AN ABSTRACT OF THE THESIS OF

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Title: FINITE ELEMENT ANALYSIS OF CULVERTS UNDER EMBANKMENTS WITH TIME DEPENDENT AND NON-LINEAR PROPERTIES

Abstract approved: Redacted for privacy

William H. White

A computer program was developed for analyzing culverts under earth embankments constructed of compacted cohesive soil using the finite element method. The culvert-embankment system is represented as a two-dimensional plane strain problem. The plane of investigation is taken normal to the culvert axis. The soil-culvert interface is assumed to be perfectly rough, with no possibility of slippage. The analysis is divided into two separate parts, to determine culvert forces and deformations during construction and after construction.

The culvert is analyzed for forces and deformations at the end of every construction step due to the gravity effects of the fill material. The construction pore pressure in the embankment is estimated by the Hilf method, and zero dissipation of pore pressure during
embankment construction is assumed.

It is assumed that the soil is saturated and drainage occurs in the vertical direction after the embankment construction is finished. The culvert forces and deformations are calculated at any time after construction by the modified finite element method, assuming that changes in the culvert-embankment system occur because of consolidation of the embankment.

The nonlinear, stress-dependent properties of soil are included in the study by considering the nonlinear problem as piecewise linear during each construction step or time increment.

The computer program can be used to analyze circular or elliptical culverts under homogeneous embankments placed on a rigid foundation. It has been found that for high fills the program requires considerable computer memory locations. For complete analysis of culverts after construction, running time for the program is large.
Finite Element Analysis of Culverts Under Embankments with Time Dependent and Non-Linear Properties

by

Voravit Lertlaksana

A THESIS submitted to Oregon State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy June 1973
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Date thesis is presented ____________
Feb. 22, 1973

Typed by Muriel Davis for Voravit Lertlaksana
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FINITE ELEMENT ANALYSIS OF CULVERTS UNDER EMBANKMENTS WITH TIME DEPENDENT AND NON-LINEAR PROPERTIES

I. INTRODUCTION

To successfully design a culvert under an embankment of compacted cohesive soil, it is necessary to have information about culvert forces and deformations which develop during and after construction. Although culverts under earth embankments have been constructed for hundreds of years, it is only with the past few decades that rational methods of structural design of this type of structure have become available. These methods are based on results from field observations and theoretical analyses. They take the time dependent properties of soils into account only empirically.

1.1 Statement and Scope of the Problem

Analyzing the soil-culvert interaction problem is difficult because almost all cohesive soils have time-dependent and non-linear, stress-dependent properties. Even if the soils were ideally linear, stress-independent and time-independent, the problem would still be a very difficult one to solve. The difficulties are threefold. First, the soil-culvert system is a composite one; second, the geometry of the problem is such that straight-forward solutions are not easy to find; and third, the boundary conditions between the fill and both the foundation and the culvert are complicated.
The object of this research was to combine existing theories methods in mechanics in a way which would make them useful for the analysis of culverts under earth embankments. The time-dependent and non-linear, stress-dependent properties of cohesive soils are included in the study. The step-by-step construction process is taken into account.

To deal with the complexity of the problem, certain assumptions have been introduced. They are as follows:

1. The actual three dimensional system can be represented as a two-dimensional plane strain problem. The plane of investigation is taken normal to the culvert axis.
2. The compacted soil is homogeneous and isotropic.
3. The time-dependent characteristics of soil are controlled by the dissipation of excess pore pressures.
4. No drainage is allowed during the embankment construction.
5. Drainage occurs in a vertically upward direction only, irrespective of the shape of the structure.
6. The embankment foundation is rigid, impervious and rough.
7. The soil-structure interface is perfectly rough, with no possibility for slip.

The scope of the study included development of a method for analyzing soil-culvert interaction problems and preparation of a
computer program to make the necessary calculations.

An important method of design for culverts was introduced by Marston and Spangler (22). The well known method was based on arching theories and empirical formulas which resulted from experiments and observations of the performance of actual culverts under embankments. The method attempted to determine the vertical loads to which the culverts would be subjected in service and their supporting strengths. There is a great deal of uncertainty regarding the empirical factors used in the method. Most of these factors depend on field conditions and can not be determined in advance. The proposed method is limited to circular, small culverts.

White and Layer (29) proposed ring compression theory for analyzing flexible, ring shaped culverts. The purpose of the theory was to determine the compressive force in the ring. The ring compression approach was based on the assumption that the vertical component of the compression can be computed by taking one-half of the span of the culvert and multiplying it by the height of cover times the soil density.

Brown (4, 5) attempted to determine the pressure distribution around culverts due to the gravity effects of the fill material and study the interface conditions between the culverts and fills by using the finite element method. The investigations were based on the assumptions that the problems can be represented by a plane strain system
and the material properties are linearly elastic. The construction process was included in the studies. Brown found that the distribution and magnitude of predicted pressures were not sensitive to the interface conditions in the rigid culvert case, and the using of the interface condition of no-slip in the flexible culvert case gave a good agreement between the predicted and field measured results. Brown did not consider the time-dependent properties of soil and presented the comparisons between the predicted and field measured results only during construction.

Trollope, Speedie and Lee (26) published field measurements of the vertical pressures acting on a rigid culvert under an earth dam. It was indicated that the pressures increased with time after the construction. He presumed that the pressures on the culvert increased due to continued settlement of the fill with consequently increased arching effects.

1.2 Method of Solution

The finite element method developed by Clough (9) will be used in the study. The basic concept of the numerical method is that a continuum may be represented by an assemblage of a finite number of elements interconnected at the element nodal points. After the stiffness characteristic of each element is obtained, the structure is analyzed by the standard stiffness method (15).

The soil-culvert system is not a continuum so an assumption
concerning the characteristics of the soil-culvert interface is needed. It is assumed that the interface is perfectly rough, with no possibility for slip between the culvert and the soil (4, 5). A symmetrical structure is used in the study in order to keep the number of elements minimum. The soil-culvert system can be composed of curved beam elements and triangular or quadrilateral elements as shown in Figure 1.1. Face 1 is assumed fixed. For the symmetrical case, there is no vertical shear force or deflections in the x direction along face 2. Face 2 must therefore be kept on rollers. If face 3 is assumed to be far from the axis of symmetry oy, the deflections in the x direction along the boundary are negligible. Face 3 can be assumed to be on rollers.

There are two consecutive analyses in the study:

(1) Incremental construction analysis which is carried out in terms of total stresses. It yields the distributions of stresses and excess pore pressures in the fill, and also forces and deformations in the culvert at any stage of construction prior to dissipation of any excess pore pressure resulting from construction.

(2) Time-dependent analysis which starts at the end of construction in terms of effective stresses by taking the results from incremental construction analysis as the initial conditions.
Figure 1.1. Finite element simulation of a soil-culvert system
The basic finite elements used in the study are a two-dimensional plane strain element and a curved beam element. The embankment is idealized by triangular and quadrilateral plane strain elements connected at nodal points. A culvert is approximately represented by a number of circular beam segments connected at nodal points. Since most compacted, cohesive soils have non-linear, stress-dependent stress-strain properties, these properties must be taken into account.

2.1 Plane Strain Matrix Equations

A triangular plate element is used in the study as shown in Figure 2.1. A global coordinate system $x, y$ is selected as a referenced coordinate system. By equating the external virtual work and the internal virtual work due to the nodal forces, body forces and stresses acting in the element, the following matrix equation is obtained:

$$
\begin{bmatrix}
  u_1 \\
  v_1 \\
  u_2 \\
  v_2 \\
  u_3 \\
  v_3 
\end{bmatrix}
= 
\int 
\begin{bmatrix}
  \varepsilon_x & \varepsilon_y & \gamma_{xy} \\
  \sigma_x & \sigma_y & \tau_{xy} \\
  \varphi_x & \varphi_y & \varphi_{xy} 
\end{bmatrix}
\begin{bmatrix}
  F_x \\
  F_y \\
  F_z 
\end{bmatrix}
dV
$$

(2.1)
The volume of the element is indicated by \( V \). The terms \( X_i \) and \( Y_i \) represent the horizontal and vertical components of nodal force of the nodal point \( i \) which takes on values 1, 2 and 3. The body forces \( F_x \) and \( F_y \) are positive if they act in the direction of positive \( x \) and \( y \), respectively. Admissible forms for the inplane element displacements \( u \) and \( v \) are chosen so that displacement continuity is maintained across element boundaries. Therefore, assume as trial functions

\[
\begin{align*}
  u &= \alpha_1 + \alpha_2 x + \alpha_3 y \\
  v &= \alpha_4 + \alpha_5 x + \alpha_6 y
\end{align*}
\]  

(2.2a)  

(2.2b)

Let \( u_i \) and \( v_i \) refer to the \( x \) and \( y \) components of the displacements, respectively, of any node \( i \) of the element. By using the displacement boundary conditions for the horizontal displacement \( u \) at each corner, the following matrix expression is obtained:

\[
\begin{pmatrix}
  u_1 \\
  u_2 \\
  u_3
\end{pmatrix} =
\begin{bmatrix}
  1 & x_1 & y_1 \\
  1 & x_2 & y_2 \\
  1 & x_3 & y_3
\end{bmatrix}
\begin{pmatrix}
  \alpha_1 \\
  \alpha_2 \\
  \alpha_3
\end{pmatrix}
\]  

(2.3)
The constant $a_i$ are obtained by inversion as

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} x_2y_3-x_3y_2 & x_3y_1-x_1y_3 & x_1y_2-x_2y_1 \\ y_2-y_3 & y_3-y_1 & y_1-y_2 \\ x_3-x_2 & x_1-x_3 & x_2-x_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

(2.4)

where $A$ is the area of the triangular element. If Equation (2.4) is rewritten as follows

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

(2.5)

and substituted into Equation (2.2a), the displacement $u$ becomes
where

\[
\begin{align*}
  n_1 &= a_1 + b_1x + c_1y \\
  n_2 &= a_2 + b_2x + c_2y \\
  n_3 &= a_3 + b_3x + c_3y.
\end{align*}
\]

Similarly, the displacement \( v \) may be written

\[
v = \frac{1}{2A} (n_1 v_1 + n_2 v_2 + n_3 v_3) \quad (2.7)
\]

The displacements at any point in the triangular element can be written in terms of the nodal displacements as

\[
\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{2A} \begin{bmatrix} n_1 & 0 & n_2 & 0 & n_3 & 0 \\ 0 & n_1 & 0 & n_2 & 0 & n_3 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix} = [N] \begin{pmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix} \quad (2.8)
\]

The chosen displacement functions in Equation (2.8) guarantees continuity of displacements with adjacent elements because the displacements vary linearly along any side of the triangular element and, with identical displacements imposed at the nodes, the same displacement will exist all along an interface.

By differentiating Equations (2.6) and (2.7) the strains can be found by

\[
\epsilon_x = \frac{\partial u}{\partial x}
\]
The strain-displacement relation can be written as

\[ \epsilon_y = \frac{\partial v}{\partial y} \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]

The strain-displacement relation can be written as

\[
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{pmatrix}
= \frac{1}{2A}
\begin{bmatrix}
b_1 & 0 & b_2 & 0 & b_3 & 0 \\
0 & c_1 & 0 & c_2 & 0 & c_3 \\
c_1 & b_1 & c_2 & b_2 & c_3 & b_3
\end{bmatrix}
\begin{pmatrix}
u_1 \\
u_2 \\
u_3 \\
v_2 \\
v_3
\end{pmatrix}
\]

(2.10)

where \( b_1, c_1 \) etc. are given by Equation (2.5).

Equation (2.10) can be written symbolically as

\[
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{pmatrix}
= [B]
\begin{pmatrix}
u_1 \\
u_2 \\
u_3 \\
v_2 \\
v_3
\end{pmatrix}
\]

(2.11)

The stress-strain relationship for an isotropic material with Young's modulus \( E \) and Poisson's ratio \( v \) in a state of plane strain is

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}
= \frac{E}{(1+v)(1-2v)}
\begin{bmatrix}
1-v & v & 0 \\
v & 1-v & 0 \\
0 & 0 & \frac{1-2v}{2}
\end{bmatrix}
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{pmatrix}
= [D]
\begin{pmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{pmatrix}
\]

(2.12)

Substituting Equation (2.9) into Equation (2.12), the stresses become
If Equations (2.8), (2.11) and (2.12) are substituted into Equation (2.1), a relation between the nodal forces and the nodal displacements is obtained

\[
\begin{bmatrix}
X_1 \\
Y_1 \\
X_2 \\
Y_2 \\
X_3 \\
Y_3 \\
\end{bmatrix} = \left( \int [B]^T [D] [B] dV \right) \begin{bmatrix}
u_1 \\
v_1 \\
u_2 \\
v_2 \\
u_3 \\
v_3 \\
\end{bmatrix} - \int [N]^T \begin{bmatrix} F_x \\ F_y \end{bmatrix} dV \quad (2.14)
\]

After an integration is taken over the volume of the triangular element, Equation (2.14) can be written symbolically as

\[
\{ F \} = [k] \{ \delta \} + \{ F_0 \} \quad (2.15)
\]

where

\[
[k] = h A [B]^T [D] [B]
\]

is the element stiffness matrix of the triangular element with thickness \( h \) and

- \( \{ F \} \) - nodal force vector
- \( \{ \delta \} \) - nodal displacement vector
- \( \{ F_0 \} \) - nodal force vector due to body forces.
2.2 Quadrilateral Element

One of the drawbacks to the triangular element is that if it is very large the determined stresses may not represent the actual stresses anywhere in the element. In order to circumvent this difficulty the quadrilateral element was developed by Wilson (30).

In the quadrilateral element a fictitious nodal point is found shown in Figure (2.2). The coordinates of the nodal point are computed as the average of the four corner point coordinates.

A 8-degree-of-freedom quadrilateral element matrix is formed by first combining the four 6-degree-of-freedom triangular element matrices into a 10-degree-of-freedom element matrix. Using a process of static condensation (32) the two internal displacements are eliminated, resulting in a 8-degree-of-freedom quadrilateral element matrix.
The four triangular element stiffness matrices are combined by the code number technique (15, 18, 25). If the load vectors \( \{F_0\} \) for each of the triangular elements are similarly superimposed, the partitioned matrix equation is obtained

\[
\begin{pmatrix}
F_a \\
0
\end{pmatrix} =
\begin{bmatrix}
 k_{aa} & k_{a5} \\
 k_{a5} & k_{55}
\end{bmatrix}
\begin{pmatrix}
\delta_a \\
\delta_5
\end{pmatrix} +
\begin{pmatrix}
F_{oa} \\
F_{o5}
\end{pmatrix}
\]

(2.16)

where  \( a \) and 5 indicate the quantities associated with the external and internal nodes, respectively.

The zero on the left-hand side of Equation (2.16) is due to the fact that the internal forces balance at the internal node.

By eliminating the internal unknowns \( \{\delta_5\} \) the quadrilateral element matrix becomes

\[
\{F_a\} = [k^*]\{\delta_a\} + \{F_{oa}^*\}
\]

(2.17)

where

\[
[k^*] = [k_{aa}] - [k_{a5}][k_{55}]^{-1}[k_{5a}]
\]

\[
\{F_{oa}^*\} = \{F_{oa}\} - [k_{a5}][k_{55}]^{-1}\{F_{o5}\}
\]

2.3 Curved Beam Element

The culvert is represented by a number of circular beam segments connected at the nodal points. The geometric properties of the circular
beam segment can be determined by using the properties of a circle passing through three consecutive points as shown in Figure 2.3.

![Figure 2.3. Forming culvert elements](image)

The equation of the culvert element \( i \) can be written as

\[
(x - P_i)^2 + (y - Q_i)^2 = R_i^2
\]

(2.18)

The circular member has 6 degrees of freedom as shown in Figure 2.4.

![Figure 2.4. Circular beam element](image)

Note: Quantities as shown are considered positive
The local coordinate system $\mathbf{x}, \mathbf{y}$ is selected so that the $\mathbf{x}$ and $\mathbf{y}$ axes are tangent and perpendicular to the culvert member, respectively. The central angle $\beta$ is found by the dot product of the unit normal vectors at nodal points 1 and 2 and the result can be written in a form as

$$\cos \beta = \frac{1}{2R^2} [(x_1 - P)(x_2 - P) + (y_1 - Q)(y_2 - Q)] \quad (2.19)$$

The desired stiffness matrix equation is written in the partitioned form as

$$\begin{bmatrix} k_{aa} & k_{ab} \\ k_{ba} & k_{bb} \end{bmatrix} \begin{bmatrix} d_a \\ d_b \end{bmatrix} = \begin{bmatrix} q_a \\ q_b \end{bmatrix} \quad (2.20)$$

where $\mathbf{a} = 1, 2, 3$ and $\mathbf{b} = 4, 5, 6$

$q_1, q_4$ - axial force at nodes 1 and 2
$q_2, q_5$ - shearing force at nodes 1 and 2
$q_3, q_6$ - bending moment at nodes 1 and 2
and $\mathbf{d}$ represents the displacements corresponding to the member forces.

The stiffness matrix is obtained by using the inversion of the flexibility matrix $[f]$ and the equilibrium conditions of a cantilever circular member as shown in Figure 2.5.
The matrix $[k_{aa}]$ is obtained by the inversion of the flexibility matrix $[f]$ as

$$[k_{aa}] = [f]^{-1}$$

(2.21)

The flexibility coefficients are as follows

$$f_{11} = \frac{R^3}{EI} \left( \frac{3}{2} \beta - 2 \sin \beta + \frac{1}{2} \sin \beta \cos \beta \right) + \frac{R}{EA} \left( \frac{\beta}{2} + \frac{\sin \beta \cos \beta}{2} \right)$$

$$f_{12} = f_{21} = \frac{R^3}{EI} \left( -\frac{\sin^2 \beta}{2} - \cos \beta + 1 \right) - \frac{R}{2EA} \sin^2 \beta$$

$$f_{13} = f_{31} = \frac{R^2}{EI} (\beta - \sin \beta)$$

$$f_{22} = \frac{R^3}{EI} \left( \frac{\beta}{2} - \frac{1}{2} \sin \beta \cos \beta \right) + \frac{R}{EA} \left( \frac{\beta}{2} - \frac{1}{2} \sin \beta \cos \beta \right)$$

$$f_{23} = f_{32} = \frac{R^2}{EI} (-\cos \beta + 1)$$

$$f_{33} = \frac{R}{EI} \beta$$
where

\[ E \] - Young's modulus
\[ I \] - moment of inertia
\[ \beta \] - central angle in radian
\[ A \] - cross sectional area

By the equilibrium conditions, the matrix \([k_{ba}]\) is obtained in terms of the stiffness coefficients of the matrix \([k_{aa}]\). The coefficients of the matrix \([k_{ba}]\) are given as

\[
\begin{align*}
    k_{41} &= -k_{11} \cos \beta + k_{21} \sin \beta \\
    k_{42} &= -k_{12} \cos \beta + k_{22} \sin \beta \\
    k_{43} &= -k_{13} \cos \beta + k_{23} \sin \beta \\
    k_{51} &= -k_{11} \sin \beta - k_{21} \cos \beta \\
    k_{52} &= -k_{12} \sin \beta - k_{22} \cos \beta \\
    k_{53} &= -k_{13} \sin \beta - k_{23} \cos \beta \\
    k_{61} &= -k_{11} R(1-\cos \beta) - k_{21} R \sin \beta - k_{31} \\
    k_{62} &= -k_{12} R(1-\cos \beta) - k_{22} R \sin \beta - k_{32} \\
    k_{63} &= -k_{13} R(1-\cos \beta) - k_{23} R \sin \beta - k_{33}
\end{align*}
\]

By using the symmetrical properties of the stiffness matrix and the circular member, the matrices \([k_{ab}]\) and \([k_{bb}]\) are obtained

\[
[k_{ab}] = [k_{ba}]^T
\]
In order to determine the stiffness property of the complete structure, the stiffness matrix of the culvert element must be referenced to a global coordinate system $x, y$ as shown in Figure 2.6.

\[
[k_{bb}] = \begin{bmatrix}
k_{11} & -k_{12} & k_{13} \\
-k_{21} & k_{22} & -k_{23} \\
k_{31} & -k_{32} & k_{33}
\end{bmatrix} \tag{2.23}
\]

Figure 2.6. Local and global coordinate system

The culvert stiffness matrix based on the global system, $[k^g]$, is obtained by the following relationship (17)

\[
[k^g] = [\lambda]^T [k] [\lambda] \tag{2.24}
\]

where $[\lambda]$ is the displacement transformation matrix relating the local displacements to the global displacements. The matrix $[\lambda]$
can be determined by calculating the direction cosines of the angles between the local and global coordinate systems and it yields as

\[
\begin{bmatrix}
\frac{(y_1 - Q)}{R} & \frac{-(x_1 - P)}{R} & 0 & 0 & 0 & 0 \\
\frac{(x_1 - P)}{R} & \frac{(y_1 - Q)}{R} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{(y_2 - Q)}{R} & \frac{-(x_2 - P)}{R} & 0 \\
0 & 0 & 0 & \frac{(x_2 - P)}{R} & \frac{(y_2 - Q)}{R} & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(2.25)

2.4. Non-Linear Stress-Strain Laws

Most cohesive soils have a stress-strain relationship that is non-linear and depends on the state of stress. Woodward (31) proposed an incremental type of non-linear analysis for estimating stresses and deformation in earth dams during construction. The method treats the non-linear problem as piecewise linear with the soil properties modified after each increment of load in accordance with the state of stress computed in the element. The tangent modulus \(E_t\) at the beginning of a load increment is calculated based on the existing deviator stress \((\sigma_1 - \sigma_3)\) and confining stress \(\sigma_3\) and it is
used during the load increment as shown in Figure 2.7. Woodward used data obtained from laboratory tests of soil samples to represent stress-strain relation of soil. He could not express the relation in mathematical form.

Duncan and Chang (10) developed a simplified, practical procedure for representing non-linear, stress-dependent soil stress-strain behavior in a form which is convenient for use in incremental finite element analysis. The non-linear stress-strain curves of soil may be approximated by hyperbolas with a high degree of accuracy as shown in Figure 2.8. The hyperbolic equation proposed was

\[ (\sigma_1 - \sigma_3) = \frac{\epsilon_1}{a + b\epsilon_1} \]  

(2.26)
Asymptote = \((\sigma'_{1} - \sigma'_{3})_{ult.} = \frac{1}{b}\)

Figure 2.8. Hyperbolic stress-strain curve

Figure 2.9. Transformed hyperbolic stress-strain curve
where \( a \) is the reciprocal of the initial tangent modulus \( E_i \), and \( b \) is the reciprocal of the asymptotic value of deviator stress \( (\sigma_1 - \sigma_3)_{\text{ult.}} \).

The values of \( a \) and \( b \) may be determined if the stress-strain data are plotted on transformed axes as shown in Figure 2.9. It is commonly found that the value of the asymptotic value \( (\sigma_1 - \sigma_3)_{\text{ult.}} \) is larger than the compressive strength of the soil \( (\sigma_1 - \sigma_3)_f \) by a small amount.

\[
(\sigma_1 - \sigma_3)_f = R_f (\sigma_1 - \sigma_3)_{\text{ult.}} \tag{2.27}
\]

where \( R_f \) is the failure ratio which has been found to be between 0.75 and 1.00 for most of the soils.

By substituting the value of \( a \) and \( b \) into Equation (2.26), the equation can be written as

\[
(\sigma_1 - \sigma_3) = \frac{\epsilon_1}{1 + \frac{\epsilon_1 R_f \sigma_3}{(\sigma_1 - \sigma_3)_f}} \tag{2.28}
\]

If the value of the confining stress \( \sigma_3 \) is constant such as is normal in the case of triaxial shearing tests, the tangent modulus can be written as

\[
E_t = \frac{\partial(\sigma_1 - \sigma_3)}{\partial \epsilon_1} \tag{2.29}
\]

By performing the differentiation on Equation (2.28), the tangent modulus can be written in the convenient form
The value of compressive strength \((\sigma_1 - \sigma_3)_f\) and the initial tangent modulus \(E_i\) are dependent on the confining stress \(\sigma_3\) and they can be obtained by performing the shearing tests on the soil samples corresponding to the field conditions. Because the confining pressure at any point in the embankment changes during construction and after construction and because each point will be subjected to different stresses and deformations in general, there is no unique stress-strain relationship for the compacted soil. What is needed for this analysis is a series of shear tests performed at changing confining pressures.

Since drainage is not allowed during construction, unconsolidated-undrained shear tests could be used to predict stress-strain relationship of soil during the construction period. The typical series of curves relating the deviator stress \((\sigma_1 - \sigma_3)\) and the axial strain \(\epsilon_1\) obtained from the shear test performed at several confining pressures are shown in Figure 2.10.

From the curves in Figure 2.10, the undrained initial tangent modulus \(E_i\) and the undrained compressive strength \((\sigma_1 - \sigma_3)_f\) can approximately be obtained as a function of the confining pressure \(\sigma_3\) as shown in Figures 2.11 and 2.12, respectively. The initial tangent modulus and the compressive strength of the compacted soil increase
steadily with the confining pressure and become constant when the confining pressure required to saturate the soil $\sigma_3$ is reached. The curved portions in Figures 2.11 and 2.12 can be replaced by straight lines without causing significant errors.

![Deviator stress-axial strain curves]

Note: Each curve performed at different confining pressure, $\sigma_3$

Figure 2.10. Deviator stress-axial strain curves

Poisson's ratio used in the undrained case will be between 0.4 and 0.5 for a nearly saturated clayey soil (31).

After the embankment construction is ended, the water is assumed to begin to flow out of the soil mass. The drained initial tangent modulus $E_1$ and the drained compressive strength $(\bar{\sigma}_1 - \bar{\sigma}_3)_f$
Figure 2.11. Initial tangent modulus - confining pressure curve

Figure 2.12. Compressive strength - confining pressure curve
needed in Equation (2.30) can be obtained by performing a number of drained shear tests at different confining pressures $\bar{\sigma}_3$.

The relationship between the drained compressive strength and confining pressure may be expressed conveniently in terms of the Mohr-Coulomb failure criterion as

$$\left(\bar{\sigma}_1 - \bar{\sigma}_3\right)_f = \frac{2 \bar{c} \cos \bar{\phi} + 2\bar{\sigma}_3 \sin \bar{\phi}}{1 - \sin \bar{\phi}}$$  \hspace{1cm} (2.31)

where

$\bar{c}$ - drained cohesion

$\bar{\phi}$ - drained angle of internal friction.

The variation of the drained initial tangent modulus value with confining pressure was represented by an empirical equation suggested by Janbu (13)

$$\bar{E}_i = K \bar{p}_a \left(\frac{\bar{\sigma}_3}{\bar{p}_a}\right)^N$$  \hspace{1cm} (2.32)

where $K$ and $N$ are modulus number and exponent, respectively which are pure numbers and $\bar{p}_a$ is the value of atmospheric pressure.

The typical value of drained Poisson's ratio $\bar{\nu}$ is 0.3 (14).

2.5. System Equations and Solution Process

As the element stiffness matrices referenced to the global system are generated, they are appropriately superimposed into a system matrix $[S]$. The superimposition is accomplished by using
the code number technique (15, 18, 25).

A structural system that contains a large number of elements will involve a good deal of input data preparation. This includes a code number for each element. In order to reduce preliminary work of this nature, a subroutine was written to generate the code numbers for each element.

When the system matrix \( [S] \) and system load vector \( \{L\} \) are generated using the code numbers, the total system of simultaneous algebraic equations in the unknowns \( \{\Phi\} \) is represented as

\[
[S]\{\Phi\} = \{L\} \quad (2.33)
\]

The primary concern in the solution of this system is the conditioning of the system matrix. It is a symmetrical matrix and can be a banded or an unbanded matrix depending on the concerning problem. In the incremental construction analysis where the unknowns are only nodal displacements, the system matrix is banded. In the time-dependent analysis where nodal displacements and element pore pressures are the unknowns, the system matrix is unbanded as a result of fill-in matrices which will be explained later in Chapter IV. By taking an advantage of the symmetry, the coefficients are stored as an upper triangular matrix.

The solution process, contained in a single subroutine, is Gaussian elimination. The band-width is automatically computed prior
to solving the system as it is required input to the solution subroutine along with the system load vector.
III. INCREMENTAL CONSTRUCTION ANALYSIS

Incremental construction analysis is used to perform the following features during the construction of a culvert under an earth fill:

(1) Calculation of stresses and deformations in the fill.
(2) Calculation of construction pore pressures.
(3) Calculation of forces and deformations in the culvert.

3.1. Simulation of Construction

The step-by-step construction procedures can be easily included in the calculations. When a lift is added to a partially completed embankment it forms a new structure which must be analyzed. First, the stiffness of this new structure is determined, then the weights of the soil elements in the new lift are calculated and the weights of the soil elements used previously are set to zero. Finally, the displacements, stresses and forces in the new structure caused only by the weight of the new lift are calculated. These are then added algebraically to the displacements, stresses and forces existing in the partially completed embankment. By repeating this procedure for each successive construction lift, the complete history of the development of displacements, stresses and forces can be obtained.

As mentioned in Article 2.4 the soil is assumed to behave linearly during each construction increment, with stiffness properties
The accuracy of the results that are obtained will depend, of course, on the size of the increments which are considered, but experience indicates that relatively coarse construction increments will yield good results in the analysis of typical earth embankments. Woodward (31) found that by using lift thickness of one tenth of the embankment height acceptable solutions could be obtained.

The flow diagram for the incremental construction analysis is shown in Figure 3.1.

3.2. Pore Pressure Calculation

The construction pore pressures due to the gravity effects of the fill material are predicted by the method developed by Hilf (12). He assumed that the pore water pressure in a partially saturated soil could be related to the amount of compression by combining Boyle's law for compressibility of air with Henry's law for the solubility of air in water. The pore water pressure due to applied stresses is computed from

\[ u = \frac{p_a A e}{e_a + H e^o_w - \Delta e} \]  

(3.1)

where

- \( u \) - induced pore pressure
Read input data

Calculate and print nodal displacement numbers and code numbers for each element

Calculate and print the geometry of culvert elements

Add one layer and calculate tangent modulus for each soil element based on existing stresses

Generate system stiffness matrix and system load vector

Calculate and print nodal displacements

Solve for culvert forces and stresses, strains in soil elements

Solve for pore pressures in soil elements

Print culvert forces, stresses and pore pressures

Is there the next layer?

Yes

No

Time-dependent analysis

Figure 3.1. Simplified flow diagram for incremental construction analysis
\( p_a \) - absolute atmospheric pressure

\( \Delta e \) - numerical value of change in void ratio from initial conditions

\( e_a \) - initial void ratio of the pore air

\( e_{0a} \) - initial void ratio of pore air

\( H \) - Henry's constant of solubility of air in water

\( e_w \) - initial void ratio of pore water

Equation (3.1) is correct if the assumption of equal pore air and pore water pressures is satisfied (i.e., if surface tension can be neglected). It is apparent that as the compaction moisture content increases above optimum, the validity of the assumption increases due to the higher degree of saturation (23).

For saturation due to compression, \( \Delta e = e_{0a} \), Equation (3.1) is reduced to

\[
 u = \frac{p_a e_{0a}}{He_{0w}} 
\]  

(3.2)

The Equation (3.2) is correct if drainage is not allowed.

The change of void ratio, \( \Delta e \), can be obtained by using an iterative process and the void ratio-effective stress relationship for the soil determined in the conventional consolidation test as shown in Figure 3.2. Effective stress is defined by

\[
 \bar{\sigma} = \sigma - u 
\]  

(3.3)

where
\[ \sigma \text{ - effective stress} \]

\[ \sigma \text{ - total stress} \]

\[ u \text{ - pore pressure} \]

Figure 3.2. Void ratio-effective stress curve

The calculation of the pore pressure change due to a change in total major principal stress \( \Delta \sigma_1 \) consists of the following steps proposed by Steward (23).

(1) Assume that the change in effective stress is equal to one-half of the change in total stress

\[ \Delta \sigma (1) = \frac{\Delta \sigma_1}{2} \]  \hspace{1cm} (3.4)
The number in the bracket indicates the number of approximations.

(2) Calculate the change in void ratio due to the assumed change in effective stress

\[
\Delta e = C_c \log \left( \frac{1 + \Delta \sigma_1^{(1)}}{\sigma_1} \right)
\]  

The compression index \( C_c \) is the slope of the void ratio-effective stress curve in Figure 3.2.

(3) Add \( \Delta e \) to the sum of the previous void ratio changes and calculate the pore pressure by using Equation (3.1).

(4) Calculate the effective stress using the calculated pore pressure \( u^{(1)} \).

\[
\sigma_1 = \sigma_1 - u^{(1)}
\]  

(5) Calculate the change in effective stress \( \Delta \sigma_1 \) and obtain the second approximation of the change in effective stress \( \Delta \sigma_1^{(2)} \) by

\[
\Delta \sigma_1^{(2)} = \Delta \sigma_1^{(1)} + \frac{\Delta \sigma_1^{(1)} - \Delta \sigma_1^{(1)}}{2}
\]  

By repeating steps two through five until the change in the value of \( \Delta e \) is very small, the change in pore pressure due to the change in total stresses is obtained. It was found from experience that using ten cycles of iteration gives good results.

After the pore pressure is calculated, a check is made to see if the soil is saturated. If the soil is saturated, the pore pressure is
recalculated by adding the pore pressure at saturation calculated from Equation (3.2) to the increment of total stress from the value required to completely saturate the soil.

It was mentioned in Article 1.1 that the assumption of no drainage during the construction period will be used in the study. The assumption is justified if the following requirements are satisfied:

1. The embankment is constructed with an impervious material such as clay which has permeability less than $10^{-6}$ cm/sec.

2. The drainage path is long such as in the case of high fill.

3. The construction time is short so that there is not enough time for drainage.

The approximation of no drainage during construction is shown in Figure 3.3 by considering a soil element in the first lift. The amount of reduction in pore pressure due to drainage will decrease as the number of lift is increased, i.e., the drainage path for the soil element is also increased.

### 3.3. Initial Stresses

The initial stresses for the soil elements in the newly added layer due to the effects of compaction can be calculated by the expressions

$$
\sigma_y = \bar{\sigma}_1 + u_o
$$  \hspace{1cm} (3.8)
Figure 3.3. Approximation of no drainage during construction
\[
\sigma_y = K \sigma_y \\
\sigma_x = \sigma_y \\
\tau_{xy} = 0
\]

(3.9) 

(3.10)

where

- \( \sigma_y \) = vertical total stress
- \( \sigma_x \) = horizontal total stress
- \( \tau_{xy} \) = shearing stress
- \( \sigma_{10} \) = major principal effective stress corresponding to the initial void ratio as shown in Figure 3.2.
- \( u_o \) = initial pore pressure due to compaction
- \( K_o \) = coefficient of earth pressure at rest.

The pore pressure due to compaction \( u_o \) can be neglected when it is compared with its final value induced by the weight of high fill, especially when soil is compacted at the water content above the optimum value (3). In the study the value of coefficient of earth pressure at rest \( K_o \) is taken to be 1.0 which it is implied that stresses induced by compaction are distributed uniformly in the newly placed layer. The effects of compaction on the underlaid layers can be neglected when it is compared with the effects caused by the dead weight (31).

The initial stresses due to compaction are added algebraically to those caused by the gravity effects of the fill material.
The matrix \( \{k^t\} \) is of the form

\[
\{k^t\} = [B]^T \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} hA \tag{4.3}
\]

There are two sets of unknowns in Equation (4.2), the nodal displacement vector \( \{\delta\} \) and pore pressure \( u \). An additional equation is required in order to solve Equation (4.2). The additional equation needed is the restriction on volume change \( \Delta V \) of the soil element. The volumetric strain \( \epsilon_n \) can be written as the sum of strain in \( x \) and \( y \) directions as

\[
\epsilon_n = \epsilon_x + \epsilon_y \tag{4.4}
\]

By substituting the values of \( \epsilon_x \) and \( \epsilon_y \) from Equation (2.11), Equation (4.4) becomes

\[
\epsilon_n = \frac{1}{2A} \begin{bmatrix} b_1 & c_1 & b_2 & c_2 & b_3 & c_3 \end{bmatrix} \{\delta\} \tag{4.5}
\]

or

\[
\epsilon_n = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} [B] \{\delta\} \tag{4.6}
\]

Therefore, the change in volume \( \Delta V \) of the triangular soil element can be written as

\[
\Delta V = h A \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} [B] \{\delta\} \tag{4.7}
\]

or symbolically as

\[
\Delta V = \{k^t\}^T \{\delta\} \tag{4.8}
\]
Combining Equations (4.2) and (4.8) gives the generalized-stiffness formulation for the triangular soil element

\[
\begin{bmatrix}
  k & k' \\
  k' T & 0
\end{bmatrix}
\begin{bmatrix}
  \delta \\
  u
\end{bmatrix}
= 
\begin{bmatrix}
  F \\
  \Delta V
\end{bmatrix}
\quad (4.9)
\]

The time-dependent analysis starts at the end of construction. The pore pressures at time \( t \) are used to calculate the change of volume to be expected in each soil element during an increment of time \( \Delta t \). This volume change \( \Delta V \) can be used to solve for new forces, stresses, deformations and pore pressures in the soil-culvert system at time \( t + \Delta t \), and so on until a steady state is reached. The steady state is practically chosen at the time when the further changes in pore pressures do not cause any significant changes in the soil-culvert system.

If water is assumed to flow from the soil mass in the vertical direction only and further that the soil is saturated after construction due to compression, the change in volume of a soil element can be obtained from an equation (14)

\[
\frac{\partial V}{\partial t} = \frac{V - C}{\gamma_w} \left( \frac{\partial^2 u}{\partial y^2} \right)
\quad (4.10)
\]

where \( \gamma_w \) is the density of water and \( C \) is the coefficient of permeability of soil in the vertical direction. The validity of the assumption of saturation at the end of construction increases if the soil is placed at the water content above the optimum value (2). If the
time increment \( \Delta t \) is taken small enough, the change of volume can be written as

\[
(\Delta V)_{t \rightarrow t+\Delta t} = \frac{C}{\gamma_w} (V)_{t} \left( \frac{\partial^2 u}{\partial y^2} \right)_t \Delta t
\]

(4.11)

where the subscript \( t \) indicates the value at time \( t \). If the pore pressure \( u \) is assumed to vary in the vertical direction according to a parabolic equation

\[
u = \beta_1 + \beta_2 y + \beta_3 y^2
\]

(4.12)

the second derivative of \( u \) in Equation (4.12) at the center of element \( i \) can be obtained by using the finite difference equation (19)

\[
\left( \frac{\partial^2 u}{\partial y^2} \right)_i = \frac{1}{(\Delta y)^2} \left[ \frac{2}{\alpha(\alpha+1)} \left( \alpha u_j - (1+\alpha)u_i + u_k \right) \right]
\]

(4.13)

where \( \Delta y \) and \( \alpha \Delta y \) are the distances from the center of element \( i \) to the centers of element \( j \) and element \( k \), respectively, as shown in Figure 4.1.
Conditions of no flow across a boundary of a soil element are approximated by defining across the boundary a mirror image of the center of the element and by making the pore pressure at this point equal to that in the element inside the boundary.

The flow diagram for the time-dependent analysis is shown in Figure 4.2.

4.2. Numerical Stability and Accuracy of Solution

The use of a piecewise linear analysis with tangent modulus in each soil element defined on the basis of stress state at the beginning of each step of analysis and the use of Equation (4.11) to calculate the volume change during the step of analysis will lead to the numerical stability and accuracy problems if excessively large increments of time are used.

If Equation (4.13) is substituted into Equation (4.11), the volume change of a soil element $i$ during an increment of time $\Delta t$ can be obtained as

$$
\frac{(\Delta V)_{i, t \to t + \Delta t}}{V_{i, t}} = \frac{C}{\gamma_w} \frac{\Delta t}{\Delta y} \frac{2}{\alpha (\alpha + 1)} \left[ \alpha u_j, t - (1 - \alpha) u_{i, t} + u_k, t \right] 
$$ (4.14)

In general, the smaller time increment and finer finite element mesh will increase the accuracy of the solution. But as the accuracy increases, the computer storage locations and costs also increase.
Print nodal displacements at the end of construction

Print culvert forces, effective stresses, and pore pressures at the end of construction

Time steps > total time steps

Yes

stop

No

Calculate $\bar{E}_t$, and $\Delta t$ if time steps are greater than time steps needed to start the solution

Time steps > time steps needed to start the solution

No

Read, $\Delta t$

Yes

Print $\Delta t$ and time after construction

Generate system stiffness matrix and system load vector

Calculate and print nodal displacements, culvert forces, effective stresses and pore pressures

Figure 4.2. Simplified flow diagram for time-dependent analysis
The conditions to ensure both stability and accuracy are not readily obtainable for Equation (4.14).

Scott (20) used Equation (4.14) to solve one-dimensional consolidation problems by neglecting the strain in the horizontal direction and assuming that the change in vertical stress is equal to the change in pore pressure. The left hand side of Equation (4.14) can be written as

\[
\frac{(\Delta V)_{i,t \to t+\Delta t}}{V_{i,t}} = (\Delta u)_{i,t \to t+\Delta t} \left[ \frac{1 - 2\gamma_s(1 + \psi)}{E_t(1 - \nu)} \right] (4.15)
\]

After substituting Equation (4.15) into Equation (4.14), the finite difference equation for Terzaghi's one-dimensional consolidation solution is obtained as

\[
(\Delta u)_{i,t \to t+\Delta t} = C_v \frac{\Delta t}{(\Delta y)^2} \frac{2}{\alpha(\alpha+1)} \left[ a_{u,j} - (1-\alpha)u_{i,t} + u_{k,t} \right] (4.16)
\]

where

\[
C_v = \text{coefficient of consolidation}
\]

\[
= \frac{C}{\gamma_w} \frac{E_t(1-\nu)}{(1-2\nu)(1+\nu)}
\]

Scott rewrote Equation (4.16) in terms of time factor \( T \) and dimensionless factor \( \Delta Y \)

\[
(\Delta u)_{i,T \to T+\Delta T} = \frac{\Delta T}{(\Delta Y)^2} \frac{2}{\alpha(\alpha+1)} \left[ a_{u,j} - (1-\alpha)u_{i,T} + u_{k,T} \right] (4.17)
\]
where

\[ \Delta T = \frac{\Delta t \cdot C_v}{H^2} \]

\[ \Delta Y = \frac{\Delta y}{H} \]

\[ H = \text{total height of the embankment} \]

Scott proposed a condition that ensures both stability and accuracy for Equation (4.17). He expressed that if an error \( E \) is made in the determination of \( u_i, T \) at one cycle of solution, i.e., at a time \( T \), from any cause, then the error in the subsequent value of \( u_i, T+\Delta T \) should not be more than \( E \). If this is true, the step-by-step calculation of \( u_i \) as a function of time will not be divergent, since the computational errors do not increase in the course of the solution. For a stable solution, the inequality must hold

\[ \left( \frac{2M}{\alpha} + \left| \frac{2M}{\alpha} - 1 \right| \right) E \leq |E| \tag{4.18} \]

where

\[ M = \frac{\Delta T}{(\Delta Y)^2} \]

From Equation (4.18), it follows that

\[ \frac{M}{\alpha} \leq 0.5 \tag{4.19} \]

where \( \frac{H}{\alpha} \) is a stability number.

By trial Scott found that the use of \( \frac{H}{\alpha} = 0.5 \) results in a stably oscillating solution, which is also undesirable. In addition, the
smaller the value of $\Delta Y$ selected, the more accurate will be the solution. Barden (1) found that using $\Delta Y$ of 0.1 will give acceptable results.

It is necessary to emphasize that the stability condition given by Equation (4.19) was derived from Equation (4.17) where the change in pore pressure is directly calculated. In the present study the change in pore pressure is obtained by solving the generalized-stiffness equation (4.9) which contains very large and very small coefficients such as in the case of soil-culvert problem. It is of course desirable to carry out the solution process in double precision as suggested by Shugar (21). This consideration must, however, be weighed with program size and available computer storage. Because of the large program size and limited computer storage of the CDC 3300 computer at Oregon State University, the solution process cannot be carried out in double precision at the present time (1973). In order to keep an error in $u_{t+\Delta t}$ smaller than an error in $u_t$, the volume change of soil elements calculated from Equation (4.14) must be accurate enough to make the error in $u_{t+\Delta t}$ resulted from solving Equation (4.9) not larger than the error in $u_t$ and this can be done by using the stability number smaller than the value required by Equation (4.19). The value of the stability number required for the soil-culvert problem can be obtained by using the trial and error method.
It is advisable to begin a solution with smaller time steps, before advancing the solution to longer time intervals. The length of time intervals required at the beginning of the solution can be obtained by performing preliminary analyses with different time intervals as explained later in Chapter V.
V. TESTING OF THE COMPUTER PROGRAM

In order to verify the finite element formulation of this investigation and also the computer program, two different types of problems are analyzed and their results are compared with those of known classical solutions.

First, an analysis of a circular ring subjected to a single load as shown in Figure 5.1 is performed. Four circular beam elements are used to model a 180 degree portion of the circular ring. The results as shown in Figures 5.2 and 5.3 indicate the excellent agreement with the exact solutions (6).

Second, a one-dimensional consolidation problem is used to check the formulation of soil elements and the process of pore pressure dissipation. Five elements are used to simulate a column of soil as shown in Figure 5.4. Drainage is allowed at the top and no drainage is permitted at the bottom or sides. The soil is assumed elastic and stress-independent. The value of initial pore pressure is constant throughout the soil column.

As mentioned in Section 4.2, the change in volume of soil elements during an increment of time $\Delta t$ is obtained by using Equation (4.14). The use of the relation in Equation (4.14) will lead to numerical instability if an excessively large time interval is used. The length of time interval has to be taken small in relation to the
variations in the rate of change of pore pressure. The length of time steps needed at the beginning of the solution are obtained by performing preliminary analyses with different time intervals. The variations of pore pressure with time in element 5 which has the fastest change of pore pressure are shown on Figure 5.5. They indicate that the time intervals of 2 days are needed to ensure numerical stability and accuracy. It was decided to use time steps of 2 days up to the time of 20 days. The length of time steps used to continue the solution are determined by using the value of stability number of 0.4. The variations of pore pressure with time in the soil elements shown in Figure 5.6 indicate a smoothly decaying pore pressure with no sign of any superimposed oscillation at any stage. The distributions of pore pressure over the soil column are compared with the well known Terzaghi's solution (24). They indicate a very good agreement as shown in Figure 5.7.
Figure 5.1. Finite element model of circular ring

\[ R = 4.5 \text{ ft.} \]
\[ E = 30 \times 10^3 \text{ ksi} \]
\[ I = 1.99 \text{ in.}^4 /\text{ft.} \]
\[ \theta = 50.5 \text{ degree} \]
\[ P = 2 \text{ k} \]
Figure 5.2. Deformed shape for circular ring

Figure 5.3. Bending moment diagram for circular ring
$C = 0.034$ inch/day
$H = 8.65$ ksf
$v = 0.3$
Initial pore pressure, $u_0 = 1000$ psf

Figure 5.4. Finite element model of soil layer
Figure 5.5. Numerical instability due to large time interval
Figure 5.6. Plot of pore pressure vs. time, linear consolidation problem
Terzaghi's solution (24)

Finite element value

Figure 5.7. Pore pressure distribution in soil layer
VI. ANALYSIS OF SOIL-CULVERT PROBLEMS

Two analyses of a soil-culvert system are presented in this chapter in order to compare the behaviors of rigid and flexible culverts under an earth embankment. As previously mentioned, the developed computer program is applied to the analysis of symmetrical soil-culvert system in order to use symmetrical properties to reduce the number of unknowns. The proposed configuration for the system is illustrated in Figure 6.1. The boundary line in Figure 6.1 was selected 18 ft. from the axis of symmetry. The selection was based on the limiting capacity of the computer. The distance of 18 ft. is justified by observing the deformed shapes in Figures 6.6 and 6.7 which indicate the uniform settlement of the soil elements along the boundary line. This indicates that the selected boundary line is far enough from the culvert. The circular culvert was used in the study and the embankment was assumed to be built in six lifts. The rigid and flexible culverts are made of concrete and steel, respectively.

Because of the assumption that drainage occurs in the vertical direction, it is convenient to represent the embankment by a series of vertical columns of soil in order to avoid mathematical difficulties in obtaining the second derivative of pore pressure used in Equation (4.11). A 42-element model was generated for the structure and is shown in Figure 6.2.
The soil proposed for the construction of Bully Creek Dam which is on Bully Creek about nine miles west of Vale, Oregon was used for the simulated embankment. Laboratory tests of the soil were performed at Denver Earth Laboratory, Denver, Colorado (27). The stress-strain parameters derived from triaxial shear tests and mechanical properties of the soil are summarized in Tables 1 and 2 and Figures 6.3, 6.4, and 6.5. Because of the lack of test data, some parameters have to be assumed. The failure ratio \( R_f \) used in Equation (2.30) can be assumed to be 0.83 (11). Poisson's ratio was assumed to have values of 0.45 and 0.30 for undrained and drained conditions, respectively.

The computer results are shown in Figures 6.6 through 6.18. One computer run was made in order to show the application of the developed computer program to the time-dependent problem. The flexible culvert-embankment system was used as the example of the time-dependent analysis. The results at the end of construction were used as an initial condition for the analysis. Because of an abrupt change of pore pressure in the soil at the sides of the culvert as shown in Figure 6.15, the rate of change of pore pressure in the soil elements along the culvert walls is very fast at the beginning of the solution. The time interval required to start the solution and the stability number used to continue the solution were found by trial and error method as explained in Chapter V to have the values of 0.1 day.
and 0.1, respectively. Ten time steps of 0.1 day are used from the end of construction up to the time of 1 day. The time intervals required to continue the solution is determined automatically by the computer by using the value of stability number of 0.1. Because of the very high cost of a computer run, the time dependent analysis was carried out only 40 days after construction. The vertical deflection at the crown of the flexible culvert was plotted against time in Figure 6.17. The pore pressure in soil elements was plotted against time in the dimensionless form as shown in Figure 6.18. An attempt was made to estimate the approximate value of the crown deflection at the time that the pore pressure dissipation is completed. If the rate of change of the deflection is assumed to vary linearly with the rate of change of pore pressure in soil elements, the deflection-time relationship can approximately be found by using Terzaghi's pore pressure-time relationship (24) shown in Figure 6.19 and the computed deflections in Figure 6.17. The approximate crown deflection-time relationship was shown in Figure 6.20.
Concrete:
- \( E = 3 \times 10^3 \) ksi
- \( I = 512 \text{ in.}^4/\text{ft} \)
- Wall thickness = 8 inches

Steel:
- Number 12 gage
- 6 in. x 2 in. corrugations
- \( E = 30 \times 10^3 \) ksi,
- \( I = 0.725 \text{ in.}^4/\text{ft} \)
- Wall thickness = .1046 in.

Figure 6.1. Properties of the simulated soil-culvert system
Figure 6.2. Soil-culvert finite element model
### Table 1. Initial conditions for tested soil samples

<table>
<thead>
<tr>
<th>Density lb/cu. ft.</th>
<th>Optimum water content %</th>
<th>Water content %</th>
<th>Degree of saturation %</th>
<th>Void ratio</th>
<th>Specific gravity</th>
<th>Coefficient of permeability in./day</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>26.6</td>
<td>29.4</td>
<td>86.3</td>
<td>.882</td>
<td>2.59</td>
<td>.0023</td>
</tr>
</tbody>
</table>

### Table 2. Drained stress-strain parameters

<table>
<thead>
<tr>
<th>$\bar{c}$ psf.</th>
<th>$\phi$ degree</th>
<th>K</th>
<th>N</th>
<th>$R_f$</th>
<th>$\bar{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>29.25</td>
<td>276</td>
<td>1.07</td>
<td>.83</td>
<td>.30</td>
</tr>
</tbody>
</table>
Figure 6.3. Initial tangent modulus - confining pressure for undrained condition

Figure 6.4. Compressive strength - confining pressure for undrained condition
Soil compacted to 95% standard Proctor maximum density.

Figure 6.5. Void ratio - effective stress curve of Bully Creek soil.
Figure 6.6.  Deformed shape at the end of construction, flexible culvert
Figure 6.7. Deformed shape at the end of construction, rigid culvert
Figure 6.8. Plot of vertical deflection at the crown of culverts vs. height of fill
Figure 6.9. Plot of vertical pressure at the crown of culverts vs. height of fill.
Figure 6.10. Radial pressure on flexible culvert at the end of construction
Figure 6.11. Radial pressure on rigid culvert at the end of construction
Note: Positive moment causes tension inside culvert

Figure 6.12. Plot of bending moment during construction period, flexible culvert
Figure 6.13. Plot of bending moment during construction period, rigid culvert

Note: Positive moment causes tension inside culvert
Figure 6.14. Plot of maximum bending moment vs. height of fill
Figure 6.15. Pore pressure contours at the end of construction, flexible culvert
Figure 6.16. Pore pressure contours at the end of construction, rigid culvert.
Figure 6.17. Plot of vertical deflection at crown of flexible culvert vs. time after construction

Stability number, $\frac{M}{\alpha}$, = 0.1
Stability number, $\frac{M}{\alpha} = 0.1$

Figure 6.18. Plot of the pore pressure vs. time after construction
Figure 6.19. Plot of Terzaghi's pore pressure-time relationship

---

Terzaghi's solution (24)

Computed result

---

Pore pressure/initial pore pressure

Time after construction, days
Figure 6.20. Plot of the approximate vertical deflection at crown of flexible culvert vs. time after construction.
VII. DISCUSSION OF RESULTS

Figure 6.6 shows the deformed shape of the embankment-flexible culvert system at the end of construction. The embankment settles almost uniformly. The deflection at the crown is 0.3 inch. The deformed shape of the embankment-rigid culvert system is shown in Figure 6.7. The soil prism which is adjacent to the culvert settles more than the soil prism that is directly over the culvert. This illustrates the arching effects explained by Spangler (22). The deflection of the rigid culvert is very small compared with that of the flexible culvert.

Figure 6.8 shows the vertical deflection at the crown of the culverts during construction period. At the beginning of the filling process the top of the culverts deflect vertically upward due to the effects of lateral soil pressure acting on the culvert walls. After the fill height is increased above the crown, the vertical deflection varies linearly with the height of fill over the culvert. The vertical deflection of the flexible culvert is about ten times larger than the one of the rigid culvert.

The vertical pressure at the top of the culvert is plotted in Figure 6.9. It indicates that the vertical pressure varies linearly with the height of fill above the culverts. Linear pressure-height function was confirmed by field measurements (4). The vertical pressure acting at the top of the flexible culvert is approximately equal
to the overburden pressure. Due to the arching effect that is caused by unequal settlements of the central soil prism in relation to that of the exterior adjacent soil prisms, the vertical pressure at the crown of the rigid culvert is larger than the overburden pressure. In the study case the vertical pressure is about 1.13 of the overburden pressure.

The results from the study and the field measurements obtained by Trollope (26) are compared with the ones obtained by using Spangler's formulae and compression ring theory as shown in Table 3. The vertical pressure acting at the top of the rigid culverts obtained by Trollope and the present study are about 0.65 of the values obtained from Spangler's formulae. The calculated vertical pressure at the top of the flexible culvert is in good agreement with the values obtained from Spangler's formula and compression ring theory. The deflection at the crown of the flexible culvert is ten times smaller than the value obtained from Spangler's formula. The difference in the deflection may be explained by the use of the approximate value of modulus of soil reaction and the assumed pressure distribution on culverts proposed by Spangler. The maximum axial force in the flexible culvert is about 15 percent higher than the value obtained from compression ring theory.

Figures 6.10 and 6.11 show the distributions of radial pressure on the culverts at the end of construction. The radial pressure
Table 3. Comparisons of results at the end of construction

<table>
<thead>
<tr>
<th>Sources</th>
<th>Culvert</th>
<th>$\frac{\sigma_y \text{ at crown}}{(\sigma_y) \text{ overburden}}$</th>
<th>Deflection at crown inch.</th>
<th>Maximum axial force lb./ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trollope (26)</td>
<td>Rigid</td>
<td>1.2 *</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Spangler</td>
<td>Rigid</td>
<td>1.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lertlaksana</td>
<td>Rigid</td>
<td>1.13</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Spangler</td>
<td>Rigid</td>
<td>1.75</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lertlaksana</td>
<td>Flexible</td>
<td>0.995</td>
<td>0.3</td>
<td>11970</td>
</tr>
<tr>
<td>Spangler</td>
<td>Flexible</td>
<td>1.0</td>
<td>3.24 **</td>
<td>-</td>
</tr>
<tr>
<td>Ring theory</td>
<td>Flexible</td>
<td>1.0</td>
<td>-</td>
<td>10400</td>
</tr>
</tbody>
</table>

* Field measurement

** Use modulus of soil reaction of 700 psi suggested by Spangler.
distributes uniformly on the flexible culvert and the magnitude is equal to the overburden pressure at the top of the culvert as proposed by White (29). The radial pressure is maximum at the crown in the case of rigid culvert and distributes almost uniformly on the culvert walls with the magnitude of 0.77 of the value at the crown. The shapes of the pressure distributions are confirmed by the results obtained by Brown (5).

Figures 6.12, 6.13 and 6.14 show the variations of bending moment in the culverts during the construction period. Actually, the bending moment diagrams should be smooth curves instead of connected straight lines. The maximum bending moment in the flexible culvert is about 25 percent of that in the rigid culvert. The bending moment obtained by using five culvert elements indicates the very high and unrealistic value especially in the flexible culvert. In order to obtain a more realistic representation of soil pressure on the culvert, it is necessary to subdivide the culvert into a larger number of elements. By increasing the number of elements, the moment diagram will be smoothed and the maximum moment in the culvert should be reduced. The translational displacements at the current nodal points are not expected to change significantly due to the increased number of elements. The deflected shape between current nodes is expected to change considerably. The slope at the current nodal points may be changed significantly. Since the soil pressure is a function of the translational displacements at the nodes, no significant change in the soil pressure is expected. It should also be noticed
that the computed soil pressures on the flexible culvert are well confirmed by the field measurements (29).

Figures 6.15 and 6.16 show curves of computed pore pressures at the end of construction. They indicate that there is almost no variation of pore pressure in the horizontal direction except at the area near the culvert walls. Because of the variations of pore pressure, the assumption of drainage in the vertical direction can be used without causing any significant errors.

Figures 6.17 and 6.18 show the vertical deflection at the top of the flexible culvert and the dissipation of pore pressure in the embankment, respectively, from the end of construction to 40 days after construction. They show that the deflection increases as the results of pore pressure dissipation.

Figure 6.19 shows the pore pressure-time relationship based on Terzaghi's linear, one-dimensional consolidation solution. The computed rate of change of pore pressure is slightly slower than the one obtained from Terzaghi's solution.

The approximate crown deflection-time relationship of the flexible culvert is shown in Figure 6.20. The approximate crown deflection based on Terzaghi's pore pressure-time relationship is slightly larger than the computed values at least 40 days after construction. The estimations of crown deflection are summarized in Table 4.
Table 4. Deflections at crown of the flexible culvert

<table>
<thead>
<tr>
<th>End of construction, ft.</th>
<th>40 days after construction</th>
<th>Predicted</th>
<th>End of pore pressure dissipation, ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computed</td>
<td>Predicted</td>
<td></td>
</tr>
<tr>
<td>2.50 x 10^{-2}</td>
<td>2.66 x 10^{-2}</td>
<td>2.75 x 10^{-2}</td>
<td>3.86 x 10^{-2}</td>
</tr>
</tbody>
</table>

Rate of change of deflection is assumed to vary linearly with rate of change of pore pressure

The value of the crown deflection at the end of pore pressure dissipation is approximately equal to 1.55 of the value at the end of construction. Spangler (22) proposed a concept of deflection lag factor to take soil time-dependent properties into account by increasing the crown deflection at the end of construction by 25% - 50%.

The computed culvert forces exhibit practically no change at 40 days after construction. As mentioned earlier, the time dependent properties of soil considered in this study is assumed to be controlled by the dissipation of pore pressure. The skeleton creep of soil after the consolidation process is neglected. This assumption is justified for most cohesive soils used for embankment construction especially when the time required to complete dissipation of pore pressure is large (28).

The results obtained in the study are based on an assumption that the axial deformation in the culverts is neglected by omitting the terms corresponding to the axial deformation in Equation (2.21). The assumption of neglecting the axial deformation may be questioned in the case of flexible culvert which has small cross sectional area A.
The results at the end of construction indicate that the computed pressure acting on the flexible culvert is in good agreement with the values obtained from Spangler's solution and compression ring theory. To neglect the axial deformation in the flexible culvert may not cause any significant errors. It is recommended to include the effects of the axial deformation in the culverts for the future analysis.
A computer program was developed using the finite element method for analyzing culverts under earth embankments constructed of compacted cohesive soil. The step-by-step construction procedure is taken into account. The construction pore pressure in the embankment is estimated by the Hilf method, and zero dissipation of pore pressure during the embankment construction is assumed. The dissipation of pore pressure is assumed to start at the end of embankment construction. The changes in the culvert forces and deformations due to the effects of consolidation of the embankment are estimated by the modified finite element method. The nonlinear, stress-dependent properties of soil are included in the study by considering the nonlinear problem as piecewise linear during each construction step or time increment.

From the study it may be concluded:

1. The pressure acting at the crowns of culverts varies linearly with the height of fill above crowns.

2. The crown deflection varies linearly with the height of fill above crown.

3. The vertical pressure at the crown of the rigid culvert was found to be overestimated by Spangler's solution.

4. An equation proposed by Spangler for calculating the crown
deflection of the flexible culvert at the end of construction was found to be very conservative.

5. The range of values of deflection lag factor from 1.25 to 1.50 suggested by Spangler is found to be reasonable.

6. No conclusions can be reached about the changes of culvert forces due to the effects of embankment consolidation from the computed results obtained at an early state of consolidation.


APPENDICES
APPENDIX A

USER'S MANUAL FOR SYMMETRIC
SOIL-CULVERT PROGRAM

The program consists of a main program and 19 subroutines. This appendix is included in order to explain to users the capability of the computer program and restrictions concerning the preparations of the finite element model and input data.

The FORTRAN IV program was written for the CDC 3300 computer at the computer center of Oregon State University.

A.1 Program Capability

The computer program is used to analyze a culvert under an earth embankment of homogeneous, cohesive soil. The culvert and embankment are placed on a rock foundation. Because of the computer capacity, the application of the computer program is limited to a symmetrical soil-culvert problem. The culvert can be circular or elliptical. The program does compute culvert forces, deformations, stresses and pore pressures in the soil at any stage of construction due to the gravitational effects of the fill, and at any time after construction due to the effects of pore pressure dissipation. The pore pressure dissipation was assumed to begin at the end of construction. By using piecewise linear analysis, non-linear, stress-dependent stress-strain behavior of the soil can be accommodated.
The program can also analyze an arch subjected to external forces, one-dimensional consolidation with non-linear, stress-dependent stress-strain behavior and one-dimensional consolidation with linear stress-strain behavior.

Present limitation on the finite element model's size are as follows: 52 nodal points, 5 culvert elements, 37 soil elements, and 20 constrained nodal points.

A.2 Preparation of the Finite Element Mesh

By taking advantage of the symmetry of the soil-culvert system, only one-half of the structure is used. The structure shown in Figure A.1 is used as an example to show how to prepare the finite element model. The embankment is represented by a series of soil columns as shown in Figure A.1 in order to avoid mathematical difficulties in obtaining the second derivative of pore pressure with respect to the vertical distance used in Equation (4.11). Three kinds of elements are used to construct the model; curved beams for the culvert, and quadrilateral and triangular elements for the embankment. The majority will be quadrilateral elements as they are the most efficient; that is, the quadrilateral element is subdivided by the program into four triangles. Therefore, a quadrilateral element will give the same results as four triangular elements without increasing the storage locations in the computer.
Figure A.1. Example finite element mesh showing node number scheme
Numbering the nodal points should begin after the finite element mesh has been established. The numbering system in this computer program was designed in such a manner that numbering of these nodes must start along the culvert and then proceed to the fill in a layer by layer manner as shown in Figure A.1. The numbering system was found to be convenient for the step-by-step analysis during construction even if the obtained structural matrix may have larger band width than that using other systems.

Once the numbering of the nodes has been accomplished, all elements must be numbered. The numbering must start along the culvert and then proceed to the fill in a layer by layer manner. The culvert elements have to be numbered in the clockwise direction.

A.3 Preparation of Input Data

The following information describes the data cards which form the necessary input data for the program.

A.3.1 Problem Type Card (15)

Columns 1-5 \[ \text{PROB} \]

Place a 1 for soil-culvert problem, both construction and consolidation periods are needed.
Place a 2 for soil-culvert problem, only construction period is needed.
Place a 3 for an arch subjected to external forces.
Place a 4 for soil-culvert problem, only consolidation period is needed.

Place a 5 for one-dimensional consolidation with non-linear, stress-dependent stress-strain behavior.

Place a 6 for one-dimensional consolidation with linear stress-strain behavior.

A. 3.2 Control Card (515)

Columns 1-5 Total number of elements (42 max.)

6-10 Total number of nodal points (52 max.)

11-15 Total number of constrained boundary points (20 max.)

16-20 Total number of culvert elements (5 max.)

21-25 Total number of layers

A. 3.3 Coordinate Cards (15, 3F15. 3)

This data set describes primarily the overall geometry of the structure in terms of two coordinates x and y for each nodal point.

In addition the number of unknowns to be assigned the nodes are described in this data set.

Columns 1-5 Nodal point number

6-20 The x coordinate of the node (feet)

21-35 The y coordinate of the node (feet)
This entry indicates the degree of freedom the program will assign the node. A 2 is assigned all nodes except the nodes on the curved beam elements which are assigned a 3.

The coordinate cards must be in order, beginning with nodal point number 1.

A.3.4 Connectivity Cards (15, 5X, 615)

This data connects the element number with the nodal point numbers on the element's perimeter. The nodal points for a quadrilateral and triangular element are listed counterclockwise sequentially around the element. The nodal points for a curved beam element are listed clockwise. Besides the above information, the element numbers of the elements which are directly above and below a given element are indicated in the data set. The two element numbers are used for calculating the distances between the center of the element under consideration and the centers of the elements above and below. These distances are used for calculating the second derivative of pore pressure used in Equation (4.11)

Columns 1-5 Element number
11-15 Nodal point i
16-20 Nodal point j
21-25 Nodal point k. Columns are left blank if it is
a curved beam element.

26-30 Nodal point 1. Columns are left blank if it is a curved beam element or a triangular element.

31-35 Element number of the element that is below a given element. Columns are left blank if the given element is a curved beam element, or it is on the foundation or the culvert.

36-40 Element number of the element that is above the given element. Columns are left blank if the given element is a curved beam element or it is a soil element at the top layer.

The element cards must be in order, starting with element number 1.

A. 3.5 The Constant Cards (415)

This data applied primarily to nodal points on the boundaries of the finite element model. A 1 is used to indicate the constraint of a nodal point.

Columns 1-5 Nodal point number of constrained nodal point. \( K_{\text{NOVA}}(1,1) \)

6-10 x direction constraint (1 or blank) \( K_{\text{NOVA}}(1,1) \)

11-15 y direction constraint (1 or blank) \( K_{\text{NOVA}}(1,2) \)

16-20 Rotational constraint (1 or blank, enter 1 if the nodal point is not on the curved beam elements). \( K_{\text{NOVA}}(1,4) \)
A.3.6 Culvert Properties Card (15, 2F10.2, 2F10.3)

Columns 1-5 Enter a 1 if the culvert is circular.
Enter a -1 if the culvert is elliptical.

6-15 Culvert radius in feet or blank if the culvert is elliptical. The radius of the circular beam segments is calculated by using the properties of a circle passing through three consecutive points in the case of elliptical culvert as shown in Figure 2.3.

16-25 Modulus of elasticity of the culvert (ksi)

26-35 Cross sectional area (in.²/ft.)

36-45 Moment of inertia (in.⁴/ft.)

A.3.7 Soil Properties Card (2F10.2, F10.4)

Columns 1-10 Total stress coefficient of earth pressure used for initial stresses in elements newly added to embankment

11-20 Unit weight of soil (lb./cu. ft.)

21-30 Coefficient of permeability (in./day)

A.3.8 Undrained Stress-Strain Parameters Card (4F12.2, 2F5.2)

Columns 1-12 Initial tangent modulus at zero confining pressure (psf.)
13-24 Initial tangent modulus at saturation (psf.)
25-36 Maximum deviator stress at zero confining pressure (psf.)
37-48 Maximum deviator stress at saturation (psf.)
49-53 Undrained failure ratio
54-58 Undrained Poisson's ratio

A. 3. 9 Drained Stress-Strain Parameters Card (6F10.2)

Columns 1-10 Drained cohesion (psf.)
11-20 Drained angle of internal friction (radian)
21-30 Modulus number, K
31-40 Exponent, N
41-50 Drained failure ratio
51-60 Drained Poisson's ratio

A. 3. 10 Soil Pore Pressure Properties Card (8F6.3, 3F10.2)

Columns 1-6 Initial void ratio
7-12 Initial air void ratio
13-18 Initial water void ratio
19-24 Void ratio corresponding to effective preconsolidation stress
25-30 Void ratio at full saturation
31-36 Initial degree of saturation (decimal)
37-42 Compression index in the over-consolidation region

43-48 Compression index in the normally consolidation region

49-58 Effective stress corresponding to initial void ratio (psf.)

59-68 Effective preconsolidation stress (psf.)

69-78 Effective stress corresponding to void ratio at saturation (psf.)

A. 3.11 Linear Stress-Strain Parameters Card (2F12.2)

This data card gives an information about stress-strain properties of soil for a one-dimensional consolidation problem which has linear, stress-independent stress-strain behavior.

Columns 1-12 Undrained elastic modulus (psf.)

13-24 Drained elastic modulus (psf.)

This card is left blank if the soil is inelastic and stress-dependent.

A. 3.12 Output Control Card (315)

Columns 1-5 Enter a 1 if tangent modulus is needed to be printed out, otherwise, the columns are left blank
6-10 Enter a 1 if the volume of soil elements during the consolidation process is needed to be printed out, otherwise, the columns are left blank.

11-15 Enter a 1 if the output is needed to be punched on cards, otherwise, the columns are left blank.

A.3.13 Layer Data Cards (415)

The data cards are not needed if the given problem is a soil-culvert problem that needs only consolidation period or it is a one-dimensional consolidation problem.

Columns 1-5 Layer number

6-10 Number of elements up through this layer

11-15 Number of nodal points up through this layer

16-20 Element number of the first soil element in this layer

The layer data cards must be in order, starting with layer number 1.

A.3.14 Arch Problem Card (15)

The card is omitted if it is not an arch subjected to external forces problem.

Columns 1-5 Total numbers of applied forces
A. 3.15 *External Forces Cards (I5, F12. 2)*

The cards are omitted if it is not an arch subjected to external forces problem.

Columns 1-5 Displacement number at the loading point corresponding to the direction of the applied force

6-17 External force (lb. or lb.-ft.)

A. 3.16 *Initial Displacement Cards (I5, 3E15.5)*

The cards are needed only if the problem is a soil-culvert problem that needs only consolidation period or if it is a one-dimensional consolidation problem.

Columns 1-5 Nodal point number

6-20 Initial displacement in x direction (ft.)

21-35 Initial displacement in y direction (ft.)

36-50 Initial rotation (radian). The columns are left blank if the nodal point is not on the culvert

A. 3.17 *Initial Stress Cards (I5, 3E15.5, /, 4E15.5)*

The cards are needed only if the problem is a soil-culvert problem that needs only consolidation process or it is a one-dimensional consolidation problem.

The first card (I5, 3E15.5):

Columns 1-5 Element number
6-20  Axial force at node i (lb.) if the given element is a curved beam element. Effective normal stress, \( \sigma_x \) (psf.), if the given element is a soil element.

21-35  Shear force at node i (lb.) if the given element is a curved beam element. Effective normal stress, \( \sigma_y \) (psf.), if the given element is a soil element.

36-50  Bending moment at node i (lb.-ft.) if the given element is a curved beam element. Shear force, \( \tau_{xy} \) (psf.), if the given element is a soil element.

The second card (4E15.5):

Columns 1-15  Axial force at node j (lb.) if the given element is a curved beam element. Effective major principal stress, \( \sigma_1 \) (psf.), if the given element is a soil element.

16-30  Shear force at node j (lb.) if the given element is a curved beam element. Effective minor principal stress, \( \sigma_3 \) (psf.), if the given element is a soil element.

31-45  Bending moment at node j (lb.-ft.) if the given element is a curved beam element.
Maximum shear force, \( T_{max} \) (psf.), if the given element is a soil element.

46-60 Excess pore pressure (psf.). The columns are left blank if the element is a curved beam element.

**A. 3.18 Volume Change Cards (I5, E15.5)**

The cards are needed only if the problem is a soil-culvert problem that needs only consolidation period or it is a one-dimensional consolidation problem. These data cards together with initial displacement cards and initial stress cards will provide the computer program the capability to begin the consolidation process at any time other than at the end of construction.

Columns 1-5 Element number

6-20 Total volume change (cu. ft.) due to consolidation effects from the end of construction. The columns are left blank if the solution starts at the end of construction or the element is a curved beam element.

**A. 3.19 Number of Time Increment Card (2I5)**

The card is needed only if the consolidation process is wanted.

Columns 1-5 Total number of time increments
6-10 Number of time increments needed to start the solution (see Section 4.2)

A. 3. 20 Stability Number Card (F6. 2)

The card is needed only if the consolidation process is wanted.

The stability number is the value of $M/\alpha$ used in Equation (4.19).

Columns 1-6 Stability number

A. 3. 21 Time Increment Cards (F10. 3)

The cards are needed only if the consolidation process is wanted.

Columns 1-10 Length of time increment (days)

The number of time increment cards are equal to the number of time steps used to start the consolidation process (see card A. 3.19)
APPENDIX B
DESCRIPTION OF COMPUTER PROGRAM

Nineteen subroutines comprise the body of the computer program. They are controlled by call statements which comprise the main program. A brief description of the function of each subroutine is given below.

1) Subroutine INPUT. This subroutine reads into storage the input data.

2) Subroutine SUBCOD. This subroutine generates the 4 x 6 matrix of code numbers used subsequently to combine four triangular elements into one quadrilateral element.

3) Subroutine GENCOD. This subroutine generates a code number for each element in the finite element model. These are subsequently used to assemble the system matrix of algebraic equations.

4) Subroutine INIT. This subroutine initializes a given problem to the initial conditions.

5) Subroutine D2Y. This subroutine calculates the vertical distances from the center of a given soil element to the centers of soil elements that are just above and below it. These distances are used to calculate the second derivative of pore pressure used in Equation (4.11).
6) Subroutine MODULUS. This subroutine computes tangent modulus of each soil element and the length of time interval used in consolidation process.

7) Subroutine CULPRO. This subroutine computes center point, radius and central angle of each culvert element.

8) Subroutine CULK. This subroutine generates 6 x 6 stiffness matrix of each culvert element referenced to the local coordinate system shown in Figure 2.4.

9) Subroutine CULTRN. This subroutine generates 6 x 6 displacement transformation matrix of each culvert element. The matrix is used to transform the culvert stiffness matrix referenced to a local coordinate system to the one referenced to a global coordinate system.

10) Subroutine TRIK. This subroutine generates 6 x 6 stiffness matrix of each triangular element.

11) Subroutine QUAD. This subroutine computes the center point of each quadrilateral element and then uses code numbers to assemble four triangular elements into a quadrilateral element. The 10 x 10 quadrilateral stiffness matrix and associated load vector are obtained.

12) Subroutine STIFF. This subroutine performs the static condensation process which reduces the 10 x 10 quadrilateral stiffness matrix to a 8 x 8 element matrix. The subroutine also transforms

\text{CALL \text{QUAD}(I)}
a culvert stiffness matrix to the one referenced to the global coordinate system.

13) Subroutine BGK. This subroutine uses the code numbers to assemble the element matrices into a system matrix.

14) Subroutine FVEC. This subroutine uses the code numbers to assemble the system load vector.

15) Subroutine BANWID. This subroutine computes the bandwidth of the system stiffness matrix in the case of incremental construction analysis prior to the solution process.

16) Subroutine SOLVE. This is a subroutine used to solve the system of simultaneous equations by Gaussian elimination.

17) Subroutine MATINV. This is a standard matrix inversion subroutine. It is used only to invert a flexibility matrix during a generation of culvert stiffness matrices.

18) Subroutine PORE. This subroutine computes pore pressures in each soil element during construction.

19) Subroutine RESULT. This subroutine computes displacements, stresses, forces and pore pressures for the elements in the finite element model. It also computes the principal stresses.

CALL CULK(1)
CALL CULTRN(1)
CALL TRICK
APPENDIX C

PROGRAM LISTING
C PROGRAM CULVERT
C
COMMON /A1/ NUMNPT,NUMELT,NCULET,NCONP,NLAY
COMMON /A2/ CCOR(153,31),CCOR(142,6)
COMMON /A3/ KNOA(124,14),KNOA(123,3)
COMMON /A4/ LADO
COMMON /A5/ AODE(14,6)
COMMON /A6/ NCODE(142,6)
COMMON /A7/ NLNCASE,NUMNP(NUMNPT-1,*),NMPNT
COMMON /B1/ KTYPE,RADIUS,VE,VITC
COMMON /B2/ ACUL(15),ACUL(15),ACUL(15),ACUL(15),ACUL(15),ACUL(15),ACUL(15),ACUL(15)
COMMON /C1/ ET(42),PR
COMMON /C2/ PR1,PR2,PR3,PR4,PR5
COMMON /C3/ VK,DENS
COMMON /C4/ CAYO
COMMON /C5/ EVO,S,EVE,S,ESM,SENO,SENO,SIGN,SIGN,SIGN
COMMON /D1/ AREA(42)
COMMON /D2/ S(665),SSK(1040),S(10)
COMMON /D3/ A(1403)
COMMON /D4/ B(119)
COMMON /D5/ NO
COMMON /D6/ NB
COMMON /D7/ ALPHA(42),HI
COMMON /E1/ Disp(52),Stress(42,6)
COMMON /F1/ DT
COMMON /F2/ F(164)
COMMON /F3/ DELV(142)
COMMON /G1/ KPR1,2,KPR1,2

1 READ(60,310)NLPROD
616 FORMAT(15)
   CALL INPUT
   CALL SUBCOD
   CALL GENCOD
   CALL D2Y
   CALL CULPRO
   CALL FVEC(I)
   CALL INIT(LPROB)
5 CONTINUE
C ** PLACE A NEW LAYER
6 NL=NL+1
C ** CHECK THE LAST LIFT
IFNLPROB .LT. NLAY00 TO 615
101 FORMAT(*, THE LAYER NUMBER = +15)
   WRITE(61,102)NL
102 FORMAT(*)
   WRITE(60,103)NL5,NUMELT,NUMNP,NSELT
620 FORMAT(15)
   WRITE(60,621)K,DELV(I)
C ** LIFT DATA
READ(60,622)NSELT,NUMNP,NMPNT
C ** LIFT DATA
READ(60,623)NSELT,NUMNP,NMPNT
908 FORMAT(*, LAYER NUMBER = +15/*, TOTAL NUMBER OF ELEMENTS UP + 1 +15/*, TOTAL NUMBER OF NODAL POINTS UP + 1, THOUGH THIS LAYER = +15/*, TOTAL NUMBER OF FIRST SOIL + 1, ELEMENT IN THIS LAYER = +15 + 1, DO 1 +1, NUMELT*14)
916 FORMAT(15,E15.5)
GO TO 614
C
** TRANSFORM THE TOTAL STRESSES TO EFFECTIVE STRESSES
614 DO 111 N= 1,NUMELT
STRESS(N,1)=STRESS(N,1)POREPR(N)
STRESS(N,2)=STRESS(N,2)POREPR(N)
STRESS(N,4)=STRESS(N,4)POREPR(N)
IF(LPROM.EQ.2 .OR. LPROM.EQ.3)GO TO 12
614 TIME=0.0
WRITE(61,112)TIME
112 FORMAT('node 03X0 XDISP .8X0 YDISP ',6X,
' ROTATION .')
DO 400 I=1.NUMNPT
WRITE(61,401)I0DISP(I),J=1,3)
401 FORMAT(/.15.3E16.5)
WRITE(61,402)
402 FORMAT(//0
ELE. SIGMAXX ..5)( SIGMAYY .05X. 
' SIGMAXY ../.9X0 AND ..10X0 AND '1,1030 AND ../.
27X.' MAX.S1GMA '0430 MIN.SIGMA ..4X.. MAX.SHEAR 
3AX+ + PORE PRES. N)
DO 403 I=1.NUMELT
WRITE161,404I10STRESS(I,J).J=1.6).POREPRIN
404 FORMAT(15.3E15.5./.4E15.5)
ND=LADOM+NSELT
NDT=ND.IND+11/2
READ(60,904)NTIME,NTIME1
904 FORMAT(I2)
WRITE(61,909)NTIME,NTIMEI
909 FORMAT(' TOTAL NUMBER OF TIME INCRE/ENTS = .15./.
' NUMBER OF STARTING TIME INCREMENTS = .15)
READ(60,912)STABNO
912 FORMAT(I2)
WRITE(61.913)5TABN0
913 FORMAT(' STABILITY NUMBER = '.F6.2)
DO 114 11=1oNTIME
CALL MODULUS(LPROB.INNTIMEI.ST,BNO)
114 CONTINUE
IF(KPRIN3 .EQ. 0)00 TO 914
914 STOP
END
C
** SUBROUTINE MODULUS(LPROB.INNTIMEI.ST,BNO)
COMMON /A1/ NUMNP,NUMELT,NUMNPT,NCONF,NLAY
COMMON /A7/ NL,LCASE,NUMNL,NUMMP,SEL1
COMMON /D7/ ALPHA1(2).HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
1 COMMON /C2/ PR1,PR2,EIO,E1U,DVSTFU,DSIGST
1 COMMON /C3/ VEVDR(2),VEVDR(2),VEVDR(2),VEVDR(2),
1 COMMON /C5/ SIG,STRESS(24)
1 COMMON /D7/ ALPHA1(2),HSTRE51(2),SIG1,SG1,SIGN.SIGS.
**FORTRAN 77 Source Code**

```fortran
IF(RF*SIG1*SIG3)DVSTF1606,635,605
606 ET(I)=11.,RF*SIG1*SIG3/DVSTF1606
605 ET(I)=12.,RF*SIG1*SIG3/DVSTF1606
GO TO 10

10 IF(KP%IN3) E0,1 WRITE(61,61) E(I)
IF(KP%IN2) WRITE(61,62) E(I)
TEMP=ALPHA(I)*HDIS(I)*HDIS(I)/ET(I)
IF(TEMP .LE. DTTEMP)
TEMP=TEMP
CONTINUE

PR=PR2
CONST=V0.(1.PR)/(1.-2.*PR)/(1.+PR)/748.8
DT=STABNO*DTTEMP/CONST
C
7 RETURN

5 FORMAT(/915,5X,' TANGENT MODULUS = '.F12.3.' KS!. .)
6 FORMAT(/915,5X,' TANGENT MODULUS = '.F12.3.' PSF. .')
800 FORMAT(/915,5X,' ELEMENT ',I50,' FAILED')

END

C .**************.***..**************.*****4,****************************

SUBROUTINE INPUT
C
COMMON /A1/ NUMNPT,NUMELT,NCULET,NCONP,NLAY
COMMON /A2/ COOR(53,3),1CONN(42,6)
COMMON /A3/ KNOVA(2094),NODOM(529,31
COMMON /A4/ KPRIN1,KPRIN2,KPRIN3
READ(609102) NUMELT,NUMNPToNCONP,NCULE79NLAY
101 FORMAT(515)
DC 102 I=1,NOMNPT
102 READ(60,103)K,(COOR(I,J)9J=1.3)
103 FORMAT(1593E15.3)
DO 104 I=1,NUMELT
114 READ(609105)Kt(ICONNII,J)9J=116)
105 FORMAT(1595X,615)
DC 106 I=1,NCONP
106 READ160+107)(KNOVA(IIJ),J=194)
107 FORMAT(4151
READ(6091C8)KTYPE,RADIUSINE9CAREA,V1
108 FORMAT(15,2F10.2,2F5.2)
READ(609110)CAYO9DENS9VK
109 FORMAT(2F10.2,F1004)
READ(609111)EIO,E(U,DVSTEO,DVSTFU,RF1,PRI
110 FORMAT(4F12.2,2F5.2)
READ(609113)CEE,PHI,VMODNO,VN,RF2
111 FORMAT(6F10.2)
READ(609113)CEE,PHI,VMODNO,VN,RF2
112 FORMAT(8F6.3,F10.2)
READ(60,112)VELIN1,VELIN2
113 FORMAT(315)
WRITE(61911
1 FORMAT( /.' TOTAL NUMBER OF ELEMENTS .)
WRITE(61950)NUMELT
WRITE(61,3)
3 FORMAT( /s' TOTAL NUMBER OF NODAL POINTS .)
WRITE(61,50) NUMNPT
WRITE(61,4)
4 FORMAT(/,' TOTAL NUMBER OF LAYERS .)
WRITE(61,50) NLAY
WRITE(61,51
5 FORMAT( /.' TOTAL NUMBER OF CONSTRAINED BOUNDARY',.. POINTS .)
WRITE(61,50) NCONP
WRITE(61,51
6 FORMAT(/,' THE X,Y ORDINATES IN (FT.) AND THE',.. DEGREE OF FREEDOMS OF EACH ../.. NODAL POINT '.)
WRITE(61,50) NUMNPT
WRITE(61,8)
8 FORMAT( /,' NODAL POINT NUMBERS OF NODES I,J,K,L',.. AND IF IT IS A SOIL ELEMENT,.. THE NUMBERS OF THE',.. SOIL ELEMENTS UNDER AND ABOVE IT ARE NEEDED .)
DO 302 I=1,NUMELT
302 WRITE(619303)1,11CONN( 19J),J=1,61
303 FORMAT(/915,2X,6(5)
WRITE(61910)
10 FORMAT(//,' CONSTRAINED NODE POINTS )91,
113X,' NODE..6X-9. X',6X,' Y.96X,. R.)
DO 308 I=1,NCONP
308 WRITE161.30911,(KNOVA(IsJ),J=194)
309 FORMAT(/915,7)(11418)
WRITE(61914)
14 FORMAT(/,' THE RADIUS OF THE CIRCULAR CULVERT.FT. .)
WRITE(61,51) RADIUS
51 FORMAT( /,F12.3)
WRITE(61,15)
15 FORMAT( /,' THE MODULUS OF ELASTICITY,KS. .)
WRITE(61,51) VE
41 FORMAT( /,' THE MOMENT OF INERTIA,(INCH**4)/FT. .)
WRITE(61,51) VI
42 FORMAT(/,' THE CROSS SECTIONAL AREA ,(INCH * *2) /FT. .)
WRITE(61051)CAREA
WRITE(61938)
38 FORMAT(/.. TOTAL STRESS COEFFICIENT OF EARTH PRESSURE .)
WRITE(61,51) CAY0
WRITE161,800)
800 FORMAT(/,' THE CROSS SECTIONAL AREA OF ELEMENT .)
WRITE(61,51) CAREA
WRITE(61,38)
38 FORMAT( /,F12.5)
WRITE(61,17)
17 FORMAT( /,' THE MOMENT OF INERTIA (INCH**4) .)
WRITE(61,51) VE
18 FORMAT( /,' THE UNIT WT. OF SOIL.PCF. .)
WRITE(61,180)
180 FORMAT(/,' THE UNIT WT. OF SOIL.PCF. .)
WRITE(61,18)
18 FORMAT( /,' INITIAL TANGENT MODULUS,PSF. WHEN .)
```

---

**Notes:**
- The provided code is a FORTRAN 77 subroutine for inputting data for a structural analysis program.
- It includes reading input data such as nodal points, elements, and material properties.
- The code is structured to handle the input of data for a 2D or 3D structural model.
```fortran
NODOM:L+1+K
11 CONTINUE
12 CONTINUE
LADOM=(K
DO 13 L=1,NUMELT
9=0
IFICONN(L,3) .EQ. 0 GO TO 18
IFICONN(L,4) .EQ. 0 GO TO 19
NP=4
GO TO 20
19 NP=3
20 K=N
DO 14 LDUM=1,NP
NP=ICONN(L,LDUM)
DO 15 L=1,NP
NDUM=1+K
15 NODE(L,NDUM)=NODOM+INTEMP+1)
K=K+2
14 CONTINUE
GO TO 17
18 NP=2
K=N
DO 17 LDUM=1,NP
NP=ICONN(L,LDUM)
DO 16 L=1,NP
NDUM=1+K
16 NCODE(L,NDUM)=NODOM+INTEMP+1)
K=K+3
13 CONTINUE
17 CONTINUE
C WRITE(61,50)
50 FORMAT(//,15X,9- 'NODE '918X9- 'DISPLACEMENT '918X9- 'R ')
12X4XK .4X4X Y .4X4X R
DO 51 J=1,NUMPT
51 WRITE(61,52)+(NODE(J,1),J=1,3)
52 FORMAT(//,15,16X,15,12X)
C WRITE(61,53)
53 FORMAT(//,5X,9- 'ELEMENT '915X9- 'CODE NUMBER ')
DO 54 J=1,NUMELT
54 WRITE(61,55)+(NCODE(J,J),J=1,8)
55 FORMAT(//,15,5X,8B5)
C RETURN
END
```

**SUBROUTINE D2Y**

```fortran
COMMON /A1/ NUMPT,NUMELT,NODEL+1,ICONN,NLAY
COMMON /A2/ COOR(53,3),ICONN(62,6)
COMMON /A3/ ALPHA(42),HDIS(42)
DIMENSION NE(3),Y(4)
DO 50 J=1,NUMELT
ALPHA(J)=0
HDIS(J)=0
IFICONN(J,3) .EQ. 0 GO TO 50
IFICONN(J,5) .NE. 0 .AND. ICONN(J,6) .NE. 0 GO TO 12
IFICONN(J,4) .EQ. 0 GO TO 23
L=4
GO TO 14
```

C CONTINUE
13 LN=4
14 YTEMP=0.0
DO 5 K=1,LN
NP=ICONN(K,1)
Y(K)=COOR(NP,2)
15 YTEMP=TEMP(Y(K)
VLN=LN
YCENT=TEMP/YLN
LMN=LN-1
DO 4 K=4,LNM
7(K)=K+1
DO 4 L=KP+1,LN
IF(Y(L), .GE. Y(K)) GO TO 51
GO TO 4
51 YTEMP=Y(K)
Y(K)=Y(L)
Y(L)=YTEMP
4 CONTINUE
IFICONN(J,5) .EQ. 0 .AND. ICONN(J,6) .EQ. 0 GO TO 15
IFICONN(J,3) .EQ. 0 GO TO 16
NNE=ICONN(J,5)
GO TO 17
16 NNE=ICONN(J,6)
17 IFICONN(NNE,4) .EQ. 0 GO TO 18
J=ICONN(J,1)
K=ICONN(J,2)
L=ICONN(J,3)
M=ICONN(J,4)
YTEMP=(COOR(J,2)+COOR(K,2)+COOR(L,2)+COOR(M,2))/4.
GO TO 19
18 J=ICONN(J,1)
K=ICONN(J,2)
L=ICONN(J,3)
M=ICONN(J,4)
YTEMP=(COOR(J,2)+COOR(K,2)+COOR(L,2)+COOR(M,2))/3.
19 IFICONN(J,5) .EQ. 0 GO TO 20
YBOTT=YTEMP
YTOP=Y(J)+Y(L)+Y(M)/2.
DY=ABS(YTOP-YCENT)
DYD=ABS(YCENT-YBOTT)
GO TO 5
20 YTOP=YTEMP
YBOTT=Y(J)+Y(L)+Y(M)/2.
DY=ABS(YTOP-YCENT)
DYD=ABS(YCENT-YBOTT)
GO TO 5
15 YTOP=Y(L)+Y(M)/2.
YTOP=M/2.
DY=ABS(YTOP-YCENT)
DYD=ABS(YCENT-YBOTT)
GO TO 5
12 N(J)=1
NE(1)=ICONN(J,5)
NE(3)=ICONN(J,6)
DO 2 N=1,3
NNE=NE(J)
IFICONN(NNE,4) .EQ. 0 GO TO 43
J=ICONN(NNE,1)
K=ICONN(NNE,2)
L=ICONN(NNE,3)
M=ICONN(NNE,4)
Y(N)=COOR(J,2)+COOR(K,2)+COOR(L,2)+COOR(M,2))/4.
GO TO 2
```
**SUBROUTINE INIT(LPROB)**

- Common variables declared:
  - NUMNPT, NUMELT, NCULET, NCONF, NLAY
  - COOR(I,3), ICONN(N,3)
  - KTYPE, RADIUS, VE, VI, CAREA
  - ACUL(I), BCUL(I), RCUL(I), BETA(I), TRAN(I,6)
  - ET(I,42), S(I,6), SYSK(I,10), QA(I,10)
  - F(I,6)

**DO 1 I=1,NCULET**

- **IF(I,LPROB .GT. 2)**
  - TEMH/(X1*X1+Y1*Y1+XJ*XJ+YJ*YJ)
  - TEMH/TEML

**DO 5 N=1,NUMELT**

- **IF(LPROB .GT. 2)**
  - TEMH/(X1*X1+Y1*Y1+XJ*XJ+YJ*YJ)
  - TEMH/TEML

**RETURN**

**END**

**SUBROUTINE COLCUL(I)**

- Common variables declared:
  - NUMNPT, NUMELT, NCULET, NCONF, NLAY
  - COOR(I,3), ICONN(N,3)
  - KTYPE, RADIUS, VE, VI, CAREA
  - ACUL(I), BCUL(I), RCUL(I), BETA(I), TRAN(I,6)

**WRITE(6,20)**

- **FORMAT(/// ELEMENT RAJUS(Ft), *TX, ** ANGLE **TX**, CMH **TX**, CENT **)**

**DO 11 I=1,NCULET**

- **IF(I,LPROB .GT. 2)**
  - TEMH/(X1*X1+Y1*Y1+XJ*XJ+YJ*YJ)
  - TEMH/TEML

**RETURN**

**END**
2
+0.001*RCUL(I)*0.5*BETA(I)*SINB/COSB/(ET(I)*CARA)

F(1,2)=0.144*RCUL(I)**2.*(-0.5*3INB*SINB-COSB+1.0)/(ET(I)*VI)

F(3,3)=0.144*RCUL(I)**3.*(0.5*BETA(I)-0.5*SINB*COSB)/(ET(I)*VI)

RETURN

END

C **************************************** * ** * * * * ** * * * * ** * * ** * * ** * ** ** **

C INVERT THE ABOVE FLEXIBILITY MATRIX

CALL MATINV(F,3,6,6,DETERM)

DO 3 JJ=1,3

3 XI=COOR(JJ,1)

YJ=COOR(JJ,2)

DO 40 H=1,3

TRAN(H,JJ)=TRAN(JJ,H)

30 S(H,JJ)=0.0

DO 40 L=1,6

DO 30 K=1,6

S(K,L)=S(L,K)

C(1,1)=AK

C(1,2)=0.0

C(1,3)=-AK

C(2,1)=0.0

C(2,2)=B/J

C(2,3)=-B/J

C(3,1)=0.0

C(3,2)=AK

C(3,3)=-B/J

RETURN

END

C SUBROUTINE CULTRN(I)

C COMMON /A2/ COOD(5,5)+ICONN(4,6)

COMMON /B2/ ACUL(5)+BCUL(I)+RCUL(I)+BETA(I)+TRAN(I,6,6)

XI=ICONN(I,1)

JJ=ICONN(I,2)

X=COORD(I,1)

Y=COORD(I,2)

JY=COORD(JJ,2)

DO 1 L=1,6

DO 1 K=1,6

TRAN(K,L)=TRAN(L,K)

11 +0.001*RCUL(I)*0.5*BETA(I)+SINB/COSB/(ET(I)*CARA)

F(1,2)=0.144*RCUL(I)**2.*(-0.5*3INB*SINB-COSB+1.0)/(ET(I)*VI)

F(3,3)=0.144*RCUL(I)**3.*(0.5*BETA(I)-0.5*SINB*COSB)/(ET(I)*VI)

RETURN

END

C **************************************** * ** * * * * ** * * * * ** * * ** * * ** * ** ** **

C SUBROUTINE TRIK(I,II,JJ,KX)

C DIMENSION (F3,6,6),DAREA(6,6)

A(J)=COOR(JJ,1)-COOR(II,1)

B(K)=COORD(KK,2)-COORD(II,2)

S(A,J)=0.0

S(B,K)=DAREA(A,J)*SINB-S(2,J)*COSB

S(3,1)=-S(1,1)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(1,2)=0.0

S(1,3)=0.0

S(2,1)=0.0

S(2,2)=S(1,1)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(2,3)=0.0

S(3,1)=0.0

S(3,2)=0.0

S(3,3)=S(1,1)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(4,1)=-S(1,1)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(4,2)=-S(1,2)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(4,3)=-S(1,3)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(5,1)=-S(2,1)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(5,2)=-S(2,2)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(5,3)=-S(2,3)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(5,4)=-S(2,4)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(5,5)=S(2,5)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(5,6)=-S(2,6)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(6,1)=-S(3,1)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(6,2)=-S(3,2)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(6,3)=-S(3,3)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(6,4)=-S(3,4)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(6,5)=-S(3,5)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

S(6,6)=S(3,6)*RCUL(I)*(1.-COSB)-S12,1)*RCUL(I)*SINB-

RETURN

END

C **************************************** * ** * * * * ** * * * * ** * * ** * * ** * ** ** **

C SUBROUTINE CULTRN(I)

C COMMON /A2/ COOD(5,5)+ICONN(4,6)

COMMON /B2/ ACUL(5)+BCUL(I)+RCUL(I)+BETA(I)+TRAN(I,6,6)

XI=ICONN(I,1)

JJ=ICONN(I,2)

X=COORD(I,1)

Y=COORD(I,2)

JY=COORD(JJ,2)

DO 1 L=1,6

DO 1 K=1,6

TRAN(K,L)=TRAN(L,K)

11 +0.001*RCUL(I)*0.5*BETA(I)+SINB/COSB/(ET(I)*CARA)

F(1,2)=0.144*RCUL(I)**2.*(-0.5*3INB*SINB-COSB+1.0)/(ET(I)*VI)

F(3,3)=0.144*RCUL(I)**3.*(0.5*BETA(I)-0.5*SINB*COSB)/(ET(I)*VI)

RETURN

END
SUBROUTINE QUAD(I)
C
COMMON /A1/ NUMNP,NUMELT,NCPNLAY
COMMON /A2/ COORD(42,6)
COMMON /A5/ ICODE(4,6)
COMMON /A7/ NL,NCASE,NUMEL,NUMNP,NSEL1
COMMON /C1/ ET(42).PR
COMMON /C3/ VIC+DEN
COMMON /D2/ SYSK(10,10).0(10)
DIMENSION NP(51,5)
AREA(I)=0.0
DO 10 M=1,4
10 NPIM=ICONN(1,M)
NP(1)=NUMNPT+1
DO 15 M=1,10
Q(M)=0.0
DO 15 MM=1,10
15 SYSK(M,MM)=0.0
J=NP(1)
K=NP(2)
L=NP(3)
M=NP(4)
KK=NP(5)
COOR(J,KK)=COOR(J)+COOR(K)+COOR(M)/4.
COOR(J,KK)=COOR(J)+COOR(K)+COOR(L)+COOR(M)/4.
DO 130 NG=1,6
130 IF(NP(NG))=0 TO 17
IF(NP(NG))=0
17 CONTINUE
IF(CASE(.EQ.2)GO TO 7
C*.
IC** IF IT IS THE TIME DEPENDENT ANALYSIS. NCASE+2
C** CALCULATE DEAD Wt. FORCES. NEWLY ADDED SOIL ELEMENTS
I=ICONN(1,1)
J=ICONN(I,2)
K=ICONN(I,3)
CALL TRIK(I,J,J,K)
AREA(I)=AREA(I)+SMQ(I)
GO TO 19
18 CONST=500.00
SMQ(I)=CONST*(COOR(J)+COOR(K)+COOR(M))/4.
DO 19 M=1,6
19 CONTINUE
C** USE CASE NO. TECHNIQUE TO ASSEMBLE QUADRILATERAL ELEMENT
IF(CASE(.EQ.2)GO TO 17
L=ICONN(I,M)
IF(CASE(I)M(1:6) TO 17
L=ICONN(I,M)
SYSK(L,M)+SYSK(L,M)+SMQ(I)
17 CONTINUE
10 CONTINUE
RETURN
C
******************************************************************************
SUBROUTINE STIFF(I)
C
COMMON /A1/ NUMNP,NUMELT,NCPNLAY
COMMON /A2/ COORD(42,6)
COMMON /A5/ ICODE(4,6)
COMMON /A7/ NL,NCASE,NUMEL,NUMNP,NSEL1
COMMON /C1/ ET(42).PR
COMMON /C3/ VIC,DENS
COMMON /D2/ SYSK(10,10).0(10)
COMMON /F2/ SMAREA
COMMON /F4/ F(6)
IF(ICONN(1,3) .EQ. 0) TO 7
IF(ICONN(I,4) .EQ. 0 TO 8
C** THIS ELEMENT IS A QUADRILATERAL ELEMENT
CALL OUAD(I)
DO 4 II=1,9
4 SYSK(II,10)=SYSK(II,10)CC*SYSK(II,II)
DO 5 JJ=1,9
5 SYSK(II,II)=SYSK(II,II)+SYSK(II,II)CC*SYSK(II,II)
GO TO 10
7 CONTINUE
C** THIS IS A CULVERT ELEMENT
CALL CULK(I)
CALL CULTRN(I)
DO 2 L=1,6
2 F(L,M)=0.0
DO 3 M=1,6
3 F(L,M)=F(L,M)+TRAN(N)*SIN(N)*M)
DO 2 L=1,6
2 CONTINUE
3 SYSL(M)+SYSL(M)+F(L,M)*TRAN(N)*SIN(N)
GO TO 10
8 CONTINUE
C** THIS IS A TRIANGLE
I=ICONN(I)
J=ICONN(I)
K=ICONN(I)
CALL TRIK(I,J,J,K)
AREA(I)=SMQ(I)
GO TO 19
17 CONTINUE
10 CONTINUE
18 CONTINUE
C** IF IT IS THE TIME DEPENDENT ANALYSIS. NCASE+2

115
IF(NCASE .EQ. 2) GO TO 9

CALCULATE LEAD WT. FORCES FOR NEWLY ADDED ELEMENT

IF(1 .LT. NSEL1) GO TO 10

DL = AREA(I) * DENS / 3.

O(1) = 0.0
O(2) = PL
O(3) = 0.0
O(4) = DL
O(5) = 0.0
O(6) = DL

GO TO 10

9 CONST = 5000.0

O(1) = 0.5 * CONST * (COOR(I,2) - COOR(J,2))
O(2) = 0.5 * CONST * (COOR(K,2) - COOR(L,2))
O(3) = 0.5 * CONST * (COOR(M,2) - COOR(N,2))
O(4) = 0.5 * CONST * (COOR(P,2) - COOR(Q,2))
O(5) = 0.5 * CONST * (COOR(R,2) - COOR(S,2))
O(6) = 0.5 * CONST * (COOR(T,2) - COOR(U,2))

10 RETURN

END

SUBROUTINE BGK(I)

COMMON /A1/ NUMNPT, NUMELT, NSEL1, NCONP, NLAY
COMMON /A2/ COOR(3,3), ICONN(4,6)
COMMON /A4/ LADOM
COMMON /A6/ NCODE(42)
COMMON /A7/ NCASE, NUMEL, NUMNP, NSEL1
COMMON /C3/ VK, DENS
COMMON /C4/ AREA(42)
COMMON /C6/ 5(6,6), SYSK(10,10)
COMMON /C7/ ALPHA(42), HDIS(42)
COMMON /D1/ DISP(52,3), STRESS(42,6), POREPR(42)
COMMON /D5/ DELV(42)
COMMON /D7/ KPRIN1, KPRIN2, KPRIN3

IF(ICONN(1,4) .EQ. 0) GO TO 10

IF(NCASE .EQ. 16) GO TO 11

IF(ICONN(1,4) .NE. 0 AND ICONN(1,5) .NE. 0) GO TO 12

POREC = POREPR(NE3)

IF(ICONN(1,5) .NE. 0 AND ICONN(1,6) .EQ. 0) GO TO 13

IF(NCASE .EQ. 1) GO TO 14

CONTINUE

14 CONTINUE

CONTINUE 

END

SUBROUTINE FVEC(I)

COMMON /A1/ NUMNPT, NUMELT, NSEL1, NCONP, NLAY
COMMON /A2/ COOR(3,3), ICONN(4,6)
COMMON /A4/ LADOM
COMMON /A6/ NCODE(42)
COMMON /A7/ NCASE, NUMEL, NUMNP, NSEL1
COMMON /C3/ VK, DENS
COMMON /C4/ AREA(42)
COMMON /C6/ 5(6,6), SYSK(10,10)
COMMON /C7/ ALPHA(42), HDIS(42)
COMMON /D1/ DISP(52,3), STRESS(42,6), POREPR(42)
COMMON /D5/ DELV(42)
COMMON /D7/ KPRIN1, KPRIN2, KPRIN3

IF(NCASE .EQ. 16) GO TO 11

IF(ICONN(1,4) .EQ. 0) GO TO 10

IF(NCASE .EQ. 16) GO TO 11

IF(ICONN(1,4) .NE. 0 AND ICONN(1,5) .NE. 0) GO TO 12

POREC = POREPR(NE3)

IF(NCASE .EQ. 1) GO TO 14

CONTINUE

14 CONTINUE

CONTINUE 

END
SUBROUTINE BAWCD
C
COMMON /A6/ NCODE(42,8)
COMMON /67/ NLCASE,NUM,EL,NUMNP,NSL:
COMMON /D6/ N8
J=0
DC 259 N=1oNUMEL
DO 258 1=1,8
IF(NICODE(No1) oEU. 0)06 TC 258
DO 257 L=1,8
IF(NCODE'NoL) oEQ. 0/20 TO 257
KK=1A0S(NCODE(No1)NCODE(N,L)/
FIKKJ1257,256
256 J=KK
257 CONTINUE
258 CONTINUE
259 CONTINUE
NB=J+1
RETURN
END
C
SUBROUTINE SOLVF_
C
COMMON /D3/ A(7140/
COMMON /04/ 0(1191
COMMON /05/ ND
COMMON /06/ NB
C
DIMENSION F(I35)
NW =ND
N=0
50G N=N+1
C..FIND THE FIRST TERM OF 9312 N
N1=(N-1) .:2*NDN+2)/2+1
C..
DIVIDE RIGHT SIDE BY DIAGONAL ELLMEIIT
E(N)=8(N) /A(N1)
HECK FOR LAST EQjATION
FIN .EQ. ND/GO TO 700
IF(NB .E2. ND)GO TO 11
L=NDNB+1
IF(N .GT. L)GC TO 11
NW.NB
GO TO 12
11 NW=NW-1
C**
DIVIDE N TH EQUATION BY DIAGONAL ELEMENT
12 DO 600 K=2oNW
NK=N1+K-1
F(K) =A(INK)
6CC A(NK)=A1NK1/A(N1/
C
C**
REDUCE REMAINING EQUATION
DO 660 L.21NW
I=N+L-1
11=41-11*(2*ND-1+2)/2+1
J=0
DO 650 K=LoNW
J=J+1
IJ.11+0-1
NK=N1+K-1
650 A(1J)=6(1J1F(L).A(NK(1)
10(1)=B(1)F(L)*E(N)
66C CONTINUE
GO TO 500
C
C **
BACK SUBSTITUTION
700 NW=1
701 N=N-1
FIN .F:0.. 0(30 TO 900
N1=(N-1)*(2.NDN+21/2+1
(FINS oE0o ND1GO TO 13
L=NDNE+1
IF(N .GT. L)G0 TO 13
NW=NB
GO TO 14
1E NW=NW+1
14 DC 800 K=2,NW
L=N+KI
NK=N1+K-1
Ei(N)=B(N)A(NK1*B(L)
800 CONTINUE
GO TO 701
900 RETURN
C
SUBROUTINE RESULT
C
COMMON /A1/ 1.LI9NPT,NUMELToNCULETINCONP NLAY
00134013 /A2/ 0001(53,31,10JNN)42.53
COMMON /A3/ KNOVA(20,41oNiD0F(52*3)
COMMON /A4/ LADOM
COMMON /A5/ IITODE(4.6)
CO:19LN /AA/ NCODE(42,8)
COkAION /67/ NLoNCASE,NUM:L,NC'9NPoNSEL1
CC"WON /81/ KTYPEoRADIUSoVE.VI,CAPEA
20:IDON /02/ ACQL(5)o3CUL(5),RCUL(5),ATAL,I,TRP '6,6'

COMMON /01/ ET(42)
COMMON /03/ VA.FOENS
COMMON /05/ EV3
COMMON /06/ M3(42)
COMMON /07/ S(6)
COMMON /08/ B(110)
COMMON /31/ AREA(42)
COMMON /02/ 5(6,6)
COMMON /D4/ 8(119)
COMMON /101/ DISP(52,3)
COMMON /101/ POREPR(42)
COMMON /F3/ ST(6)
COMMON /F4/ F(6,6)

DIMENSION TDISP(10)
WRITE(61.502)

502 FORMAT(//, ' NODE 8X, XDISP, 8X, YDISP, 8X, ROTATION')

IFINCASE .50. +1)00 TO 301
NUMNP=NUMNP

301 DO 50 C 1=1,NJMNP
       DO 4 J=1,3
       IF(NODOM(IsJ) .EQ. 0)00 TO 4
       K=N300M(1,1)
       DISP(IsJ)=DISP(IsJ)+8(K)
       4 CONTINUE

      I,(DISP(1.0)

501 FORMAT( //,15E16.5)

50C CONTINUE

WRITE(61,503)

503 FORMAT(//, 'ELE.

105 TDISP(K)=0.0
12 CONTINUE
11 INCASE = EQ. 2)00 TO 106
GO TO 107
106 =LADOM+L-WCULET
DO 12 L=1,6
   13 Q(L)-Q(L)*S(6)
107 DO 14 = K(6,6)
        DO 14 L=1,6
       14 (K(L,J)+G(L,N)+TRAN(N,J))

15 ST(N)=0.0
DO 16 NOUAD=1,4
      II=ICONN(I,NOUAD)

16 STRESS(ISN)=STRESS(ISN+1)
GO TO 18

17 STRESS(ISN)=STRESS(ISN+1)

18 CONTINUE
**MATINV subroutine**

**EQUATIONS OF THE FORM AX = B.**

JORDAN'S METHOD

**INVERSE OR SOLUTION**

**DETERM is the location in which the determinant is stored.**

**RETURN**

**SEARCH FOR PIVOT ELEMENT**

**RETURN**

**MAX Nicar**

**DELET = CCH + ALOG(10)*Y3**

**IF (IPIVOT(j) == 1) GO TO 13**

**DO 105 IPIVOT(j) = 2**

**DO 100 IPIVOT(j) = 1, N**


**C**

**IN NC RANGE**

**Y3*SIG2/SIG2**


**C**

**IN NC RANGE**

**11 CONTINUE**

**GO TO 13**

**12 CONTINUE**

**13 CONTINUE**

**RETURN**
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
IF (IFROM<>ICOLUMN) (IFROM, 260, IFROM)
DO 200 L=1,N
SWAP=A(IFROM,L)
A(IFROM,L)=A(ICOLUMN,L)
200 A(ICOLUMN,L)=SWAP
IF (IFROM) 260, 260, 210
SWAP=A(IFROM,L)
A(IFROM,L)=A(ICOLUMN,L)
210 A(ICOLUMN,L)=SWAP
INDEX(1,1)=IFROM
INDEX(1,2)=ICOLUMN
PIVOT1=A(ICOLUMN,ICOLUMN)
DETERM=DETERM*PIVOT1
C DEVIDE PIVOT ROW BY PIVOT ELEMENT
A(ICOLUMN,ICOLUMN)=1.0
DO 350 L=1,N
A(ICOLUMN,L)=A(ICOLUMN,L)/PIVOT1
350 IF (INDEX(1,1)) 380, 360, 360
360 DO 370 L=1,M
370 A(L,ICOLUMN)=A(L,ICOLUMN)/PIVOT1
380 IF (INDEX(1,2)) 390, 400, 400
400 DO 450 L=1,N
450 A(L,ICOLUMN)=A(L,ICOLUMN)*PIVOT1
460 DO 500 L=1,N
500 A(L,ICOLUMN)=A(L,ICOLUMN)/PIVOT1
550 CONTINUE
C REDUCE NON-PIVOT ROWS
DO 550 L=1,N
IF (INDEX(1,1)) 530, 510, 510
510 DO 520 L=1,N
520 A(L,ICOlUM)=A(L,ICOLUMN)-A(L,ICOLUMN)*A(L,ICOLUMN,ICOLUMN)
530 CONTINUE
C INTERCHANGE COLUMNS
DO 710 L=1,N
710 IF (INDEX(1,1)) 720, 710, 710
710 IF (INDEX(1,2)) 730, 710, 710
730 CONTINUE
710 CONTINUE
700 RETURN
END

C *******************************************************