THE CONTROL OF SEEPAGE UNDER DAMS ON PERVIOUS FOUNDATIONS

by

MAHIR UDDIN CHOWDHURY

A THESIS

submitted to the

OREGON STATE COLLEGE

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

July, 1948

APPROVED:

Redacted for Privacy

Head of Department of Civil Engineering

Redacted for Privacy

Chairman of School Graduate Committee

Redacted for Privacy

Dean of Graduate School

ACKNOWLEDGMENT

The writer owes a debt of gratitude to Dr. C. A. Mockmore, Head of the Department of Civil Engineering, Oregon State College, for his constructive criticism, helpful suggestions and advice in the preparation of this thesis.

Due acknowledgment is made to Mr. S. H. Poe, acting Chief Technical Editor, Denver Hydraulic Experiment Station, U. S. Bureau of Reclamation for his kind help and supplying some data on experiments conducted by the Research Engineers of the Bureau of Reclamation and other valuable suggestions.

Appreciation is also expressed to the department of Civil Engineering for the excellent facilities without which this work would have been impossible.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

Table | Page

LIST OF CHARTS AND PLATES

Chart

THE CONTROL OF SEEPAGE UNDER DAMS ON PERVIOUS FOUNDATIONS

CHAPTER I

INTRODUCTION

When a masonry dam or weir is constructed on pervious foundation or earth, some of the water impounded in the reservoir percolates beneath it and appears on the downstream side. So, in addition to the usual structural stresses as in an ordinary dam on impervious solid rock foundation, it is subjected to (1) the hydrostatic pressure along the foundation contact, (2) the hydraulic or exit gradient with which the water escapes upward at the toe and (3) the approximate leakage under the structure.

The problems encountered in the design of a masonry dam on a pervious soil foundation are:

1. To provide safe conduct for seepage water under and around it, so that neither foundation, nor bank material will be removed. If the velocity of flow where the seepage water emerges is sufficient and the exit gradient is beyond the safe limit for the foundation soil, flotation will occur and the particles of the foundation material will be disturbed and carried away. This will decrease the resistance to percolation and accentuate disturbance of the soil particles. The result is a progressive erosion and formation of cavities, below the base of the dam, which

in turn will ultimately cause formation of a channel or pipe beneath the floor. This may enlarge rapidly and cause failure of the structure. To prevent piping it is necessary to design the dam, so that the exit gradient of the seepage water at the toe is insufficient to remove the foundation material.

2. To reduce seepage losses under and around the dam to a reasonable and economic minimum, consistent with construction economy.

3. To obtain stability of the structure under all conditions of loading, uplift, overflow, tail-water fluctuations, etc.

4. To design the structure so as to distribute the load and to minimize settlement and also to permit it to accommodate itself to unavoidable settlement so as to prevent internal stresses and cracking.

5. To destroy the energy of the waste water at the toe of the spill-way in such a manner as to avoid injury to the apron and to minimize the erosion of the stream bed and banks.

6. To anticipate, in the design, the possible limit of degradation of river channel below the dam and consequent subsidence of tail-water level as affecting (a) undermining and stability of the structure, (b) functioning of spillway and (c) loss of draft head.

History of the Development of This Phase of Dam Design.

The first rational basis for the design of masonry dams on earth foundations seems to have been developed in India, as a result of the investigations of Col. Clibborn and Mr. Beresford.[8] Col. Clibborn was at one time principal of the Thomason Civil Engineering College, India. There were a number of weirs founded on light sand in the United Provinces, India, which had often given trouble. He carried out a classical set of experiments on the law of flow through sand. From these experiments, Mr. Beresford concluded that the Narora weir on the Ganges River was unsafe, because of excessive upward pressure on the apron and made a report to that effect. At the time no special trouble had been experienced with the dam, but as a result of his report pressure pipes were placed in the apron to indicate the uplift pressure beneath it. The pressure as indicated in these pipes confirmed Mr. Beresford's conventions and by coincidence, the next day after the readings were taken (March 30, 1898), the apron at another part of the dam was blown up, resulting in a breach of the dam. The failure of an important structure, following so promptly after the declaration of its instability, profoundly impressed the engineers of India and the "hydraulic gradient theory" became generally accepted about 1898.

The first edition of "Practical Design of Irrigation Works" by W. G. Bligh appeared in 1907, in which the theory was advanced that the stability of a weir on a porous foundation depended on the weight of the structure and not on the ratio of the percolation distance to the head. In his second edition published in 1910, Bligh admitted the fallacy of his original contentions and explained his well-known theory that the safety of masonry dams on earth foundations depended on the length of the percolation path which is along the line of contact of the structure and its foundation. Bligh's theory was accepted widely because of its simplicity.

In 1911, a paper was prepared by Mr. Arnold C. Koenig[24] giving rules for the design of masonry dams on earth foundations which contains a number of valuable ideas.

Coleman[5] carried out, for the first time in 1915, tests with models of dams resting on sand, to determine the distribution of pressure under the base and the relative effect of sheet piling at the heel and toe of the dam. These experiments demonstrated the greater relative effectiveness, length for length, of vertical contacts as against horizontal ones. But much advance in determining conclusive results for design was not made.

Little appeared in technical literature for many years after these publications to aid the engineer in the practical design of masonry dams on earth foundations.

In 1920, Pavlovsky[34] approached the problem of the flow of water through sub-soils of hydraulic structures from the analogy of flow of electricity through a conductor. The work was published in Russian and he achieved success in solving a number of problems.

In 1922, a notable contribution came from Charles Terzaghi[37]. He stated and proved by laboratory experiments that failures occurred by undermining, if the hydraulic gradient at exit was greater than the "flotation gradient" for the particular foundation material.

In 1926-27, trouble in some structures in the Punjab, India, became acute. Cracks appeared at the upstream and downstream aprons due to undermining of the sub-soil. Repairs were carried out on the then accepted Bligh theory, but the trouble persisted. A set of pressure pipes with well points were inserted in the floors of these structures and the observations disclosed that the pressures indicated by these piezometer pipes had absolutely no relationship with those calculated from the Bligh theory. These researches were carried out by Khosla and embodied in the Punjab Engineering Congress Papers Nos. 138 and 142 of 1930.

About the same time, investigation of uplift pressures under dams was being actively pursued in the United States of America. Parsons[33] and J. Hinds[17] presented two interesting papers before the American Society of Civil Engineers in 1928.

In 1929 it was decided to extend the Panjnad Weir in the Punjab (India). This afforded the opportunity of putting in a comprehensive set of pressure pipes and of conducting full scale experiments. This was the first full size experiment in the world and the results obtained in 1932[18] paved the way to the reliable solution of the problem. It was confirmed that the seepage flow of water through the sub-soil is in stream lines and therefore is susceptible of mathematical treatment and also experiments on hydraulic or electric analogy models.

In 1932, Prof. Warren Weaver,[40] Head of the Department of Mathematics at the University of Wisconsin and at the time working with the Rockefeller Foundation, developed his mathematical treatment of the flow of water through the permeable sub-soils under dams. Weaver's work provided an inspiration for co-relating the experimental results with the mathematical solutions, thus leading to confirmation and complete solution of the problem.

The years between 1932 and 1935 were marked by special activity in respect of the study of sub-soil hydraulics in relation to dam design in India, the United States of

America and Europe.

In September 1934, Prof. E. W. Lane[25] analyzed a large number of dams in the United States and many others almost all over the world. As a result of these investigations, he evolved his "Weighted Creep Theory" which in effect might be taken as the Bligh creep theory corrected for vertical cut-offs and sloping faces. While this theory was an improvement on the original Bligh theory, it was still empirical and lacked the back-ground of rational basis for design.

A number of useful papers on the subject were presented at the International Commission on dams in July, 1933.

In May, 1934, Mr. D. J. Hebert[16] carried out experiments for various dams on the electric models. His conclusions are that the application of electric models to problems of practical designs must be made with caution, until the point is reached at which it is possible to investigate all the geoligical features of a sub-soil and to reproduce them in an electric set-up.

In the same year, Mr. F. F. Haigh[10] and Mr. L. F. Harza[12] independently produced two very useful papers on almost similar lines. They took note of the exit gradients as a controlling factor in stability and discussed the distribution of pressure which could be considered as safe.

Harza got agreement between theoretical values of uplift pressures and those obtained from the electric analogy models for some of the simple cases dealt with by Prof. Weaver in his mathematical solutions. He also carried out experiments on electric analogy models for determination of the characteristics of pressure distribution in stratified foundation and also in cases of provision for filter drainage through the downstream apron for the purpose of reducing uplift pressure and exit gradient at the toe.

About the same time Dr. Vaidhianathan[38] obtained results on the electric models and also the Panjnad Weir model which showed conclusively:

(a) The distribution of pressures under structures on sand foundations can be reproduced on hydraulic or electric models.

(b) All seasonal and other variations from the normal conditions can be reproduced on hydraulic models by superimposition of silt, temperature or both and by simulating the stratification.

(c) The problem is susceptible of mathematical treatment.

After the conclusive agreements in the investigations conducted on the subject of sub-soil seepage flow by the various well-known engineers in the different countries, the theory of safe exit gradients has been generally accepted in the modern practice of design for dams on pervious foundations. Of-late the technique of drainage at the toe of a dam for safe escape of the seepage water has been greatly improved and is being widely practiced, especially in the case of earthen dams and flood control levees on the Mississippi River.

In 1945, Mr. P. T. Bennet[1] gave an interesting mathematical treatment of the effect of blankets on seepage through pervious foundations. He has presented the general principles involved in the design of blankets by mathematical analysis. The application of these principles to blanket problems, although not leading to exact results, provides, in many cases, a satisfactory method of estimating blanket performance.

In 1946, Messrs. T. A. Middlebrooks and W. H. Jarvis[29] developed certain design criteria for systems of drainage relief wells which provide deep drainage. Their paper has outlined the general method of design together with the mathematical and empirical back-ground.

CHAPTER II

BLIGH'S CREEP THEORY

Mr. Bligh[3] treated the course of seepage beneath a dam on the assumption that the water follows a path along the line of contact of the dam foundation (including the cut-off walls) with the foundation material. This contact between the dam and foundation is called the line of creep and the method may be called the line of creep method. The head is assumed to drop along a straight line from the head-water to the tail-water (if there is any) in proportion to the distance along the line of creep.

According to Darcy's Law of sub-soil stream line flow,

$$v = k\frac{h}{L}, \quad \ldots \ldots \ldots \ldots \ldots \text{(II- 1)}$$

where k = coefficient of permeability,

h = head under which the water is flowing,

L = length of path,

v = net effective velocity.

$$L = k\frac{h}{v}, \quad \ldots \ldots \ldots \ldots \ldots \text{(II- 2)}$$

For a given class of material, there is a definite maximum velocity, v_m at which the water can emerge below the dam without carrying away the foundation material and causing the failure of the structure. Combining this value of v_m with k which also depends on the foundation material to form a new coefficient $C = \frac{k}{v_m}$, the expression is:

$L_n = Ch$, in which L_n is the minimum safe length of travel path and C is a coefficient depending on the foundation material.

Mr. Bligh further assumed that the horizontal length of contact was just as effective as a vertical length. In other words, for a dam with horizontal floor of length b and an upstream cut-off wall of depth d_1, $L_n = b + 2d_1 = CH$ . . . . . . (II-3)

For a dam with horizontal floor length b and upstream and downstream cut-off walls of depths d_1 and d_2 respectively, $L_n = b + 2d_1 + 2d_2 = CH$, ... (II-4)

BLIGH'S COEFFICIENTS OR PERCOLATION FACTORS

1. River beds of light silt or sand of which 60% passes the 100- mesh sieve, as those of the Nile or the Mississippi Rivers 18
2. Fine micaceous sand of which 80% passes a 75- mesh sieve, as in the Himalayan Rivers and in such rivers as the Colorado 15
3. Coarse grained sands, as in central and south India 12

4. Boulders or shingle mixed with gravel
   and sand . . . . 5 to 9

In an abridged paper Mr. Griffith[9] gave the following values of the coefficient of creep or the ratio of creep distance to head which had been found sufficient to ensure stability in the United Provinces of India:

Material	Limiting Safe Value of "C"
1. Fine micaceous sand	$14\frac{1}{2}$ to 16
2. Fine quartz sand	$12\frac{1}{2}$ to 14
3. Coarse quartz sand	10 to 12
4. Shingles	8
5. Boulders	4

Concerning shingles and boulders, Mr. Griffith stated that the question of loss by leakage might make higher values of $\frac{L}{H}$ advisable in these cases. He also, however, suggested a 20% reduction in the values given above where reliable vertical staunching of 10 feet depth was used.

Bligh's theory was accepted widely because of its simplicity and his empirical assumption and coefficient were generally accepted until Prof. E. W. Lane[26] modified it and introduced his empirical "Weighted-Creep Theory".

LANE'S WEIGHTED-CREEP THEORY

Prof. E. W. Lane made an extensive study of the designs of a large number of dams in the United States and many other countries almost all over the world. His study of all the available records of percolation distances in the existing masonry dams on earth foundations and in those that have failed from piping has brought out three important conclusions:

1. Several dams have failed from undermining, with percolation distance which, judged by the ordinary standards, should be safe.
2. Many dams have stood successfully with percolation distances much less than those previously recommended.
3. The dams which failed had very little of their creep paths along vertical or steeply sloping surfaces, while those that stood, with much smaller distances, had a considerable proportion of such creep.

Prof. Lane's attention was first directed to the greater value of vertical creep by the difficulty of explaining the stability of the Prairie duSac Dam in Wisconsin with its extremely low plain creep ratio of 4.3 on a coarse sand foundation. According to him, this idea apparently was first expressed by Griffith who suggested that a reduction could be made in his recommended creep-head values for reliable vertical staunching.

On the basis of the study, Prof. Lane has developed his empirical "Weighted-Creep" theory. In this theory, it is assumed that the line of flow will follow the line of contact between a dam and its foundation; but the vertical contact is more effective than the horizontal contact and the coefficient C differs from that given by Bligh. He defines that the creep along a vertical face or surface sloping more than 45° with the horizontal should be treated as "vertical creep" and other surfaces, "horizontal creep". He has recommended that a unit horizontal length of creep should be considered to be one-third as effective as a unit length of the vertical creep.

For a dam with horizontal floor of length b and upstream and downstream cut-off walls of depths d_1 and d_2,

$$L_n = \frac{b}{3} + 2d_1 + 2d_2 = CH \quad \ldots\ldots \quad \text{(II- 5)}$$

TABLE I

PROF. LANE'S WEIGHTED-CREEP RATIO $= C^{(27)}$

(weight of horizontal creep, one-third)

Material	Lane's safe weighted-creep ratio	Bligh's value
Very fine sand or silt	8.5	18
Fine sand	7.0	15
Medium sand	6.0	--
Coarse sand	5.0	12
Fine gravel	4.0	--
Medium gravel	3.5	--
Gravel and sand	--	9
Coarse gravels including cobbles	3.0	--
Boulders with some cobbles and gravels	2.5	--
Boulders, gravel and sand	--	4 to 6
Soft clay	3.0	--
Medium clay	2.0	--
Hard clay	1.8	--
Very hard clay or hard pan	1.6	--

In order to use these values with safety, the cut-offs are to be built of solid masonry, built in contact with the earth sides of the trench, or of inter-locking steel or concrete piling driven so that the interlock is not broken and it is satisfactorily embedded at the top in the masonry structure. According to this theory, the uplift pressure to be used in design may be estimated by assuming that the drop in pressure from head-water to tail-water along the contact line of dam and foundation is proportional to the weighted-creep distance.

About the same time, Charles Terzaghi,[37] L. F. Harza,[13] A. N. Khosla[19] and many other engineers carried out experiments on the subject of sub-soil seepage with hydraulic and electric-analogy models. They do not seem to agree with Prof. Lane and according to them, Lane's weighted-creep ratio is not strictly correct under all circumstances.

Harza mentions that rational analysis by mathematics and its equivalent, the electric analogy, proves that even in a homogeneous foundation in perfect contact with the structure, the apparent horizontal creep resistance along the base may have a value much less than one-third the average resistance to vertical seepage around the cut-offs, the actual value being related to the relative depth of cut-offs in proportion to their distances apart.

As a result of his model experiment with electric analogy method, the following horizontal creep ratios have been derived:

Example: In case of both heel and toe cut-offs of equal depth d and horizontal distance b, the hydraulic gradient or creep resistance per unit distance along the base as compared with the creep resistance per unit distance along the vertical faces of cut-offs (equivalent to Lane's weighted-creep) are shown as below:

Space b between heel and toe cut-offs.	Approximate weighted value of horizontal creep
5.0 d	1/3
2.5 d	1/4.2
2.0 d	1/7
1.5 d	1/9

As per the mathematical analysis and results of experiments conducted by the aforesaid engineers, a new theory of safe exit gradients at the toe as related to the safety and design of dams on pervious foundations has been developed. This is treated in the next chapter.

CHAPTER III

THEORY OF SEEPAGE FLOW THROUGH SOIL

Darcy's Law:

In 1856 Darcy[6] carried out some experiments with soil and found that the velocity of seepage is proportional to the hydraulic gradient. Thus, water flowing with a net effective velocity, v through a length of path, l and under a head, h is given by,

$$v = K \frac{h}{l} \quad \text{. . . . .} \quad \text{(III- 1)}$$

where K is known as the coefficient of permeability or transmission constant, that is, it may be expressed as the net effective velocity of seepage under a hydraulic gradient of unity. The constant, K is dependent upon temperature, effective size and shape of soil grains, density, type of packing and porosity.

The law is based on the principle of slow laminar viscous flow through the soil mass in contrast to the open channel turbulent flow. The experimental investigation conducted by Welitsch-Kowsky,[41] Allen Hazen,[15] Prof. C. S. Slichter[36] and G. Von Heidiken of the Royal Technical University, Stockholm, tends to set the limitations for the above law as regards the high head. As per the experimental evidence about the range of validity of Darcy's Law, the upper limit of the velocity should be 0.01 to 0.015 ft/sec. and the maximum grain size of the medium between 1.5 and 2.0 mm.

SLICHTER'S THEORETICAL TREATMENT OF SUB-SOIL FLOW

If water is percolating through a homogeneous mass of soil in such a manner that the voids of the soil are completely filled with water and no change in the size of the voids takes place, the quantity entering from one or several directions into a small element of volume of soil must be equal to the amount of water flowing out on the other faces of this element of volume during any given time, or in other words the flow is continuous. This condition, which is a statement of the fact that both water and soil are incompressible, can be expressed for the three dimensional case by the following equation:

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (III\text{-} 2)$$

This is known as the equation of continuity. In this equation, u, v, and w are the three components of the discharge velocity v_o. If $\frac{dh}{dl}$ represents the hydraulic gradient in the direction of flow, and $\frac{dh}{dx}$, $\frac{dh}{dy}$ and $\frac{dh}{dz}$ are its components, then Darcy's Law can be expressed by the following equations:

$$v_o = K\frac{dh}{dl} \quad . . . . (III- 3)$$

$$\left.\begin{aligned} u &= K\frac{dh}{dx} \\ v &= K\frac{dh}{dy} \\ w &= K\frac{dh}{dz} \end{aligned}\right] \quad . . . . (III- 4)$$

Therefore, substituting the equation (III- 4) in equation (III- 2) one arrives at:

$$\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} + \frac{d^2h}{dz^2} = 0 \quad . . . . (III- 5)$$

This is the general differential equation for the steady flow and has the form of the well-known Laplace's equation. The pressure function which satisfies this differential equation is called the "potential function".

It is well known that the same equation of Laplace governs the steady flow of heat, electricity through a conductor, or a perfect fluid. It is thus clear that the problem of flow of water through the sub-soil is analogous to that of the flow of a viscous fluid or that of electricity through a conductor.

The potential law which has been derived from Darcy's Law will, however, be subject to all the limitations of the latter law. These are:

1. The velocities in the medium should not exceed 0.015 ft/sec. and the maximum grain size should not be more than 2 mm. in diameter.

2. The soil medium in which the flow takes place must be completely saturated with water.

In our problem of seepage under dams, we have to deal with only the two-dimensional case which is satisfied by the equation:

$$\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (III-6)$$

This equation represents two families of curves intersecting at right angles. In hydro-mechanics these curves are known as the "Flow lines" and the "Equipotential lines" or lines of equal pressure.

Although the general differential equation has solved only for some simple cases of seepage flow, the geometric properties of the flow lines and the equipotential lines permit graphical or electric analogy methods of solutions for practically all two-dimensional seepage problems.

The graphical method was first devised by Prof. Forchheimer.[7] The electric analogy method was first tried by Prof. N. N. Pavlovsky[34] in Russia in 1920. The technique was subsequently verified with model experiments by Dr. V. I. Vaidhianathan in the Punjab Irrigation Research Institute, India,[38] and Mr. L. F. Harza, M. Am. Soc. C. E.[12] The pressure observations in the hydraulic models and electric analogy tray gave results which had very satisfactory agreement with those derived mathematically.

MODEL CHARACTERISTICS IN SUB-SOIL WORK

Experiments have been conducted in the Punjab Irrigation Research Institute with the hydraulic and electric analogy models. The results in both the cases have proved that the flow net of equi-pressure and flow lines, for any particular dam profile on homogeneous soils, is independent of:

(a) Class and structure of sub-soil so long as it is homogeneous.

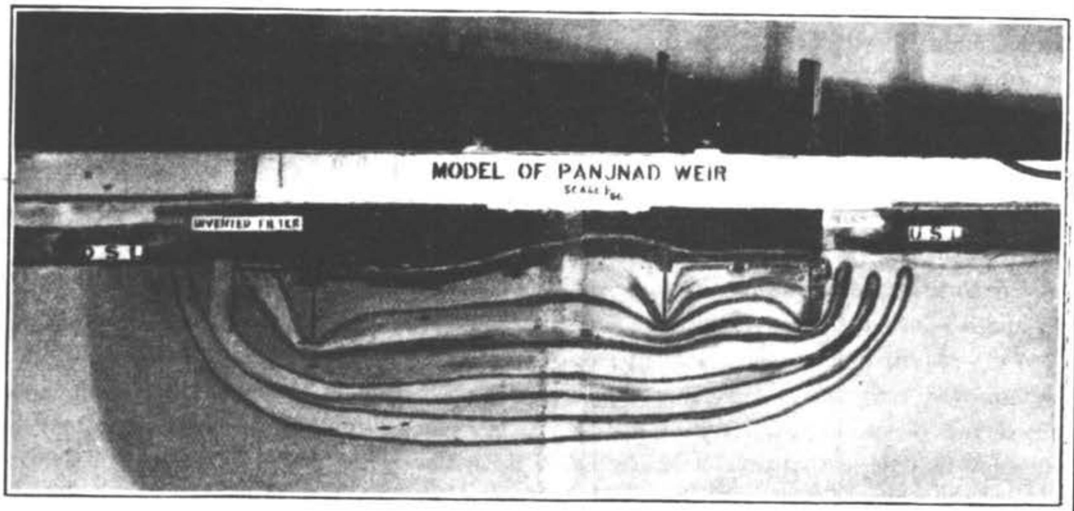
(b) Scale ratio.

(c) Temperature so long as it is uniform throughout the medium.

(d) Applied head or pressure.

(e) Upstream and down-stream water levels.

(a) Model of the Panjnad weir showing stream lines in the foundation.



(b) Stream lines shown in the work done by Ionides of the Irrigation Directorate, Baghdad (Mesopotamia) in 1932.

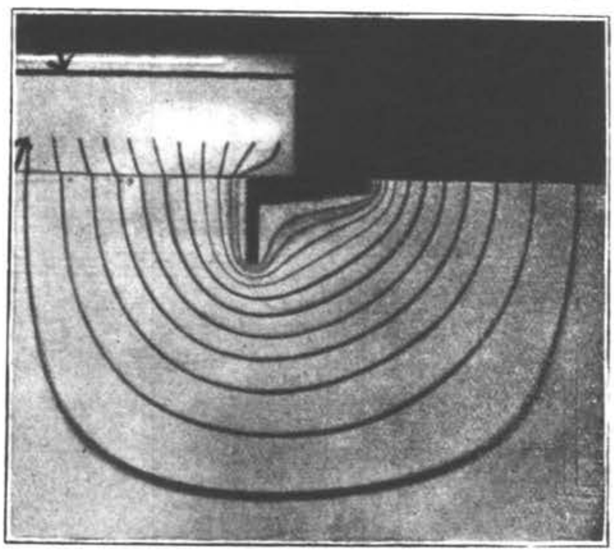


Fig. 1 - Models showing the sub-stratum seepage flow patterns under dams.

EXIT GRADIENTS AS RELATED TO DESIGN OF DAMS

The failure of a dam by seepage flow can occur by:

(a) Undermining of the sub-soil.

(b) Uplift pressure under the dam floor being in excess of the weight of the floor.

Therefore, the conditions requiring determination in order to design or test a dam on porous foundation are:

1. The uplift pressure under the base of the dam as related to its weight.
2. The vertical exit gradient at the toe of the dam as related to the physical characteristics of the foundation material.

The failure by under-mining is the most common, so that a knowledge of its causes and of the measures to prevent it, is of utmost importance both for design of new works and for ascertaining the safety of existing ones.

The undermining of the sub-soil starts from the tail end of the dam. It begins at the surface due to the residual force of seepage water at this end being in excess of the restraining forces of the sub-soil which tend to hold the latter in position. According to the commonly accepted ideas of Bligh and Lane, this undermining is supposed to result from what is known as "piping", that is, the erosion of sub-soil is caused by the high velocities of flow of water through it, when such velocities exceed a certain limit. This conception of undermining is incomplete.

The conception of undermining by "flotation" was put forward by Prof. Charles Terzaghi in 1922[37] and independently arrived at by F. F. Haigh in 1930.[11] Prof. A. Casagrande[4] has mentioned that it was demonstrated by Terzaghi convincingly by theory as well as by experiments that it is the magnitude and distribution of seepage forces near the toe of water impounding structures (particularly the discharge gradient at the toe) which determines the degree of safety against piping or under-ground erosion.

Prof. Charles Terzaghi also has mentioned that when he derived the equation for the flotation gradient, he attempted at once to determine by experiment to what extent this factor can be used as a criterion for the safety of dams against piping. The results of experiments with perfectly homogeneous sands showed that the escape gradient at which piping occurred is practically equal to the flotation gradient. Thus, if the base of a dam rests on the horizontal surface of the sand without any sheet piles intercepting the flow of seepage, the escape gradient is equal to infinity irrespective of the creep-head ratio, L/H. Hence, if the escape and the flotation gradients are really identical, piping should occur at any hydraulic head. This actually took place and as a consequence he was compelled to establish the base of his flat-bottom model dams at a certain depth below the surface, in order to be able to produce any hydraulic head at all. For a single row of sheet-pile with a depth, d, the escape gradient is approximately 2/3 H/L = 2/3.H/2d = H/3d. Piping occurred in his tests at an escape gradient which was a trifle less than the flotation gradient. But for practical purposes, this small difference may be neglected.

THEORY OF FLOTATION GRADIENT

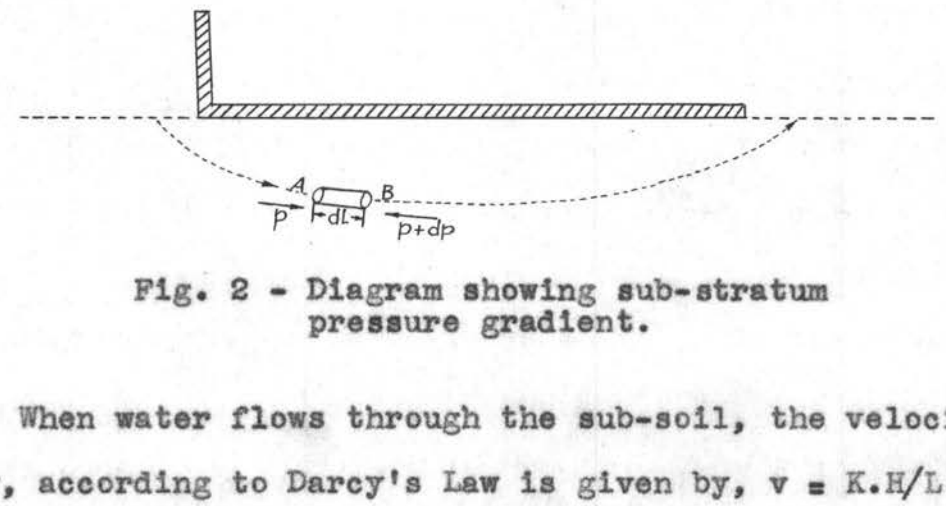


Fig. 2 - Diagram showing sub-stratum pressure gradient.

When water flows through the sub-soil, the velocity of flow, according to Darcy's Law is given by, v = K.H/L. This water exerts a force (F) on the soil medium along its line of flow. Besides this force, the particles of the soil medium are subjected to two other forces:

1. The force of gravity or weight of sand particles which acts downwards.
2. The force of buoyancy which acts opposite to gravity.

These two forces can be combined to represent the buoyed weight of the loose soil particles.

Let S = specific gravity of soil particles,

W = weight of unit volume of water,

P = ratio of pore space in the material.

Then, the weight of soil particles per unit volume

$= W(1-P)S$

The weight of displaced water $= W(1-P)$

Therefore, the buoyed weight of the soil particles per unit volume $= W(1-P)\ (S-1) = W_s$.

In Fig. 2, let a cylinder of soil be assumed along one of the stream lines,

da = area of cylinder section

dL = length of cylinder

pda = pressure at A

$(p+dp)da$ = pressure at B

The force acting on the cylinder along the direction of the stream line $= pda - (p+dp)da = -dp.da$

The force per unit volume $= -dp.da/da.dL = -dp/dL$

The action of water at any point in the soil medium through which it percolates can be given by the force (F) per unit volume. Therefore, $F = -dp/dL$ = Pressure gradient at the point. At the toe of the dam, (F) acts vertically upwards. So the resultant force, $R = F - W_s$. The soil particles will start to be lifted up or float when (F) becomes greater than W_s. The critical condition occurs when the force of percolating water = F(critical) = W_s

$$\text{or} \quad dp/dL = W(1-P)(S-1) \quad . \; . \quad \text{(III- 7a)}$$

The "flotation gradient" = G_f = $(1-P)(S-1)$ . . . (III- 7b)

The flotation gradient will vary with the pore space of the sub-soil and the specific gravity of the soil particles. The density of the soil particles, though generally in the neighborhood of 2.6 to 2.7 for ordinary natural sand formations, may vary in extreme cases from 1.8 to 2.8. Similarly the pore space, which ranges between 37 and 42 per cent, may be as low as 20 per cent and as high as 45 per cent. Working on these limits, the extreme values of G_f may range from 0.44 to 1.44.

"Flotation" is the cause and "piping", the effect. If flotation is prevented, piping can not occur. Therefore, a designer is concerned as to safety only with the conditions obtaining at the point of escape of the seepage water.

Uncertainties enter into the computation of the critical conditions of flotation at the toe because cross-flow may occur by convergence towards the easiest point of escape where the material is non-homogeneous and the hydraulic gradient, dH/dL at the point of escape is seldom equal to the average gradient, H/L even in homogeneous material. Furthermore, if lenses or strata of porous material are inter-bedded with finer material, the relative permeability may encourage concentration of flow from the finer into the coarser material near the outlet, giving the latter more than its proportionate quantity and a higher rate of loss of head at the point of exit. It is the function of the factor of safety to allow for such conditions and other uncertain factors in the sub-soil. This has been discussed by Prof. Terzaghi under the subject of "Minor Geologic Details," although it is often of major importance to the success of the structure. It does add to the difficulties of predicting the local flotation gradient.

Considered as a purely laboratory problem, the determination of critical or flotation gradient for a given material is reasonably definite once the characteristics of the material are known. In Prof. Terzaghi's experiments on varied materials the measured internal pressure to cause flotation nearly equalled the theoretically computed critical pressure.

Terzaghi in his experiments used sands of specific gravity of 2.65 and found the following values of G_f.

TABLE II

Pore space = P	G_f(observed)	G_f(computed) $(1-P)(S-1)$
0.43	0.95	0.94
0.355	1.11	1.06
0.430	1.13	0.94
0.410	0.96	0.97
0.440	1.05	0.92
0.350	1.08	1.07

Dr. C. A. Mockmore and J. W. Dougherty(31) carried out some experiments at the Oregon State College with water flowing vertically through beds of different kinds of granular materials and the results are shown in Table III.

TABLE III

Material	G_f(observed)	G_f(theoretical) $(1-P)(S-1)$
1. White filter sand	0.814	0.97
2. Columbia River sand	0.812	1.05
3. Willamette River sand	0.878	1.07
4. Lead shot	6.00	5.88

Dr. Vaidhianathan(39) carried out some very careful experiments in 1935, with Punjab sands of different orders of packing and size distributions. These results are tabulated below:

TABLE IV

Specific gravity of sand (S)	Pore space (P)	Transmission constant (K)	G_f(observed)	G_f(theo.) $(1-P)(S-1)$
2.68	0.413	0.0167	1.00	0.99
2.69	0.385	0.0414	0.85	1.04
2.69	0.403	0.0422	1.00	1.01
2.64	0.399	0.0434	1.00	0.99
2.65	0.415	0.0778	0.95	0.97
2.68	0.410	0.0854	0.95	0.99
2.67	0.381	0.0945	1.00	1.03
2.69	0.382	0.0948	1.25	1.03
2.70	0.388	0.1023	1.25	1.04
2.69	0.392	0.1521	1.00	1.04
2.68	0.390	0.2219	1.00	1.02
2.69	0.384	0.3002	0.95	1.04

Dr. Vaidhianathan's experiment has shown interesting characteristics of the flow of water through sand medium. In downward flow, the transmission constant or the coefficient of permeability remains absolutely steady irrespective of the pressure gradient. In upward flow, it remains constant to about a gradient of 1:1; beyond that it increases rapidly. If after that, the gradient is again lowered, the transmission constant remains much above its previous value. When the gradient is again increased, the transmission constant increases first slowly and then rapidly till at a gradient of about 1.7:1 the soil definitely blows off and finally breaks up (Fig. 3).

These characteristics or curves have great practical significance. If the sub-soil at the tail end of a structure has once been subjected to a gradient more than the critical value, the whole soil mass gets disturbed as a result of flotation and does not settle back to its normal consolidation even though the gradient is reduced. Thus once the stage of flotation is crossed, the mischief is done.

This also explains the phenomenon of "quick sand". Quick sand is primarily a condition characterized by the flow of seepage water upward through the sand, rather than a characteristic of the material itself. This is the case because fine sand, when once expanded with an excess of pore water and consequent voids due to flotation, retains this condition for some time even with removal of pressure because of the greater resistance to the escape of the contained water through the small pores. The jetting of piles, as an example, is merely the artificial creation of a "quick" condition to make the sand more fluid.

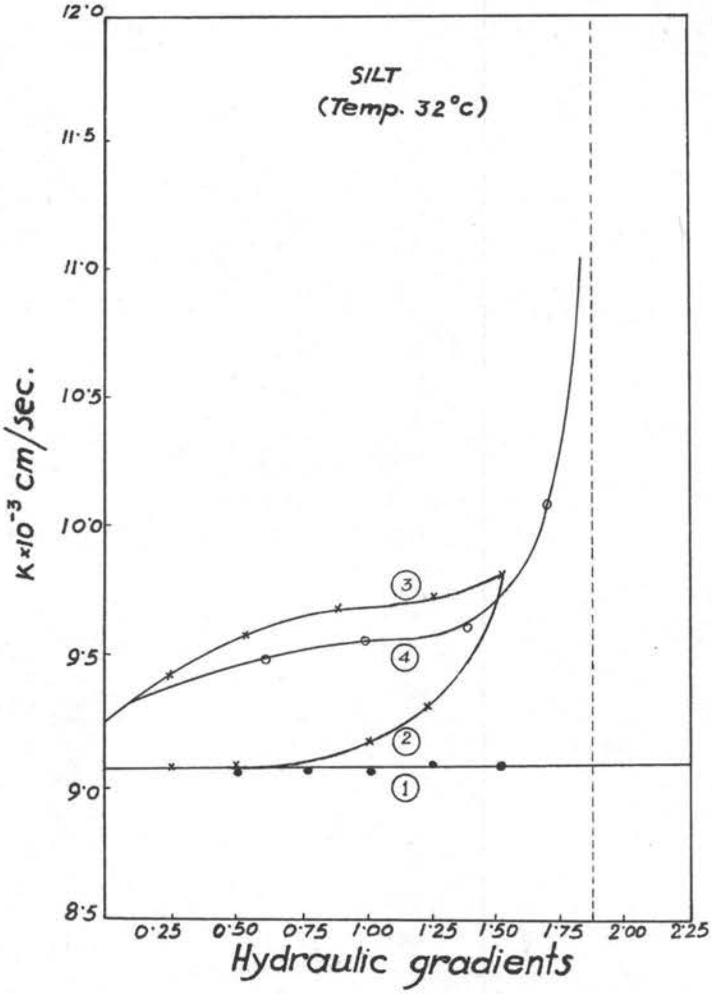


Fig. 3 - Characteristic curves for hydraulic gradient and coefficient of permeability.

Factor of Safety:

The factor of safety is to be chosen with reference to the geological details, uniformity of the foundation soil, stratified condition and extent of knowledge of the foundation conditions. It will then be dependent on judgement of field conditions and empirical data. Besides there are other conditions to be considered, such as formation of scour hole at the toe of the spill-way dam due to surface flow which may increase the exit gradient beyond the critical value. The exit gradient at the toe may also be increased many times by the occurrence of local surges or waves which in action resemble the working of an intermittent pump.

MATHEMATICAL TREATMENT

Determination of Uplift Pressures under the Base of the Dams and Exit Gradients.

The general equation for the potential function of two-dimensional seepage flow is given by:

$$\frac{d^2p}{dx^2} + \frac{d^2p}{dy^2} = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (\text{III- } 8)$$

The first solution of a problem of two-dimensional motion, in which the motion is bounded partly by fixed plain walls, was given by Helmholz. A solution of equation (III- 8) for any boundary conditions has been made possible by the method of conformal transformations devised by Schwarz and Christoffel in 1869.

General Form:

If, under a floor, a vertical obstruction like a line of sheet piles or wells is introduced, the configuration of stream lines and potential lines will be distorted. But it is possible to bring back this distortion to the normal configuration as under a simple floor by means of Schwarz Christoffel transformation and the solution for pressure distribution under the floor can be arrived at.

Solution in Case of no Sheet Piling:

In Fig. 4a, when u is the stream line function,

$$\frac{(x - b/2)^2}{(b/2\cosh u)^2} + \frac{y^2}{(b/2\sinh u)^2} = 1 \quad \ldots \text{(III- 9)}$$

When v is the pressure function,

$$\frac{(x - b/2)^2}{(b/2\cos v)^2} - \frac{y^2}{(b/2\sin v)^2} = 1 \quad \ldots \text{(III- 10)}$$

Prof. Warren Weaver[40] has derived simple solutions from these equations.

The pressure at any point under the dam floor is,

$$p(x,0) = \frac{H_o}{\pi} \cos^{-1} \left[\frac{2x - b}{b}\right] \quad \ldots \text{(III- 11)}$$

Total pressure under base,

$$F = \frac{H_o}{\pi}\int_o^b \cos^{-1}\left(\frac{2x-b}{b}\right) dx = \frac{H_o.b}{2} \quad . . . . \text{(III- 12)}$$

= average value of straight line assumption.

Moment of resultant pressure =

$$\frac{H_o}{\pi}\int_o^b x \cos^{-1}\left(\frac{2x-b}{b}\right) dx = \frac{3b^2.H_o}{16} \quad . . . \text{(III- 13)}$$

$$\text{Moment arm} = \bar{X} = \frac{3b^2.H_o}{16} \cdot \frac{2}{H_o b} = \frac{3}{8} b \quad . . . . \text{(III- 14)}$$

In case of straight line assumption, it is b/3.

<u>Dam with Stepped Floor and Sheet Piling at Any Point under the Base:</u>

The configuration of stream and pressure lines will be distorted in such case. It is possible to bring back this distortion to the normal configuration as shown in Fig. 4a, by means of Schwarz Christoffel transformation.

The fundamental general equation for pressure distribution under the foundation profile is as given below:

$$x + iy = \frac{d_1+d_2}{\pi K}\sqrt{(\lambda\cos v-\lambda_1)^2-1} - \frac{d_1-d_2}{\pi}\log\left[(\lambda\cos v-\lambda_1)+\sqrt{(\lambda\cos v-\lambda_1)^2-1}\right]+i(d_1-d_2)$$

$$. . . . \text{(III- 15)}$$

in which, $k = \cos 0$

$$\lambda = \frac{L_1+L_2}{2}, \qquad \lambda_1 = \frac{L_1-L_2}{2}$$

$$L_1 = \cosh \gamma_1 \ , \ L_2 = \cosh \gamma_2 \ , \ \delta = \frac{d_2}{d_1-d_2}$$

$$\sinh \gamma_1 + k\,\gamma_1 = -\pi k \,\frac{b_1}{d_1-d_2} = -\pi k\,\delta_1,$$

$$\sinh\gamma_2 - k\gamma_2 = \pi k\frac{b_2}{d_1-d_2} = \pi k\delta_2 \text{ and } \tan\theta - \theta = \pi\delta$$

Pressure at E = $P_E = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1-1}{\lambda}\right)$ . . . . (III- 16a)

Pressure at C = $P_C = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1+1}{\lambda}\right)$ ..... . (III- 16b)

Pressure at D = $P_D = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1+k}{\lambda}\right)$ . . . . (III-16c)

<u>Solution for Floor with Pile Line Any Where Along the Floor:</u>

Prof. Weaver's solution:

Origin ($x = 0$) is taken at the position of pile line. In Fig. 4c, pressure at point $p(x,0) = \frac{H}{\pi} v(x,y)$

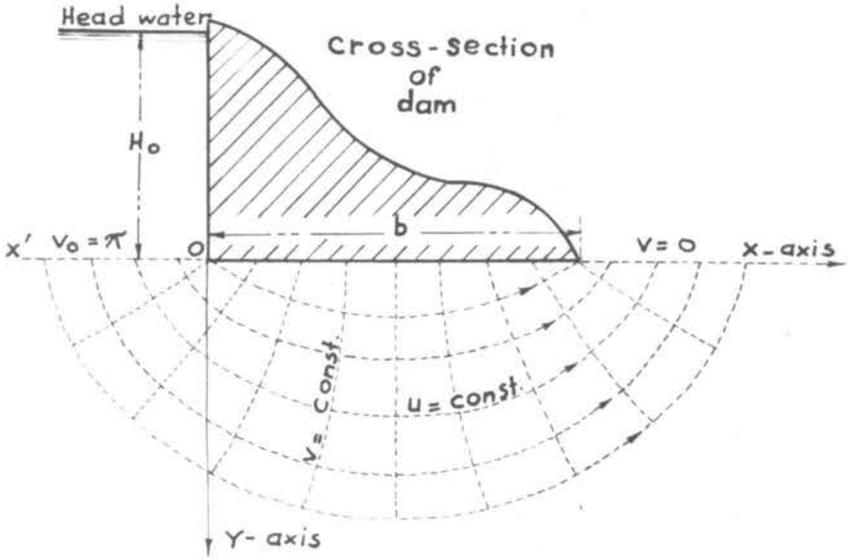
in which, $= \frac{H}{\pi}\cos^{-1}\left[\frac{\lambda_1 d \pm \sqrt{d^2+x^2}}{d\lambda}\right]$ . . . (III - 17)

$$\alpha_1 = \frac{b_1}{d}, \ \alpha_2 = \frac{b_2}{d}$$

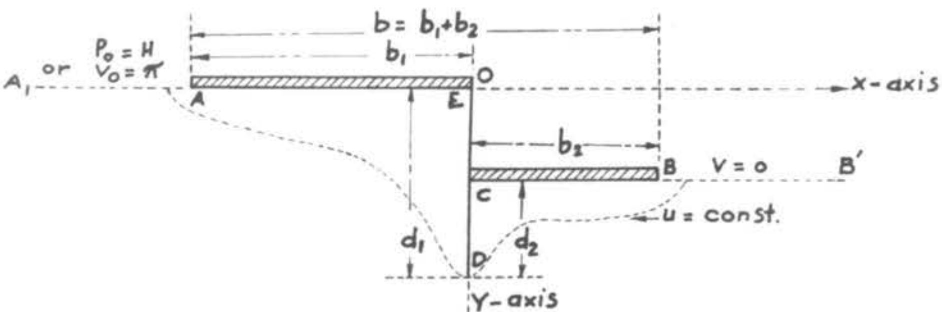
$$L_1 = \sqrt{1+\alpha_1^{\ 2}} \text{ and } L_2 = \sqrt{1+\alpha_2^{\ 2}}$$

$$\lambda = \frac{L_1+L_2}{2} = \frac{\sqrt{1+\alpha_1^{\ 2}}+\sqrt{1+\alpha_2^{\ 2}}}{2}$$

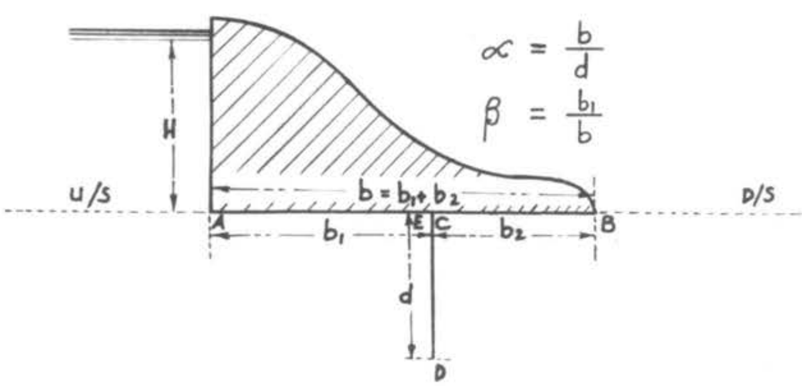
$$\lambda_1 = \frac{L_1-L_2}{2} = \frac{\sqrt{1+\alpha_1^{\ 2}}-\sqrt{1+\alpha_2^{\ 2}}}{2}$$



(a) - Diagram of dam with no sheet piling, showing Stream and equi-potential lines.



(b) - Dam with stepped floor and sheet piling at any point under base.



(c) - Dam with pile line any where along the floor.

Fig. 4.

Pressure along the sheet pile = p(0,y)

$$= \frac{H}{\pi} \cos^{-1} \left[\frac{\lambda_1 d \pm \sqrt{d^2 - y^2}}{d\lambda} \right] \quad . . . . (III- 18)$$

The drop of pressure at the sheet-piling, on the basis of the line of creep assumption, is $\left(\frac{2H}{2+\alpha} \right)$ and is independent of the location of pile line.

As per the mathematical solution, the line of creep theory gives results too small for $\beta < 0.5$ and too large for β greater than 0.5, as regards the total uplift force. For $\beta = 0$, the error ranges from 20% for $\alpha = 1$, to 10% for $\alpha = 8$. At $\beta = 0.5$, the error is zero for any value of α. At $\beta = 1.0$, the error is about 8% and does not depend very sensitively on the value of α.

Solution When the Sheet-piling Is at the Heel:

Pressure at point $p(x,y) = \frac{H}{\pi} \nu(x,y)$

$$= \frac{H}{\pi} \cos^{-1} \left[\frac{d(1-\lambda) + \sqrt{d^2 + x^2}}{d\lambda} \right] \quad . . . . (III- 19)$$

where $\lambda = \frac{1 + \sqrt{1 + \alpha^2}}{2}$ and $\alpha = \frac{b}{d}$.

The total uplift force on the base of the dam is given by,

$$F = \frac{H}{\pi} \int_0^b \cos^{-1} \left[\frac{d(1-\lambda) + \sqrt{d^2+x^2}}{d\lambda} \right] dx = \frac{bH}{2} \sqrt{\frac{\lambda - 1}{\lambda}} . \quad (III- 20)$$

$$\text{Moment} = \frac{H}{\pi}\int_0^b x \cos^{-1}\left[\frac{d(1-\lambda) + \sqrt{d^2+x^2}}{d\lambda}\right] dx$$

$$= \frac{b^2H}{16\pi}\cdot\left[\frac{2(3\lambda-2)}{\lambda\sqrt{\lambda-1}} + \frac{3\lambda-4}{\lambda-1}\left(\pi-2\sin^{-1}\frac{1}{\sqrt{\lambda}}\right)\right]. \quad \text{(III- 21)}$$

$$\overline{X} = \frac{b}{8\pi(\lambda-1)}\cdot\left[\frac{2(3\lambda-2)}{\sqrt{\lambda}} + (3\lambda-4)\sqrt{\frac{\lambda}{\lambda-1}}\left(\pi-2\sin^{-1}\frac{1}{\sqrt{\lambda}}\right)\right]. \quad \text{(III- 22)}$$

The above formulas permit one accurately to discuss the errors involved in the assumption of linear distribution of pressure and the line of creep theory of Bligh. As regards pressures, the line of creep theory assigns, in the case of piling at the heel, a pressure which drops linearly to zero from the initial value:

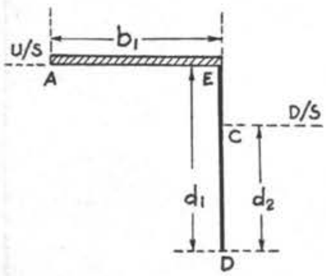
$$\frac{H\,b}{2d-b} = \frac{H\alpha}{2+\lambda} \quad \text{. . . . . . . (III- 23)}$$

This initial value is uniformly larger than that given by the accurate formula. For $\alpha = 1$, the error is + 22.4%, for $\alpha = 2$, the error is + 18%, while for higher values of α, the error fluctuates slowly about + 16%. Total uplift force corresponding to the line of creep distribution is $\frac{\alpha}{2+\alpha}\cdot\frac{bH}{2}$, which is different from the accurate value.

The ranges of error in total uplift and moment depend on the depth of piling as compared to the width of base. If piling were driven deeper than the base width ($\alpha < 1$), the errors in the line of creep assumption would be larger than those quoted above.

Determination of Exit Gradients and Uplift Pressures Under the Bases of Dams of Standard Forms[21].

CASE I.



Floor with pile line at down-stream end with step.

$$\text{Pressure at } E = P_E = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1 - 1}{\lambda}\right) \quad \text{(III- 24a)}$$

$$= \frac{H}{\pi}\cos^{-1}\left(\frac{L_1 - 3}{L_1 + 1}\right)$$

$$\text{Pressure at } C = P_C = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1 + 1}{\lambda}\right)$$

$$= \frac{H}{\pi}\cos^{-1}(1) = 0. \quad \text{(III- 24b)}$$

$$\text{Pressure at } D = P_D = \frac{H}{\pi}\cos^{-1}\left(\frac{k + \lambda_1}{\lambda}\right)$$

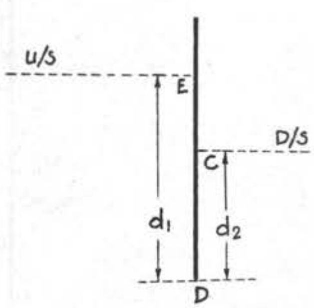
$$= \frac{H}{\pi}\cos^{-1}\left(\frac{2k + L_1 - 1}{L_1 + 1}\right) \quad \text{(III- 24c)}$$

Exit gradient at C = G (exit)

$$= \frac{H}{d_2} \cdot \frac{d_2}{d_1 - d_2} \cdot \frac{k}{1-k} \cdot \frac{1}{\sqrt{\lambda}}$$

$$= \frac{H}{(1-k)\sqrt{\lambda}} \cdot \frac{k}{(d_1 - d_2)}. \quad \text{(III- 25)}$$

CASE II.



Single pile line with fall.

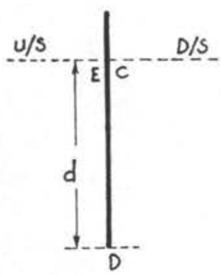
In this case, $\lambda = 1$.

$$G(\text{exit}) \text{ at } C = \frac{H . d_2}{d_2(d_1 - d_2)} \cdot \frac{k}{(1-k)}.$$

$$= \frac{H}{(1-k)} \cdot \frac{k}{(d_1 - d_2)}.$$

(III- 26)

CASE III.



$$P(x,y) = \frac{H}{\pi} \sin^{-1}\left(\frac{y}{d}\right) \quad \ldots\ldots \quad \text{(III- 27)}$$

$$G(\text{exit}) \text{ at } C = \frac{H}{(1-k)} \cdot \frac{k}{(d_1 - d_2)}.$$

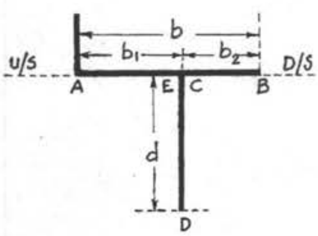
$$= \frac{H}{\pi d}, \quad \ldots \quad \text{(III- 28)}$$

Single pile line without fall.

as in the limit $d_1 = d_2 = d$,

$$\frac{k}{(d_1-d_2)} = \frac{1}{\pi d} \text{ and } k = 0.$$

CASE IV.



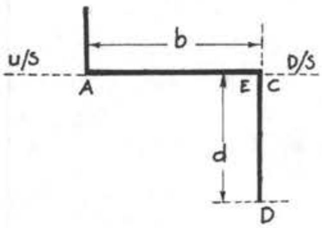
$$P_E = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda_1 - 1}{\lambda}\right) \quad \ldots \quad \text{(III- 29a)}$$

$$P_C = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda_1 + 1}{\lambda}\right) \quad \ldots \quad \text{(III- 29b)}$$

$$P_D = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda_1}{\lambda}\right) \quad \ldots \quad \text{(III- 29c)}$$

Floor with sheet-piling not at end.

CASE V.



$$P_E = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda - 2}{\lambda}\right) \quad . . \quad \text{(III- 30a)}$$

$$P_C = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda}{\lambda}\right) = 0 \quad . . \quad \text{(III- 30b)}$$

$$P_D = \frac{H}{\pi} \cos^{-1}\left(\frac{\lambda - 1}{\lambda}\right) \quad . . \quad \text{(III- 30c)}$$

G(exit) at C

Floor with pile line at end.

$$= \frac{H}{(1 - k)\sqrt{\lambda}} \cdot \frac{k}{(d_1 - d_2)}$$

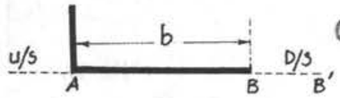
$$= \frac{H}{\pi d \sqrt{\lambda}} \quad . . . . . . \quad \text{(III- 31)}$$

Since in the limit, $d_1 = d_2 = d$

$\dfrac{k}{(d_1 - d_2)} = \dfrac{1}{\pi d}$, and $k = 0$.

CASE VI.

$$P(x,0) = \frac{H}{\pi} \cos^{-1}\left(\frac{2x - b}{b}\right) \quad . . \quad \text{(III- 32)}$$



G(exit) at B $= \dfrac{H}{\pi d} \cdot \dfrac{1}{\sqrt{\lambda}}$

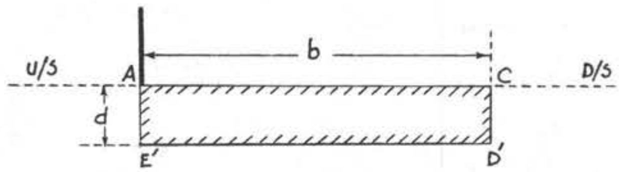
$$= \frac{H}{\pi . 0} \cdot \frac{1}{\sqrt{\lambda}} = \infty \quad \text{(III- 33a)}$$

G(exit) along BB'

$$= \frac{2H}{\pi b} \cdot \frac{1}{\sqrt{\left(\frac{2x - b}{b}\right)^2 - 1}} \quad \text{(III- 33b)}$$

Simple floor without any cut-off wall or pile line.

CASE VII.



Depressed Floor.

Mathematically, pressure at D' = $P_{D'} = \frac{H}{\pi} \cos^{-1}(k)$, where $k = \cos \theta$. The following formula has been derived from the laboratory experiments:

$$\phi D' = \phi D - 2/3(\phi E - \phi D) + 3/\alpha^2, \quad \ldots\ldots \text{(III- 34)}$$

where $\phi D'$ = percentage of pressure at D' in respect of the total initial pressure = $\frac{PD'}{H} \times 100$,

ϕD and ϕE are given by equations as in case V.

$$G(\text{exit}) = \frac{0.84}{d} \cdot P_{D'} \quad \ldots\ldots\ldots\ldots \text{(III- 35)}$$

FLOOR WITH TWO OR MORE LINES OF SHEET PILING

The theoretical solution for such cases is very complex. The values of pressure distribution under the floor in such cases can be arrived at by two processes.

1. By experiments with hydraulic or electric models.
2. By Khosla's method of independent variables[22], that is, the independent solutions for the simple standard forms and combining the effects of the same with corrections for mutual interference of the various pile lines. The values as obtained from the standard equations and the plate chart-2 are to be corrected for:
   (a) the mutual interference of piles,
   (b) the floor thickness,
   (c) the slope of the floor.

The mutual interference of piles is given by the formula,

$$C = 19\sqrt{\frac{D}{b'}} \cdot \frac{d + D}{b}, \quad \ldots\ldots\ldots \quad \text{(III- 36)}$$

where C = the correction to be applied as percentage of head,

b' = distance between the two pile lines,

D = the depth of pile whose influence has to be determined on the neighboring pile of depth d,

d = the depth of pile on which the effect of pile D is to be determined,

b = total floor length.

CHAPTER IV

MODEL EXPERIMENTS

It may be taken as established that both for purposes of design and for subsequent field analysis of data, the pressure distribution under any structure as derived mathematically or as given by the hydraulic or electric model will represent such distribution under the full size field structure. This applies to soils which are homogeneous or very nearly so. In cases of non-homogeneous soils, the results derived from mathematical solutions or models can, at best, be approximate, as it is almost impossible to ascertain in detail all the elements of non-homogeneity under any big structure and, therefore, the simulation of such conditions on the models can not in all cases be done with any great degree of accuracy.

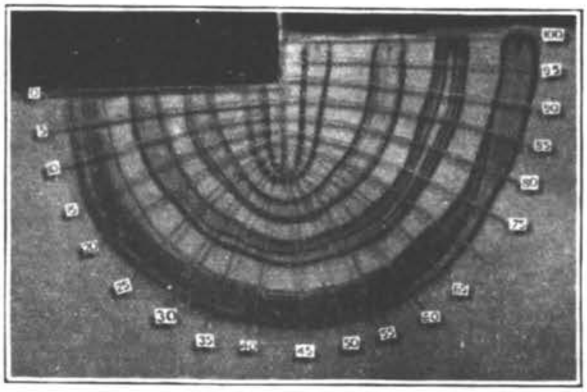
Experiments on hydraulic models and electric analogy trays were carried out in the Punjab Irrigation Research Institute (India) for several standard forms. The experimental results were compared with the theoretical values. The results obtained on models agree with those derived mathematically. Therefore, the experimental work done on these models served to give an experimental confirmation of the mathematical formulae. There are, however, many complicated floor profiles which are either mathematically indeterminate or too laborious. These cases can be solved by model experiments with a fair degree of accuracy.

The experimental data, regarding distribution of pressure under the standard forms as shown in the Figures 5, 6, 7 and 8, were obtained by Vaidhianathan[23]. The results are incorporated in Tables 5, 6, 7 and 8 (Appendix A).

The general feature which is noticeable in each case is that the insertion of a pile anywhere results in heading up or increase of pressure upstream and a draw-down or reduction of pressure downstream. The main influence is confined to a radius equal to the depth of the pile. At twice that radius, the influence becomes negligible. The hydraulic gradient line between any two corners is very nearly a straight line, in any case, the departure from a straight line is small and may be neglected in designs.

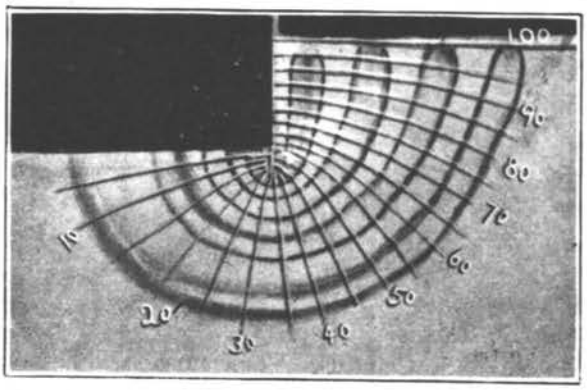
Models of single sheet pile without aprons. Unequal fill on two sides.

Stream lines and pressure contours.



(a) Single sheet pile
Length = 6
Drop = 2

Stream lines and pressure contours.



(b) Single sheet pile
Length = 6
Drop = 5

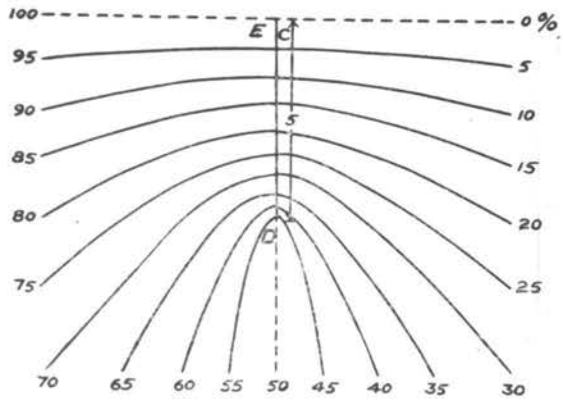
The instances for such cases are cofferdams with pumping and excavation within, ordinary wells during sinking, abutments of bridges and aqueducts. The results will also determine the stability of the walls should erosion or scour occur close to them.

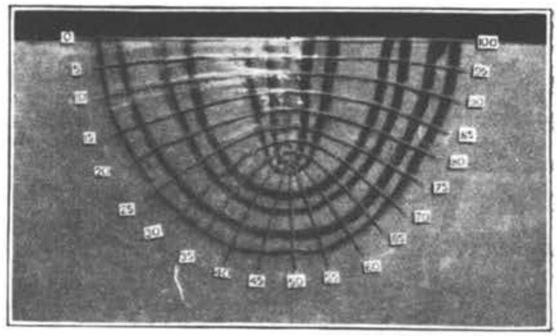
Fig. 5 - Models of sheet pile showing seepage flow and uplift pressure distribution.

(a) Single sheet pile without aprons. Equal fills on both sides.

Length of pile = 5

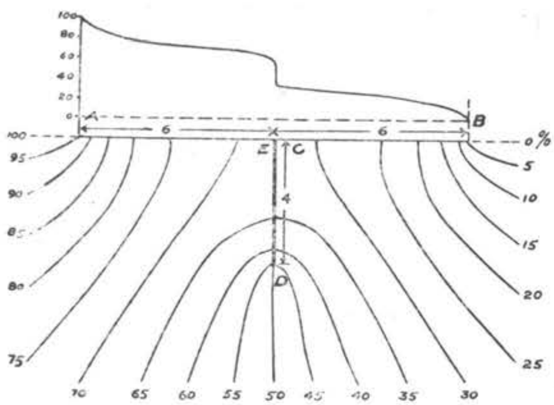
Stream lines and pressure contours.





(b) Floor with sheet pile, not at the ends.
Floor = 12
Central sheet pile = 4

Stream lines.



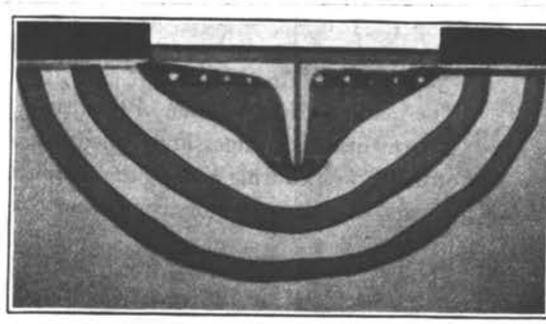
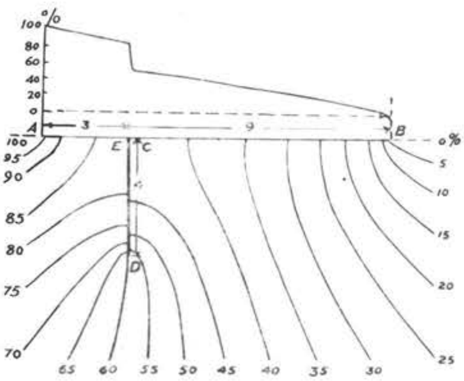


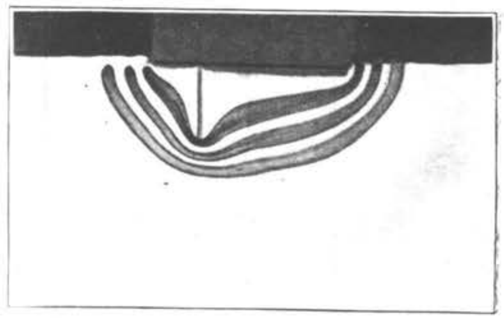
Fig. 6 - Models of simple foundation profiles of dams showing seepage flow and uplift pressure distribution under floor.

(a) Floor with sheet pile
not at ends.

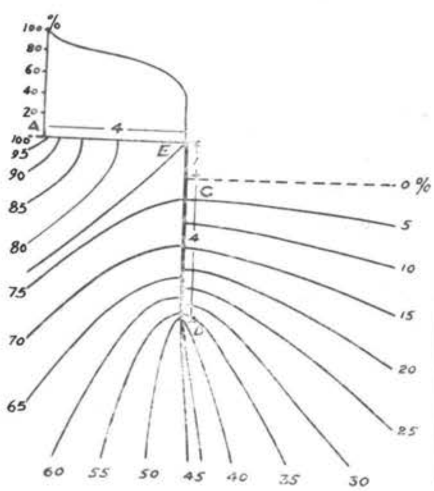
Floor = 12
Sheet pile = 4



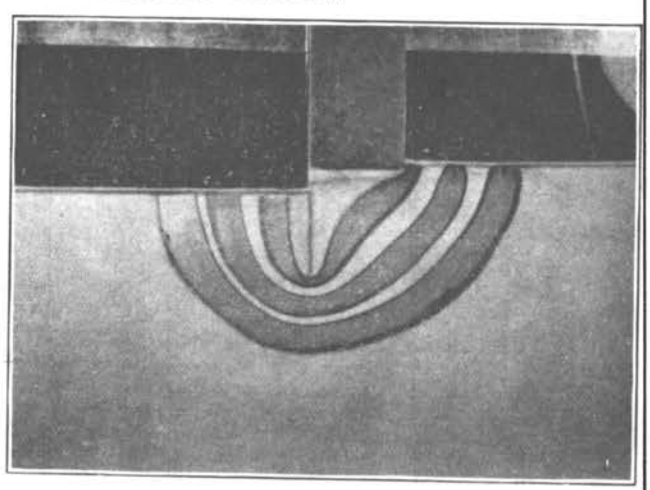
Stream lines.



(b) Floor with sheet pile at
the downstream end or
toe with step (case of
scour at the toe).



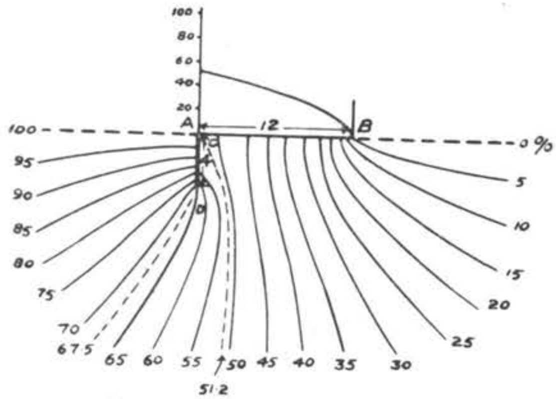
Stream lines.



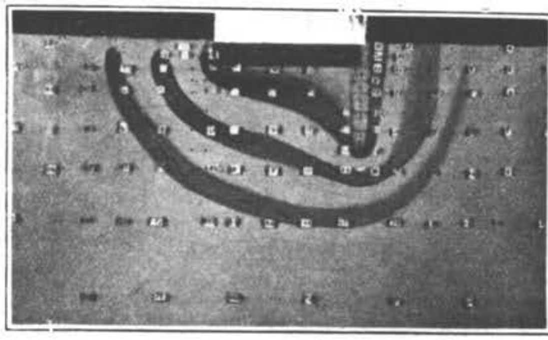
Length of floor = 4
Length of pile = 5
Depth of scour = 1

Fig. 7 - Models of dam profiles showing seepage
flow pattern and uplift pressure
distribution under floor.

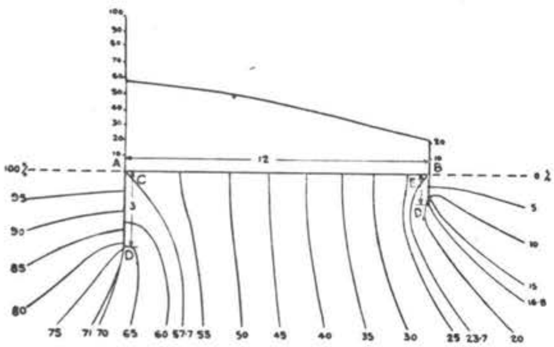
(a) Floor with sheet pile
at the heel end.
Length of floor = 12
Length of pile = 4



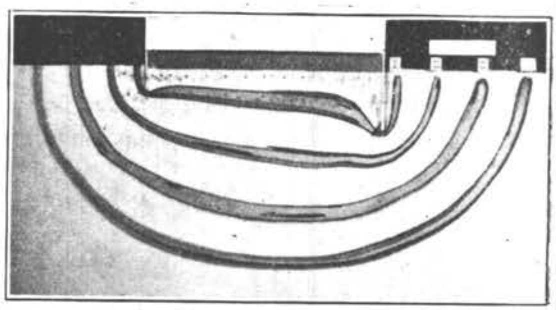
Stream lines.



(b) Floor with unequal sheet-
piles at the heel and toe.



Stream lines.



Length of floor = 12
Length of u/s sheet pile = 3
Length of D/s sheet pile = 1

Fig. 8 - Models of dams showing seepage
stream lines and uplift pressure
distribution under the floors.

Mr. Selim[35] has also carried out some experiments on the electric analogy models in the University of California. His results agreed with those of the experiments done in Punjab and by Mr. Harza.

In his experiments, the advantage of an upstream cut-off wall in reducing the uplift force has been demonstrated. A downstream cut-off wall tends to increase the uplift force and hence to off-set the value of the upstream cut-off wall. The downstream cut-off wall is necessary for safe exit gradient to prevent flotation, formation of piping and undermining beneath the dam.

When the thickness of the stratum of porous material under a dam is greater than the base width, the distribution of the uplift pressure is essentially the same as in the case of porous medium of infinite depth.

KHOSLA'S METHOD OF INDEPENDENT VARIABLES FOR DETERMINATION OF UPLIFT PRESSURE AND EXIT GRADIENTS UNDER COMPLEX FLOOR PROFILES[22]

There are complete mathematical and experimental solutions for the simple standard forms of the floor profiles, such as:

(a) A stepped horizontal floor with a pile line at the step, but no depression of the floor in the sub-soil.

(b) A horizontal floor with a pile line anywhere along its base, but no depression.

(c) A horizontal floor with depression in the sub-soil, but no pile line.

(d) A horizontal floor with no depression and no pile.

(e) A vertical pile line with equal or unequal fills on the two sides or no fill on the downstream side.

These are the basic elementary forms. Theoretical solutions will become indeterminate, if two or more of these elementary forms are grouped together. In practice, the usual dam sections rarely conform to any single elementary form. They consist of a combination of almost all the forms mentioned above.

In this method, a complex profile of a dam section is split up into its elementary standard forms. Each elementary form is then treated as independent of the others. The pressures are then read off at the key points, that is, the junction points of the floor and pile line of any particular elementary form. The readings at these junction points are corrected for:

(a) the mutual interference of piles,

(b) floor thickness, and

(c) the slope of the floor, if any.

For simplicity of calculation, the uplift pressure curves for the essential forms are grouped in one single plate (Plate 2). The calculated and experimental results are compared in Tables 9 and 10 (Appendix A).

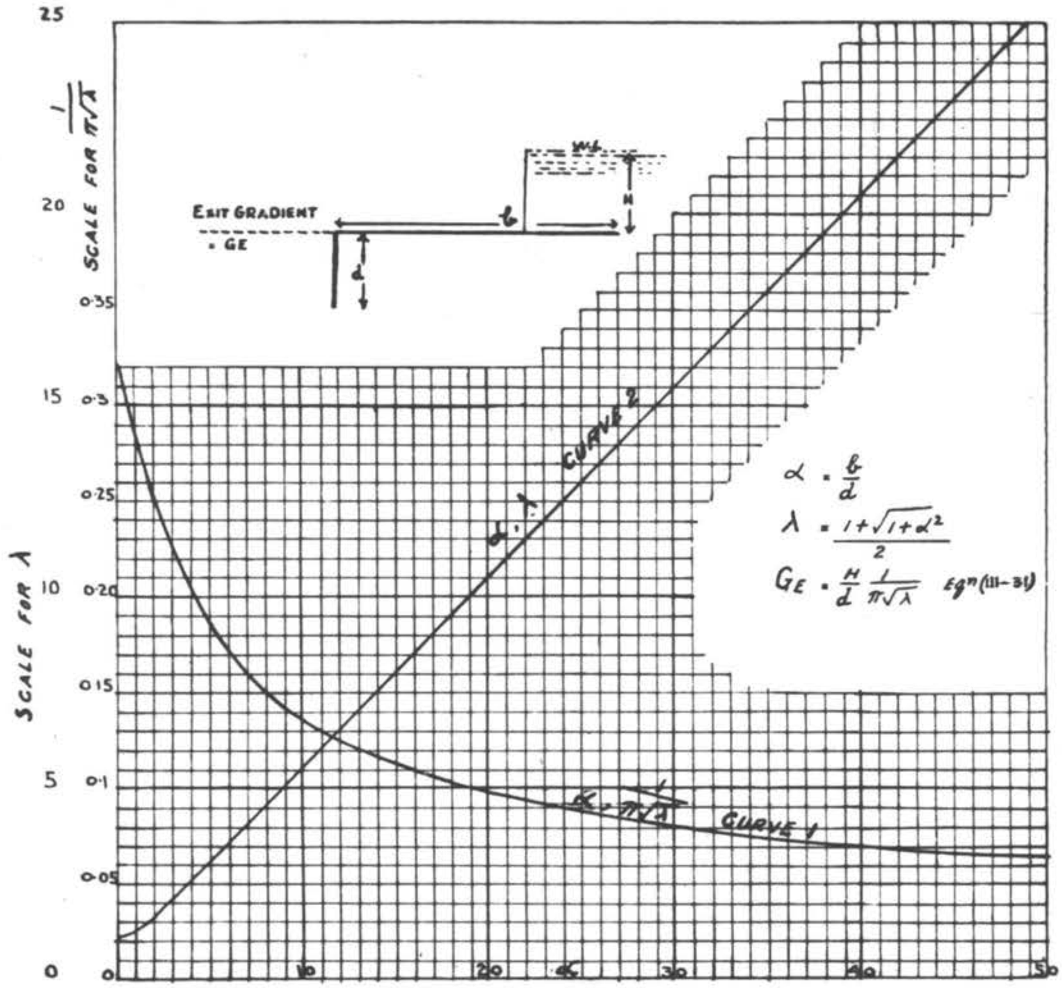


Chart I - Curves for calculating exit gradients.

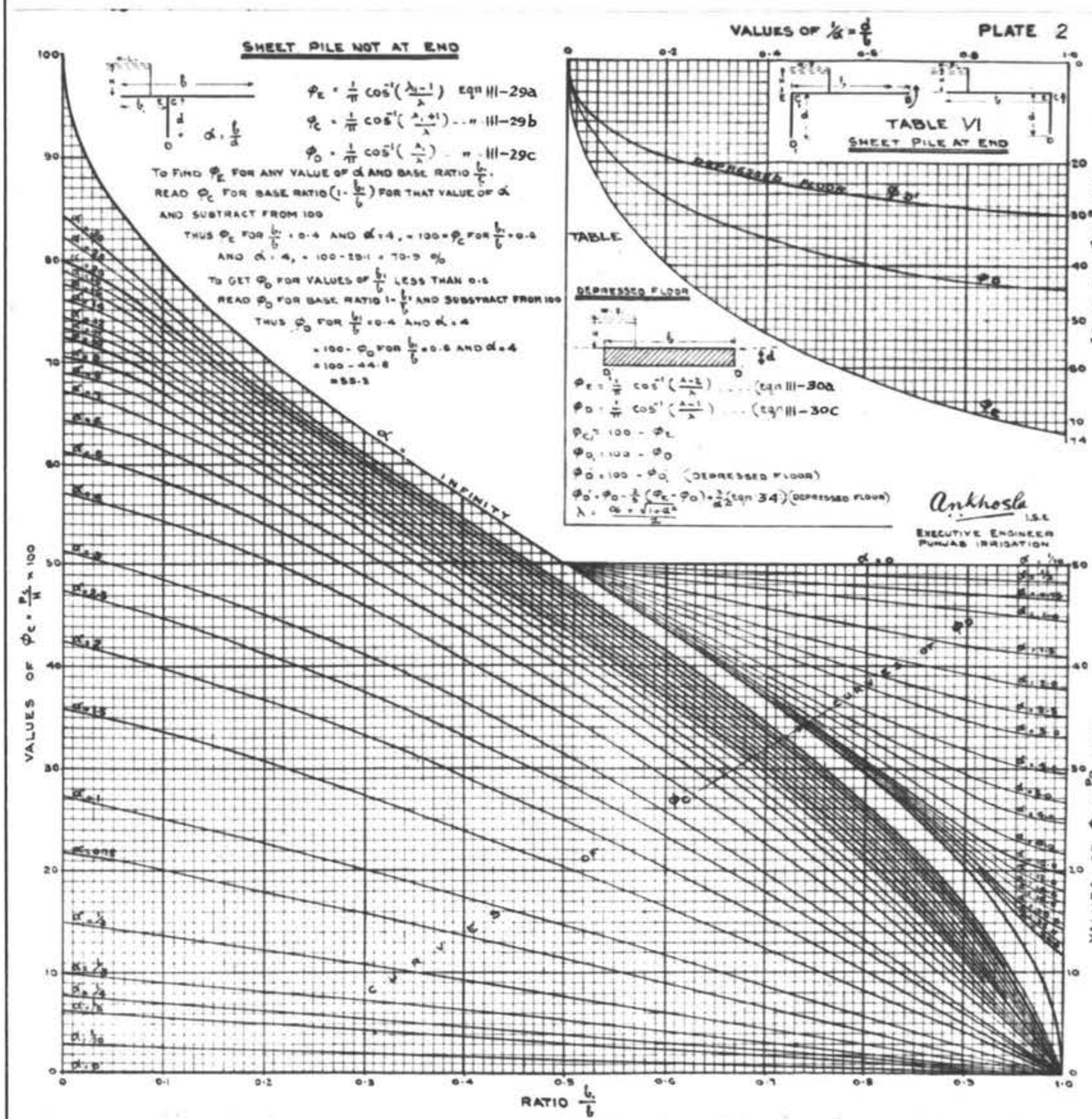


Chart II - Curves for calculating the pressure distribution at the key points under a dam profile.

DRAINAGE OF SEEPAGE FLOW

The principle of drainage can be applied both to reduce upward pressure and to increase the factor of safety against flotation at the toe. For safe discharge of the seepage water, the surface drainage must follow the principle of an inverted sanitary water filter. There should be at least two layers of sand and one of gravel screened to such sizes that the material of one layer can not penetrate into the pores of the next layer above.

The effect of surface drainage or weep-holes in the down-stream apron has been investigated by Mr. L. F. Harza[14] in the electric analogy tray. The results are shown in Fig. 9. As a result of draining the 15th inch space of a 20 inches foundation without heel or toe sheeting, the uplift pressure curve to the point of drainage is the basic S- curve with the base reduced to three-fourths length. Beyond the point of drainage a small uplift pressure is resumed with a curve tangent to the vertical at the toe. This results in an infinite upward exit gradient at the immediate toe as in the basic S-curve. In Fig. 9(b) the same conditions are repeated except with a length of toe sheeting equal to b/5. The sheeting has no effect up-stream from the drainage and serves principally to change the shape of pressure curve

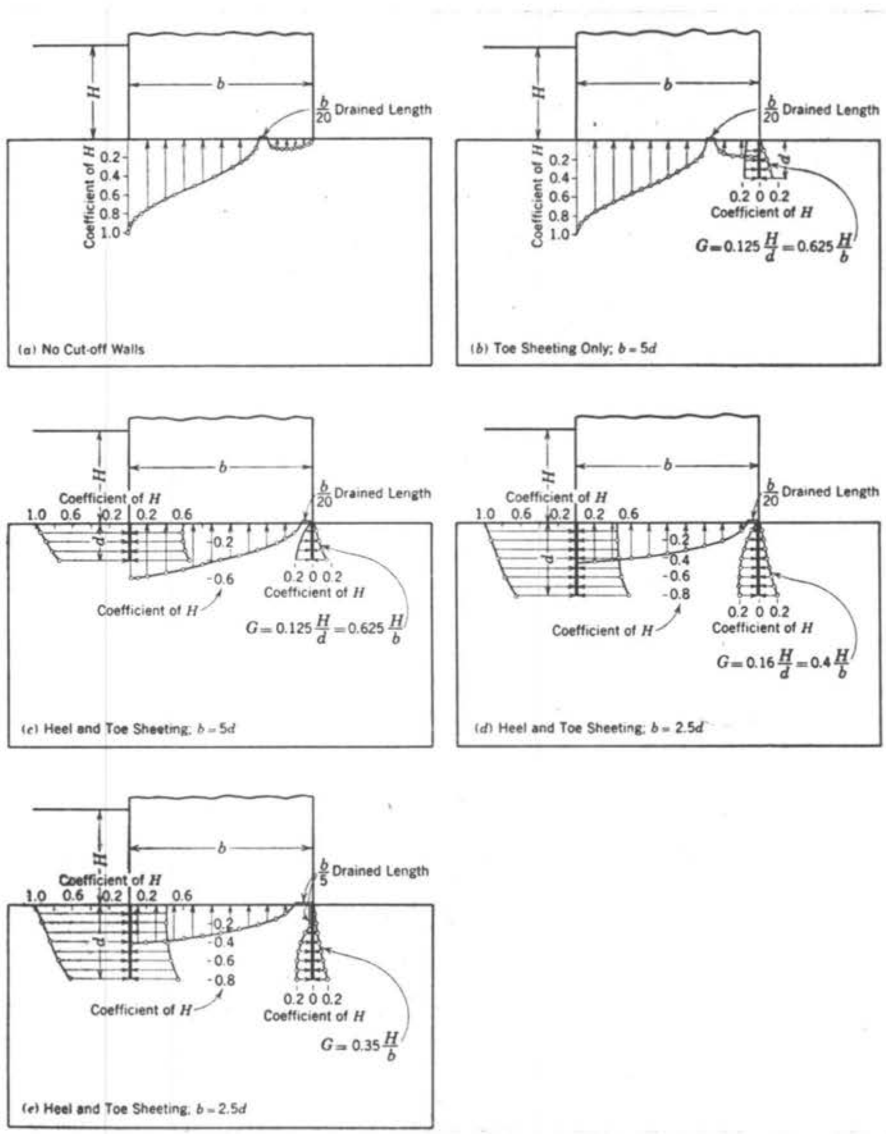


Fig. 9 - Effect of drainage on a dam with horizontal base.

at the toe to give a finite exit gradient. Comparing this with the same design without drainage, the reduction in escape gradient is found to be from 0.19H/d to 0.125H/d, or a decrease of 34%.

In Fig. 9(c) are shown heel and toe cut-offs (b=5d) with drainage one inch long immediately ahead of the toe cut-off; the resulting upward toe exit gradient is 0.125H/d. The results of the test made of a dam with 8-inch heel and toe sheetings, in which b = 2.5d, with one inch drainage strip are shown in Fig. 9(d). The toe exit gradient is 0.4H/b as compared with 0.6H/b without drainage. This test was repeated with a 2-inch drainage strip immediately above the toe sheeting which resulted in a gradient of 0.35H/b. It was again repeated with drainage along the upper 2 inches of the toe sheeting as well as 2 inches of the adjacent base width and the upward toe gradient was 0.35H/b, that is unchanged from the last one.

Regarding drainage, there are two schools of opinions, one generally for and the other, against it. But the drainage provides a useful factor of safety, especially in uncertain ground and should be utilized where complete sealing off the seepage flow regardless of construction economy is not of utmost importance.

The modern opinions tend to the principle of imposing effective resistance to the seepage through such a distance as is required until the quantity of leakage is sufficiently reduced and then providing ample artificial, permanent and well constructed drainage works which will relieve it, as much as possible through a controlled route where its escape will be easier than through the foundation material and where it can do no harm. Only the escape of seepage through the fine material is hazardous and so it is advisable to provide a safe route than through the foundation material itself.

Any pervious stratum or lense that may conduct excessive pressure beneath a dam, possibly to burst upward into tail-water, should be intercepted by cut-off wall or blanketed apron near the heel and then near the toe it should be drained as freely as possible. The line of safe escape may be created by drainage relief wells, intercepting trench, inverted filter under or in the apron or at toe of the dam along the down-stream face of the toe sheeting and the seepage water led to a drainage sump away from the toe of the dam. This will eliminate the bursting or floating tendency of the up-flow through the foundation material itself in an unstable situation requiring such precaution.

Impervious blankets, blanket and longitudinal drains, cut-offs and drainage relief wells are the seepage control expedients which an engineer should try to blend so as to form an adequate and economical design.

CHAPTER V

THE EFFECT OF BLANKETS ON SEEPAGE THROUGH PERVIOUS FOUNDATIONS

In the design of earth dams built on pervious foundations, there appears to be a trend toward the control of seepage through the foundation by construction of an upstream blanket or vertical cut-off wall connected to the impervious section of the dam and providing drainage for the downstream section of the dam by horizontal drains or by a system of vertical drainage relief wells. In dams where the control of seepage is undertaken primarily to increase the stability of the structure, rather than to reduce seepage losses to a minimum, this type of design offers several advantages over a central impervious core in the dam and a cut-off through the foundation.

In the latter design, practically all the hydrostatic pressure of the reservoir is exerted on the central core and cut-off and the downstream part of the dam must have slopes sufficiently flat to withstand the full pressure of the reservoir. If the reservoir head is dissipated through a blanket and upstream impervious section, a much larger portion of the dam is effective in offering resistance to the hydrostatic pressure. Moreover, the pervious foundation is maintained at a relatively low hydrostatic potential by the combined action of the upstream blanket and the downstream drainage system. If an excess of impervious material unsuitable for embankment construction is available, the important advantage of the blanket lies in the economic use of waste material, rather than in the use of a costly cut-off wall. Quite commonly the alluvial deposit in a river valley consists of a stratum of sand and gravel over-lain by a surface layer of relatively impervious finer soils. It is advantageous to leave such a blanket in place for seepage control.

Mr. P. T. Bennet[1] has presented the mathematical analysis of flow through blankets based on idealized assumptions as to the uniformity of permeability and thickness of the pervious stratum and blanket.

Since the natural deposits of soils cannot be expected to be uniform in thickness or permeability and since the determination of the permeability of such deposits can result at best only in a good approximation, the formulae in the paper do not represent accurately the actual conditions of flow and potential distribution under a dam. However, the same sources of error are present in the analysis of flow by models, so the analytical method for blanket studies may be preferred when the foundation consists of a single pervious layer. If the foundation is separated into two or more distinct pervious strata by impervious lenses, a model study would be the better method to investigate the resulting complex flow pattern.

The advantages of upstream blanket are reduction in the uplift pressures in the seepage emergence zone and some reduction in the quantity of seepage. These apply particularly to dams holding a permanent pool where the blanket is protected by the water above and where the head lost through the blanket exerts a force compressing the blanket and thus increasing its impermeability.

In contrast to dams holding a permanent pool, an upstream blanket is not quite as advantageous for flood control dams where water is stored only intermittently with frequent rapid draw-downs of the pool and where the volume of loss from seepage is seldom important. In these cases, thicker blankets are apt to be necessary, as the frequent exposures of the blanket during non-storage periods offer opportunities for erosion and in the colder climates, for loosening the upper part of the blanket by frost and thus substantially reducing its impermeability.

MATHEMATICAL ANALYSIS

Assumptions:

(1) The ratio of permeability of the pervious stratum to the natural or artificial top stratum of blanket is at least 10:1 or greater. If the ratio is small, the assumption that the seepage flow is vertical through

the blanket and horizontal through the pervious foundation may not be warranted.

(2) The ratio of the path of travel of seepage water to the thickness of the pervious stratum is relatively great.

(3) There is only a single pervious stratum which is largely responsible for the under-seepage flow.

(4) The horizontal flow through the embankment of dam structure is relatively small as compared with that through the pervious foundation stratum.

Notations:

X = horizontal distance measured positive in the direction of flow; dx = an element of the distance X.

X_r = resistance of a blanket, being the effective length of the path of flow under the blanket.

Z = thickness of a blanket when the thickness is variable.

Z_o = thickness of blanket at $X = 0$.

Z_b = thickness of blanket when it is constant.

Z_f = thickness of the foundation stratum.

$S = Z/X$, the slope of the surface of a triangular blanket.

K_b = permeability of the blanket material which influences the vertical seepage path.

K_f = permeability of the foundation material which influences the horizontal seepage path.

q_b = vertical flow per unit width through the blanket,

dq_b = element of vertical flow through an element dx of the blanket.

q_f = horizontal flow, per unit width, through the foundation.

q_{fo} = inflow to the foundation at the upstream end of the blanket.

h = head loss through the blanket,

= head loss at specified points in the foundation.

h_o = head loss through the blanket at point X = 0.

h_n = a constant factor for computing h, determined by the relationship between the blanket and the flow system as a whole.

n = an arbitrary constant defining the profile of curved blanket surfaces.

$\alpha = K_b/K_f \cdot Z_f$.

$a = \sqrt{\dfrac{K_b}{Z_b \cdot K_f \cdot Z_f}}$ = coefficient of blanketed transmissibility.

Referring to Chart 3, at any point X, under the blanket, the horizontal flow through the foundation is equal to the flow through the blanket upstream from that point, plus the inflow under the upstream end of the blanket.

That is, $q_f = \int_0^x dq_b + q_{fo}$ . . . . . . (V- 1)

The vertical flow dq_b through a small element of the blanket of length dx and thickness Z_b, varies, according to Darcy's Law, directly with the head loss h and inversely with blanket thickness Z_b, so that, $dq_b = \frac{kb.dx.h}{Z_b}$. (V- 2)

In the interval dx, the horizontal flow in the pervious stratum is increased by an amount equal to the vertical inflow dq_b, thus,

$$\frac{dq_f}{dx} = \frac{dq_b}{dx} = \frac{K_b.h}{Z_b} \quad \ldots\ldots\ldots\ldots \quad (V- 3)$$

$$qf = K_f.Z_f \cdot \frac{dh}{dx} \quad \ldots\ldots\ldots\ldots \quad (V- 4a)$$

$$\frac{dq_f}{dx} = K_f.Z_f \cdot \frac{d^2h}{dx^2} \quad \ldots\ldots\ldots\ldots \quad (V- 4b)$$

From equation (V- 3), $\frac{d^2h}{dx^2} = \frac{K_b.h}{Z_b.K_f.Z_f}$ . . . . (V- 4c)

Using the notation, $a = \sqrt{\frac{K_b}{Z_b.K_f.Z_f}}$ for blanket of uniform thickness and $\alpha = \frac{K_b}{K_f.Z_f}$ for blanket of variable thickness,

$$\frac{d^2h}{dx^2} = a^2h \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (V- 5a)$$

$$\text{and } \frac{d^2h}{dx^2} = \frac{\alpha h}{Z_b} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (V- 5b)$$

For a blanket of constant thickness and infinite horizontal extent, the solution of Eq. (V- 5a) is,

$$h = h_o\, e^{ax} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (V- 6)$$

in which h_o is the head loss through the blanket at point $X = 0$ which, in this case, is at the downstream end of the blanket. By differentiation,

$$\frac{dh}{dx} = ah_o e^{ax} = ah \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (V\text{- }7a)$$

$$\frac{d^2h}{dx^2} = a^2 . h_o . e^{ax} = a^2 h \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (V\text{- }7b)$$

The resistance X_r is a measure of the efficiency of the blanket. It may be defined as the length of a prism of the foundation material of thickness Z_f and permeability K_f, which, under head loss of h, would carry a flow equivalent to the flow which passes the blanket system under the same head loss.

$$X_r = \frac{h}{\frac{dh}{dx}}.$$

For this case, $X_r = \dfrac{h}{ah} = \dfrac{1}{a} = \sqrt{\dfrac{Z_b . K_f . Z_f}{K_b}}$ . . . . . . (V- 8)

The above equation may be used to estimate the effectiveness of a natural surface layer as a blanket when none of the fine material is removed for borrow pits and all the seepage enters the substratum by percolation through the blanket.

If part of the natural blanket is removed, the substratum is exposed directly to full reservoir head and a part of the flow enters the foundation sand at the upper limit of the blanket. For such case, the solution of Eq.

(V-5a) is,

$$h = h_n(e^{ax} - e^{-ax}) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (V\text{- }9a)$$

$$\frac{dh}{dx} = ah_n(e^{ax} + e^{-ax}) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (V\text{- }9b)$$

$$\frac{d^2h}{dx^2} = a^2h_n(e^{ax} - e^{-ax}) = a^2.h \quad . \; . \; . \; . \; . \quad (V\text{- }9c)$$

$$X_r = \frac{(e^{ax} - e^{-ax})}{a(e^{ax} + e^{-ax})} = \frac{(e^{2ax} - 1)}{a(e^{2ax} + 1)} \quad . \; . \; . \quad (V\text{- }10)$$

In Eqs. (V- 9a) and (V- 10), $X = 0$ at the upstream edge of the blanket and h_n is a constant depending on the total head loss of the system of which the blanket is a part and on the ratio of the blanket resistance to the remainder of the system. The value of h_n is determined after computation of a definite value of h at the lower end of the blanket using the relation between Δh, X_d and X_r indicated graphically in Chart 3.

Ordinarily for the upstream blanket, the pressure distribution under the blanket is not of great interest and the computation of constants need not be made, since determination of X_r is all that is necessary for the estimation of pressure under the dam or for computation of the approximate horizontal flow or discharge per linear foot of dam which is,

$$q_f = \frac{K_f . Z_f . \Delta h}{X_r + X_d} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (V\text{- }11)$$

The curves in Chart 3 give values of X_r computed from X and 'a' by the use of Eq. (V- 10). The curves for X_r and $\frac{Z_b \cdot x}{\alpha}$ are plotted on the a- scale and X- scale, and the curves for Z_b are plotted on the a- scale and α- scale. It is apparent from these characteristics curves that the ratio of resistance to blanket length, for any value of a, decreases rapidly as the length is extended beyond that indicated by the diagonal line $ax = \sqrt{2}$ and that very little benefit is to be gained either by leaving a natural blanket in place to great distances above a dam or by the construction of an artificial blanket of uniform thickness and excessive length.

These conclusions show that, if a blanket is to be built, its thickness probably should be reduced at the upstream where the blanket is relatively ineffective and increased near the dam interposing an additional thickness of impervious material where the head loss through the blanket is greatest.

This procedure would result in a blanket of triangular section, for which the resistance may be determined by SX for Z_b in Eq. (V- 5b), S being the slope of the blanket surface and X, the blanket length measured from its upstream edge. The solution is as follows:

$$h = h_n\left[X + \frac{\alpha X^2}{2!\,S} + \frac{\alpha^2 \cdot X^3}{2!3!S^2} + \cdots + \frac{\alpha^{(n-1)} \cdot X^n}{(n-1)!\,n!\,S^{(n-1)}}\right] \qquad \text{(V- 12a)}$$

$$\frac{dh}{dx} = h_n\left[1 + \frac{\alpha X}{S} + \frac{\alpha^2 \cdot X^2}{(2!)^2 S^2} + \cdots + \frac{\alpha^{(n-1)} \cdot X^{(n-1)}}{\left[(n-1)!\right]^2 S^{(n-1)}}\right] \qquad \text{(V- 12b)}$$

$$\frac{d^2h}{dx^2} = h_n\left[\frac{\alpha}{S} - \frac{\alpha^2 \cdot X}{2S^2} + \cdots\cdots + \frac{\alpha^{(n-1)} \cdot X^{(n-2)}}{(n-2)!(n-1)!S^{(n-1)}}\right] \qquad \text{(V- 12c)}$$

$$\frac{d^2h}{dx^2} \cdot \frac{S \cdot X}{\alpha} = \frac{d^2h}{dx^2} \cdot \frac{Z_b}{\alpha} = h \qquad \text{(V- 13)}$$

The value of $X_r = \dfrac{h}{\frac{dh}{dx}} = X - \dfrac{\alpha X^2}{2S} + \dfrac{\alpha^2 X^3}{3S^2} - \dfrac{11\alpha^3 X^4}{48S^3} + \dfrac{19\alpha^4 X^5}{120S^4}$

Therefore, Eq. (V- 12a) satisfies the Eq. (V- 14)
(V-5b) for the triangular blanket.

The numerical difficulties involved in computing the resistance of a triangular section can be avoided by substituting a vertically curved surface profile,

$$Z_b = \frac{\alpha X^2}{n(n-1)} \qquad \text{. . . . . (V- 15)}$$

for the tangent profile $Z_b = SX$.

This substitution is made only to facilitate computations and it is not recommended that the profile of the blanket should be so curved. There is very slight deviation between the curved and triangular profiles and hence the triangular section can be safely adopted.

As per Eq. (V-15), $h = h_n . X^n$ . . . . (V- 16a)

$$\frac{dh}{dx} = n . h_n . X^{(n-1)} \quad . . . . \text{(V- 16b)}$$

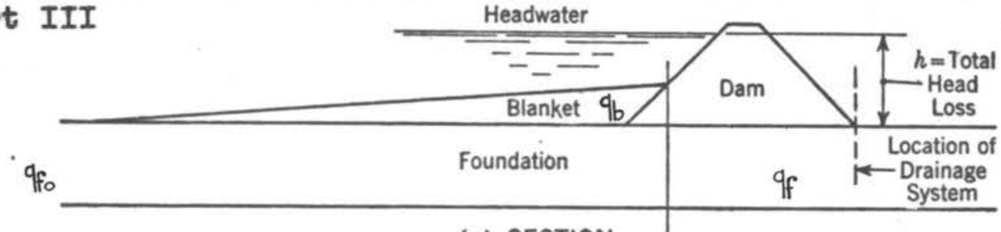
$$\frac{d^2h}{dx^2} = n(n-1) . h_n . X^{(n-2)} \quad . . \text{(V- 16c)}$$

The resistance $= X_r = \frac{h}{dh/dx} = \frac{X}{n}$ . . . . . . (V- 17)

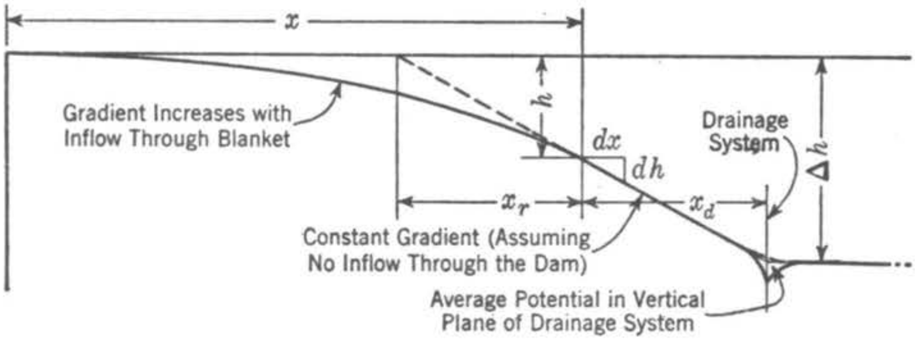
From the comparisons of the values of X_r for triangular sections with those for the uniform sections, it appears that the resistance of a triangular section is approximately equal to that of a rectangular section of the same end area, if the length of the latter is one-half the length of the triangular section. Considering the fact that the basic data used in the computations will be only approximately correct, the approximate values of the resistance of a triangular section are in sufficiently close agreement to justify the use of either Eq. (V - 10) or Eq. (V- 17) in estimating the efficiency of this type of blanket.

Since Chart 3 is approximately applicable to triangular sections if the base length is considered to be 2X, it appears that either the triangular or rectangular shape may be used with equal effectiveness. For either section a considerable departure may be made from the theoretical optimum dimensions without great loss of efficiency. Therefore, the feasibility of construction should govern the dimensions of a blanket, as long as the extremes of inefficiency represented by the ends of the X_r- curves are avoided.

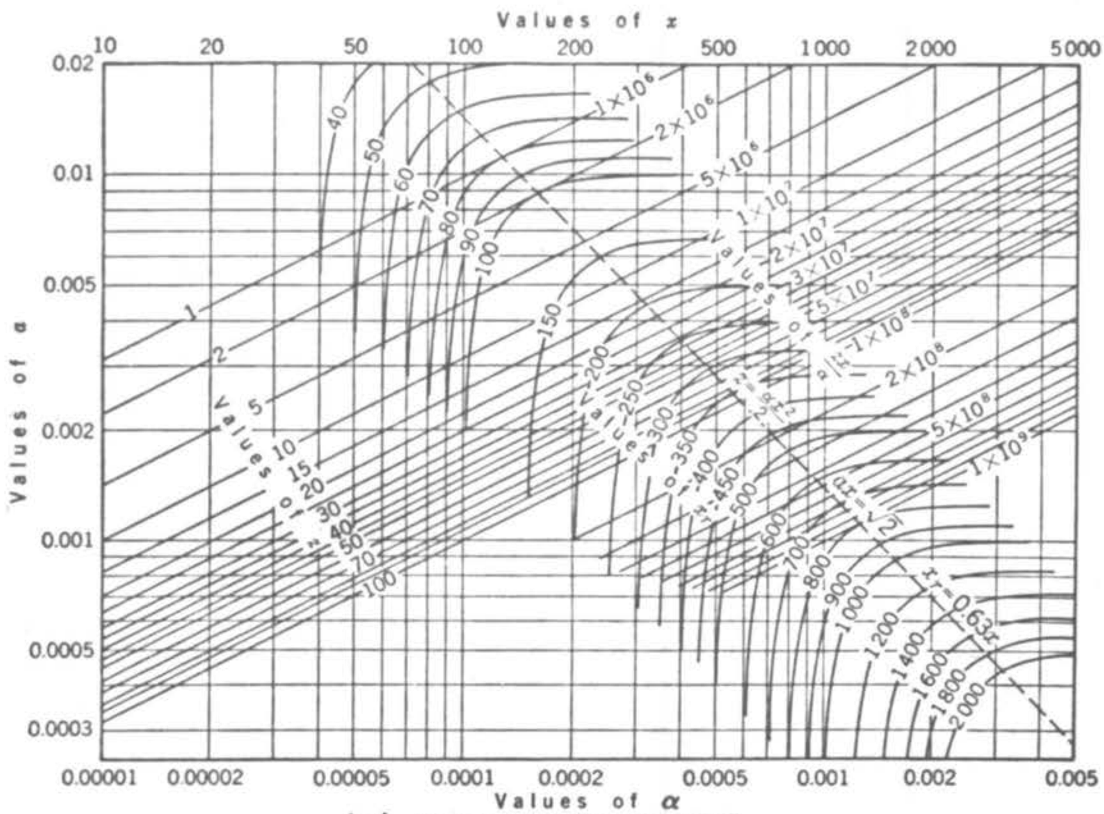
The above development neglects the fact that the seepage paths are confocal curved lines in the pervious stratum. Because of this phenomenon, the piezo-metric heads along the upper part of the pervious deposit will be somewhat less at the riverside toe of the embankment, or upstream heel of the dam and greater at the down-stream toe, as compared with the heads obtained from the mathematical studies. But the mathematical analysis presents the general principles involved in the design of blankets. The application of these principles to blanket problems, although not leading to exact results, will provide in many cases an improved method of estimating the blanket performance.



(a) SECTION



(b) POTENTIAL GRADIENT



(c) RESISTANCE CURVES

CHAPTER VI

RELIEF WELLS

In nature, the alluvial deposits usually grade from fine materials near the ground surface to coarser materials in lower portions of the strata. As a result, the lower parts of the deposits forming the foundations are much more pervious than the upper parts. Due to the stratification and excessive depth of sand formation, the installation of sheet-piling, or cut-offs only, may not be effective. In such cases, deep drainage facilities are necessary for relieving high uplift pressure downstream of the dam and thus preventing dangerous boils. After considerable experimentation both in the field and in the laboratory, the method of drainage relief wells has been adopted recently which permits exit of the water downstream of the dam in such a manner that the uplift pressures and exit gradients are reduced to safe value.

The primary function of a system of drainage wells is to provide pressure relief and drainage for water under hydrostatic pressure in a pervious substratum. For efficient operation as a means of relieving substratum pressures, the relief well must be designed to: (a) offer little resistance to the flow of water into and out of the well; (b) prevent infiltration of sand or foundation material into the well; (c) resist the deteriorative action of the water and soil; and (d) withstand the earth pressure. Some of the factors which should be considered in the selection of material for the screen and the riser pipe are: (1) corrosiveness of the soil and water; (2) expected operation of the wells; (3) method of installation; (4) size and depth of well; (5) durability under water and alternate wetting and drying; (6) crushing strength and (7) types of joints for joining the various sections. A variety of materials which may be used as drainage wells are: brass, wrought iron, aluminum, cast iron, galvanized pipe, plastic, cement asbestos, bituminous fibre, concrete, clay tile, porous concrete and wood. The materials should be used after ascertaining whether they will withstand the corrosive action of the soil and water in which they will be placed.

In view of the test experience, Mr. C. I. Mansur, Engineer, U. S. Waterways Experiment Station, Vicksburg,[28] has recommended that materials, such as brass, stainless steel, cast iron and special metal alloys, clay tile and wood are to be preferred for relief wells.

Mr. T. A. Middlebrooks [30] has mentioned that at Fort Peck, Montana, the ground water in the pervious strata was highly alkaline and was found unsatisfactory for water supply purposes before the dam was built. The slotted black pipe used in the temporary installation during World War II started failing because of complete deterioration of the metal in about two years. At this dam, slotted wooden pipes have been used in the subsequent permanent installations. He recommends that only non-corrosive materials, such as wood, chemical resistant vitrified clay, stainless steel and other highly resistant metals should be used.

The wells, including screen, riser and discharge pipe, should have an inside diameter which will permit maximum design flow without head loss of more than ½ foot to 1 foot of water, depending on the residual pressure permissible in the design. According to Mansur, the inside diameter, under no circumstances, be less than 2.5 inches. But according to Middlebrooks, large wells (6 inches minimum inside diameter) are necessary for highly efficient operation. For hydraulic considerations the screen section of wells up to 6 inches in diameter should have at least 25 holes or ten slots with an open area of at least 3 square inches to 6 square inches per linear foot of screen. For design purposes, the screen openings should be equal to or less than the 70% size of the foundation sand or filter gravel.

As per the experience and test-data based on the experiments by Mansur, the gravel filters around a perforated screen pipe should have a thickness of at least 4 inches measured radially from the outer circumference of the pipe. The filter material should be washed gravel or crushed stone composed of hard, tough and durable particles. The gradation of the filter material will depend on the foundation sand being drained. The gravel should be of a size and gradation that will prevent in-wash of an appreciable quantities of sand into the well and also of a size and gradation which will be retained by the screen section of the well. The filter should also be considerably more pervious than the foundation sand. These filter requirements may be met by a filter which conforms to the following criteria:

$$\frac{15\% \text{ size of filter}}{85\% \text{ size of foundation}} < 4 \text{ to } 5 < \frac{15\% \text{ size of filter}}{15\% \text{ size of foundation}}$$

In addition to meeting these criteria, filters should be graded uniformly without any lack or excess of particles of any particular size. To minimize segregation during placement under water, the filter should be as uniform as possible and yet meet the aforementioned criteria.

There are many pervious foundations that are sufficiently well graded and which contain sufficient gravel to permit the installation of perforated well pipe without an artificial filter. Following installation, such wells should be developed by pumping and surging so as to build up a pervious filter around the well screen. The perforations in the screen for a well of this type should be of sufficient size to permit the inwash of sand within the immediate vicinity of the well and yet to hold back the larger gravel particles, which thus build up a natural filter.

Porous concrete pipe may also be used as a filter around perforated pipe screens. If porous concrete pipe is used as a filter, it should be fitted closely around the inner pipe and should have a wall thickness of at least 1 inch and preferably greater. It should drain the foundation material freely without inwash of sand (this should be checked by laboratory tests prior to installation). The 70% size of aggregate used in the manufacture of the pipe should be greater than the perforations of the inner pipe.

DESIGN OF RELIEF WELL SYSTEMS

In view of the potential danger of seepage boils at the toe of dams and levees, a large research program on the relief of uplift pressure was undertaken by the Mississippi River commission and the U. S. Waterways Experiment Station at Vicksburg, Miss. The research consisted largely of hydraulic models and electric models both for designing well systems as a whole and for checking well inflow losses. These models yielded valuable information concerning relief well design which has been successfully applied to several earth dams.

Messrs. T. A. Middlebrooks and W. H. Jervis[29] have furnished an interesting paper on the design of relief wells for dams and levees with both the mathematical analysis and test results based on hydraulic and electric model experiments. The tests data have shown the following important facts:

1. The length of the strainer has more effect on the effectiveness of the well system than any other factor.
2. The seepage quantity falls off rapidly and the head midway between the wells rises high as the penetration drops below 25% of the thickness of the pervious stratum.
3. The well size and spacing have only minor over-all effects, within the limits studied, provided the wells are large enough to carry off the seepage quantity.

The basic design factors for an installation of wells may be summarized as follows:

1. Determination of the permissible substratum pressures at and behind the line of wells.
2. The degree of natural pressure relief resulting from seepage through the top stratum and flow of water into underground storage behind the structure.
3. The effective distance from the source of underseepage.
4. The permeability and thickness of the sub-stratum.

5. The discharge of the well system necessary to provide the desired degree of pressure relief.
6. Design of the surface storage system to take care of the well discharge.
7. Selection of an efficient and economical type of well which will handle the necessary discharge without excessive head losses and which will continue to function with a minimum of maintenance.
8. Determination of the well diameter, spacing and penetration best suited to the requirements of the particular job.

MATHEMATICAL ANALYSIS

Dr. M. Muskat [32] has determined the mathematical treatment for the design of multiple well systems and Messrs. W. H. Jarvis and P. T. Bennett[2] have developed some formulae based on Muskat's equations for the design of relief well systems and their spacing and pressure distribution.

<u>Assumptions</u>: 1. There is an infinite array of wells along the dam for the purpose of fundamental representation of the pressure distribution.

2. All the flow from a line source is

discharged through the wells and the residual gradient behind the wells is zero.

<u>Notations:</u>

a = distance between zero.

d = distance of the line of wells from the line source.

p = pressure at any point (X,Y).

x = distance measured parallel to the line of wells.

y = distance measured normal to the line of wells.

c = pressure at the line source.

r_w = well radius.

q = flux coefficient.

Q = discharge per well.

μ = viscosity of the fluid.

Δp = pressure differential between the line source and the wells = (h_1+h_2).

h_1 = the difference of head between the line source and the average head in the plane of wells.

h_2 = the difference of head between the average head in the plane of wells and the head at wells.

h_3 = the difference of head between the head at the mid-point between the wells and the average head in the plane of wells.

p_w = head in the wells.

P_a = average head in the plane of wells.

P_m = the head at mid-point between wells.

k = coeff. of permeability.

z = thickness of the pervious stratum.

$$p = C + q \log \left[\frac{\cosh 2\pi(Y-d)/a - \cos 2\pi X/a}{\cosh 2\pi(Y+d)/a - \cos 2\pi X/a} \right] \ldots (VI-1)$$

$$Q = \frac{2\pi k.Z.\Delta p/\mu}{\log(ae^{2\pi d/a}/2\pi r_w)} \ldots\ldots\ldots\ldots\ldots (VI-2)$$

$$\text{Again, } p(X,Y) = q\left[\log \cosh 2\pi(Y-d)/a - \cos 2\pi X/a\right] (VI-3)$$

$$\text{or } p = q \log 2 + 2q \log \pi r/a, \quad \ldots (VI-4)$$

where r represents the distance from the well and p, the pressure very near any particular well, such as the one lying on the Y- axis.

$$\text{Here } Q = \frac{4\pi k.Z.q}{\mu} \ldots\ldots\ldots\ldots (VI-5a)$$

$$\text{Considering } \mu = 1, \quad Q = 4\pi k.Z.q. \ldots\ldots\ldots (VI-5b)$$

$$\text{or } q = \frac{Q}{4\pi k.Z} \ldots\ldots\ldots (VI-6)$$

At a distance from the line of wells equal to the well spacing, the above expressions for p approach very closely a linear function. Therefore, below the line of wells, the pressure curve approaches the tangent,

$$p = C + q(-4\pi d/a) \quad \ldots\ldots\ldots\ldots \quad \text{(VI- 7)}$$

and above the line of wells,

$$p = C + q(-4\pi Y/a) \quad \ldots\ldots\ldots\ldots \quad \text{(VI- 8)}$$

These two tangents intersect at the line of wells,

$$\text{so } h_1 = (C-p) = \frac{4\pi d.q}{a} = \frac{Q.d}{a.k.Z} \quad \ldots\ldots\ldots\ldots \quad \text{(VI- 9)}$$

From Eq.(VI- 2), $\Delta p = (h_1 + h_2) = \frac{Q}{2\pi k.Z}\left[\log \frac{a}{2\pi r_w} + \frac{2\pi d}{a}\right]$

(VI- 10)

$$h_2 = \frac{Q}{2\pi k.Z} \log \frac{a}{2\pi r_w} \quad \ldots\ldots \quad \text{(VI- 11)}$$

The pressure at mid-point between wells, where $Y = d$ and $X = a/2$, is from Eq. (VI- 1),

$$p_m = C + q \log \frac{\cosh 0 + 1}{\cosh 4\pi d/a + 1} \quad \text{(VI- 12a)}$$

$$\text{or } p_m = C + q\left[\log 2 - 4\pi d/a + \log 2\right] \quad \text{(VI- 12b)}$$

neglecting $\cos 2\pi X/a$ in the denominator,

$$\text{or } p_m = C + 2q\left[\log 2 - 2\pi d/a\right] \quad \ldots \quad \text{(VI- 12c)}$$

From Eq. (VI- 7) the average pressure in the line of wells is,

$$p_a = C - 4\pi d.q/a \quad \ldots\ldots\ldots \quad \text{(VI - 13)}$$

Therefore, the difference is, $h_3 = p_m - p_a = 2q \log 2$ (VI- 14)

The total contributing pressure gradient (S) is,

$$S = h_1/d = 4\pi q/a = \frac{Q}{a.k.Z} \quad \ldots \quad \text{(VI- 15)}$$

Then, $h_2 = \frac{a.S}{2\pi} \log \frac{a}{2\pi r_w} = F.S$, where $F = \frac{a}{2\pi} \log \frac{a}{2\pi r_w}$.

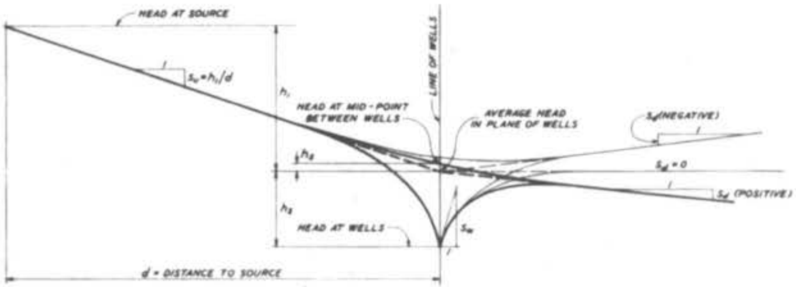
(VI- 16)

$$h_3 = \frac{a.S}{2\pi} \log 2 = 0.11a.S \quad \ldots\ldots\ldots \text{(VI- 17)}$$

PARTIAL PENETRATION OF WELLS IN THE PERVIOUS STRATUM

No exact mathematical solution is available for studying partly penetrating wells. A series of hydraulic model tests, performed at the U. S. Water-ways Experiment Station and augmented by electric analogy model tests carried out at the Vicksburg Engineer District Laboratory, have yielded empirical data from which design curves have been developed. These curves can be applied in a great number of cases by utilizing the well design in the part of seepage system through the pervious sub-stratum.

Since mathematical design formulas for relief well systems are based on idealized assumptions as to the uniformity of the pervious stratum and blanket and also the determination of permeability which is only a good approximation, the results from the formulas should not be considered as an exact design but rather as an approximation to aid the judgement of the designer. With this thought in mind, the reasonably accurate results, quickly computed, are more desirable than mathematically exact solutions, achieved through repeated trials. It is desirable to provide wells of large effective radius in order to furnish sufficient entrance face area and to keep the well screen and riser losses to a minimum.



S_u = Pressure gradient on river-side of wells.
S_d = Pressure gradient on land-side of wells.
$S = S_u - S_d$ = Net gradient normal to line of wells contributing to well discharge.
S_w = Radial flow gradient immediately adjacent to well.
h_1, h_2, h_3 as shown in figure.
a = Well spacing.
r_w = Well radius.
k = Permeability of sub-stratum.
z = Depth of sub-stratum.
z_p = Length of well screen.
$(kz)_p$ = Apparent value of kz for partial penetration well, determined by test.
Q_1 = Discharge of wells per unit length along line of wells.
$Q_w = aQ_1$ = Discharge per well.
$F = \frac{a}{2\pi} \log_e \frac{a}{2\pi r_w}$ (See graph below)

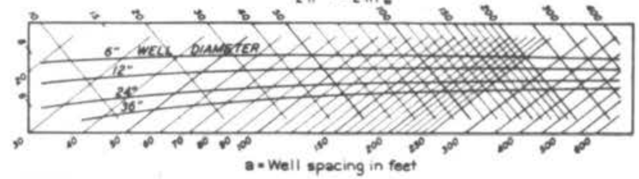
$$Q_1 = Skz$$

$$h_2 = FS\frac{kz}{(kz)_p}$$

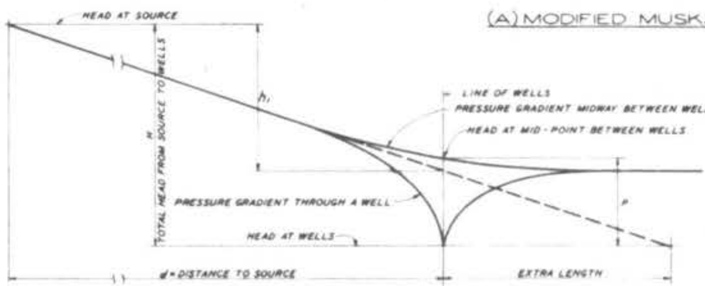
$$h_3 = 0.11\,aS\frac{kz}{(kz)_p}$$

$$S_w(\text{average}) = \frac{aSz}{2\pi r_w z_p}$$

$$F = \frac{a}{2\pi}\log_e\frac{a}{2\pi r_w}$$



(A) MODIFIED MUSKAT – JERVIS FORMULA



Q_w = Discharge per well
a = Well spacing
k = Permeability of sub-stratum
D = Depth of sub-stratum
r_w = Well radius.
"Extra Length" and "P" are obtained from design chart published in U.S.W.E.S. Technical Memorandum No. 195-1

$$Q_w = \frac{kHaD}{d + \text{extra length}}$$

(B) MUSKAT-JERVIS WELL SPACING FORMULA

Chart IV

CHAPTER VII
CONCLUSIONS

1. The hydrostatic uplift pressure under a dam acts over the entire base area, instead of over one-half or two-thirds of the area as per the old conceptions. It is the magnitude of the uplift force which undergoes changes and is uncertain, not the area of application. The pressure is applied progressively along the route of seepage and from head water to tail water.

2. Bligh's "Creep theory" appears to be the first one, developed for dealing with the problems of sub-stratum seepage flow under dams, which was subsequently modified for vertical cut-offs into Lane's "Weighted creep theory". Though Lane's theory was widely accepted, it was empirical and had no rational or logical basis.

3. The seepage flow under a dam on pervious foundation follows the principle of slow laminar viscous flow through the soil mass and is analogous to the steady flow of heat or electricity through a conductor.

4. Mathematical analysis and investigation by means of hydraulic or electric analogy models offer valuable information as to the fundamental principle of dissipation of pressure head in homogeneous foundation material, the pressure distribution under any structure and the relative ability of the different foundation types to promote the

minimum upward hydraulic gradient at the toe.

5. The extent to which "roofing", stratification of foundation materials and other effects of non-homogeneity may modify the theoretical pressure profile in practice can be observed in the hydraulic or electric analogy models by superimposition of silt and temperature and simulating stratification and also in the actual structures by pressure wells, and the values compared.

6. The flow-net of equi-pressure and flow lines for any particular dam profile on homogeneous soils is independent of:

(a) class or structure of the sub-soil so long as it is homogeneous,

(b) scale ratio,

(c) temperature so long as it is uniform throughout the medium,

(d) applied head, and

(e) upstream and downstream water levels.

7. Dissipation of head is proportionate to velocity which, in turn, is dependent on the tendency of the flow to concentrate. Such concentration occurs, especially, around the protruding corners of the structure and about the tips of cut-offs, as well as along the upstream face of heel cut-off and the downstream face of toe cut-off.

8. The problem of the safe conduct of seepage under

a dam is analogous to that of the safe conduct of water over a spill-way in the sense that it seeks means to dissipate the head without disturbing or carrying away the foundation or stream-bed material.

9. The seepage and force both act in the direction of maximum reduction of pressure. The force exerted on a volume of foundation material is, therefore, in the direction of seepage and is equal to the difference in internal pressure or head acting on the approaching and receding faces of the volume as against a solid rather than a porous surface.

10. The foundation material can not move when it is confined. No material can be carried away by seepage unless and until the material ahead of it in the direction of seepage is first removed. Thus undermining of the foundation material is caused by "flotation" at the toe, which is the cause, and "piping", the effect. If flotation is prevented, piping can not occur.

11. Analysis has indicated that the higher the upward hydraulic gradient at the toe, the less becomes the effective weight of the foundation material, until at a critical value when the exit gradient is equal to $h/L = (1-P)(S-1)$, (Eq. III- 7b), the material actually floats and may be carried away with resulting rapid crumbling from the toe backward under the dam, causing failure.

Thus the unstability is a local toe phenomenon, termed as toe "flotation" which may cause local sand "boils" and may give the impression that the sand is moving simultaneously throughout the route from head water, which is not correct.

12. A depressed toe or toe cut-off is theoretically essential for any dam, in order to avoid infinite exit gradient at the immediate toe as well as to prevent erosion by rain, current and waves. A heel cut-off in homogeneous material of relatively great depth is apparently more effective as to toe escape gradient than a heel apron.

13. Safety from toe flotation is best promoted by:

(a) Choosing a design with a depressed toe or toe cut-off and one which dissipates the head rapidly along the early part of the seepage route leaving as little residual head as possible during the upward escape into tail water.

(b) Upstream blanketed apron which will help in dissipating the uplift head in the foundation.

(c) Providing inverted filter for safe drainage under the toe of the structure or downstream of the toe cut-off.

(d) Safe drainage of the seepage water through the controlled route by means of drainage relief wells and drainage galleries at the toe.

14. The design of blanket for control of under-seepage can be developed by mathematical analysis, as well as by model studies for homogeneous foundations. The results help in approximate estimation of the effect of blanket, the advantages of which are reduction of uplift pressure in the seepage emergency zone and some reduction in the quantity of seepage.

15. The deep drainage relief wells at the toe of a dam for artificial drainage of the sub-stratum seepage flow have special advantages where the foundation soil is stratified with relatively less pervious and finer material at the top layer and excessive depth of more pervious sand formation below.

BIBLIOGRAPHY

1. Bennett, P. T. "The effect of blankets on seepage through pervious foundations" Trans. Am. Soc. C. E., vol. 111, pp. 215-228, 1946.

2. ________ "Comments on the design of relief wells", Conference on control of underseepage, Vicksburg, U. S. Waterways Experiment Station, 1945, pp. 95-101.

3. Bligh, W. G. Practical Design of Irrigation Works, 2nd Ed. (revised), 1910.

4. Casagrande, A. "Discussion on security from underseepage" Trans. Am. Soc. C. E., vol. 100, p. 1290, 1935.

5. Coleman, A. C. "The action of water under dams", Am. Soc. C. E., paper No. 1356, 1916.

6. Darcy, H. Les Fontaines Publiques de la Ville de Dijon, 1856, p. 590.

7. Forchheimer, Phillip, Hydraulik, 1930.

8. Government of India, Technical paper No. 97, 1902.

9. Griffith, W. M. "The stability of weir foundations on sand and soil subject to hydrostatic pressure", Minutes of proceedings, Inst. C. E., vol. 197, part III, p. 221, 1913-14.

10. Haigh, F. F. "Design of weirs on sand foundations" Punjab Engineering Congress, paper No. 182, 1935.

11. ________ Punjab Engineering Congress, paper No. 138, 1930.

12. Harza, L. F. "Uplift and seepage under dams on sand" Trans. Am. Soc. C. E., Vol. 100, pp. 1352-85, 1935.

13. Harza, L. F. — "Discussion on security from under-seepage" Trans. Am. Soc. C. E., vol. 100, p. 1275, 1935.

14. Harza, L. F. — "Uplift and seepage under dams on sand" Trans. Am. Soc. C. E., vol. 100, p. 1377, 1935.

15. Hazen, Allen — Report, Mass. State Board of Health, 1892.

16. Hebert, D. J. — "Hydrostatic uplift pressures under Dams on pervious earth foundations" U. S. Bureau of Reclamation, Tech. memorandum, No. 384, 1935.

17. Hinds, J. — "Upward pressures under dams", Experiments by the U. S. Bureau of Reclamation, Am. Soc. C. E., paper No. 1717, 1929.

18. Khosla, A. N. — "Pressure pipe observations in Punjnad weir", Punjab Engineering Congress, paper No. 162, 1932.

19. __________ — "Discussion on security from under-seepage", Trans. Am. Soc. C. E., vol. 100, p. 1320, 1935.

20. Khosla, A. N., N. K. Bose and E. Mckenzie Taylor, "Design of weirs on permeable foundations", Central Board of Irrigation, India, Publication No. 12, Sept., 1936, p. 28.

21. Ibid. — p. 61

22. Ibid. — p. 111

23. Ibid. — p. 101

24. Koenig, A. C. — "Dams on sand foundations", Trans. Am. Soc. C. E., vol. 73, pp. 175-89, 1911.

25. Lane, E. W. — "Security from under-seepage", Trans. Am. Soc. C. E., vol. 100, pp. 1235-72, 1935.

26. Ibid. p. 1255.

27. Ibid. p. 1257.

28. Mansur, C. I. "Discussion on Relief wells for dams and levees", Proc. Am. C. E., vol. 73 (No. 5) pp. 748-52, May, 1947.

29. Middlebrooks, T. A. and W. H. Jervis, "Relief wells for dams and levees", Trans. Am. Soc. C. E., vol. 112, pp. 1321-38, 1947.

30. ________ "Discussion on Relief wells", Trans. Am. Soc. C. E., vol. 112, p. 1400, 1947.

31. Mockmore, C. A. "Discussion on uplift and seepage under dams" Trans. Am. Soc. C. E., vol. 100, p. 1396, 1935.

32. Muskat, M. The flow of homogeneous fluids through porous media, New York, The McGraw-Hill Book Co., Inc., 1st Edition, 1937, Chap. IX, pp. 527-29.

33. Parsons, H. DeB. "Hydraulic uplift in pervious soils" Am. Soc. C. E. paper No. 1713, 1929.

34. Pavlovsky, N. N. "Motion of water under dams" paper No. 36, International commission on dams, 1933 and Eng. News-Record, vol. 112, p. 765, June 14, 1934.

35. Selim, M. A. "Dams on porous media" Proc. Am. Soc. C. E., vol. 71 (No. 10), p. 1525, December, 1945.

36. Slichter, C. S. "Water supply and irrigation papers Nos. 67 and 140, U. S. Geological Survey, 1898.

37. Terzaghi, Charles "Discussion on uplift and seepage under dams on sand" Trans. Am. Soc. C. E., vol. 100, p. 1391, 1935.

38. Vaidhianathan, I. I. "Potential distribution in infinite conductors and uplift

pressure on dams", Proceedings of the Indian Academy of Sciences, vol. II (No. 1), July, 1935.

39. ________ Memoir of the Punjab Irrigation Research Institute, vol. 5, 1935.

40. Weaver, W. "Uplift on dams", Journal of Mathematics and Physics, pp. 114-45, June, 1932.

41. Welitschkowsky Archiv fur hygiene, vol. 2, 1884.

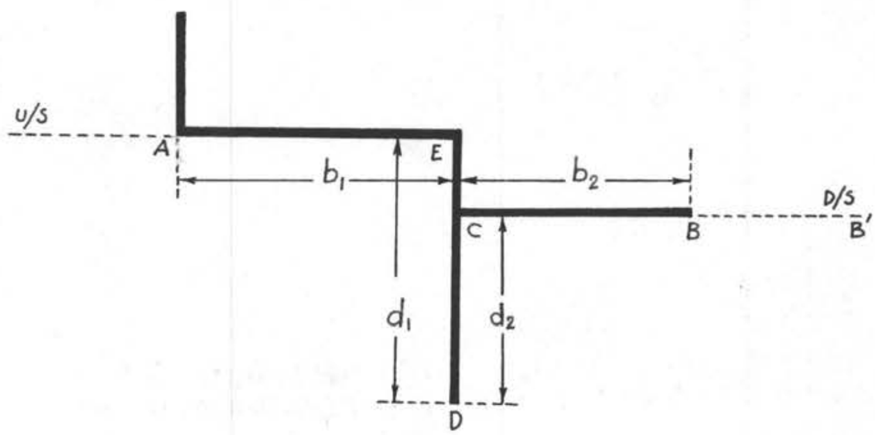
Appendix A

(Tables)

The experimental data regarding distribution of pressure under the standard forms of dam floor.

TABLE V

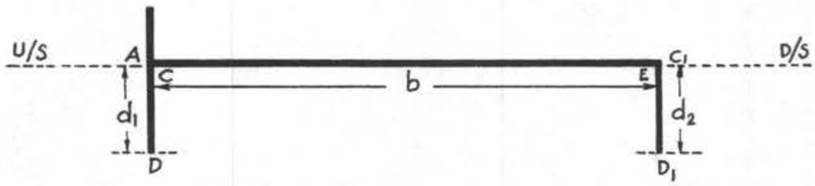
Stepped floor with pile line at the step.



$\delta = \frac{d_2}{d_1 - d_2}$	$\delta_1 = \frac{b_1}{d_1 - d_2}$	$\delta_2 = \frac{b_2}{d_1 - d_2}$	$\phi E = \frac{P_E \times 100}{H}$ Exp.	Theor.	ϕD Exp.	Theor.	ϕC Exp.	Theor.
4	4	0.0	77.0	76.3	43.4	43.8	0.0	0.0
4	4	1.0	77.1	76.5	44.0	44.3	6.8	6.8
4	4	2.0	77.4	76.9	45.6	45.6	12.4	13.7
4	4	3.0	78.2	77.1	47.1	47.3	18.7	19.4
4	4	4.0	78.6	78.1	48.9	49.2	24.2	24.6
4	4	5.0	79.3	78.8	50.5	51.2	27.9	28.9
4	4	6.0	79.9	79.5	53.3	53.1	31.9	32.7
4	4	7.0	80.7	80.0	55.4	54.8	37.1	36.0
4	4	8.0	81.7	80.8	56.5	56.4	39.4	38.9

TABLE VI

Floor with equal sheet piles at heel and toe.



b = 12

d_1	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4
d_2	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4
ϕD	85.7	81.7	78.7	76.6	73.2	71.7
ϕC	79.7	74.0	69.7	66.3	61.3	58.2
ϕE	20.3	26.4	30.7	34.1	40.0	42.1
ϕD_1	15.2	18.9	21.7	24.0	26.2	29.2

Floor with unequal piles at heel and toe.

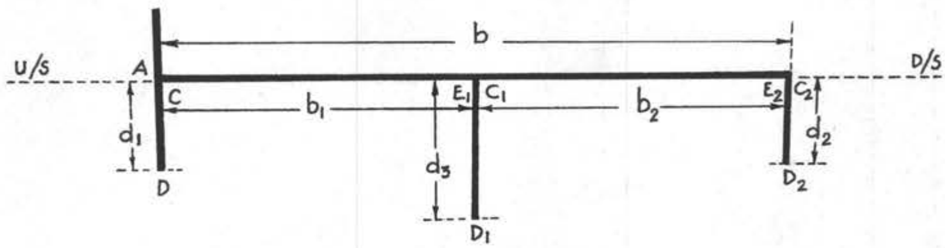
b = 12

d_1	2	3	3	3	3	3
d_2	1	2	3	4	5	6
ϕD	75.0	71.9	73.2	74.4	75.8	77.0
ϕC	64.5	59.1	61.3	63.4	65.2	67.3
ϕE	25.1	32.2	40.0	44.2	48.7	52.9
ϕD_1	18.1	22.0	26.2	29.7	32.9	35.0

TABLE VII

Floor with piles at the heel and toe and

also one line in-between them.



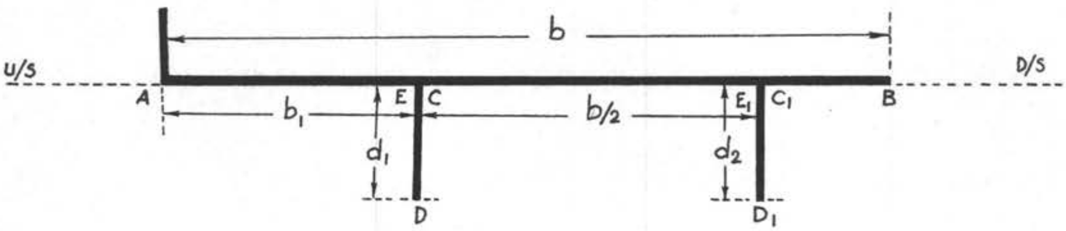
b = length of floor = 12

b_1 = distance of intermediate pile from upstream end.

b=12	$d_1=d_2$	2	2	2	2	2	2	2
	d_3	0.5	1.0	1.5	2.0	3.0	4.0	5.0
	ϕD	76.4	76.92	77.45	77.88	78.45	79.02	80.26
$b_1=9$	ϕC	66.5	66.5	66.62	67.18	68.79	69.72	71.67
$b_1/b=3/4$	ϕE_1	42.3	43.21	45.0	46.92	50.5	54.30	58.59
	ϕD_1	40.0	39.75	40.0	40.0	39.12	41.66	41.99
	ϕC_1	38.3	36.12	34.8	32.58	28.92	26.10	23.33
	ϕE_2	33.25	32.10	31.82	30.34	27.88	25.0	22.66
	ϕD_2	23.54	23.05	23.08	22.69	20.98	20.58	17.24
	ϕD	77.5	77.17	77.28	77.20	77.92	79.24	80.0
	ϕC	66.8	66.75	67.06	67.52	68.70	70.48	72.13
$b_1=7.5$	ϕE_1	47.03	48.78	50.30	52.28	55.40	59.29	62.60
$b_1/b=5/8$	ϕD_1	44.7	45.0	45.0	44.73	45.72	45.52	45.61
	ϕC_1	42.83	41.07	39.48	37.72	35.10	31.66	28.39
	ϕE_2	33.24	33.09	32.13	31.68	29.53	27.89	25.62
	ϕD_2	23.15	23.16	23.40	22.90	22.27	21.41	18.99
	ϕD	77.27	77.29	77.59	77.94	78.6	79.23	80.4
$b_1=6$	ϕC	66.52	67.32	67.32	67.90	69.3	71.43	72.97
$b_1/b=\frac{1}{2}$	ϕE_1	52.30	54.06	55.83	57.66	60.93	63.88	66.42
	ϕD_1	50.0	50.0	50.0	50.0	50.0	50.0	50.0
	ϕC_1	47.79	46.27	44.14	42.93	39.3	36.56	33.89
	ϕE_2	33.67	33.30	32.8	32.60	30.92	29.07	27.48
	ϕD_2	22.98	22.90	23.5	23.08	22.53	21.27	20.0

TABLE VIII

Floor with a pair of piles not at the ends.



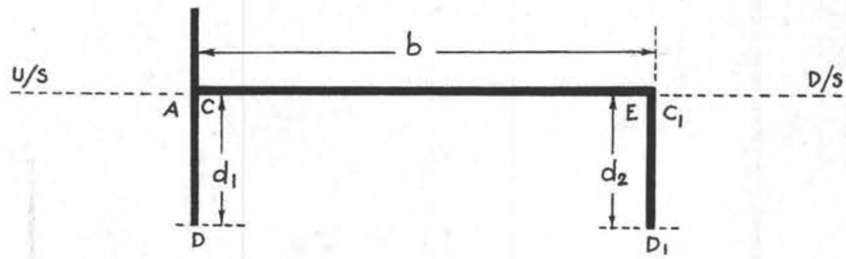
$b = 12$ and distance between piles $= b/2$

$d_1 = d_2 =$	1	1	1	1	1
$b_1 =$	6	5.5	5	4.5	4
ϕE	58.8	60.7		65.2	67.9
ϕD	53.4	55.0	56.8	59.2	61.6
ϕC	47.9	49.2	51.0	53.2	55.5
ϕE_1	27.3	28.2	30.0	32.5	35.2
ϕD_1	19.9	21.3	23.4	25.9	28.4
ϕC_1	0	6.3	11.5	15.6	19.6

Comparison of results by the different methods.

TABLE IX

Equal piles at heel and toe.



Dimensions	Percentage of initial head	Readings from Khosla's Curve	Corrected value	Theoretical value	Experimental value
$b = 12$	ϕC	81.8	82.1	82.2	79.7
$d_1 = d_2 = \frac{1}{2}$	ϕE	18.2	17.9	17.8	20.3
$b = 12$	ϕC	74.4	75.3	75.4	74.0
$d_1 = d_2 = 1$	ϕE	25.6	24.7	24.6	26.4
$b = 12$	ϕC	68.9	70.6	70.8	69.7
$d_1 = d_2 = 1\frac{1}{2}$	ϕE	31.1	29.4	29.2	30.7
$b = 12$	ϕC	64.4	67.0	67.1	66.3
$d_1 = d_2 = 2$	ϕE	35.6	33.0	32.9	34.1
$b = 12$	ϕC	57.0	61.9	62.1	61.3
$d_1 = d_2 = 3$	ϕE	43.0	38.1	37.9	40.0
$b = 12$	ϕC	51.2	58.5	58.6	58.2
$d_1 = d_2 = 4$	ϕE	48.8	41.5	41.4	42.1

TABLE X

Floor with unequal piles at toe and heel.

Dimensions	Percentage of initial head	Readings from Khosla's curve	Corrected value	Experimental value
$b = 12$ $d_1 = 2$ $d_2 = 1$	ϕC ϕE	64.4 25.6	65.8 23.7	64.5 25.1
$b = 12$ $d_1 = 3$ $d_2 = 2$	ϕC ϕE	57.0 35.6	60.2 31.6	59.1 32.2
$b = 12$ $d_1 = 3$ $d_2 = 1$	ϕC ϕE	57.0 25.6	58.8 22.4	57.7 23.7
$b = 12$ $d_1 = 3$ $d_2 = 3$	ϕC ϕE	57.0 43.0	61.8 38.3	61.3 40.0
$b = 12$ $d_1 = 3$ $d_2 = 4$	ϕC ϕE	57.0 48.8	63.4 43.3	63.4 44.2
$b = 12$ $d_1 = 3$ $d_2 = 5$	ϕC ϕE	57.0 53.8	65.2 47.5	65.2 48.7
$b = 12$ $d_1 = 3$ $d_2 = 6$	ϕC ϕE	57.0 57.6	67.1 50.5	67.3 52.9