F,I),K,N] the F-statistic is 3·03. At $\alpha = 0.05$, this result is marginal, and rejection may be assumed. The results still involve the ambiguity between the models with KI and FI terms. It may also be noted the test of significance of FI given KI in the model

Table 8

Conditional Double Partitions

Terms	MODEL (21) Maintained Hypothesis $H_o:[(F,I,K),N]$	MODEL (22) Model Hypothesis $H_o:[(K,I),F,N]$	MODEL (23) Model Hypothesis $H_o:[(F,I),K,N]$
$e'e$	2.96661	3.04214	3.20943
R^2	0.7090	0.7016	0.6852
K	14	12	12
N	88	88	88
$F = \frac{N-K}{q} \frac{e_*' e_*}{e' e} - 1$	-	0.95	3.03
Critical $F_{2,74}$ at $\alpha = 0.01$	-	4.90	4.90
Critical $F_{2,74}$ at $\alpha = 0.05$	-	3.12	3.12
Conclusion on H_o	-	Can't Reject at $\alpha = 0.01$ or $\alpha = 0.05$	Can't Reject at $\alpha = 0.01$ Marginal Rejection at $\alpha = 0.05$

is very low. A similar test of KI given FI in the model indicates that KI is significant. Thus, even though the results are still ambiguous, inseparability between K and I appears to be more likely than that between F and I. The choice of separability between F and I versus K and I must be resolved in some other way, i.e., through the imposition of further restrictions on the model. This involves nonlinear restrictions and Sadan type models.

Mitigation of Multicollinearity

The Cobb-Douglas structure for western Oregon results in negative values of the coefficients for T, I and N. It may, however, be noted that all the coefficients except that of fertilizer are insignificant at $\alpha = 0.01$. This is caused by the strong multicollinearity between K and N, K^2, N^2, etc. The inflationary effect on the variance of the estimated coefficients due to multicollinearity has resulted in imprecision in the estimated coefficients, and possibly in improper signs of some of the coefficients.

The Census data for 1954-1974 period indicates that the K/N ratio has remained relatively stable over this period (See Graph 1 in Appendix B), implying that labor services tend to be proportional to capital services for aggregate county-level data.

In the case of the eastern Oregon linearly restricted and unrestricted TRANSLOG functions, the precipitation and irrigation variables P and I are almost always significant. The restrictions have little effect on the sign and the magnitude of the coefficient of P. The sign and magnitude of the coefficient of I is not so stable, however, but the sign and magnitude remain more or less the same when the restriction includes either the KI or the FI cross-product term in the model. The sign and magnitude of the coefficient of irrigation service, I, remains areound 0·20 in various single and multiple partition models where FI occurs without KI. The coefficient is around 0·60 in models where KI occurs without FI.

The double partition model [(K,I),F,N], has only three significant variables, P,I, and KI. Again there exists multicollinearity amongst the N,K,N^2 and K^2 terms which magnify the variance of the estimated coefficients. There are various alternative methods that may be employed to resolve this problem as indicated above in Chapter II, page 36.

a) Models with Fixed K/N Ratio

Amongst the various alternatives available, such as the ridge estimators, principal component, and mixed estimate approach, the latter appears to be suitable here. The ridge estimators are disadvantageous in the present situation because of the diverse values of the coefficients. The principal component approach, on the other hand, is not attractive because the estimated components are difficult to interpret (although intuitively, the separable parts of the production function should correspond with the major components, one cannot impose the separability a priori; consequently, this approach is not feasible).

The relative stability of the K/N ratio therefore indicates the mixed estimators as a natural choice in mitigating problems of multicollinearity. The implication, in practical terms, is that one of the

K or N terms can be deleted from the estimation procedure. The interpretation of the value of the coefficient of the remaining variable, however, is changed. The coefficient now represents the effects of both the variables K and N. For example, if we assume that K/N ratio is fixed, we can delete N from the regression equation. The coefficients of all K, KI and K^2 terms then represent the joint effects of K and N, and their individual effects cannot be separated.

It was also noted that the temperature index, T, was not significant for all of the regressions. The value of the coefficient was often close to zero. These facts were considered in the following regression runs shown in Table 9. Labor service, N, has been deleted in favor of capital service, K. Though all the previous runs with single and double partitions could have been duplicated, this was felt to be unnecessary since the results of the four models presented here are not much different from the earlier ones.

Model (24) with the assumption of fixed K/N ratio, has an F-statistic of 0·6518 when the full model (2) is constrained with fixed K/N ratio and deleting T. This indicates that the fixed K/N ratio model cannot be rejected at $\alpha = 0{\cdot}01$. The main effect of deleting T and N appears to be a slight increase in the precision of the remaining coefficients. The coefficient of capital service, K, is still not significant at $\alpha = 0{\cdot}05$, however. The conclusions of the previous runs with both T and N included, does not change even after deletion of T and N. Table 9 also indicates that the ambiguity between models with FI and KI cross-product terms still persists.

Table 9

Models without T and with Fixed K/N Ratio Represented by K

Variables	MODEL (24) H_0:"/(F,I,K) Coeff.	(t-value)	MODEL (25) H_0:P/[(F,I),K] Coeff.	(t-value)	MODEL (26) H_0:P/[(F,K),I] Coeff.	(t-value)	MODEL (27) H_0:P/[(K,I),F Coeff.	(t-value)
P	0.158	(2.87)	0.136	(2.47)	0.129	(2.15)	0.148	(2.75)
F	0.384	(0.39)	0.301	(2.63)	0.718	(1.71)	0.205	(1.84)
I	0.664	(3.55)	0.187	(3.78)	0.045	(1.54)	0.760	(4.27)
K	-.733	(-.60)	0.880	(0.84)	0.474	(0.40)	-.991	(-.88)
F^2	-.006	(-.30)	-.001	(-.07)	-.015	(-.68)	0.003	(0.21)
I^2	-.003	(-.97)	0.002	(0.97)	0.005	(2.04)	-.003	(-1.07)
K^2	-.111	(-.80)	0.091	(0.88)	0.003	(0.03)	-.134	(-1.16)
FI	0.021	(1.52)	0.040	(3.31)	-	-	-	-
FK	0.038	(0.39)	-	-	0.117	(1.12)	-	-
KI	0.110	(2.69)	-	-	-	-	0.143	(4.03)
C	3.042	(1.05)	6.355	(2.35)	6.145	(2.13)	2.29	(0.81)
R^2	0.7060		0.6775		0.6386		0.6955	
e'e	2.99734		3.28729		3.68462		3.10418	
$F_{n1},n_2)$	0.6518		1.4672		2.8079		0.8494	
(n_1,n_2)	(6,77)		(8,77)		(8,77)		(8,77)	
Critical-F (α = .01)	3.04		2.74		2.74		2.74	
(n_1,n_2)	(6,80)		(8,80)		(8,80)		(8,80)	
Conclusion at α level 0.01	Can't Reject H_0		Can't Reject H_0		Reject H_0		Can't Reject H_0	

The results are not dramatically different when we delete T and N variables, probably because K^2 is still highly correlated with K,F^2 with F and I^2 with I. The results could improve if the square terms F^2, K^2 and K^2 are deleted. The deletion of F^2 only appears to be permissible because its coefficient is small and insignificant at $\alpha = 0{\cdot}01$ in all equations of models (24), (25), (26) and (27).

b) Models with only KN Term

Yet another approach to reduce multicollinearity problems is as follows. The variables K and N, and their squares, were eliminated and only the cross-product term KN preserved. This approach emphasizes the possible nonlinear nature of the effect of K and N inputs upon yield. The multicollinearity is smaller between the rest of the variables as indicated by the correlation coefficients. The results are indicated in Table 10 below.

The models tested are again limited to those with FI and KI variables. Results similar to the previous ones are obtained. Model (30) with KI does better than Model (29) with FI, as before. The variable T is insignificant, so could have been eliminated without changing the results. The coefficients for other variables P, F, I, are significant in all three models. But the values of the coefficients of I are rather unstable, though the coefficients for P and F are stable. This type of pattern was also exhibited in the constant $\frac{K}{N}$ ratio model discussed previously.

The coefficient for NK is small but negative and significant at $\alpha = 0{\cdot}05$ in Model (30) with KI term. It is positive and not significant

Table 10

Models with Cross-product Term KN

Null Hypotheses	MODEL (28) $H_o: \beta_{FI}, \beta_{KI} \neq 0$		MODEL (29) $H_o: \beta_{FI} \neq 0$		MODEL (30) $H_o: \beta_{KI} \neq 0$	
Variables	Coeff.	(t-values)	Coeff.	(t-values)	Coeff.	(t-values)
P	0.159	(2.94)	0.130	(2.37)	0.161	(2.90)
T	-.006	(-.18)	0.019	(0.59)	-.022	(-.69)
F	.299	(2.78)	0.280	(2.52)	0.299	(2.70)
I	.545	(3.67)	0.190	(3.82)	0.572	(3.76)
KN	-.010	(-1.47)	0.001	(0.26)	-.015	(-2.13)
F^2	.002	(0.17)	-0.002	(-.16)	0.009	(0.66)
I^2	-.002	(-.60)	0.003	(1.17)	-0.001	(-.45)
FI	.030	(2.35)	0.041	(3.34)	-	-
KI	.080	(2.53)	-		0.105	(3.48)
C	4.409	(15.22)	4.088	(15.19)	4.509	(15.30)
R^2	0.6999		0.6753		0.6787	
e'e	3.05933		3.30991		3.27552	
F	0.7356		1.4233		1.3163	
d.f.(n_1,n_2)	7,71		8,71		8,71	
Critical F ($\alpha = .01$)	(2.91) $\alpha = 0.05$ (2.14)		$\alpha = 0.05$ (2.07)		$\alpha = 0.01$ (2.77)	
d.f. (n_1,n_2)	(7,70)		(8,70)		(8,70)	

at α = .05 in Model (29) with the FI term included, and negative and marginally significant at α = 0.05 in models with both FI and KI. The coefficient for I^2 is generally not significant in all three models, the value is small, negative in Model (30) with KI and positive in Model (29) with FI.

Model (28) with both FI and KI term has the lowest F-statistic and the highest R^2. Coefficients of both FI and KI terms are positive and significant at α = 0.01. But T, F^2, I^2 and NK have coefficients that are insignificant at α = 0.01 level. The coefficients T, I^2, NK are negative. The next best model appears to be the Model (30) with KI. The coefficients of T, F^2, I^2 terms are not significant at α = 0.05. The parameters are nearly equal in Model (28) with only KI term. This indicates that models with only the KI term are more acceptable than those containing FI.

c) Models with N or N^2 and K or K^2 Only

Yet other models were tested using combinations of N, K, N^2 and K^2 terms. The deletion of N^2 and K^2 was expected to reduce multicollinearity to some extent. Again the tests were limited to the two competing models, one with the FI term and the other with the KI term. The results indicate that models with KI do consistently better than the models with FI. The coefficients of T, F^2, K^2, N^2 or N are insignificant at α = 0.01 in all six models. The models with KI have significant coefficients for K or K^2 but not for N^2 and N, at α = 0.01.

Amongst models (32), (35) and (36), the model with the smallest sum of squared errors is Model (35) (with K^2 instead of K), and here

Table 11

Models with Single Terms with N and K-variables

	MODEL (31)		MODEL (32)		MODEL (33)	
	$H_o:\beta_B2=\beta_{KI}=\beta_{KI}=0$ $\beta_{FI}\neq 0.$		$H_o:\beta_N2=\beta_K2=\beta_{FI}=0$ $\beta_{KI}\neq 0$		$H_o:\beta_N2=\beta_K=\beta_{KI}=0$ $\beta_{FI}\neq 0$	
Variables	Coeff.	(t-values)	Coeff.	(t-values)	Coeff.	(t-values)
P	0.134	(2.35)	0.144	(2.58)	0.135	(2.38)
T	0.163	(.50)	-0.011	(-0.35)	0.015	(0.46)
F	0.296	(2.39)	0.205	(1.71)	0.301	(2.44)
I	0.193	(3.78)	0.672	(4.22)	0.194	(3.81)
N or N^2	0.011	(0.17)	-0.029	(-0.48)	0.014	(0.24)
K or K^2	-0.051	(-0.35)	0.364	(2.33)	0.006*	(0.44)
F^2	-0.001	(-0.08)	0.002	(0.13)	-0.001	(-0.06)
I^2	0.003	(1.12)	-0.003	(-0.87)	0.003	(1.10)
FI	0.041	(3.30)	-	-	0.042	(3.33)
KI	-	(-)	0.126	(3.95)	-	-
C	3.956	(8.43)	5.642	(9.73)	4.086	(13.27)
R^2	0.6757		0.6920		0.6760	
e'_*e_*	3.30595		3.13975		3.30298	
$F_{n1,n2}$	1.6125		1.0216		1.6020	
(n_1,n_2)	(7,71)		(7,71)		(7,71)	
Critical F (α = .01) (n_1,n_2)						
Conclusion	Can't Reject H_o		Can't Reject H_o		Can't Reject H_o	

*means K^2 instead of K.

Table 11 (Cont.)

	MODEL (34)		MODEL (35)		MODEL (36)	
	$H_o:\beta_{K2}=\beta_N=\beta_{KI}=0$		$H_o:\beta_K=\beta_{N2}=\beta_{FI}=0$		$H_o:\beta_N=\beta_{K2}=\beta_{FI}=0$	
	$\beta_{FI} \neq 0$		$\beta_{KI} \neq 0$		$\beta_{KI} \neq 0$	
Variables	Coeff.	(t-values)	Coeff.	(t-values)	Coeff.	(t-values)
P	0.134	(2.35)	0.145	(2.62)	0.144	(2.56)
T	0.017	(0.51)	-0.009	(-0.29)	-0.014	(-0.43)
F	0.294	(2.43)	0.200	(1.67)	0.221	(1.88)
I	0.193	(3.79)	0.705	(4.30)	0.670	(4.20)
N or N^2	-0.001	(-0.18)*	-0.028	(-0.48)	0.0004*	(0.07)
K or K^2	-0.049	(-0.37)	-0.038	(-2.44)*	0.318	(2.18)
F^2	-0.001	(-0.09)	0.002	(0.12)	0.003	(0.21)
I^2	0.003	(1.13)	-0.003	(-0.96)	-0.003	(-0.88)
FI	0.041	(3.31)	-	-	-	-
KI	-	-	0.132	(4.04)	0.125	(3.93)
C	3.933	(7.43)	4.750	(13.80)	5.580	(9.05)
R^2	0.6757		0.6940		0.6911	
$e'_* e_*$	3.30586		3.11923		3.14886	
$F_{n1,n2}$	1.6122		0.9486		1.0539	
(n_1,n_2)	(7,71)		(7,71)		(7,71)	
Critical F (α=.01) (n_1,n_2)						
Conclusion	Can't Reject H_o		Can't Reject H_o		Can't Reject H_o	

*represents N^2 or K^2 as the case may be.

the coefficient of K^2 is negative and significant at $\alpha = 0.05$. However, models (2) and (6) (with K and N) have positive and significant (at $\alpha = 0.05$) coefficients for K. The latter two models may thus be preferred to Model (35). In all six models, coefficients of P, F, I and the constant C are positive and significant at $\alpha = 0.01$.

Nonlinear Computational Procedure

TRANSLOG models with nonlinear restrictions were estimated using the Time Series Processor (TSP) package. The nonlinear least squares estimation procedure used in TSP, or any other package, must utilize some kind of numerical iterative procedure. There are many types of numerical methods available which can be used to minimize the sum of squares (or some such objective criterion) to determine the nonlinear least squares estimates. What is required is that the functional form be known explicitly.

The diagram below indicates the procedure. Starting with an initial parameter vector β_0, a step length, t, is chosen. The next value of the parameter vector is given according to the vector equation, $\beta_{n+1} = \beta_n + t\,\xi_n$; where β_n is the n^{th} approximation of the true parameter β and ξ_n is the step direction vector chosen at the n^{th} iterative step. Unless a termination criteria is fulfilled at the n^{th} step, a new β_{n+1} is generated and a new β_{n+2} computed, and so on.

In the TSP the gradient approach is taken to decide the new step direction, whereas the step length is determined on the basis of a specified tolerance limit. The new β-vector must not exceed the tolerance limit plus or minus the previous β-vector, for convergence to be achieved; and at the same time, the sum of the squared errors must diminish (or stay the same).

Let us define the sum of the squared error terms as $S(\beta) = e'e = [Y - f(\beta,X)]' [Y - f(\beta,X)]$; then an appropriate step length, t, and direction, ξ, is such that:

$$S(\beta_n + t\xi) < S(\beta_n).$$

The direction vector ξ is such that $S(\beta_n + t\xi)$ is a decreasing function of t, where t is a small scaler. Thus we must have, $\left.\frac{dS}{dt}\right|_{t=0} < 0$, i.e.,

$$\left.\frac{dS(\beta_n + t\xi)}{dt}\right|_{t=0} = \left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right] \cdot \frac{d(\beta_n + t\xi)}{dt} = \left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right] \cdot \xi$$

is negative. Thus the gradient $\left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right]$ of the sum of the squared errors, S, may be utilized to define the new step direction vector, ξ, as follows:

$$\xi = -P_n\left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right]$$

where P_n is any positive definite matrix such that

$$\left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right]' P_n \left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right] > 0.$$

This assures us as required that,

$$\frac{dS}{dt} = \left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right]\xi = \left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right]' P_n \left[\left.\frac{\partial S}{\partial \beta}\right|_{\beta_n}\right] < 0,$$

Now the question arises as to the choice of P_n.

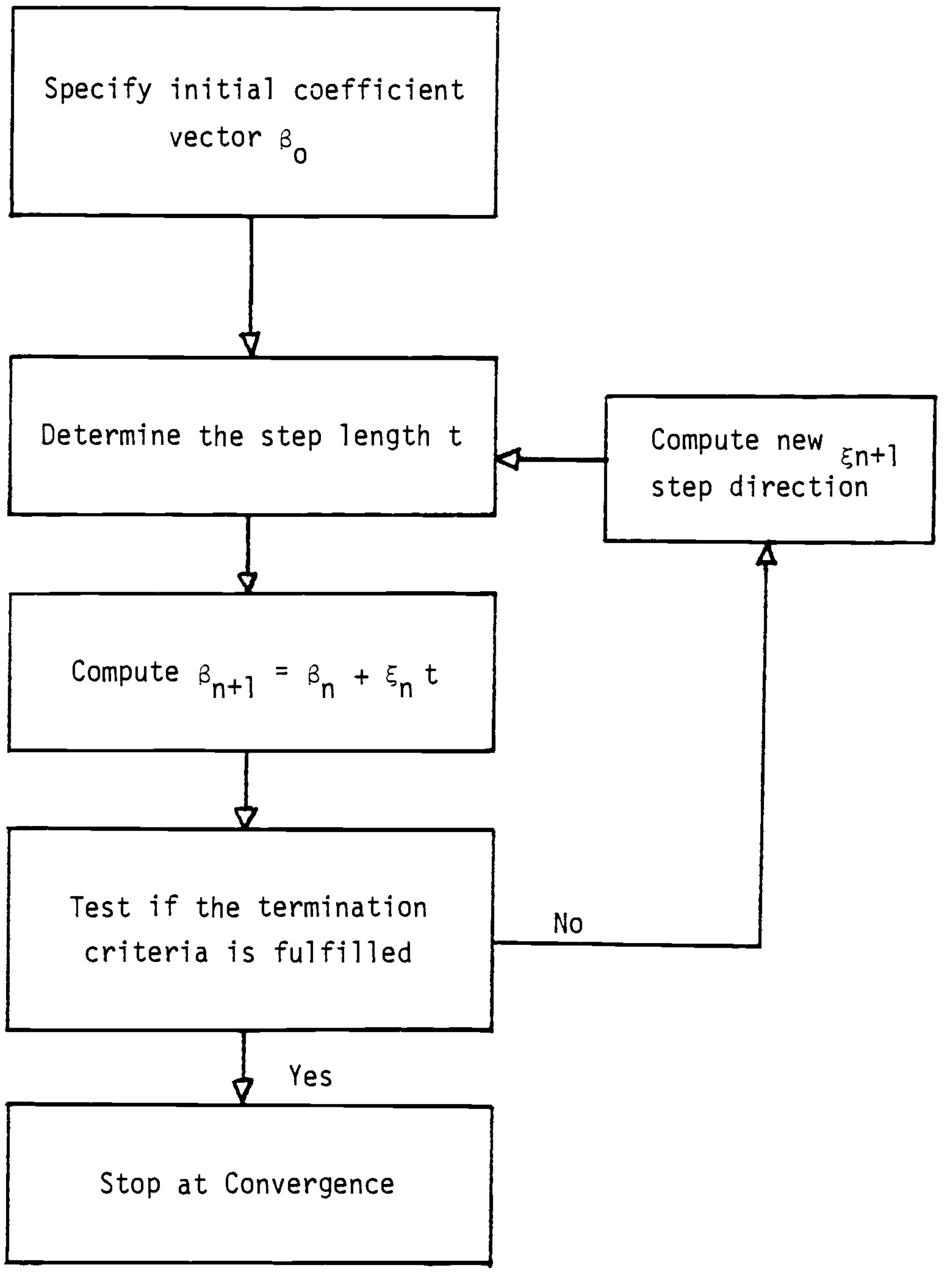


Figure 11: Iterative Nonlinear Least Squares Algorithms

The Gauss-Newton method involves approximating a positive definite P_n with the Hessian of S. Since, $S(\beta) = (e'e) = (y-f(\beta,X))'(y-f(\beta,X))$ the Hessian of $S(\beta)$ is given as,

$$H(S(\beta)) = H(\beta) = Z(\beta)'Z(\beta) - \Sigma\, (y - f(\beta,X\ ) \left[\frac{\partial^2 f}{\partial\beta\partial\beta'}\right]$$

where, $Z(\beta) = \dfrac{\partial f(\beta, X_t)}{\partial\beta}$. Assuming that we are near the true value, the second term on the right [$(y_t - f(\beta, X_t)$ is close to zero] can be ignored. Thus the choice of the positive definite matrix P_n is provided by,

$$P_n \simeq [Z(\beta)'\ Z(\beta)]^{-1}.$$

Therefore,

$$\beta_{n+1} = \beta_n + t\ \ \xi = \beta_n + t\ (-P_n)\left[\frac{\partial S}{\partial\beta}\Big|_{\beta_n}\right] .$$

And for a unit step length, t = 1, we get,

$$\beta_{n+1} = \beta_n + [Z(\beta_n)'\ Z(\beta_n)]^{-1}\left[\frac{\partial S}{\partial\beta}\Big|_{\beta_n}\right]$$

or,

$$\beta_{n+1} = \beta_n + [Z(\beta_n)'Z(\beta_n)]^{-1} \cdot Z(\beta_n)'\ [y-f(\beta_n,X)]$$

or,

$$\beta_{n+1} = [Z(\beta_n)\ Z(\beta_n)]^{-1} \cdot Z(\beta_n)'\ [y-f(\beta_n,X) + Z(\beta_n)\beta_n]$$

(The last expression simplifies to the last but one, after some multiplication and expansion).

Thus β_{n+1} may be thought of as being the OLS estimator of the model, $\check{y} = Z(\beta_n)\cdot\beta + \varepsilon$, where, $\check{y} = y-f(\beta_n,X) + Z(\beta_n)\cdot\beta$. This can be recognized as the linear pseudo model at β_n, approximating the non-linear model, $y=f(\beta,X)+\varepsilon$, with β_n (instead of the true parameter β). The Gauss-Newton algorithm can be interpreted as a sequence of OLS estimations. In each step of computing β_{n+1}, one computes the least squares estimator for a linear approximation (based on the previous

β_n, of course) of the true non-linear model [Judge et al (1980), pp. 735].

The $Z'(\beta)Z(\beta)$ matrix should be non-singular in general, so that, the new β can be computed. This, however, may not always be true; $Z'(\beta)Z(\beta)$ can be singular and therefore not positive-definite. In such a situation, convergence may be rather slow and this should be considered in estimating large nonlinear models.

Model-A Nonlinear Restrictions

It has already been indicated the nonlinear weak separability restriction allows one to write the TRANSLOG function as a contracted polynomial, i.e., one can express the quadratic part of the TRANSLOG as a quadratic in linear functions (for each separable group). The resulting contracted quadratic has the following form,

$$y = \alpha_0 + \alpha_1(\tilde{\theta}'\tilde{X}) + \gamma_{11}(\tilde{\theta}'\tilde{X})^2 + \alpha_N(\hat{\theta}'\hat{X}) + \gamma_{NN}(\hat{\theta}'\hat{X})^2 + \gamma_{IN}(\tilde{\theta}'\tilde{X}) \cdot (\hat{\theta}'\hat{X}).$$

The $\alpha_1\tilde{\theta}$ and $\alpha_N\hat{\theta}$ are linear coefficient vectors, which also generate the quadratic terms $(\tilde{\theta}'\tilde{X})^2$, $(\hat{\theta}'\hat{X})^2$ and $(\tilde{\theta}'\tilde{X}) \cdot (\hat{\theta}'\hat{X})$. Substituting $\alpha_1\tilde{\theta}'\tilde{X} = f(X)$, $\alpha_N\hat{\theta}'\hat{X} = g(X)$, we have,

$$y = \alpha_0 + f(\tilde{X}) + g(\hat{X}) + a f^2(\tilde{X}) + b f(\tilde{X}) \cdot g(\hat{X}) + c g^2(\hat{X})$$

which indicates a contracted quadratic in f and g or the ϕ-function, under the non-linear restrictions of weak separability.

Within Model-A, F,I,N and K can possibly generate as many nonlinear separability partitions as was enumerated with linear separability partitions. The iterative estimations required in nonlinearly restricted TRANSLOG however indicates that an exhaustive analysis analogous to the linear restrictions would not be appropriate. Thus to simplify the computational complexities and costs, it was necessary to reduce the number of variables as well as the number of restrictions to be evaluated. The tolerance for convergence was maintained at 0.005.

The temperature index, T, was deleted since it was not significant at all in all previous cases. The number of variables was also reduced in the F, I, N, K group by assuming that (K/N) ratio was fixed and so (N,K) could well be represented by K. Thus the model chosen, on which to test the nonlinear restrictions on, was TRANSLOG with estimated form:

$$y = \beta_0 + \beta_p P + \beta_F F + \beta_I I + \beta_K K + \beta_{FF} \cdot F^2 + \beta_{II} \cdot I^2 + \beta_{KK} \cdot K^2 + \beta_{FI} F \cdot I + \beta_{FK} F \cdot K + \beta_{KI} \cdot KI$$

Table 12 indicates the results. Model (37) was estimated using the expression,

$$y = a + a_P \cdot P + a_F \cdot F + a_I \cdot I + b\,(a_F \cdot F + a_I \cdot I)^2 + c\,K^2 + d\,(a_F F + a_I I)\,K$$

and models (38) and (39) were also estimated using similar expressions. Model (37) can be rejected because assymptotic F-statistic of 7.7812 implies the rejection of H_0: [(F,I),K] at $\alpha = 0.01$. The result indicates that P and F coefficients are significant at $\alpha = 0.01$. The coefficients of K,I, $(a_F F+a_I I)^2$ and $(a_F F+A_I I)K$ are not significant at $\alpha = 0.05$.

Model (38) has P,I and F coefficients that are significant at $\alpha = 0.05$, $= 0.01$ and $= 9.10$, respectively. The coefficents of $(a_F F+a_K K)^2$, I^2 and $(a_F F+a_K K)$ are insignificant at $\alpha = 0.05$.

Table 12

Non-linear Weak Separability Restrictions

	TRANSLOG with K/N ratio fixed and T deleted			Cobb-Douglas Process Functions					
	MODEL (24)			MODEL (37)		MODEL (38)		MODEL (39)	
Null Hypothesis	H_o:P/[F,I,K]			H_o:[(F,I),K]/P		H_o:[(F,K)I]/P		H_o:[(K,I),F]/P	
Variables	Coeff.	(t-values)	Terms	Coeff.	(t-values)	Coeff.	(t-value)	Coeff.	(t-value)
P	0.158	(2.87)	P	0.12	(2.00)	0.12	(2.17)	0.12	(2.51)
F	0.384	(0.39)	F	0.88	(2.10)	0.24	(1.64)	0.50	(1.98)
I	0.664	(3.55)	I	-.002	(-.03)	0.24	(2.70)	0.20	(3.74)
K	-.733	(-.60)	K	1.03	(0.90)	0.07	(0.75)	0.39	(0.92)
F^2	-.006	(-.30)	$(a_F F+a_I I)^2$	-.02	(-0.90)	-		-	
I^2	-.003	(-.90)	$(a_F F+a_I I)K$	0.17	(3.48)	-		-	
K^2	-.111	(-.80)	K^2	0.05	(0.36)	-		-	
FI	0.021	(1.50)	$(a_F F+a_K K)^2$	-		-.06	(-0.22)	-	
FK	0.038	(0.40)	$(a_F F+a_K K)I$	-		0.16	(1.55)	-	
KI	0.110	(2.69)	I^2	-		0.002	(0.72)	-	
			$(a_I I+a_K K)^2$	-		-		0.06	(1.37)
C	3.042	(1.05)	$(a_I I+a_K K)F$	-		-		0.16	(2.38)
			F^2	-		-		0.01	(-.70)
			C	7.82	(2.8)	4.54	(8.53)	5.76	(3.62)
R^2	0.7060			0.6169		0.6753		0.6787	
e'e	2.99734			3.90603		3.30971		3.27528	
F_{n_1,n_2}				7.7812		2.6749		2.3800	
(n_1,n_2)				(3,77)		(3,77)		(3,77)	
Critical-F	α = 0.01			4.04		4.04		4.04	
(n_1,n_2)				(3,80)		(3,80)		(3,80)	
				Reject H_o		Can't Reject H_o		Can't Reject H_o	

Using the F-statistics of 2.67 one cannot reject the H_0:[(F,K),I) at $\alpha = 0.01$. Model (39) on the other hand has P, F and I coefficients significant at $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.01$, respectively. However, the coefficients of $(a_I I + a_K K)^2$ and F^2 are not significant even at $\alpha = 0.05$; the coefficient of $(a_I I + a_K K)F$ is significant at $\alpha = 0.05$. The coefficient of K, though positive, is not significant at $\alpha = 0.05$. Using the assymptotic F-statistic of 2.38, the null hypothesis H_0 = [(K,I),F] cannot be rejected.

In conclusion, there appears to be some ambiguity amongst weak nonlinear separability between (K,I) and F and that between (K,F) and I. There is, indeed, very little difference between these two models from the standpoint of R^2 or e'e. But the [(K,I),F] model does marginally better and the coefficient of the cross-product term $(a_I I + a_K K)F$ is significant at $\alpha = 0.01$.

Quadratic, Cubic and Quartic Approximation to Sadan Model

The results from linear and nonlinear weak separability restrictions indicate that a model with the KI term provides the best explanation of the data. Though this conclusion appears reasonable, there is still some ambiguity. In the case of the linear restriction models, those with FI terms are strong contenders. In the case of the nonlinear restriction models, the model with the FK term is the closest contender. This ambiguity can be resolved either by utilizing more data or by imposing more restrictions on the production function.

The latter approach is methodologically more appropriate because it can deal with ambiguous situations that may persist in spite of the data set extensions. Sadan's thesis of perfect process complementarity provides us with a set of further restrictions.

As discussed in Chapter III, though the Sadan Model is discontinuous in its parts f and g, it may be approximated through the use of upside down parabolic cylinders or simple saddle surfaces in the y-f-g space. When f and g are linear in variables, the parabolic cylinder, $y = a + f + g + \eta\ (f-g)^2$, becomes a quadratic in F, I and K.

When f and g are themselves quadratic because of KI or FI terms, the parabolic cylinder $y = a + f + g + \eta\ (f-g)^2$ becomes a quartic in F, I and K. When the exponent of (f-g) is 1.5, the expression for y in terms F, I and K becomes a cubic. Alternatively, the cubic of simplest form in F, I and K can also be equivalent to a saddle surface $y = a + f + g + \eta fg$. Though it appears that the dimension of the production function has increased (to a cubic or quartic from quadratic), the functional form is more constrained due to the specific parabolic or saddle surface specified in the y-f-g space.

In the case of the inverted parabolic surface $y = a + f + g + \eta(f-g)^2$, the coefficient η should be negative to support Sadan's thesis. In the case of the saddle surface, $y = a + f + g + \eta fg$, the η coefficient should be positive and small. Both of these surfaces implicitly imply that there is an optimum process-mix such that yield y is maximized along the 45° line in f-g plane. Though the 45° line optimum focus is not necessary, it makes it convenient to compare and interpret the results.

Quadratic Approximations to the Sadan Model

The quadratic approximations to the Sadan Model is tested using linear separable parts in F, I and K. The (K/N) ratio is assumed constant as before. Models (40) and (41) represent models with separability [(K,I),F] and [(F,K),I), respectively. These two models are chosen because the test of nonlinear separabilities were not rejected in these cases. For [(F,I),K] and [(K,I),F], linear weak separability was not rejected, and so Model (42) tests for separable parts ($a_K K$) and ($a_F F + a_I I$) in a Sadan model. However, as in Models (40) and (41), the optimal process-mix is constrained along a 45° line in the f-g plane.

Assuming that the general unconstrained Model (40) in Table 13 is correct, the assymptotic F-statistics were computed for these cylindrical surfaces in y-f-g space, using the formula,

$$y = a + f + g + \eta\ (f-g)^2.$$

The results of Table 13 indicate that Sadan type perfect complementary process functions which are linear in F,I or K are incompatible with the data. The rejection can occur because of (i) the linear process-functions imply strong separability between the grouped inputs, and no such strong separability actually exists, or (ii) because the process-complementarity is not as high as is required by the Sadan model. Because of this, the present results cannot be taken as a rejection of the Sadan type model. Actually, since strong/pairwise separabilities have been rejected before, the present result is likely to be caused by assuming linear process-functions, f and g.

The significant parameter in Model (40), (41) and (42) in Table 13

Table 13

Inverted Cylinder, $y = a + f + g + \eta\,(f-g)^2$.
(Quadratic Approximation to Sadan Model)

Hypothesis H_o	MODEL (40) $f = a_K \cdot K + a_I \cdot I$, $g = a_F \cdot F$		MODEL (41) $f = a_F \cdot F + a_K \cdot K$, $g = a_I \cdot I$		MODEL (42) $f = a_F \cdot F + a_I \cdot I$, $g = a_K \cdot K$	
Variable Parameters	Coeff.	(t-value)	Coeff.	(t-value)	Coeff.	(t-value)
a_P	0.10	(1.8)	0.10	(1.8)	0.03	(0.40)
a_F	0.37	(2.9)	0.39	(3.1)	-0.09	(-0.60)
a_I	-0.01	(-1.5)	0.01	(1.7)	0.01	(0.50)
a_K	0.01	(0.2)	0.06	(0.4)	0.06	(0.94)
a	4.33	(11.7)	4.57	(7.1)	4.28	(12.70)
η	+0.12	(3.0)	0.10	(2.6)	-2.02	(-0.45)
R^2	0.6050		0.6060		0.5354	
$e'e$	4.02655		4.01686		4.73648	
Assymptotic F-statistics d.f. (n_1,n_2)	5.29 (5,77)		5.24 (5,77)		8.94 (5,77)	
Ciritcal-F	3.30		3.30		3.60	
$df_1(n_1,n_2)$	(5,77)		(5,77)		(4,77)	
Conclusion	Reject H_o		Reject H_o		Reject H_o	

*Assymptotic F-statistics computed assuming that the Model (24) with hypothesis to be correct (see Table 9 or Table 12) is the constant term a. The coefficients of P and F are significant at $\alpha = 0.05$ and $\alpha = 0.01$, respectively. The most inconsistent result from Models (40) and (41) with respect to the Sadan model is that the coefficient of $(f-g)^2$ is positive and significant at $\alpha = 0.01$.

Cubic and Quartic Approximations to Sadan Model

The rejection of log-linear process-functions (or the separable parts of the production function) was indicated as a likely possibility in explaining the failure of parabolic cylinders as approximations to the Sadan Model. This opened up the possibility of using nonlinear parts as process-functions, and to seek to refute the Sadan hypothesis. The linear and nonlinear weak separability results were reviewed in determining what kind of nonlinear process-functions to use in approximating the Sadan Model.

There appeared to be three possibilities for the model under fixed K/N ratio assumption with F, I, K inputs. These three models represent the three process-function possibilities represented by three inseparabilities (F,I), (K,I) and (F,K). The first two inseparabilities appeared to be successful in the linear restriction tests and the last two in the nonlinear restriction tests performed previously.

Table 14 indicates the results for a process-function with a nonlinear KI term. That is, the process-function for the husbandry process here is assumed to be, $g = a_K K + a_I I + a_{KI} KI$, and the biological process function here is, $f = a_F F$. The results of Model (43) in Table 14 indicates that all the parameters of the model are significant at

Table 14

Approximations of Sadan Model using KI Nonlinear Term in the Process Function, g

$Y = a + f + g + \eta\,(f-g)^r,\ f = a_F F + a_P P,\ g = a_K K + a_I I + a_{IK} IK$

	MODEL (43)		MODEL (44)		MODEL (45)	
Hypothesis	H_o: r= 2 (Quartic)		H_o: r = 4 (8th Order)		H_o: r = 1.5 (Cubic)	
Variable or Terms	Parameters	(t-values)	Parameters	(t-values)	Parameters	(t-values)
a	5.30	(12.7)	5.30	(13.6)	5.30	(12.5)
a_P	0.13	(2.6)	0.14	(2.7)	0.13	(2.6)
a_F	0.23	(7.3)	0.22	(8.1)	0.23	(7.5)
a_I	0.51	(5.0)	0.52	(5.4)	0.51	(5.0)
a_K	0.24	(4.4)	0.25	(4.2)	0.24	(4.6)
a_{KI}	0.09	(5.5)	0.09	(5.5)	0.09	(5.1)
η	-0.21	(-.9)	-0.23	(-.7)	-0.21	(-1.01)
R^2	0.6910		0.6946		0.6903	
$e'e$	3.15035		3.11336		3.15710	

$\alpha = 0.01$ and positive except for η, the coefficient of $(f-g)^2$, which is negative though not significant at $\alpha = 0.01$.

Model (43) is a quartic model since KI is squared in this case. It should be noted that the inverted parabolic cylinder in Y-f-g space is indicated by this model; however, it is not confirmed as η is negative but not significant. The parabolic cylinder, however, may not quite have the 'sharp-ridge' property necessary in mimicking the Sadan Model:

$$Y = \min (f,g).$$

Model (44) indicates the sharp-ridge parabolic cylinder: $y = a + f + g + \eta (f-g)^4$ [Figure 12 (a)]. The standard errors of the parameters decrease at the cost of standard error of η. This result may be interpreted to indicate that the parabolic cylinder fits the observations better because it has "flattened" near the ridge and falls off rapidly with movements away from it. Because the "flattened" surfaces do better, the standard error of η increases indicating increased uncertainty about the surface being "inverted."

Model (45) on the other hand, attempts to approximate a "ridge" more closely away from the "ridge" and only approximately at the "ridge" itself [Figure 12 (b)].

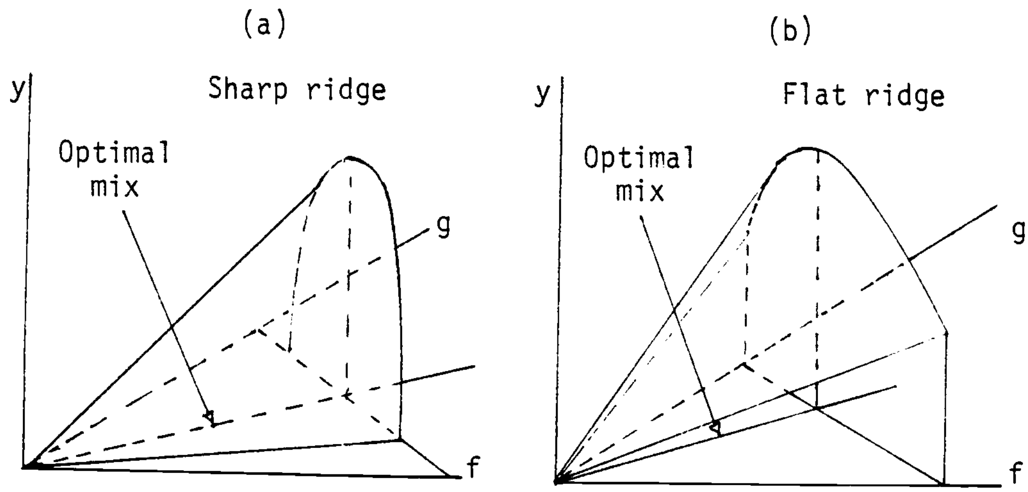


Figure 12: a) Eighth Order Approximation b) Cubic Approximation

All the coefficients in Models (43), (44), (45) are positive and significant at $\alpha = 0.01$ except the parameter η. It is clear, though, that for the computed f and g, the observed-computed coordinates (y,f,g) tended to lie more or less along a straight line, in y-f-g space as shown below:

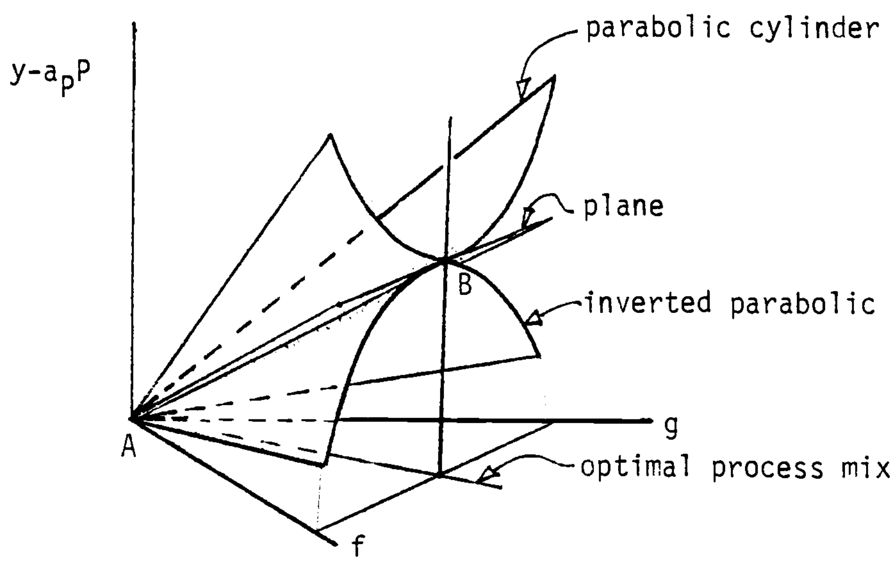


Figure 13: Surfaces to Fit Along Scattered Points Around AB

Table 15

Approximation of Sadan Model using KI Term in the Process Function g

$$Y = a + f + g + \eta\,(f-g)^r$$

Hypothesis Terms	MODEL (46) H_0:r=6 Parameters	(t-value)	MODEL (47) H_0:r=1.3 Param.	(t-value)	MODEL (48) H_0:r=1.1 Param.	(t-value)
a	5.25	(13.85)	5.33	(12.43)	5.34	(12.43)
a_p	0.14	(2.74)	0.13	(2.58)	0.13	(2.58)
a_F	0.22	(9.44)	0.23	(7.53)	0.23	(7.56)
a_I	0.50	(5.93)	0.51	(4.98)	0.51	(4.96)
a_K	0.24	(4.03)	0.24	(4.74)	0.24	(4.90)
a_{IK}	0.09	(5.98)	0.09	(5.05)	0.09	(5.02)
η	-0.37	(-0.49)	-0.21	(-1.02)	-0.21	(-1.01)
R^2	0.6993		0.6900		0.6897	
$e'e$	3.06519		3.15997		3.16318	

Table 15 (cont.)

	MODEL (49) r=1.60		MODEL (50) r=1.75	
Terms	Parameters	(t-values	Parameters	(t-values)
a	5.32	(12.52)	5.32	(12.59)
a_p	0.13	(2.58)	0.13	(2.59)
a_K	0.23	(-1.44)	0.23	(7.38)
a_I	0.51	(4.99)	0.51	(15.00)
a_K	0.24	(4.57)	0.24	(4.50)
a_{IK}	0.09	(5.06)	0.09	(5.06)
η	-0.21	(-1.01)	-.21	(-0.98)
R^2	0.6905		0.6906	
$e'e$	3.15575		3.15375	

The results for the other values of the exponent r of $(f\text{-}g)^r$ are shown in Table 15. The results indicate that the data is explained better by a model with $r \geq 2.0$ rather than by Model (45), where r = 1.5. However, the coefficient η gets progressively worse as r is increased. This indicates and supports the diagramatic representation of the Sadan thesis in Figure 13. The other two models with FI and FK terms in the process-function are shown in Table 16. They did not do as well as the previous models from (40) to (50). It may be indicated that in Model (51) the coefficient of FK is negative and significant at $\alpha = 0.05$ in spite of being negative. Furthermore, the parameter, η, is large, positive and significant at $\alpha = 0.01$. This model contradicts both the production theory and the Sadan thesis, because a_F, a_I, a_K, a_{FK} are all significantly negative and η is significantly positive.

Similarly, in Model (52) with FI term in the process-function, the coefficient of K alone is insignificant at $\alpha = 0.01$. The parameter, η, is positive and significant in this model at $\alpha = 0.01$, indicating that the inverted parabolic cylinder is not inverted at all. Thus, both Model (51) and Model (52) fail to support the Sadan thesis; Model (51) also appears to contradict production theory. Statistically, the cubic model with KI term in the process-function, g, seems to have the highest R^2, smallest sum of squared error-term, and almost all the coefficients a, a_p, a_F, a_I, a_K significant at $\alpha = 0.01$. Though η is negative, it is not significant at $\alpha = 0.05$.

We may conclude that the Sadan thesis is not rejected because η is not positive and not significant. If the optimum efforts are applied, in both processes, we would expect that most of the farmers

Table 16

Approximation of Sadan Model using FI and FK Terms in the Process Function g

$$Y = a + f + g + \eta\ (f-g)^2$$

	$g=a_F F + a_K K + a_{FK} FK$		$g=a_F F + a_I I + a_{FI} FI$	
	MODEL (51)		MODEL (52)	
Terms	Param.	(t-values)	Param.	(t-values)
a	2.42	(10.40)	4.22	(11.28)
a_p	0.12	(2.00)	0.15	(2.80)
a_F	-0.10	(-1.69)	0.47	(3.47)
a_I	-0.04	(-3.40)	0.25	(2.98)
a_K	-0.13	(-4.14)	-0.05	(-0.75)
a_{KI}		-		-
a_{FI}		-	0.07	(3.06)
a_{FK}	-0.04	(-2.78)		-
η	3.03	(1.85)	0.79	(2.55)
R^2	0.6328		0.6748	
$e'e$	3.74361		3.31533	

would operate along the "ridge" as shown in Figure 13. Thus, as long as the points $(y-a_pP,f,g)$ in the process outputs-yield space are scattered tightly along the line AB (which has a slope of 45° with respect to both f and g axes) the parameter η can be varied considerably without changing the results too much. This is indeed what is suggested by the results of Models (43) to (50).

The fact that the estimated coefficients a, a_P, a_F, a_K, a_{KI}, a_I are very insensitive to the actual value of the parameter, η, can only be explained by assuming that the values of the variables, $Y-a_pP,f,g$, lie sufficiently close to AB. What is happening in Models (43) to (50) can then be explained. It is noted that the points lying along a straight line can be contained in a plane, a cone or a parabolic cylinder of proper orientation. In the case of the parabolic cylinder, one can change from a large positive to a large negative value and thus invert the cylinder. Alternatively, one can "flatten" or "sharpen" the ridge of the cylinder by raising or lowering the value of the parameter r. The results suggest that values of the variables $Y-a_pP$, f and g lie along a "ridge" (AB), thus indirectly supporting Sadan's thesis.

Negative Error Models and Process Functions

On the basis of previous results, it was concluded that there exists perfect process complimentarity between the husbandry and the biological processes. The biological process was characterized by the inputs, fertilizer, F and rain, P. The husbandry process on the other hand, was characterized by the inputs, capital services K, and the irrigation services, I. The other methods of combining the various inputs, such as F and I in the biological process description and K in the husbandry process, were not as successful in terms of R^2's.

Under Sadan complimentarity, it is appropriate to use the negative error model directly to estimate the process-functions. To make the results of the negative error models comparable to the previous models, the K/N ratio was again assumed constant. The temperature index T was again deleted from the regression.

The results from the negative error models for the husbandry process function and the biological process function are indicated in Tables 17 and 18. In Table 17, the husbandry process function has inputs I, K and also P, whereas the biological process function has P and F. The precipitation input is common to both the process-functions primarily because it represents a weather variable, separable from both the husbandry inputs K,I and biological input, F.

The results in Table 17 indicate that the estimated husbandry process function has $R^2 = 0.4642$, whereas the biological process function has $R^2 = 0.5918$. This indicates that both the fertilizer input, F, and the capital and irrigation service variables, K and I,

Table 17

Negative Error Models with KI Term in Husbandry Process Function

Variables	Husbandry Process Function (53)		Biological Process Function (54)	
	Coeff.	(t-values)	Coeff.	(t-values)
P	0.09	(1.38)	0.10	(1.87)
F	-		0.23	(11.00)
I	0.88	(5.70)	-	
K	0.65	(6.98)	-	
KI	0.16	(5.74)	-	
C	6.76	(12.80)	4.06	(27.41)
R^2	0.4642		0.5918	
e'e	5.46261		4.16175	
d.f.				
Durbin-Watson	1.31		1.71	

Table 18

Ninety-five Percent Confidence Limits on the Coefficients of Cubic Approximation Model and Negative Error Process Function Models

	Coefficients of Cubic Approximation Model			Coefficients of Husbandry Process			Coefficients of Biological Process		
Variables	Mid-Value	99%	C.I.	Mid-Value	99%	C.I.	Mid-Value	99%	C.I.
Constant, a	5.30	4.88 -	5.72	6.76	6.23 -	7.29	4.06	3.89 -	4.21
P	0.13	0.08 -	0.18	0.09	0.02 -	0.16	0.10	0.09 -	0.11
F	0.46	0.40 -	0.52	-		-	0.23	0.21 -	0.25
I	1.02	0.82 -	1.22	0.88	0.73 -	1.03	-		-
K	0.48	0.37 -	0.59	0.65	0.55 -	0.75	-		-
KI	0.18	0.14 -	0.22	0.16	0.13 -	0.19	-		-
η	-0.42	-0.82	0.00	-		-	-		-
R^2	0.6903			0.4642			0.5918		

respectively, explain roughly the same amount of variance in yield and all of the coefficients in the process-functions are positive and significant at $\alpha = 0.01$. In comparing these results with the cubic and higher order approximations to Sadan type complementarity, one must be careful. That is, if the Sadan thesis is correct, then the process-outputs f,g are equal; therefore, the approximation reduces to $y = a + f + g + \eta\ (f-g)^r \equiv\ a + 2g$. Thus, the coefficients computed using the cubic, quartic and other higher order approximations must be doubled when they are to be compared with the negative error models.

The constant terms computed using the negative error models are 6.76 and 4.06, which compare well with the values of a in Models (43) through (50). Similarly, the coefficient of P is 0.10 which compares well with 0.13 in Models (43) through (50). Table 18 indicates the range of coefficient values implied by the cubic approximation model and those computed using the negative error model or process functions.

The results indicate that the estimated 99% Confidence Interval (C.I.) overlap each other to some extent for all variables except F. In the case of I the cubic approximation coefficient is higher than the direct process function estimate while the case is reversed for K. For KI the estimated coefficients are within the C.I. from Models (53) and (54) at $\alpha = 0.01$. Thus, except for F, there appear to be no major differences in direct and cubic approximation estimates.

In contrast to the process functions of Models (53) and (54), the negative error models were also attempted using a biological process function with inputs P, F, I (and the term FI) and a husbandry process function with inputs P and K. The results are given in Table 19. Though the biological process function here explains 67 percent of the variation in yield, the husbandry process can explain only 25 percent of the variation. More importantly, the effect of the weather variable, P, appears to be different in the two process-functions. The effect of P is significant in the biological process function, and insignificant in the husbandry process at $\alpha = 0.01$. Model (55) has a coefficient for F equal to 0.31 and that of I is 0.17, both of which are

Table 19

Negative Error Models with FI Term in Husbandry Process Function

Variables	Biological Process Function (55)		Husbandry Process Function (56)	
	Coeff.	(t-values)	Coeff.	(t-values)
P	0.13	(2.61)	-0.003	(-0.04)
F	0.31	(11.50)	-	
I	0.17	(3.94)	-	
K	-		0.40	(5.17)
FI	0.04	(4.25)	-	
C	4.26	(29.90)	5.59	(11.61)
R^2	0.6683		0.2513	
$e'e$	3.38191		7.63	
d.f.	5,82		5,85	
Durbin Watson	1.73		1.26	

significant at $\alpha = 0.01$. The coefficient of the FI term is only 0.05, although significant at $\alpha = 0.01$.

Thus, as far as negative error models are concerned, the process-functions appear to support the Sadan complementarity hypothesis better than do the process-functions (Table 19). Under Sadan complementarity, both biological and husbandry processes must be optimized so as to result in equal contribution to yield. This is indicated by the rather close R^2's in case of the process-functions (53) and (54). In the case of (55), (56) process functions, the R^2's are diverse and the coefficient on precipitation, P, is not the same in these two functions. Thus again, negative error model estimates indicate that the husbandry process with inputs K, I and term KI are more consistnet with the notion of complementary processes than those with merely K.

Additive Error Model

A cubic or a higher approximation to Sadan model (with perfectly complementary process functions) is a multiplicative error model:

$$Y = e^{\ln y} = e^{a + f + g + n|f-g|^r} e^{\varepsilon}$$

where, $\varepsilon \sim N(0,\sigma^2)$, $f = a_F \ln F$, $g = a_I \ln I + a_K \ln K$, etc. The additive error model on the other hand is:

$$Y = e^{\ln y} = e^{a + f + g + n|f-g|^r} + \varepsilon, \ \varepsilon \sim N(0,\sigma^2).$$

The nature of the true model decides which of these models is better. In the case of Cobb-Douglas true model, use of log-linear form with the multiplicative error terms may be expected to do better. This is because the error terms are small in magnitude and distributed very closely to the normal with zero mean. On the other hand, if the true model is not Cobb-Douglas then this superiority of multiplicative error model may not always hold.

It is possible that the true model is the nonlinear logarithmic model such as TRANSLOG; if so, the additive error terms model does better than the multiplicative error term model.

The result of the additive error model is shown in Table 20. It indicates that the coefficients are rather stable between additive and the multiplicative error term models. The coefficients are all significant at $\alpha = 0.01$, except for the parameter, η. This parameter is negative, though with a very large standard deviation, making its sign to be insignificant at $\alpha = 0.01$. It should be noted that the unadjusted R^2 has increased slightly, indicating that the additive error model does slightly better than the multiplicative error model. Though this result is unlike the results obtained when using a log-linear Cobb-Douglas model, the difference could very well arise from the fact that the model is log-nonlinear cubic. The log-nonlinear model can be better when the model has additive error terms, probably because the additive errors are closer to a normal with zero mean.

One important distinction is that the additive error model has a substantially less negative value for parameter η and the standard error of η is increased. This means that there is less certainty about the "downward inverted" nature of the parabolic cylindrical surface in the y-f-g space as indicated before. This, however, is as required. If the process-mix is optimal, the observations must lie along the "ridge" as already discussed. So the data should be indifferent between planes, cylindrical surfaces and conic surfaces as long as they contain the "ridge" line (say line AB in Figure 13). This indeed appears to be true for the additive error model, more so than for the multiplicative error models.

Table 20

Additive Error Model for Cubic Approximation to Sadan Type Process Complimentarity

$$(57)\ Y = e^{\ln y} = e^{a + f + g + \eta[f-g]^{1.5}} + \varepsilon\ ,\ \varepsilon \sim N\ (0,\sigma^2)$$

Terms	Estimated Coefficient	t-Statistics
a	5.29	(14.66)
a_b	0.09	(2.39)
a_f	0.23	(4.15)
a_i	0.54	(3.79)
a_k	0.22	(3.56)
a_{ki}	0.10	(3.89)
η	-0.09	(-0.32)
R^2	0.7002	
$e'e$	3650.12	
d.f.	88-7	
Durbin-Watson Statistics	1.66	

Model-B Results

The maintained hypotheses of the previous model was that the weather variables P and T were separable from other input variables, such as F, I, K, N, etc. Furthermore it was assumed that the management process was perfectly complimentary to the farming process, constituting of both the biological and the husbandry process. The former assumption relates to the independence of the precipitation and temperature index variables from other farming inputs. The latter relates to the behavioral assumption of optimization in addition to separability, and so is distinct from the previous one.

Though weather variables are not possible to be controlled at present, weather forecasting has advanced considerably. This means that many decisions in farming are done better now, in terms of timing of inputs. This of course does not necessarily imply that the inputs and weather variables are interrelated in the sense the other controllable inputs are interrelated. But because of weather variables being potentially capable of affecting input decisions, it was felt advantageous to relax this assumption of separability between P,T and F,I,N,K group. The fact that management inputs are not known, does make a difference. The previous assumption that the management process be considered perfectly complimentary to the 'other' process does also appear realistic and necessary as before. Though weather variables are forecastable, the accuracy of forecasts are not high enough to influence day-to-day process-control. So the management process may be assumed independent of weather variables.

Thus Model-B was estimated maintaining the assumption that the management process was a perfect compliment to the 'other' process, where 'other' stands for the whole gamut of weather, biological and the husbandry inputs and the processes represented by them. The purpose of the model is to test if the weather variables are indeed separable from F,I,N,K the biological and husbandry process inputs.

The separability was tested using only linear restrictions. In the presence of linearly restricted separability, it was considered unnecessary to test for non linear restrictions, because linear restrictions suffice one to confirm the Model-A assumptions. It must however be noted that for computational ease, PK, PN, TK, TN terms were deleted from the regression maintaining the hypothesis of separability between weather inputs P and T with capital service K and labor service N. This appears to be a reasonable assumption, because the magnitudes of K and N are not usually influenced by weather variables. Though irrigation service I could be influenced. And because terms FN, FK were not found significant in the previous linear restricted models, it was deleted maintaining the hypotehsis of separability between F and N, K also.

The results are indicated in Table 20. Only the coefficients of P and I are significant at $\alpha = 0.01$ and both are positive. The Model (58) is the general translog function with the maintained hypothesis that F, P, T are separable from K and N. Taking this as the general model, we can put further linear restrictions so that we delete PT, PF, PI, TF, TI, FI, FK, FN, P^2 and T^2. This means Model (58) is transformed due to linear restrictions into Model (9): [F,(K,I,N)].

The validity of this linear restriction implies that P,T are separable from [F,(K,I,N)].

The F-statistics of the restriction discussed above was computed at 2.20. And because the critical F-statistics at $\alpha = 0.01$ is 2.80 with (8,66) degrees of freedom, we can safely conclude that the assumption of linearly restricted weak-separability of (P,T) with [F(K,I,N)] is valid and not rejected. A further linear restriction could be employed on Model (9): [F(K,I,N)] by deleting the cross-product terms, NI, KN, so that the new model becomes Model (19): [F,N,(K,I)] with double partitions. The F-statistics of this final model, with respect to the general Model (58) is computed at 2.00. Because the critical F-statistics at $\alpha = 0.01$ is 2.60 with (10,66) degrees of freedom, we cannot reject the null hypothesis of separability of P,T,F from [(K,I),N].

Thus, in conclusion, we can confirm that the (P,T) are weakly separable from [F,N,(K,I)]as was assumed in models with KI terms. This result can also be extended to separability between (P,T) and (F,I,N,K), in general, by expanding Model (58) to include cross-products terms FN,FK, etc. and re-computing the model. The new F-statistics computed is however expected to confirm this, so it was not actually computed. In any case, because the relevant model is Model (19): [F,N,(K,I)] amongst Model-A results, we see that (P,T) is separable from [F,N,(K,I] and so the results of Model-A remain unchanged, even after losening the constraint of weak separability between (P,T) and [F,N,(K,T)].

Table 21

Separability of Weather Variables P,T and inputs F,I
or Separability Test of [P,T,(F,(K,I,N))]

	MODEL (58)		MODEL (9)		MODEL (19)	
	[P,T,(F,(K,I,N)]		[P,T,(F,K,I,N)]		[P,T,(F,N,(K,I))]	
Variables	Coeff.	(t-value)	Coeff.	(t-value)	Coeff.	(t-value)
P	1.630	(2.77)	0.170	(2.85)	0.173	(2.93)
T	-0.180	(-0.58)	-0.009	(-0.27)	-0.007	(0.22)
F	-0.058	(-0.16)	0.121	(0.86)	0.132	(0.99)
I	0.878	(2.28)	0.836	(3.79)	0.682	(3.57)
N	-0.882	(-0.81)	-0.435	(-0.39)	-0.533	(-1.22)
K	2.816	(1.00)	0.573	(0.19)	1.595	(0.67)
P^2	-0.138	(-1.28)	-		-	
T^2	0.089	(2.42)	-		-	
F^2	-0.009	(-0.52)	-0.006	(-0.35)	-0.006	(-0.41)
I^2	0.001	(0.17)	-0.003	(-0.87)	-0.003	(-0.95)
N^2	0.012	(0.09)	-0.008	(-0.06)	0.053	(1.20)
K^2	0.295	(0.70)	0.0016	(0.04)	0.113	(0.49)
PT	-0.169	(-1.63)	-		-	
PF	-0.031	(-0.44)	-		-	
PI	0.012	(0.27)	-		-	
TF	0.054	(0.94)	-		-	
TI	-0.065	(-1.49)	-		-	
FI	0.023	(1.39)	-		-	
NI	-0.026	(-0.80	-0.041	(-1.33)	-	
KI	0.111	(1.34)	-0.195	(3.93)	0.128	(3.37)
NK	-0.159	(-.39)	-0.040	(-0.10)	-	
C	8.174	(1.49)	5.281	(0.95)	7.550	(1.48)
R^2	0.7709		0.7095		0.7016	
$e'e$	2.33566		2.96114		3.04214	
F-statistics (df)	None		2.20 (8,66)		2.00(10,66)	
D.f.	88-22		88-14		88-12	

VI. SUMMARY OF RESULTS

The results of the Chow test indicated that eastern and western Oregon indeed have two distinct wheat production functions. The factor productivities are also different in these two regions. Water productivity appears to be small or near zero in western Oregon. Fertilizer and precipitation both appear to influence the yield in a positive manner in both regions. The coefficients also appear to be comparable to one another for these two variables when we compare Model (6) with Models (43) through (50). This is remarkable considering the fact that the types of wheat grown in eastern and western Oregon differ significantly from one another. The coefficients for capital service, K, also are comparable in value between Model (6) and Models (43) through (50). Model (6) however, suffers extensively from multicollinearity between K and N; consequently, the results may change somewhat if multicollinearity is reduced when K/N is fixed as before.

The test of linear homogeneity of the production function failed for eastern Oregon. This was, of course, anticipated, since we did not expect that the "per unit land" model itself could be linearly intensified. Linear homogeneity of the per unit production function would imply that there is increasing or decreasing returns to scale. The exact value of returns to scale is of some interest, although the local nature of the result reduces its value considerably. Thus returns to scale here would not be difficult to interpret, as it would refer to scale implying process intensities. But the computation of returns to scale or intensities has not been attempted presently.

The conclusion derived from the previous results chapter is that the linear restriction of weak separability is valid between (P,T) and (F,I,N,K) in eastern Oregon. Thus the maintained hypothesis of Model-A is a valid hypothesis. Within the Model-A results, the test of single, double and pairwise partition using linear restrictions indicates that not all F,I,N,K are strongly separable. The linear restrictions where FI and KI terms were absent were usually rejected under the appropriate F-test. Thus the ambiguity arose as to the empirical superiority between log-quadratic models with FI and KI terms. The model with KI term has a slight superiority in terms of the sum of the squared errors compared to one with the FI term, as indicated by the results of the Models (14), (9), (19), (27) versus those of Models (13), (10), (16), (25), respectively.

But statistically speaking the F-test was still ambiguous in the sense that both these models could not be rejected on statistical grounds at $\alpha = 0.01$. The ambiguity persists in all the log-quadratic models with linear weak separability restrictions.

The nonlinear weak-separability restriction tests using assymptotic F-distributed statistics again gave similarly ambiguous results as regards Model (39) where K and I were not nonlinearly separable and Model (38) where F and K were not nonlinearly separable, as shown in Table 12. Thus, the best process-functions could not be decided upon, as ambiguity presented in both the linear and nonlinearly restricted models. This led to the further restriction of Sadan perfect process-complimentarity, to be applied to the log-quadratic, TRANSLOG. However as indicated by the models (40), (41) and (42) of Table 13, the hypothesis of perfect-process complimentarity using the Cobb-Douglas approximation to the process functions were rejected quite strongly, in all cases.

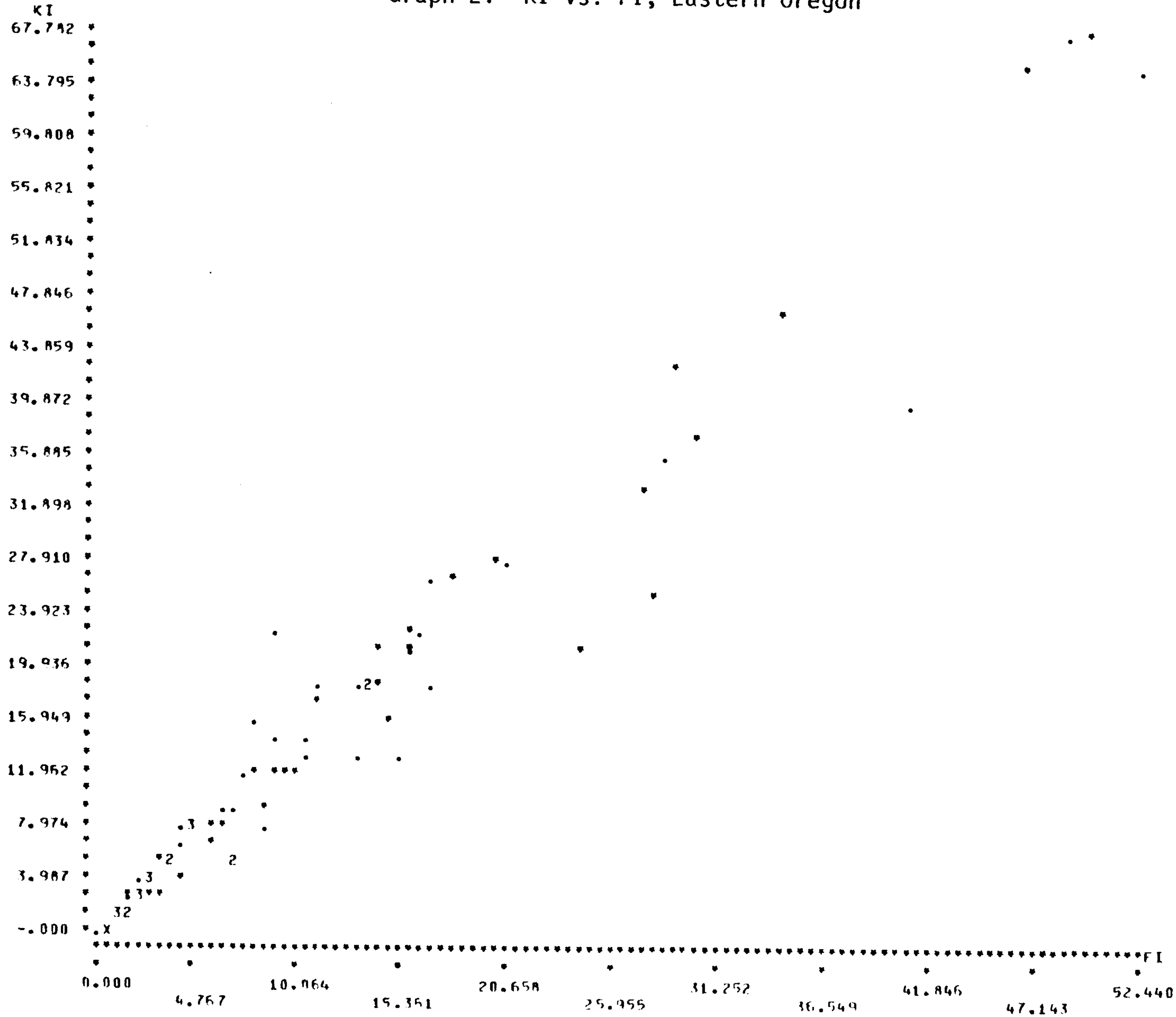
As already explained this rejection implies that either the Cobb-Douglas process-functions were invalid or the process complimentarity was low. The previous rejections of pairwise strong separability, however, seemed to indicate that the assumption of Cobb-Douglas process-functions was responsible for this rejection. Thus other log-quadratic process-functions were necessary to explain the data better. Three models with log-nonlinear process functions were then attempted using cross-products (FI), (KI), (FK) in the process-functions. Models (51) and (52) for FK and FI, respectively, in Table 16 did rather poorly in approximating the Sadan model, which is also inconsistent with general production theory. On the other hand, Models (43) through (50) in Tables 14 and 15 strongly support the Sadan thesis of perfect complementarity, where the husbandry process consists of a simple log-quadratic in K and I and the biological process is a simple Cobb-Douglas in P and F.

The coefficients of Models (43) through (50) are all positive and significant except for the parameter η which was negative and not significant at $\alpha = 0.01$. This indicated that the observed data lie along a straight line "ridge" characteristic of the Sadan function, Y = Min (f,g). Thus the higher-order functions of Models (43) through

(50) have enough flexibility to depict a situation which represents possibly a situation of near perfect process complementarity. The difficulty of attempting to resolve the close competition between models with FI and KI terms can best be understood with the help of Graph 2 between FI and KI given below. As can be seen, the log-quadratic forms with FI would do nearly as well as that with the KI term, because the two terms are very strongly correlated. But the cubic and higher-order functions used to approximate the perfect process complementarity also have highest R^2 or least sum of the squared errors, when the process-functions have KI term.

The validity of Sadan's perfect process-complementarity allowed the use of direct estimation of the process-functions using the framework of negative error-models. Again, the results from the estimated process-functions are consistent with the cubic approximation model in general. The implied true coefficients are obtained from the cubic approximation model by doubling them (except for P and the constant) so that they are comparable to the process-function coefficient estimates. Except for the coefficient of F, the results can be considered to be mutually supportive of each other. The cubic approximation model does, however, have a coefficient of F twice as large as the coefficient in the direct estimation of the process-function.

Graph 2: KI vs. FI, Eastern Oregon



Total and Marginal Productivities based on the Cubic Approximation to Sadan's Perfect Process Complimentary Model

The cubic approximation in Model (45) was used to compute the various input-output relations conditional upon other factor inputs being fixed. For Figure 14, precipitation was assumed to be fixed at 10 inches and capital at $10.00/acre. Irrigation service level I was given one of the three values 1.0, 0.5 or 0.07 and the yield computed for varying levels of fertilizer inputs. Figure 14 shows that the marginal physical product changes continuously for fertilizer from more than 5 bushels/lb. to 0.25 bushels/lb. Fertilizer input has a diminishing marginal product. The figure indicates various yield curves at differing levels of irrigation service. The level I = 0.07 is taken merely to indicate the sample mean situation. The curves are unrealistically reduced to the F-axis if I = 0.0.

To indicate the effect of increasing capital service on yield, another set of fertilizer yield response curves were computed using Model (45) as shown in Figure 15. The important fact here is that at lower levels of irrigation, say at I = 0.07, the fertilizer response-function does not change at all from the previous curve in Figure 14. This is due to the fact that at low or no irrigation, increasing the capital service alone does not improve yield. The husbandry process of seed bed preparation, tillage and harvesting does not improve yield without increasing the irrigation service. Similar results would be obtained for I at low levels of K. However, when the irrigation is

Fertilizer Input, Marginal Product and Expected Wheat Yield
Based on Cubic Approximation
Model (45): P = 10 inches, K = $10.00/Acre.

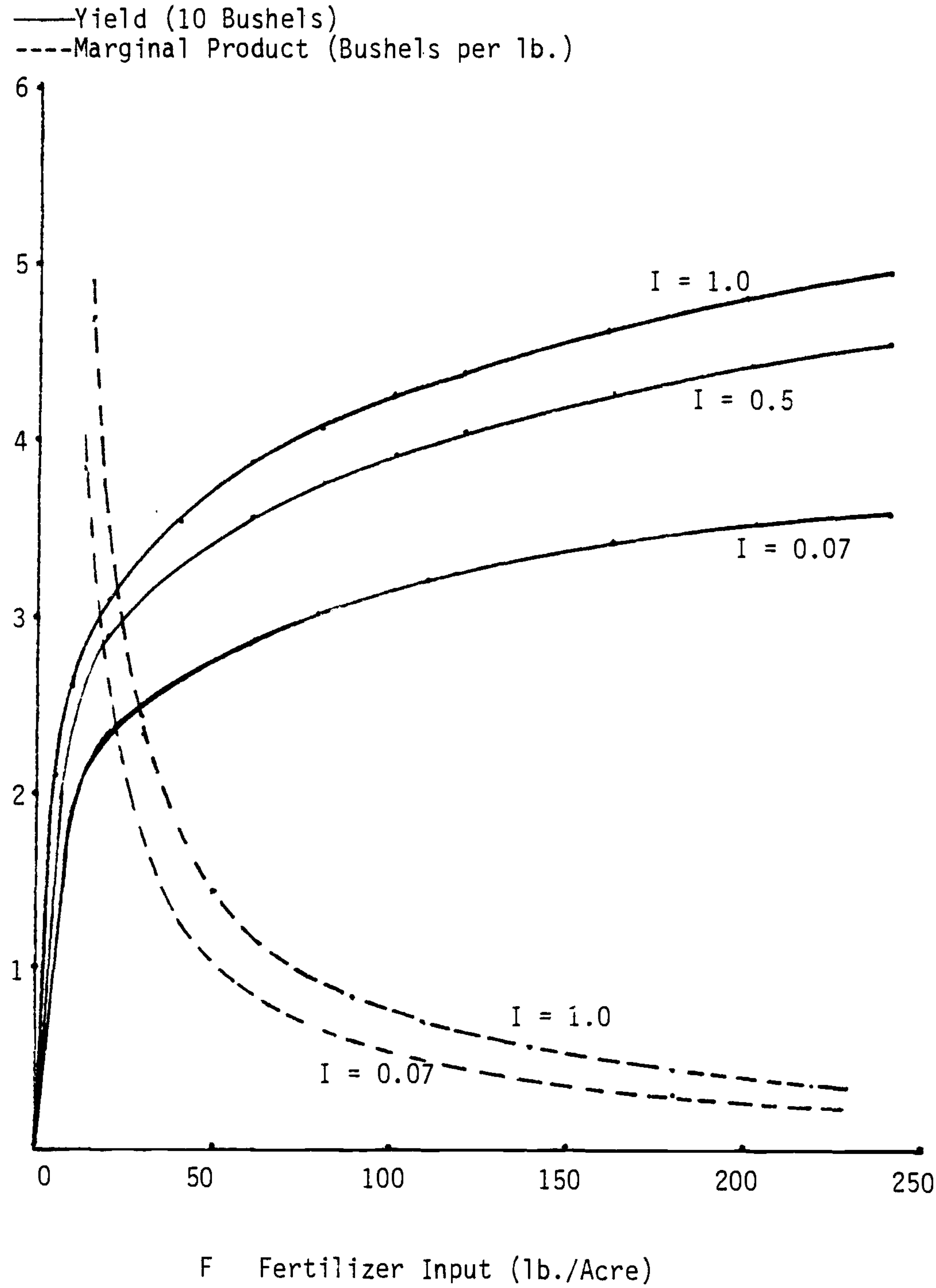
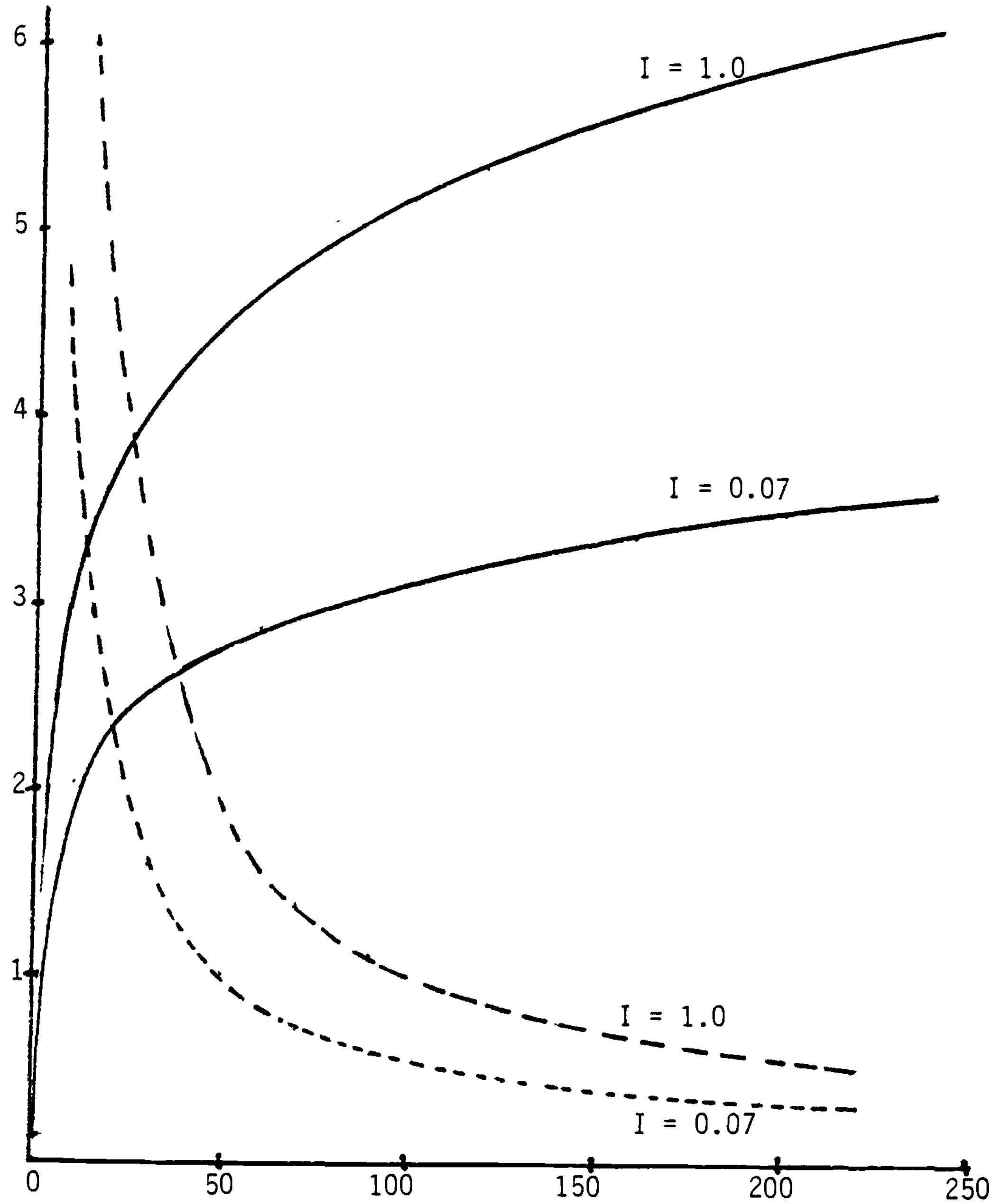


Figure 14

Fertilizer Input, Marginal Product and Expected Wheat Yield
Based on Cubic Approximation
Model (45): P = 10 inches, K = $20.00/Acre
- Yield (10 Bushels) --- Marginal Product (Bu./lb.)



F Fertilizer Input (lb./Acre)

Figure 15

present, I = 1.0, the effect on yield is substantial. For comparison, in the usual range of fertilizer application in eastern Oregon, say 50-100 lbs/acre, the marginal product of fertilizer is in the range of 1.75-1.10 bu./lb. when K = \$20.00 per acre. The corresponding values when K = \$10.00/acre were 1.3-0.75 bu./lb. The curves of Figure 14 and Figure 15 cannot be directly used to compare marginal productivities of irrigation service I. At K = \$10.00, the total gain in yield from irrigation appears to be in excess of 10 bu./ acre beyond 40 lb./acre fertilization. Similarly at K = \$20.00, the total gain in yield from irrigation appears to be in excess of 15 bu/ acre beyond 40 lb./acre. At higher level of fertilization, F = 120 lb/acre, the total gains in yield are 15 and 20 bu./acre, respectively at K = \$10.00 and K = \$20.00. The total gains in yield are relevant here because the decisions relate either to have irrigation or not to have irrigation. However, the cubic approximations are local approximations and therefore the results are to be interpreted accordingly, with care taken not to extend the results too far beyond the locality of the approximation. The case in point is the value of I = 0.0, though this value is legitimate ordinarily, the log-transformation of this value tends to be negative infinity; and so the lowest value for the curves has been taken as I = 0.07, the sample midpoint for irrigation. This value should correspond more or less to the situation of dryland wheat production in eastern Oregon.

The negative error models could also be used. However, the total yield estimated using such a model will be definitely biased. The marginal products for the negative error models therefore cannot be computed without knowing the unbiased yield, since the marginal product terms contain yield in their expression. The elasticities of substitution could also be determined for the inputs using the process functions; however, later on they will be shown to be unnecessary for the present case. It may be noted here that direct estimation of process-functions does not fulfill the purpose of measuring marginal productivities unless there are means by which the yield can be estimated without bias. Such an unbiased estimate of yield has been obtained by using the cubic approximation here.

Though the previous figures indicate that there are diminishing returns for all inputs F, K, I, the substitutions are difficult to stipulate using those "conditional" curves. To simplify matters, the precipitation and irrigation levels were fixed at P = 10 inches, I = 1.0. Then using a price for fertilizer of about $400.00 per ton, the expenditure on the fertilizer and the capital service was fixed at $10.00/acre and $20.00/acre. It should be noted that the cost of irrigation, etc. are ignored at this point for simplicity, because their explicit consideration does not change the conclusions drawn. Figure 17 indicates the result of changing the K/F ratio on total yield (keeping the expenditure constant). It may be noted that for the maximum yield the K/F ratio is around 0.6. The maximum yield that was attainable at $20.00/acre expenditure was 37.60 bu./acre and the maximum yield that can be achieved at $10.00/acre was 26.2 bu./acre.

Since, keeping I = 1.0 implies that, K can represent the husbandry process-function (or its monotonic transformation in this case, g) and F represents the biological process-function (or its transformation)

Cross-section of the Cubic Approximation Function
Yield vs. K/F Ratio
Expenses on F and K Fixed at $10.00 and $20.00
Assumption: I = 1.0 and P = 10.0 inches.

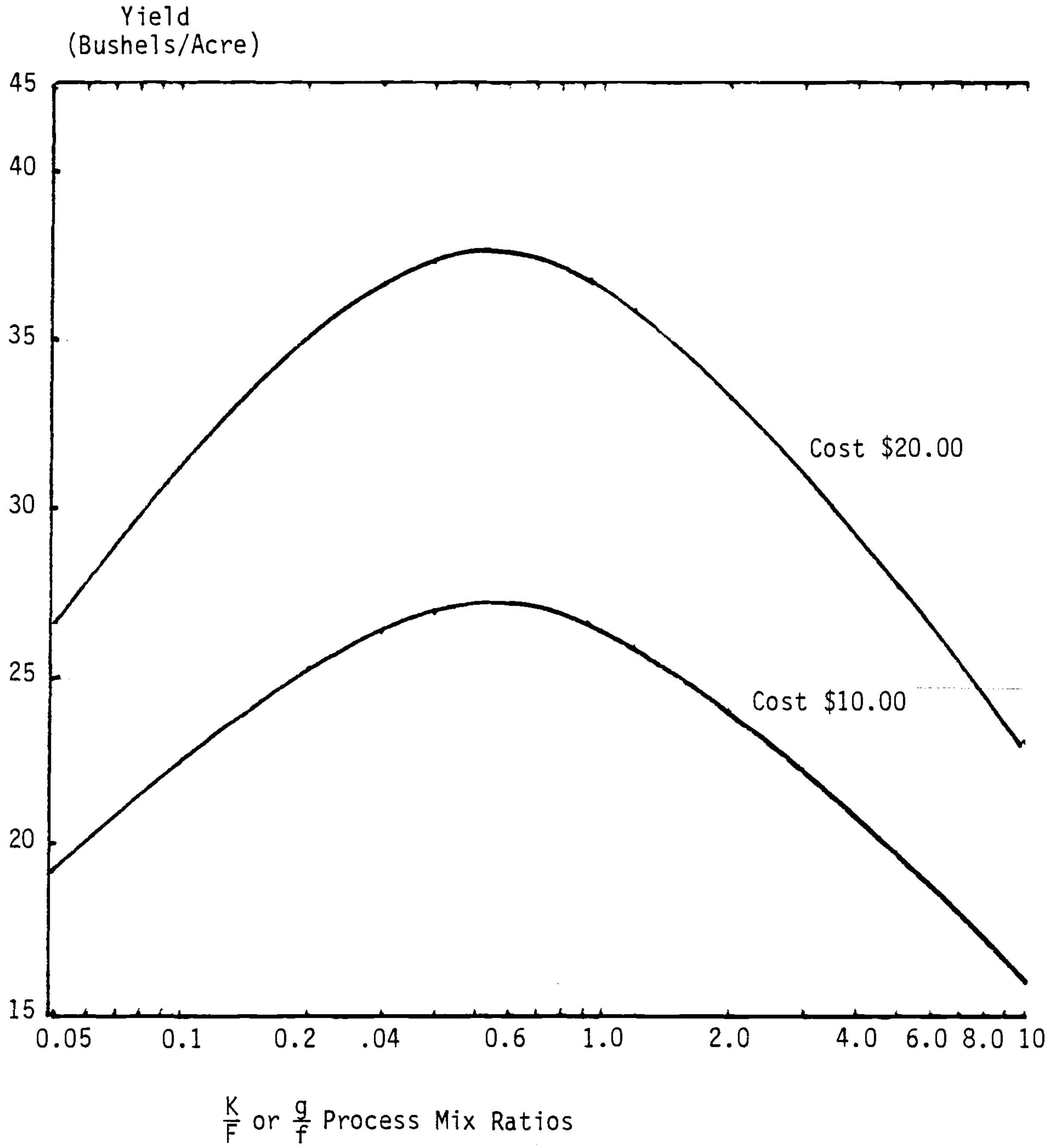


Figure 16

f, the curves in Figure 17 can also be regarded as the cross-sectional profile of the inverted parabolic cylinder in the Y-f-g-space. One peculiarity of this form of the cubic approximation is that the between process-substitution is highly elastic near optimal process-mix and highly inelastic away from the optimal process-mix. This can be observed from the yield - K/F ratio curves in Figure 17. The maximum yield attained changes rather slightly between 0.4-0.9 of K/F ratio, but begins to fall off rapidly beyond these ratios. The closer inspection of the cubic approximation function, $Y = a + f + g + \eta\ (f-g)^r$, reveals the reason. For the above functional form it can be shown that the process-substitution between f and g, σ_{fg}, takes on the following simple form:

$$\sigma_{fg} = - \frac{Y_f Y_g\ (Y_f \cdot f + Y_g \cdot g)}{f \cdot g\ (Y_f - Y_g)^2 Y_{fg}}$$

where, $(Y_f - Y_g)^2 = r^2\ \eta^2 (f-g)^{2(r-1)}$, $Y_{fg} = r(r-1)\eta\ \ (f-g)^{r-2}$, and $Y_f Y_g = 1 - r^2\ \eta^2 (f-g)^{2(r-1)}$.

The denominator term tends towards zero when f and g are close together, so that near optimal process-mix, σ_{fg} tends to infinity. On the other hand, as f and g diverge from one another, the $Y_f Y_g$ term becomes small and σ_{fg} tends towards zero.

Within Process-Substitutions

Within the framework of Sadan's perfect process complimentarity and the assumption of optimizing behavior, we can write down the process-functions as follows:

(i) Biological Process Function

$$\overline{Y} = q + b\overline{P} + h\overline{F}, \text{ and}$$

(ii) Husbandry Process Function,

$$\overline{Y} = a + b\overline{P} + c\overline{I} + d\overline{K} + g\overline{KI},$$

where, $\overline{Y}$, $\overline{P}$, $\overline{F}$, $\overline{I}$, $\overline{K}$ are all in logarithms and a,b,c,.... are coefficients. The within process substituability between inputs in a Cobb-Douglas production function is known to be unity. Thus for the biological process function, since there is only one man made input F, unitary substitution elasticity with respect to P does not have any practical significance (since P is not a controllable input). On the other hand in the husbandry process function, the substitutability between K and I is of some interest.

The husbandry process-function, though a simple log-quadratic, does have unit elasticity of substitution between K and I just as in the Cobb-Douglas function form. This may be shown by computing the elasticity of substitution between K and I, σ_{KI}. We note that,

$$\sigma_{KI} = - \frac{G_K G_I (G_K K + G_I I)}{KI\ (G_I^2 G_{KK} - 2G_I G_K G_{IK} + G_K^2 G_{II})}$$

where, Y = G(P,I,K) represents the process function in original variables, denoted without a bar on top of the respective log-form variables. G_K and G_{KI} represent the derivatives of G with respect to K and with respect to K and I, respectively. Simiarly for other derivatives, G_I, G_{II}, and G_{KK}, etc. it may be noted that,

$$G_K = (d+g\ln I)(Y/K), \quad G_I = (c+g\ln K)(Y/I)$$

$$G_{KK} = (d+g\ln I)(d+g\ln I-1)(Y/K^2) = (G_K^2/Y) - (G_K/K)$$

$$G_{II} = (c+g\ln K)(c+g\ln K-1)(Y/I^2) = (G_I{}^2/Y)-(G_I/I)$$

$$G_{IK} = (c+g\ln K)(d+g\ln I)(Y/KI) = (G_I G_K/Y)$$

So that, the numerator $-G_K G_I(G_K K+G_I I)$ in the expression for σ_{KI} is as follows:

$$-G_K G_I\ [(d+g\ln I)(Y/K)\ K + (c+g\ln K)(Y/I)I]$$

$$= \quad -G_K G_K\ [(c+d) + g(\ln K + \ln I)]\ Y$$

The denominator KI $[G_I{}^2 G_{KK}-2G_I G_K G_{IK}+G_K{}^2 G_{II}]$ in the expression for σ_{KI} takes the following form after simplification:

$$-(IK)(c+g\ln K)(d+g\ln I)[(c+d)+g(\ln K+\ln I)](Y^3/I^2K^2)$$

Thus the numerator and denominator are equal, implying σ_{KI} = 1.0 within the husbandry process.

Similarly it can be shown that the elasticity of substitution between P, and I and that between P and K is unity. Thus, within process substitution elasticities in this Sadan model are just like those in Cobb-Douglas production function.

CONCLUSION

The conclusion of the present study can now be reiterated as follows:

a) There exists weak separability amongst the biological, weather and husbandry inputs in both eastern and western Oregon wheat production functions. The implication is thus clear that in aggregating into higher level of aggregate production functions, the preferred approach would be aggregation along these implied processes rather than across farms.

b) There also exists empirical support to the notion that there are some processes in agriculture which are nearly perfect compliments to one another.

c) There are major differences in production functions in eastern and western Oregon, resulting from the differences in precipitation and other climatic factors.

For the eastern Oregon data, set (K,I) has been found inseparable, in the sense that the hypothesis of separability of K and I is consistently rejected in both linear and nonlinear restrictions tests. Thus K and I belong to a single process within which they interact conducively to increase yield. They can therefore be aggregated with one another but not with F. The effects of K and I on yield can be summarized with a single index g constructed from K and I and incorporated into an aggregate production function, $Y = \phi(f(F),g(K,I)$. This kind of aggregation would not result in any aggregation bias. In other words, if the aggregation is performed using K and F where they constitute an aggregate "expenditure" there is a good possibility of aggregation bias. On the other hand, in the case of the western Oregon data there appears to be more than one good way to aggregate inputs to make an aggregate index.

Thus additive separability of western Oregon Cobb-Douglas function indicates that an aggregate 'expenditure' constructed from K and F is valid, and results in no aggregation bias in estimating the ϕ-function. There are of course other equally valid aggregation possible, such as aggregation of K and I, and of I and F.

Also, it may be noted that in eastern Oregon the husbandry process with function, g(K,I), and the biological process with function f(F), are highly complementary to one another, as Sadan has hypothesized. The process-functions when estimated directly using the negative error model gave rise to similar coefficients for the variables as was obtained in the cubic approximation model. However, though this provides some support to Sadan's perfect process compl mentarity, the biased nature of the intercept terms disqualifies these models for estimating marginal productivities directly, unless the yields are known without bias a priori.

Thus the conclusion that can be drawn is that Sadan's thesis of complimentary processes appear plausible and is at least not rejected. The important consideration here is that the cubic approximation was necessary to attempt to refute Sadan's thesis. The results are strictly local. When refutation of a thesis is sought, this can be attempted locally since local refutation is a valid refutation.

On the other hand, the refutation may not occur outside of the locality. Thus Sadan's model must not be regarded as a global result for eastern Oregon. The same conclusion also holds for the weak separability tests for eastern and western Oregon wheat production functions.

LIMITATIONS AND RECOMMENDATION

The analysis in this present study has maintained a few hypotheses and the result of the analysis are conditionally valid upon them. The important assumption has been that the "management process" is perfectly complimentary to "other processes" and that farmers behave optimally in equating these process-outputs. This is Sadan's thesis with respect to intangible inputs such as "management". If Sadan's process-complimentarity does not hold here, then there certainly will exist some specification error in the present estimation of the production function. Since, test of Sadan's thesis in this context must await the solution to the measurement of management variables, the present study results must be viewed with some skepticism. The specification error may be quite large. This is the major methodological limitation of the present study.

Related to this Sadan type process complementarity assumption is the assumption of linear homogeneity or physical replicability of the wheat production in space. When the "management" inputs are the bottleneck to expansion and/or when technological change allows conserving management inputs, the assumption of linear-homogeneity may not hold. This is particularly true when the "management" inputs are not available for "hire". Thus, Sadan type complementarity which implies weak separability as well as perfect process complementarity, may not hold. Linear-homogeneity may be physically possible, but not so mangerially; this may result in specification error.

The tests of separability and linear homogeneity and process-complementarity are all based on the quadratic and cubic approximations to a general functional form at a point. The results are therefore only true locally and may not hold far beyond the point of approximation. This is a serious limitation of the present methodology. In the absence of a priori knowledge of the form of the production function, this limitation appears very difficult to overcome.

Another equally important limitation of the present study stems from the nature of the county level data. Since the data is already aggregated amongst farms in the county, the relevant question that can be asked of this data related to aggregation beyond the county level data. It is felt, however, that when the data is available at lower levels of aggregation, the present methodology can be re-employed to see if the presently used aggregation to the county level data has results in significant aggregation bias.

The analysis would have considerably benefited if disaggregate county level data could have been obtained for totally, partially and not irrigated farms. This would have permitted greater accuracy in the measurement of irrigation service, I. The measurement error of capital and labor services utilized in this study is quite high, since total fuel and oil expenses and hired labor wages were used. Disaggregated data specific to wheat production activities would be advantageous. The problem of multicollinearity between K and N, however, may still persist requiring the use of the fixed K/N ratio assumption or ridge-regression techniques in the analysis. Similarly a better temperature index is needed which relates to the minimum temperature in eastern Oregon.

In view of the above discussion the following recommendations are appropriate:

a) The analysis of production relations at a given level of aggregation can and should be conducted to test for weak and strong separabilities. This opens up the possibility of identifying or ascertaining various salient features of production technology, such as its analysis into distinct and separate processes and sub processes. The identification of such separabilities or processes comprises the first step towards valid aggregation at higher level of analysis;

b) The present analysis can be extended to other crops and activities of the farm. When various processes are identified within each activity, the appropriate aggregate index for each can be formulated so that aggregate production functions for cropping, dairy and other major activities can be estimated without too much aggregation bias;

c) The use of a cubic approximation to the Sadan model has indicated that the nonlinear-in-parameter forms are economical in parameters. Thus beyond one or two variables cases, this economy in parameter becomes advantageous from the computational standpoint in providing necessary functional flexibility. The perfect process-complementarities are of some interest from the standpoint of simplifying production function estimation procedures. The use of higher order approximations beyond quadratic can be used in the analysis of high process complementarities;

d) The use of separability tests on second order approximations to a general production function at a point, can lead to ambiguities regarding the process-functions. The option is either to extend the analysis to higher order-approximations or to proceed with notions such as "perfect complements" or "perfect substitutes" amongst processes. The use of the latter is to provide the simplest means of upgrading from second order to third, or higher order approximations that may become desirable, without exhausting all of the higher order approximation possibilities. This approach leads to choosing a functional form dictated by the economic characteristics of the production process and not vice versa.

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APPENDICES

APPENDIX A

Theorem 1:

If a general function $y = F(X_1,\ldots,X_N)$ is weakly separable, such that,

$$y = F(X_1,\ldots,X_N) = \Psi\{L^1(X_1^1,\ldots,X_{N_1}^1),\ldots,L^r(X_1^r,\ldots,X_{N_r}^r)\}$$

and if $F(\cdot\cdot)$ can be expressed as $G(x_1,\ldots,x_N)$, where x_l is a monotonic transformation of X_l, i.e. $x_l = x_l(X_l)$ and $x_l^{'} = \frac{\partial x_l}{\partial X_e} > 0$ or $\frac{\partial x_l}{\partial X_e} < 0$, for all $l = 1,\ldots,N_1,\ldots,N_r=N$, then, $y=G(x_1,\ldots,x_N)$ is weakly separable, such that,

$$y = G(x_1,\ldots,{}_N) = \phi\{f^1(x_1^1,\ldots,x_{N_1}^1),\ldots,f^r(x_1^r,\ldots,x_{N_r}^r)\}$$

Proof:

Weak separability of $F(X_1,\ldots,X_N)$ has the necessary and sufficient condition (due to Leontief) as follows:

(A) $$\frac{\partial}{\partial X_k}\left(\frac{\partial F/\partial X_i}{\partial F/\partial X_j}\right) = 0$$

where, $X_i, X_j \in X^r$ (i.e. X's in r^{th} separable group) and $X_k \in X^s$ where $r \neq s$. The equation (A) can be further simplified as,

(B) $F_i F_{jk} - F_j F_{ik} = 0$, where $F_i = \frac{\partial F}{\partial X_i}$, $F_{ik} = \frac{\partial^2 F}{\partial X_k \partial X_i}$ etc.

If we represent, $\frac{\partial x_l}{\partial X_l} = x_l^{'}$ for all l, then, differentiating G with respect to X_l's we obtain,

$$F_i = G_i x_i^{'},\ F_j = G_j x_j^{'},\ F_{ik} = G_{ik}\, x_i^{'} x_k^{'},\ F_{jk} = G_{jk} x_j^{'} x_k^{'}$$

and, from (B),

(C) $$F_i F_{jk} - F_j F_{ik} = (G_i G_{jk} - G_j G_{ik})\, x_i^{'} x_j^{'} x_k^{'} = 0$$

Since, $x_l^{'} \neq 0$ for all the values of l's, we conclude that,

(D) $$G_i G_{jk} - G_j G_{ik} = 0.$$

This is the necessary and suggificient condition for $G(x_1,\ldots,x_N)$ to be weakly separable, so we can write,

$$y = G(x_1,\ldots,x_N) = \phi\{f^{'}(x_1^{'},\ldots,x_{N_1}^{'}),\ldots,f^{r}(x_1^{r},\ldots,x_{N_r}^{'})\} .$$

Q.E.D.

Theorem 2:

If $y = G\,(x_1,\ldots,x_N)$ is a second order polynomial (quadratic) in X_1 and $G(\cdot\cdot) = \phi\{f^{'}(x_1^{'},\ldots,x_{N_1}^{'}),\ldots,f^{'}(x_1^{r},\ldots,x^{r}_{r})\}$ that is, $G(\cdot\cdot)$ is weakly separable, then, either

PART-I

ϕ is linear in $f^r(x_1^r,\ldots,x_{n_r}^r)$ where $f^r(\cdot\cdot)$ are second order polynomial (quadratic) in x_1^r's, or

PART-II

ϕ is 2nd order polynomial (quadratic) in $f(\cdot)^r$'s where $f^r(\cdot)$ are linear in x_1^r's.

Note: Proof is provided for two separable groups here, $\tilde{x} = x_1,\ldots,x_i,\ldots,x_K$ and $\hat{x} = \{x_{k+1},\ldots,x_p,\ldots x_N\}$. The proof can be extended into three or more partitions by induction. Consider indices $i,j = 1,\ldots,k; p,q = k+1,\ldots,N$ and $m,l = 1,2,\ldots,K,\ldots N$.

Since $G(x_1,\ldots,x_N)$ is quadratic we can express $G(\cdot)$ as,

$$y = \alpha_o + \alpha' X + X'\gamma X$$

(E) $$= \alpha_o + \sum^N \alpha_1 x_1 + \sum_1^N \sum_m^N \gamma_{1m} x_1 x_m$$

where, $\alpha' = (\alpha,\ldots \alpha_1 \ldots \alpha_N)$ and $\gamma = \begin{vmatrix} \gamma_{1n}\cdots\gamma_{1N} \\ \gamma_{N1}\cdots\gamma_{kN} \end{vmatrix}$,

and γ is symmetric square matrix.

The Leontief condition for Weak Separability is,

(F) $$G_i G_{jk} - G_j G_{ik} = 0.$$

Note that, for a quadratic $G(\cdot)$, we have,

$$G_i = (\alpha_i + \Sigma_l \gamma_{il} X_l), \quad G_j = (\alpha_j + \sum_l^N \gamma_{jl} X_l)$$

$$G_{ik} = \gamma_{ik}, \quad G_{ik} = \gamma_{jk}$$

So that (F) implies that,

(G) $$(\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik}) + \sum^N (\gamma_{il}\gamma_{jk} - \gamma_{jk}\gamma_{ik}) x_l = 0.$$

The necessary and sufficient condition for (G) to hold is, either, the LINEAR RESTRICTIONS:

(H) $$\gamma_{ik} = \gamma_{ik} = 0 \text{ , for all } \begin{cases} i,j=1,\ldots,k \\ k=k+1,\ldots,N \end{cases},$$

or, the Non-Linear Restrictions:

(I) $$\frac{\alpha_i}{\alpha_j} = \frac{\gamma_{il}}{\gamma_{jl}} \text{ and } \frac{\alpha_p}{\alpha_l} = \frac{\gamma_{pm}}{\gamma_{qm}} \text{ for} \begin{cases} i,j = 1,\ldots,k \\ p,q=k+1,\ldots,N \\ l,m = 1,\ldots,N \end{cases}$$

Linear Restrictions give rise to the validation of PART-I

Proof: PART-I

Rewirte (E) as follows:

$$y = \alpha_0 + \sum_i^K \alpha_i X_i + \sum_{p\,k+1}^N \alpha_p X_p + \sum_i^K \sum_j^K \gamma_{ij} X_i X_j$$

$$+ \sum_i^K \sum_{p\,k+1}^N \gamma_{ip} X_i X_p + \sum_{p\,k+1}^N \sum_{q\,k+1}^N \gamma_{pq} X_p X_l$$

Note that Linear Restrictions implies that $\gamma_{ip} = 0$ for all i and p. So we can rewrite the expression in terms of $\tilde{X} = (X_1,\ldots X_k)$, $\tilde{\alpha} = (\alpha_1\ldots\alpha_k)$, $\tilde{\gamma} = [\gamma_{ij}]$, $\hat{\gamma} = [\gamma_{pq}]$, etc. as,

$$y = \alpha_0 + \tilde{\alpha}'\tilde{X} + \tilde{X}'\tilde{\gamma}\tilde{X} + \hat{\alpha}'\hat{X} + \hat{X}'\hat{\gamma}\hat{X}$$

where, $\tilde{\alpha}'\tilde{X} = \sum_i^K \alpha_i X_i$, $\tilde{X}'\tilde{\gamma}\tilde{X} = \sum_i^K \sum_j^K \gamma_{ij} X_i X_j$ etc.

Note that, $\tilde{\alpha}'\tilde{X} + \tilde{X}'\tilde{\gamma}\tilde{X} = \tilde{a}$ (say) is a quadratic in $\tilde{X} = (x_1, \ldots, x_N)$, similarly, $\hat{a} = \hat{\alpha}'\hat{X} + \hat{X}'\hat{\gamma}\hat{X}$ and, we can express,

$$y = \alpha_0 + \tilde{\beta}\tilde{a} + \hat{\beta}\hat{a}, \text{ where } \beta\text{'s are some coefficients.}$$

Thus $G(\cdot)$ has a linear form in $\tilde{a}$ and $\hat{a}$ and a's are quadratic in X's.

Q.E.D.

Non-linear restrictions lead to PART-II.

Proof: PART-II

From (I), we can write,

(J) $$\frac{\alpha_i}{\alpha_j} = \frac{\gamma_{i1}}{\gamma_{j1}} \ldots = \frac{\gamma_{im}}{\gamma_{jm}} = \ldots . \frac{\gamma_{iN}}{\gamma_{jN}} = \theta^i_j \text{ (say), } i,j=1,\ldots,k.$$

(K) $$\frac{\alpha_p}{\alpha_q} = \frac{\gamma_{p1}}{\gamma_{q1}} = \ldots . \frac{\gamma_{pm}}{\gamma_{qm}} = \ldots . = \frac{\gamma_{pN}}{\gamma_{qN}} = \theta^p_q \text{ (say), } p,q=k+1,\ldots,N$$

Consider (J) with i=1, j=j, then,

(L) $$\frac{\alpha_j}{\alpha_1} = \frac{\gamma_{j1}}{\gamma_{11}} = \ldots . = \frac{\gamma_{jm}}{\gamma_{1m}} = \ldots . = \frac{\gamma_{jN}}{\gamma_{1N}} = \theta^j \text{ (say)}$$

From (L), we get, $\gamma_{jm} = \theta^i \gamma_{1m}$ and $\gamma_{ji} = \theta^j \gamma_{11}$ for all $j=1,\ldots,k$.

Now let m=i then above two results become

(M) $$\gamma_{ji} = \theta^j \gamma_{1i} \text{ , and } \gamma_{j1} = \theta^j \gamma_{11} \equiv \gamma_{i1} = \theta^i \theta^j \gamma_{11} \quad i,j=1,\ldots,k,$$

by substituting for $\gamma_{i1} = \theta^i \gamma_{11}$.

Similarly from (K) we can show that,

(N) $$\gamma_{pq} = \theta^p \theta^q \gamma_{NN}, \; p,q = k+1,\ldots,N$$

where, $\frac{\alpha_p}{\alpha_N} = \theta^p$ for all $p = k+1,\ldots,N$.

Consider (L) again, so that

$$\gamma_{jm} = \theta^{j} \ \gamma_{1m} \quad m=1,\ldots,N$$

Now limit m=p where p=k+1,...,N, then

(O) $$\gamma_{jp} = \theta^{j} \gamma_{1p} = \theta^{j} \gamma_{p1} \quad \text{, from symmetry, } \gamma_{1p} = \gamma_{p1}.$$

Consider (K), and allowing for q=N, we get,

$$\frac{\alpha_p}{\alpha_N} = \frac{\gamma_{p1}}{\gamma_{N1}} = \ldots\ldots = \frac{\gamma_{p1}}{\gamma_{N1}} = \ldots\ldots = \frac{\gamma_{pN}}{\gamma_{NN}} = \theta^{p},$$

from which we get,

(P) $$\gamma_{p1} = \theta^{p}\gamma_{N1} \quad \text{, by choosing } 1 = 1 \text{ above.}$$

From (O) and (P), we see that,

(Q) $$\gamma_{jp} = \theta^{j} \gamma_{p1} = \theta^{j} \theta^{p} \gamma_{1N}$$

Let us substitute (M), (N) and (θ) into the vectors $\tilde{\alpha}$ and matrix $\tilde{\gamma}$. So that, we get, $\tilde{\alpha} = (\alpha_1 \ldots \alpha_k)$, $\tilde{\gamma} = [\gamma_{ij}]$, so, $\tilde{\alpha} = \alpha_1(\theta^{'} \ldots \theta^{k})$,

$$\tilde{\gamma} = \tilde{\gamma}_{11}\begin{bmatrix} \theta^{'}\theta^{'} \ldots & \theta^{'}\theta^{k} \\ \vdots & \vdots \\ \theta^{k}\theta^{'} \ldots & \theta^{k}\theta^{k} \end{bmatrix} = \gamma_{11}\begin{pmatrix} \theta^{'} \\ \\ \theta^{k} \end{pmatrix} (\theta^{'} \ldots k)$$

note that, $\tilde{\gamma} = \gamma_{11} \tilde{\theta} \tilde{\theta}'$

$\tilde{\alpha} = \alpha_1 \ \tilde{\theta}'$, where $\tilde{\theta}' = (\theta^{'} \ldots \theta^{k})$.

Also note that the cross-partials matrix, $\tilde{\gamma} = [\gamma_{jp}] = \gamma_{1N}\begin{bmatrix} \theta^{1}\theta^{k+1} \ldots & \theta^{1}\theta^{N} \\ \vdots & \vdots \\ \theta^{k}\theta^{k+1} \ldots & \theta^{k}\theta^{N} \end{bmatrix}$

can be written as, $\bar{\gamma} = \tilde{\theta}\hat{\theta}'$, where, $\theta' = (\theta^{k+1} \ldots \theta^{N})$. Similarly,

$\alpha = \alpha_N (\theta^{k+1} \ldots \theta^{N}) = \alpha_N \hat{\theta}'$ and $\hat{\gamma} = \gamma_{NN} \ \hat{\theta}'\hat{\theta}$, where

$\hat{\alpha} = (\alpha_{k+1}, \ldots, \alpha_N)$ and $\gamma = [\gamma_{pq}]$.

The quadratic expression could be written as,

$$y = \alpha_0 + \tilde{\alpha}'\tilde{X} + \tilde{X}' \tilde{X} + \alpha' X + \hat{X}'\hat{\gamma}\hat{X} + \tilde{X}'\bar{\gamma}\hat{X}$$

$$= \alpha_0 + \alpha_1(\tilde{\theta}'\tilde{X}) + \gamma_{11}\tilde{X}'(\tilde{\theta}\tilde{\theta}')\tilde{X} + \alpha_N(\hat{\theta}'\hat{X}) + {}_{NN}\hat{X}'(\hat{\theta}\hat{\theta}')\hat{X}$$

$$+ \gamma_{1N}\tilde{X}'(\tilde{\theta}\hat{\theta}')\hat{X}$$

$$= \alpha_0 + \alpha_1(\tilde{\theta}'\tilde{X}) + \gamma_{11}(\tilde{\theta}'\tilde{X})'(\tilde{\theta}'\tilde{X}) + \alpha_N(\hat{\theta}'\hat{X}) + \gamma_{NN}(\hat{\theta}'\hat{X})'$$

$$(\hat{\theta}'\hat{X}) + \gamma_{1N}(\tilde{\theta}'\tilde{X})'(\hat{\theta}'\hat{X})$$

This expression is recognized as a quadratic in $(\tilde{\theta}'\tilde{X})$ and $(\hat{\theta}'\hat{X})$. Thus, under Non-linear Restrictions, we have, $G(X_1,\ldots,X_N)$ expressed as a quadratic in linear functions $\tilde{\theta}'\tilde{X}$ and $\hat{\theta}'\hat{X}$.

Q.E.D.

✡ Lemma: If $X_1 = \ln X_1 \ \forall\ 1$, then $\theta' X$ is a Cobb-Douglas process function so that $\ln\tilde{f} = \tilde{\theta}'\tilde{X}$, etc. and writing the quadratic expansion of $\ln Y$, in terms of these Cobb-Douglas forms, we get,

$$\ln Y = \alpha_0 + \alpha_1 \cdot (\tilde{\theta}'\tilde{X}) + \gamma_{11}(\tilde{\theta}'\tilde{X})' \cdot (\tilde{\theta}'\tilde{X})$$

$$+ \alpha_N \cdot (\hat{\theta}'\hat{X}) + \gamma_{NN}(\hat{\theta}'\hat{X})' \cdot (\hat{\theta}'\hat{X})$$

$$+ \gamma_{1N}(\tilde{\theta}'\tilde{X})' \cdot (\hat{\theta}'\hat{X}).$$

i.e. TRANSLOG: $\ln y = \alpha_0 + \alpha_1 \ln\tilde{f} + \gamma_{11}(\ln\tilde{f})'(\ln\tilde{f})$

$$+ \alpha_N \ln\hat{f} + \gamma_{NN}(\ln\hat{f})'(\ln\hat{f})$$

$$+ \gamma_{1N}(\ln\tilde{f})'(\ln\hat{f}).$$

Theorem 3A

If $G(x_1,\ldots,x_N)$ is quadratic in x_i's and if G is weakly separable with Nonlinear Separability Restrictions, then G is quadratic in linear functions, f and g of $\tilde{x}'$s and $\hat{x}'$s, respectively. The Elasticity of substitution between f and g is variable but finite, the elasticity of substitution between $X^{'}$s or $X^{'}$s is however infinite.

<u>Proof</u>: $$\sigma_{fg} = \frac{-\phi_f\phi_g\,(f\phi_f + g\phi_g)}{fg\left\{\phi_{ff}\,\phi_g^2 - 2\cdot\phi_{fg}\phi_f\phi_g + \phi_{gg}\phi_g^2\right\}} \qquad \ldots\ldots\ldots\ldots(1)$$

Note that, from <u>Theorem 2 (PART II)</u>,

$$\phi = \alpha + \beta_1 f + \beta_2 f^2 + \gamma_1 g + \gamma_2 g^2 + \delta fg$$

where, f,g are linear in $X^{'}$s and $X^{'}$s, respectively.

Also,
$$\phi_f = \beta_1 + 2\beta_2 f + \delta g, \qquad \phi_{ff} = 2\beta_2$$
$$\phi_g = \gamma_1 + 2\gamma_2 g + \delta f, \qquad \phi_{gg} = 2\gamma_2$$
$$\phi_{fg} = \delta.$$

The terms within $\{\cdots\}$ brackets must be negative (i.e. non-zero) for convexity of ϕ- isoquandts in f-g space. Therefore, we have, σ_{fg} a finite function of f and g in general. The linear f or g however have infinite elasticities of substitution between $\tilde{X}^{'}$s or $\hat{X}^{'}$s, respectively as shown in <u>Theorem 3B</u>.

<u>Theorem 3B</u>

If G $x_1, \ldots, x_N$ is quadratic, then a linear WS restriction implies that G is linear in quadratic process functions. i.e. $G(\cdot\cdot) = a + bf(\tilde{x}) + cg(x)$ where a,b,c, are parameters and $\tilde{x}$, x are vectors representing two separable groupds of process inputs. Thus linear WS restriction implies that the elasticity of process-substitution σ_{fg} is infinitely large.

<u>Proof</u>: The elasticity of process-substitution is shown in (1), where, $G(\cdot\cdot) = \phi(f,g)$ and $f = f(\tilde{x})$ and $g = g(x)$

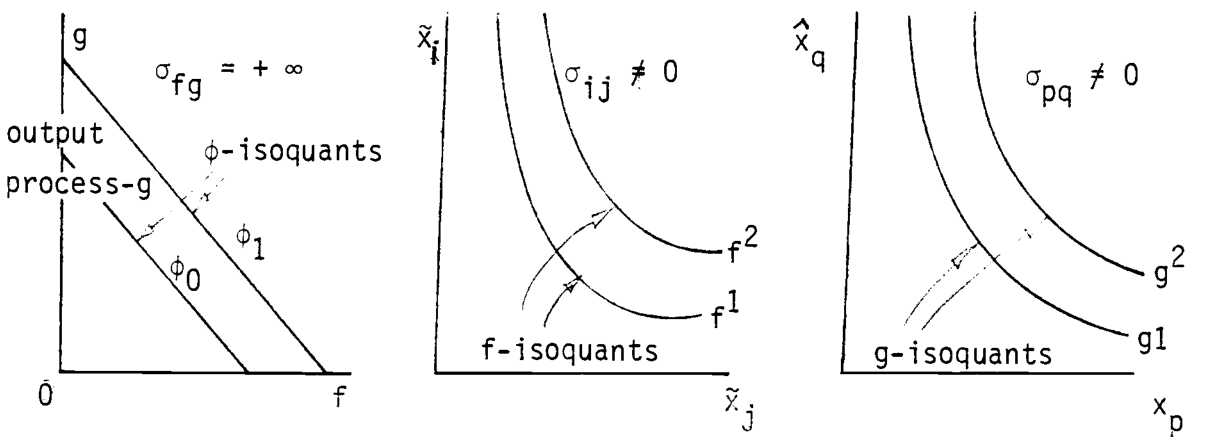
are process functions, and $\{\cdots\}$ is negative. From Theorem 2 (PART-I) we know that linear WS restriction gives rise to, $\phi = a+bf(\tilde{x}) + cg(\hat{x})$, so that

$$\phi_f = b,\ \phi_g = c \text{ and } \phi_{ff} = \phi_{gg} = \phi_{fg} = 0.$$

Thus, $$\sigma_{fg} = \frac{+\ bc\ (bf + cg)}{fg\ (0)} = +\ \infty.$$

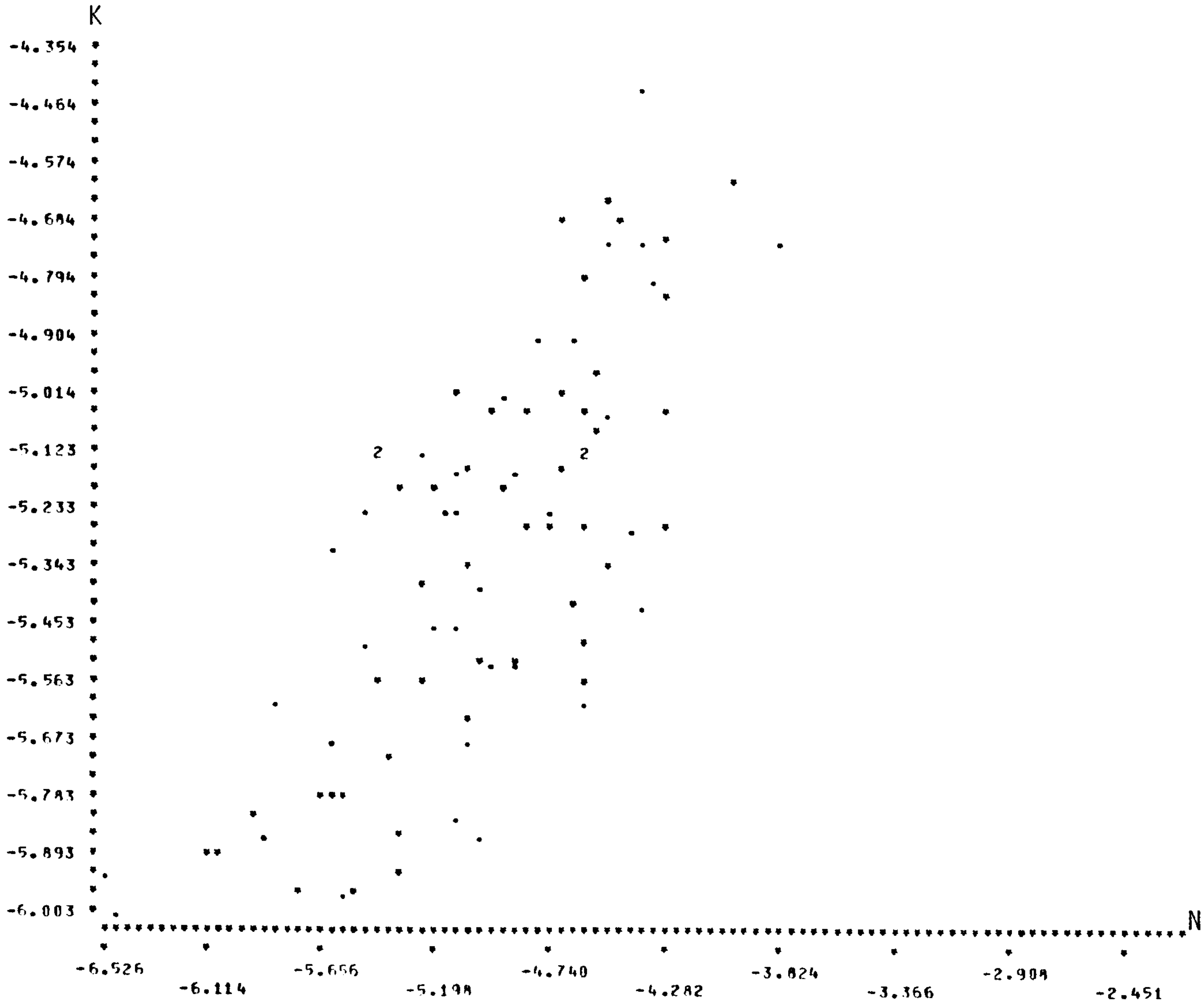
Q.E.D.

<u>Note</u>: The ϕ-isoquandt in f-g space is straight lines with negative slopes as shown below contrasted with f and g-isoquandts.



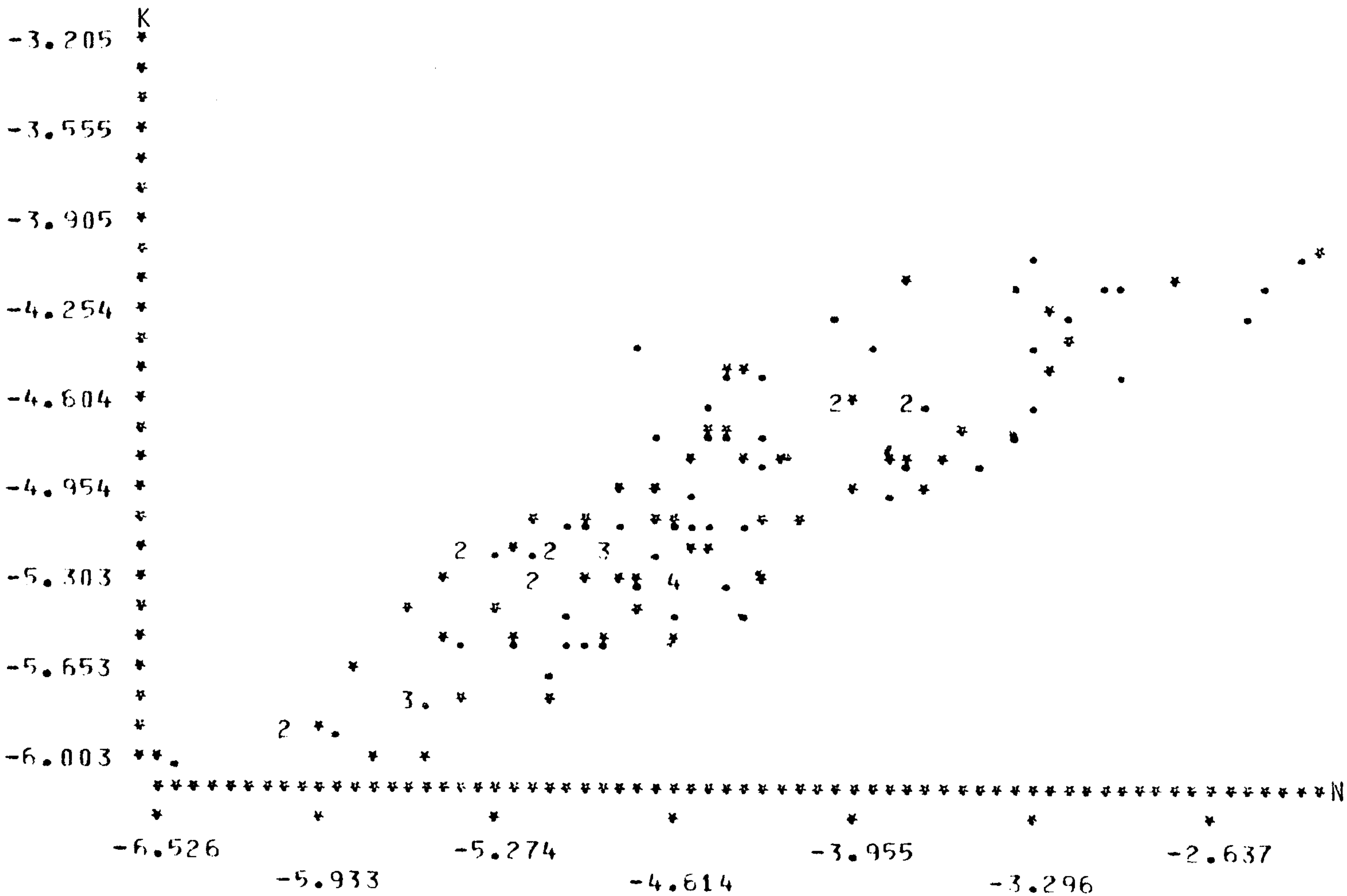
APPENDIX B

Graph 1: Capital Service K vs. Labor Service N - Eastern Oregon (Logarithms of $1000/acre)



APPENDIX B

Graph 2: Capital Service K vs. Labor Service N
All Oregon (logarithms of $1000/acre)



APPENDIX C

List of Weather Stations Chosen

County	Station	County	Station
Baker	Baker CAA Airport	Linn	Albany
Benton	Corvallis	Malheur	Malheur Br. Exp. Stn.
Clackamas	Canby	Marion	Salem
Clatsop	Astoria AP	Multnomah	Bonneville Dam
Coos	North Bend	Polk	Dallas
Crook	Prineville	Sherman	Kent
Curry	Brookings	Tillamook	Tillamook
Deschutes	Redmond	Umatilla	Pendleton
Douglas	Roseburg	Union	LeGrande AP
Gilliam	Arlington	Wallowa	Wallowa
Grant	John Day	Wasco	The Dalles
Harney	Burns WB City	Washington	Canary
Hood River	Pardale	Wheeler	Fosill
Jackson	Medford Exp. Station	Yamhill	McMinville
Jefferson	Madras		
Josephine	Grants Pass		
Klamath	Klamath Falls AP		
Lake	Paisley		
Lane	Eugene		
Lincoln	Newport		

APPENDIX D

Data Summary of Log-transformed Variables

a) Eastern Oregon

Variables	Means	Standard Deviations
Y	3.58	0.43
I	-2.96	2.10
F	-2.54	0.91
L	-3.85	0.71
K	-4.67	0.41
P	5.18	0.19
T	3.85	0.46

Correlation Matrix

	Y	I	F	L	K	P	T
Y	1.00	-0.14	0.48	-0.19	-0.14	0.28	0.11
I		1.00	-0.32	-0.52	-0.42	-0.80	-0.45
F			1.00	-0.11	-0.11	0.21	0.18
L				1.00	0.74	0.08	0.08
K					1.00	0.06	0.20
P	symmetrical					1.00	0.04
T							1.00

APPENDIX D (Contd.)

Data Summary of Log-transformed Variables

b) Western Oregon

Variables	Means	Standard Deviations
Y	3.50	0.34
I	-2.68	2.71
F	-3.46	1.12
L	-4.93	0.72
K	-5.26	0.43
P	2.44	0.45

Correlation Matrix

	Y	I	F	L	K	P
Y	1.00	0.28	0.76	0.24	0.50	0.12
I		1.00	0.45	0.41	0.59	-0.02
F			1.00	0.27	0.62	-0.01
L				1.00	0.81	0.32
K	Symmetrical				1.00	0.26
P						1.00

APPENDIX E

T.S.P. Output

TIME SERIES PROCESSOR D.P. VERSION 3.3 OCT,1977 CYBER 73 81/04/27. 15.35.34. RUN55

EQUATION 1
****
ORDINARY LEAST SQUARES

DEPENDENT VARIABLE: Y

RIGHT-HAND VARIABLE	ESTIMATED COEFFICIENT	STANDARD ERROR	T-STATISTIC
P	.136729	.511437E-01	2.67344
F	.196576	.256568E-01	7.66177
I	.589921	.124186	4.75031
K	.269431	.873099E-01	3.08591
KI	.105930	.217784E-01	4.86398
C	5.27635	.449438	11.7399

LOG OF LIKELIHOOD FUNCTION = 21.1844
R-SQUARED = .6877
DURBIN-WATSON STATISTIC (ADJ. FOR 4. GAPS) = 1.7481
SUM OF SQUARED RESIDUALS = 3.18355
STANDARD ERROR OF THE REGRESSION = .197038
SUM OF RESIDUALS = .343903E-11
NUMBER OF OBSERVATIONS = 88.
MEAN OF DEPENDENT VARIABLE = 3.49789
F-STATISTIC(5., 82.) = 36.1178

APPENDIX E
(Continued)

ESTIMATE OF VARIANCE-COVARIANCE MATRIX OF ESTIMATED COEFFICIENTS

	P	F	I	K	KI	C
P	.261568E-02	.152512E-03	.140747E-02	-.904958E-03	.230927E-03	-.102607E-01
F	.152512E-03	.658269E-03	-.966841E-03	-.127941E-02	-.165693E-03	-.496513E-02
I	.140747E-02	-.966841E-03	.154221E-01	.536967E-02	.269605E-02	.229432E-01
K	-.904958E-03	-.127941E-02	.536967E-02	.762302E-02	.100511E-02	.374041E-01
KI	.230927E-03	-.165693E-03	.269605E-02	.100511E-02	.474300E-03	.436429E-02
C	-.102607E-01	-.496513E-02	.229432E-01	.374041E-01	.436429E-02	.201995
	1	2	3	4	5	6

APPENDIX F

Data Set in Logarithms of Variables

The variables are given as follows. The row with five elements represents Y, I, F, N and K, respectively. The next row with two elements represents the corresponding T and P, respectively.

Y T	I P	F	N	K
-1.2114557	-.01584552	-4.5117851	-4.3024779	-5.4963679
4.3174881	1.9459101			
-.83715977	-.58746389	-3.0292034	-4.3044121	-5.0313566
1.3862944	1.9459101			
-.91406919	-.14660578	-2.7230173	-4.6767571	-4.6399106
3.2188758	1.9459101			
-1.0944523	-6.5206315	-4.6110106	-5.3756432	-5.9095270
5.0172798	2.0794415			
-1.5751248	-2.6136405	-5.8190933	-4.5219567	-5.3195007
4.9836066	2.1972246			
-2.3984625	-.95092527	-7.0501225	-5.0355665	-5.8488636
4.5432948	2.0794415			
-1.0652763	-1.9459101	-4.3438054	-1.9925722	-4.1156164
4.6347290	3.8066625			
-.78559346	-.79090346	-2.8960582	-4.5139145	-5.1008726
4.6347290	1.9459101			
-1.2759614	-1.0562379	-3.4953384	-4.2356326	-5.2339010
4.5325995	2.4849066			
-1.4934544	-2.3987596	-5.3120629	-5.1066384	-5.6062066
4.3820266	2.3025851			
-.53433943	-.17421476	-2.6964120	-3.3330855	-4.7086257
4.8520303	1.9459101			
-1.1967259	-6.9361395	-5.6527947	-5.3693296	-5.8378352
4.8202816	2.4849066			
-1.1363101	-11.479383	-4.3285035	-5.5962485	-5.9622718
4.8202816	2.3025851			
-.91483943	-3.9114734	-3.9953566	-4.6366762	-5.5355479
5.1704840	2.3025851			
-1.0463236	-2.3725165	-4.0565466	-5.1166062	-5.4413481
4.5538769	2.9957323			
-1.2355626	-1.7439758	-3.5317650	-5.0562283	-5.3132601
3.5263605	2.6390573			
-1.2144975	-4.9481232	-3.5459303	-4.6119475	-5.5893696
5.1704840	2.4849066			
-1.4252366	-4.2702271	-5.6266685	-4.7542593	-5.2315017
2.6390573	2.5649494			
-1.7913842	-3.2035627	-2.7764984	-4.3346146	-5.2570169
5.0369526	3.8286414			
-1.3848371	-5.2227350	-2.8721672	-3.7238094	-4.7054974
5.1704840	3.7376696			
-.69314718	0.	0.	-3.2560702	-4.3383639
5.1929569	3.9318256			
-1.2092026	-5.9026333	-5.9026333	-3.8877139	-4.6558962
5.1704840	4.3174881			

Data Set in Logarithms of Variables (continued)

-1.0986123	-1.0986123	-1.0986123	-4.0212756	-4.4019539
5.5645204	4.1431347			
-1.5861360	-3.5835189	-3.5160777	-4.3264335	-4.6525250
4.6362819	3.4011974			
-1.6393940	-1.5116740	-4.0375917	-3.1312313	-4.4376759
4.3675345	2.3903718			
-1.4199676	-.67998910	-2.7140947	-3.3706417	-4.4649436
4.8263137	3.4965076			
-1.6049677	-3.7210487	-2.9347454	-3.7070427	-4.6997190
5.0304379	3.6375862			
-1.6853663	-4.1706084	-3.1643001	-4.7939152	-5.4157994
5.1704840	3.8230414			
-1.4363042	-3.9024560	-3.3043671	-3.6047504	-5.0529902
5.1647860	3.7612001			
-1.3602827	-5.4134590	-1.9591117	-2.5523309	-4.6162200
5.3612922	4.2626739			
-1.5984859	-4.4816426	-3.2942626	-4.1142566	-5.3014514
5.0369526	4.0775374			
-1.6040248	-4.4494179	-3.4537736	-3.6335907	-4.9788351
5.0369526	3.9318256			
-1.1322939	-6.1151919	-3.3704257	-3.3021057	-5.1742163
4.8903491	3.7841896			
-.97347505	-.45912907	-3.4158242	-4.3427512	-5.2446477
3.3322045	2.1972246			
-.67549007	-.38310602	-2.5834644	-4.5591739	-4.9505366
2.3978953	1.9459101			
-1.0547136	-.73191802E-01	-2.1273155	-4.5211979	-4.7087855
4.0775374	1.6094379			
-.90950723	-7.3627523	-4.2738729	-5.5491567	-5.9410599
5.0238805	1.9459101			
-1.2972143	-3.5099855	-4.6746962	-4.7808211	-5.2336262
4.5108595	2.4849066			
-2.3555007	-.95594966	-6.8167359	-5.0361459	-5.6154504
4.7095302	2.3025851			
-1.1274306	-4.6539604	-2.9492123	-2.5310975	-4.2152471
4.8121844	3.4965076			
-.76615592	-.33850722	-2.6016933	-4.5039173	-5.0904844
4.4543473	2.0794415			
-1.2693765	-.65533123	-3.2527510	-4.5258119	-5.2353625
4.5951199	1.9459101			
-1.3857344	-1.3703606	-4.5207175	-5.0960359	-5.6592331
4.7535902	1.7917595			
-.38996234	-.12062594	-2.3193801	-4.0283468	-4.5352155
5.1179936	2.1972246			
-.91767774	-6.2292303	-4.5287833	-5.6688715	-5.7607305
4.9972123	2.3978953			
-1.0525942	-11.454523	-4.2232357	-5.7357303	-5.9436751
4.9558271	2.1972246			
-.61070279	-4.0454466	-3.7972802	-4.9094634	-5.5094906
5.2882670	2.4849066			
-.76648204	-2.8476320	-3.6354659	-5.2362831	-5.3573969
5.1059455	2.3903713			
-1.1950043	-1.8947506	-3.2356199	-5.1365370	-5.2323661
3.2133759	2.9444390			
-1.0498702	-4.9613795	-3.3607520	-4.5226693	-5.4669819

Data Set in Logarithms of Variables (continued)

5.4110401	2.1972246			
-1.2643539	-3.3239117	-3.5304325	-4.5730417	-5.0535674
3.4339872	2.3978953			
-1.7082812	-4.7694732	-2.6045095	-4.3116856	-5.1708828
5.2832037	3.4965076			
-1.3035224	-5.9436341	-2.8608905	-3.6593801	-4.7449679
5.3276762	4.2046926			
-2.5538995	-1.6094379	-1.6094379	-3.2577512	-4.4095914
5.1474945	3.8066625			
-1.0268639	-1.6332486	-2.5257286	-3.7562413	-4.5813200
5.3798974	4.3040651			
-1.5694154	-4.2877160	-3.5457766	-4.4115678	-4.4254078
5.2781147	3.2580965			
-1.3073443	-2.1301416	-2.7063171	-3.2172132	-4.2920085
4.3040210	2.4849066			
-1.6985361	-1.6796422	-4.7706846	-2.7319988	-4.0367055
5.2781147	3.0445224			
-1.5257110	-2.9259302	-2.3393057	-3.3310524	-4.9449068
5.2832670	3.4965075			
-1.7008777	-3.7675688	-2.5163769	-4.6745766	-5.1979874
5.3278762	2.0794415			
-1.3040684	-4.2350323	-2.7478045	-3.5447951	-4.8185174
5.2625912	3.5553481			
-1.1155740	-6.3866793	-2.3978953	-2.2197996	-4.1022358
5.5093883	4.3820266			
-1.4276404	-5.3931224	-2.6136132	-4.3164836	-5.1898607
5.1873858	3.7612001			
-1.4397477	-4.7563213	-2.9032613	-3.9372489	-4.8685791
5.2832037	4.3944492			
-.98025157	-5.3534766	-2.7885272	-4.1762584	-4.9823372
5.1673658	3.6375862			
-.64482336	-.53510170	-3.4095431	-4.9313855	-5.1718441
4.5538769	2.7080502			
-.55342663	-.14590998	-2.0230645	-4.7268950	-5.1336671
2.3978953	2.4849066			
-.48253679	-.88375036	-2.3146855	-4.5970593	-4.6642404
2.3978953	2.3025851			
-1.1515514	-11.347485	-4.6212512	-5.5984759	-5.7474386
4.9558271	2.4849066			
-1.5592673	-2.6695059	-3.8784662	-4.9001315	-5.1562247
4.6634391	2.7080502			
-1.7936655	-.37967118	-3.7640824	-5.4092464	-5.6560040
4.5325995	2.7725887			
-.55859486	0.	-3.0910425	-2.4311710	-4.1299626
4.8675345	3.9120230			
-.56598594	-.65956409	-2.4727008	-4.4167463	-4.7079577
4.1588831	2.3025851			
-.79013379	-.54085351	-2.9914125	-4.6422449	-5.0210979
4.3174881	2.9444390			
-1.0090605	-2.3550167	-3.8343765	-5.2599674	-5.5304377
4.4426513	2.7725887			
-.39275856	-.12935187	-2.5051390	-4.3125659	-4.6920819
4.0823019	2.5649494			
-1.1757383	-6.0197993	-4.5006845	-5.5123555	-5.6563903
4.6751973	2.5649494			

Data Set in Logarithms of Variables (continued)

-1.1225174	-11.410282	-4.0536371	-5.9598144	-5.7696978
4.9698133	2.5649494			
-.78638640	-4.0152885	-3.5072935	-5.0377066	-5.4949265
5.2832037	2.3025851			
-.76095259	-2.3791742	-2.9000423	-5.1472593	-5.2299562
4.7791235	3.0910425			
-.61755184	-1.7918169	-2.9860055	-5.1226392	-5.1528071
3.9702919	3.1354942			
-1.1944620	-4.4907077	-3.5173968	-4.4023487	-5.2631111
5.3518581	2.8903718			
-1.4144760	-5.2529561	-5.3439279	-4.9394336	-5.0203861
3.4011974	2.7725887			
-1.2767923	-3.0563162	-1.8159881	-4.3093689	-5.3020008
5.0998604	3.8066625			
-1.0125543	-5.3592436	-2.6011830	-3.3602574	-4.5971989
5.1059455	4.4543473			
-2.1972246	-4.0943446	-4.0943446	-3.0112901	-4.4431013
5.0689042	3.7612001			
-.54865560	-5.2526902	-1.3308646	-4.0256062	-4.5557952
5.2522734	4.3040651			
-1.3600487	-2.5366604	-3.6098321	-4.3130291	-4.4509266
5.2832037	3.7612001			
-1.2423872	-2.6355238	-4.2573842	-3.1799392	-4.2183345
4.9904326	3.3672958			
-.90367017	-.78845736	-3.7841896	-3.4185600	-4.1354683
5.0998664	3.6375862			
-1.0657293	-2.8680179	-1.9664954	-3.7923962	-4.7972857
5.1704840	4.0604430			
-1.1301110	-3.7612001	-3.7612001	-4.7348473	-4.3305447
5.3375381	4.3438054			
-1.1765251	-3.5196971	-2.0946873	-4.9256359	-5.4220832
5.0998664	3.8066625			
-.98749180	-3.2943066	-2.2731601	-3.3635225	-4.7046457
4.9767337	3.7612001			
-1.2095167	-4.5643482	-2.8903718	-2.2063925	-3.9432634
5.3565863	4.4426513			
-1.2718209	-6.9144550	-2.2949437	-4.6598070	-5.2841168
5.3181200	3.9889840			
-1.1535824	-3.9715420	-2.4601252	-3.7296129	-4.8665314
5.1059455	4.4188406			
-.77862697	-4.1946593	-2.4839087	-4.3980523	-5.0267854
4.9836066	3.3712010			
-.84989751	-.43165607	-2.8359246	-5.0949477	-5.1356386
4.6202816	2.3978953			
-.53619029	-.23905901	-2.0483510	-4.5413283	-5.0472263
4.2341065	2.3025851			
-.27332205	-.15332665	-1.9763307	-4.5322242	-4.6274472
4.5325995	2.3025851			
-1.0236866	-4.8814692	-4.1762978	-5.8930102	-5.8460874
5.1416636	1.9459101			
-1.4638394	-4.4074812	-1.9462910	-5.2076274	-5.1635496
4.8040210	2.6390573			
-1.4560027	-.63975065	-4.4773368	-5.6325982	-5.7520721
4.6202816	2.4849066			
-.62825748	-1.2909047	-2.2284477	-4.2350787	-4.6069590

Data Set in Logarithms of Variables (continued)

4.6634391	2.6390573			
-.67133858	-.36701652	-2.3373143	-4.7461938	-4.9938496
4.9126549	2.7725887			
-1.0215021	-1.4046384	-4.1443655	-5.4805296	-5.4717501
4.6051702	2.4849066			
-.28747628	-.24144010	-2.0421382	-4.3593302	-4.7758929
5.0625950	2.3025851			
-1.1135929	-3.2381083	-4.0497136	-6.0729154	-5.8607503
5.0369526	2.7080502			
-1.3084728	-4.8202472	-4.0812905	-6.4973331	-6.0027848
5.1929569	2.3025851			
-.63811713	-2.7537547	-3.1963436	-5.0025440	-5.5113199
5.0304379	2.5649494			
-.54995528	-1.6181432	-2.8560790	-5.6105237	-5.3037456
4.9052748	2.7725887			
-.91835989	-1.5635634	-2.7716165	-5.4843725	-5.2262237
4.2484952	2.7080502			
-1.0149561	-3.5156427	-3.0648351	-4.3918701	-5.4018623
5.3033049	2.3978953			
-.95400375	-3.3279497	-4.4406467	-4.9591938	-4.9990123
4.8121344	2.7080502			
-1.3516985	-3.5985777	-1.4619778	-4.5219942	-5.0815244
5.3936275	3.7841896			
-.87651730	-4.8775148	-2.0632290	-3.3032443	-4.3294182
5.2417470	4.1431347			
-.64159711	-5.1761497	-1.8803129	-3.7938066	-4.5614534
5.4722707	4.1431347			
-1.0986123	-3.9120230	-3.9120230	-3.7869882	-4.0550443
5.5872487	4.1431347			
-.68146172	-2.5821867	-2.0809963	-4.4729700	-4.4873838
5.2417470	3.4339872			
-1.0016525	-.91080459	-2.1742628	-3.0215595	-4.1356155
5.0239805	3.1354942			
-.69737646	-.83665637E-01	-2.3333566	-3.3240226	-4.0185582
5.0498560	3.4965076			
-.91032427	-3.1451560	-1.7326823	-3.5542567	-4.7227911
5.3033049	3.8712010			
-2.1431574	-2.7080502	-2.0149030	-4.0557141	-4.2024304
5.1179938	4.2904594			
-.97070031	-3.3084080	-1.7265063	-4.3008635	-5.3202551
5.2522734	3.6888795			
-.84020271	-2.7226971	-2.0661322	-3.5546806	-4.6489062
5.0369526	3.6888795			
-.65891921	-5.3318825	-2.0706824	-2.2335105	-3.9349007
5.5606816	4.2046925			
-1.1147222	-3.7851579	-2.0981519	-4.5419967	-5.2256836
5.1119878	3.8712010			
-1.0413345	-3.3376336	-2.0754522	-3.7596192	-4.7728394
5.0937502	4.4067192			
-.53220031	-3.7821757	-2.1919373	-4.3446140	-4.9844102
4.9272537	3.7841896			
-.79189188	-.60202839	-2.3550710	-5.3370519	-5.1643891
4.5747110	2.0794415			
-.52573664	-.31775041E-01	-1.7234565	-4.5103102	-4.7746055
6.3099183	4.3943446			

Data Set in Logarithms of Variables (continued)

-.90531471E-01	-.52095112E-01	-.93610045	-4.4173028	-4.4153456
4.5643482	1.6094379			
-1.1537942	-3.9251399	-4.1314529	-6.1166757	-5.8594859
5.1647860	1.7917595			
-1.2167867	-3.1373623	-2.4630649	-5.2406294	-5.1128715
4.5643482	2.1972246			
-2.0105133	-1.3054953	-3.2354051	-5.8558206	-5.5900087
4.7095302	1.9459101			
-.60946768	-.70098910	-1.8523841	-4.8161366	-4.8860341
4.5643482	1.7917595			
-.51372154	-.13271549	-2.1357431	-4.8337765	-5.0257769
4.7706846	2.3025851			
-1.3565733	-1.7696621	-3.6590195	-5.4640047	-5.5265336
4.7874917	2.1972246			
-.28615591	-.35243354	-1.8760334	-4.4719240	-4.6601758
4.8903491	1.9459101			
-1.0994649	-2.4279811	-3.5994457	-5.2238070	-5.4363901
4.9558271	2.3978953			
-1.2378863	-7.3218571	-3.9410918	-6.5261732	-5.9170268
4.9487599	2.5649494			
-.54564569	-2.4640372	-3.0099822	-5.0410882	-5.3714774
4.6539604	2.3025851			
-.59480523	-1.5642735	-2.5165444	-5.4643541	-5.1175500
4.7874917	2.7725887			
-1.0023895	-1.7435859	-2.6899290	-5.4412286	-5.1126625
4.4886364	2.3978953			
-.98881697	-3.4539788	-3.1496651	-4.6517666	-5.3821520
5.2882670	2.8332133			
-1.3285991	-3.9132223	-3.5347859	-5.1522593	-4.9835057
4.7874917	2.0794415			
-1.0400291	-3.6360594	-1.5598841	-4.5335473	-4.9642099
5.1873856	3.9318256			
-.71937203	-3.5314592	-2.1511990	-3.2912244	-4.1922216
5.1590553	4.4773368			
-.41397130	-4.1713056	-2.3150076	-3.9159740	-4.3349422
5.5333895	4.2904594			
-.82512460	-3.6085650	-2.1439015	-4.5211394	-4.5854721
5.1704840	3.5263605			
-.95086595	-1.2858399	-2.4345126	-3.0424262	-4.1098167
4.7957905	3.0910425			
-.70736786	-2.0633708	-1.6577686	-4.2516364	-4.7569725
5.1817836	4.4773368			
-.38665378	-3.1580834	-1.6531948	-4.9782904	-5.1983683
5.1590553	3.3501476			
-.77513196	-2.6433255	-1.9814465	-3.7271707	-4.5917448
4.9767337	3.8286414			
-.68026976	-7.1196356	-1.6021827	-2.3361533	-4.0013338
5.4065339	4.4308168			
-.93679347	-3.9640480	-1.9502256	-5.0118570	-5.0560277
5.1647860	4.0430513			
-.95092068	-4.9043887	-1.9249110	-3.9720348	-4.5069719
5.0698809	4.3820266			
-.49283684	-4.0433030	-1.9734765	-4.7230482	-4.6927355
4.9487599	3.9120230			