The angular distribution and interception of diffuse solar radiation below overcast skies

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SUMMARY

The angular distribution of diffuse radiation was measured from May 1976 to May 1977 with a set of purpose-built actinometers. Radiance distributions for 98 overcast hours were fitted well by an expression similar to the conventional function for a 'standard overcast sky' (SOC). However, values of the coefficient b averaged 1·23, significantly smaller than for the SOC ($b = 2$). Expressions for the interception of diffuse irradiance by sloping planes are discussed.

1. INTRODUCTION

There have been several studies of the angular distribution of radiation from cloudless skies (e.g. Kondratyev 1969; Steven 1977a), although cloudless skies occur infrequently in many regions. On the other hand, below overcast skies, which may be more common, the radiance distribution has seldom been measured, although there is a substantial literature describing luminance. Consequently, to estimate the irradiance of sloping surfaces, the angular distribution of radiance below overcast skies is commonly assumed to be either isotropic or identical with the luminance distribution. Although the isotropic assumption is mathematically convenient, it is supported neither by theory nor by observation, and in this paper we show that one consequence is a serious overestimation of the irradiance of sloping surfaces. To account for the decreasing luminance of overcast skies near the horizon, Moon and Spencer (1942) proposed a 'standard overcast sky' (SOC) distribution for luminance. The physical justification for describing radiance distributions by the same expression rests on Kondratyev's observations (1969) that distributions of spectral radiance and spectral luminance were independent of azimuth angle below overcast skies and that differences between relative radiance and luminance distributions were minimal. However, there have been few tests of this important point at other locations or over wide ranges of cloud type and turbidity. Grace (1971) compared the angular distribution of spectral radiance of overcast skies at 575 nm with the SOC distribution and found good agreement with the mean of measurements on three days; but he commented that the same close agreement might not have resulted if a broad waveband had been sampled.

This paper describes a long series of measurements of the angular distribution of sky radiance using instruments sensitive to the waveband 300 to 2800 nm. Records selected when skies were overcast are compared with the SOC distribution.

2. MEASUREMENTS

As part of a wider study, nine actinometers of a new design were established at Sutton Bonington (52·8°N 1·3°W) to measure the angular distribution of sky radiance (Steven 1977b).

The actinometers were designed to measure diffuse radiance continuously and un-attended, so that critical factors were sensitivity, zero-drift and weather-proofing. The

instruments had thermopile sensors of the 'black and white' type described by Monteith (1959), but with hot and cold junctions arranged in the form of a 2×2 chequer-board to reduce azimuthal dependence. The thermopile was mounted at the base of an aluminium tube 160 mm long, and 50 mm diameter windows of Schott WG 295 glass (transmitting from about 300 to 2800 nm) were fixed at the tube entrance and close to the thermopile. Optical black paint minimized reflection from the inner walls of the tube, and the actinometer was covered with 4 mm of expanded polystyrene and a layer of reflecting foil to minimize radiative and conductive heating.

A partial compensation was made for zero-drift, using a previously established relationship with the rate of change of instrument temperature. Typical values of the residual zero-offset, as measured at night, were 2 or $3 \mu V$, whereas instrument sensitivities were $\simeq 2 \mu V/(\text{W m}^{-2}\text{sr}^{-1})$. Skies with irradiance, S_d, less than 50 W m^{-2} were not considered in this study so the zero-offset would rarely have exceeded 10% of the total output even in the dullest conditions.

Each actinometer measured radiance, N, over a region of sky of about 0·1 steradian. The instruments were calibrated by comparison with a Linke–Feussner radiometer: the absolute calibrations had standard errors of less than 5%; standard errors of relative calibrations between instruments ranged from 1 to 3%.

Eight of the actinometers were oriented facing north, south, east and west at angles, θ, of 30° and 60° to the zenith. The ninth actinometer faced vertically upwards ($\theta = 0$). Global radiation, S_t, and diffuse radiation, S_d, on a horizontal surface were measured with Kipp solarimeters. The instruments were monitored by a data logger at ten-minute intervals over the period May 1976 to May 1977. Calibrations and small corrections for zero-offsets were applied during computer analysis.

Periods with cloudy skies were selected initially by choosing occasions when no direct radiation was recorded for an hour, i.e. the measured values of S_d and S_t were equal for six consecutive readings. A set of 355 'sunless hours' were thus obtained from the records, and hourly averages of the sky radiance measurements were calculated. Not all the sunless hours were completely overcast however: since each hour consisted of six spot measurements, some apparently sunless hours were recorded below partly cloudy skies. For example, with 6 oktas of cloud the probability that no direct sunshine will be recorded in six consecutive readings is $(6/8)^6$ or about 0·18. The proportion of non-overcast hours in the total could not be determined without knowledge of the frequency distribution of cloudiness. However, the azimuthal dependence of radiance is larger for partly cloudy skies than for overcast skies because of the influence of the sun's position, and the analysis that follows is inappropriate for such data. Hence, to eliminate spurious records, the sample of overcast hours was restricted to occasions when the variation of radiance with azimuth was minimal. In all the 99 hours selected in this way the coefficient of variation of radiance with azimuth was less than 0·1 at both $\theta = 30°$ and $\theta = 60°$. The values of S_d measured during the 99 hours ranged from 50 to 380 W m^{-2}.

3. RESULTS

Under overcast skies Grace (1971) noted rapid changes of radiance distribution. In the present work, the coefficient of variation of S_d within one hour (six observations) was typically about 0·2 and the coefficient of variation of N was typically about 0·25. Hourly average values were less variable.

Several authors have described the radiance distribution of overcast skies by an equation of the form

$$N(\theta) = N(0)(1 + b\cos\theta)/(1 + b) \quad . \qquad . \qquad . \qquad (1)$$

where b is a constant. The number $1 + b$ is the ratio of radiance at the zenith to that at the horizon. The standard overcast sky (Moon and Spencer 1942) uses this formula with $b = 2$ but Walsh (1961) suggested that $b = 1.5$ fitted the mean overcast sky more accurately. Goudriaan (1977) and Fritz (1955) gave a theoretical foundation for this form of distribution on the basis of an analysis of scattering and attenuation in clouds. Goudriaan's analysis shows that the value of b depends on surface albedo, ρ, according to $b = 2(1 - \rho)/(1 + 2\rho)$. For a typical range of surface albedo of 0.1 to 0.2, b should vary between 1.5 and 1.14. Fritz derived the relationship of b with ρ as $b = 1.5(1 - \rho)/(1 + \rho)$ and Kasten (1962) used this formula in studies of overcast skies over the Greenland icecap. With the same range of surface albedo, 0.1 to 0.2, the value of b according to the Fritz formula varies between 1.23 and 1.0. Table 1 summarizes the results of different authors.

TABLE 1. THE COEFFICIENT b IN THE ANGULAR DISTRIBUTION FORMULA FOR OVERCAST SKIES (EQ. (1)), ACCORDING TO VARIOUS SOURCES

	b
Isotropic sky	0·0
Moon and Spencer (1942)	2·0
Fritz (1955)	1·0 to 1·23*
Walsh (1961)	1·5
Goudriaan (1977)	1·14 to 1·5*
Present results	Mean 1·23
(95% fiducial limits)	(1·12 to 1·36)

*b is an explicit function of surface albedo, ρ; here $0.1 \leqslant \rho \leqslant 0.2$.

In the present study the 99 hourly mean values of radiance in 9 directions were used to test Eq. (1). Linear regressions were made of radiance, N, on $\cos\theta$ and values of b were obtained from the ratio of the slope to the intercept. One value of b was rejected because the intercept of the regression was not significantly different from zero. Fig. 1 shows the frequency distribution of the values of b. The distribution of b is positively skewed because b is the ratio of two uncertain numbers. Fiducial limits of b were calculated by Fieller's theorem (Finney 1971) and the error band widths were approximately proportional to the magnitude of b. A logarithmic transformation was therefore made to give appropriate weighting to the values of b, and the geometric mean value of b was 1.23, with 95% fiducial limits 1.12 and 1.36. Values of b exhibited a wide range on individual overcast days but no significant variation of b could be found with season or with the magnitude of S_d.
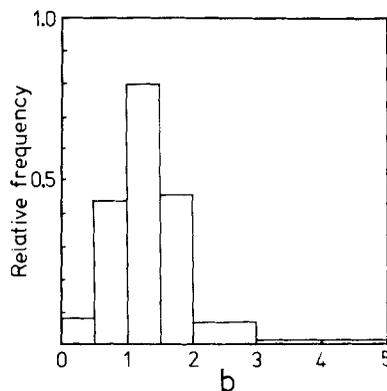

Figure 1. Frequency distribution of b for overcast hours. The frequencies are expressed per unit range of b to allow for unequal ranges.
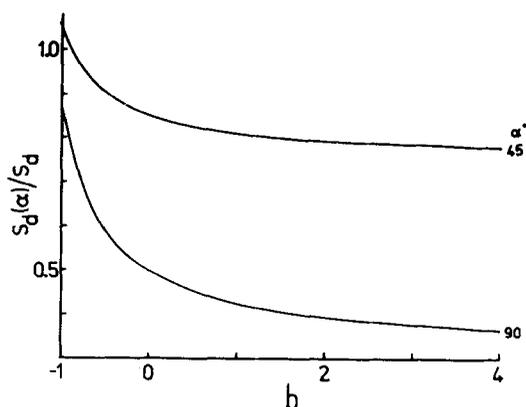
Figure 2. The relative irradiance $S_d(\alpha)/S_d$ of planes with slope $\alpha = 45°$ and $90°$ as a function of the parameter b.

4. Calculations of slope irradiance

The radiance distribution of Eq. (1) can be integrated analytically to give the diffuse irradiance $S_d(\alpha)$ of a plane surface of inclination α from the horizontal. Moon and Spencer derived the integral for the case $b = 2$. Extending their analysis to the general case, the irradiance relative to a horizontal surface is:

$$S_d(\alpha)/S_d = \cos^2(\alpha/2) + 2b\pi^{-1}(3+2b)^{-1}\{\sin\alpha - \alpha\cos\alpha - \pi\sin^2(\alpha/2)\} \qquad (2)$$

where α is in radians. When $b = 2$ this reduces to the illuminance formula of Moon and Spencer. The effect of b on the relative irradiance of inclined surfaces under an overcast sky is shown in Fig. 2, for values of α of $45°$ and $90°$. Compared with the value $b = 1\cdot23$ the isotropic assumption $b = 0$ overestimates the irradiance of vertical surfaces by almost 20%, but the maximum error in using $b = 2$ (SOC) instead of $1\cdot23$ is about 5%. Larger errors may arise when steep slopes are exposed to only part of the sky; the value of $b = 1\cdot23$ should be used, for example, when estimating radiation entering windows.

5. Conclusions

The general expression for angular distribution (Eq. (1)) fits the radiance of overcast skies well. The range of values of the coefficient b for individual overcast hours is quite large, most values lying between $0\cdot5$ and 2. Seasonal changes in albedo had no apparent effect on b and this range of values is presumably caused by differences in cloud density and height. Values of b are consistently smaller than the value $b = 2$ given for luminance distributions by Moon and Spencer, but the values suggested by Fritz ($1\cdot0$ to $1\cdot23$) and Goudriaan ($1\cdot14$ to $1\cdot5$) are close to our mean ($1\cdot23$). Errors in slope irradiance introduced by using $b = 2$ rather than $b = 1\cdot23$ may be large if only part of the sky is visible. Similarly it is desirable to use the value $b = 1\cdot23$ in calculations where radiance itself is required. These results suggest that there may be significant differences between radiance and luminance distributions of overcast skies, and a programme of spectral measurements would be desirable for fundamental applications.

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NOTE

There are two errors in our paper 'The diffuse solar irradiance of slopes under cloudless skies', **105**, 593–602.

In Appendix 1, p. 601, the function f_9 should be

$$f_9 = \sin\theta \cos\phi(5\cos^2\theta - 1)$$

and in Appendix 2, p. 602, the value of d_4 at $z = 45°$ should be

$$d_4 = -0.02 \pm 0.10.$$