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Supplement to

**BUCKLING OF FLAT PLYWOOD PLATES IN
COMPRESSION, SHEAR, OR COMBINED
COMPRESSION AND SHEAR**



**Buckling Tests of Flat Plywood Plates in Compression With
Face Grain 45° to Load -- Loaded Edges Clamped,
Others Simply Supported**

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FOREST PRODUCTS LABORATORY
MADISON 5, WISCONSIN

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE

In Cooperation with the University of Wisconsin

BUCKLING OF FLAT PLYWOOD PLATES IN COMPRESSION,

SHEAR, OR COMBINED COMPRESSION AND SHEAR¹

Buckling Tests of Flat Plywood Plates in Compression

With Face Grain 45° to Load -- Loaded Edges

Clamped, Others Simply Supported

By

Forest Products Laboratory,² Forest Service
U. S. Department of Agriculture

Summary and Conclusions

This investigation was undertaken for the purpose of developing a curve for the determination of the critical buckling loads of flat plywood plates with the grain direction at an angle of 45° to the edges and with the plates subjected to edgewise compression with their loaded edges clamped and the remaining edges simply supported.

From a plot of the buckling values of 50 such test plates, a curve was developed similar to the curve in figure 103 of Forest Products Laboratory Report No. 1316-G applying to similar plates tested with all edges simply supported.

Upon comparison, it is found that the curve developed in the present report has values for the abscissas 1.5 times those of the previous curve.

¹-The work here reported was done in cooperation with the U. S. Air Force under Order No. (33-038)45-301E. Original report published in 1949.

²-Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

Introduction

Frequently, in closely designed plywood structures, long plywood panels with the face grain at an angle of 45° to the edges are subjected to an edgewise compressive stress applied in the direction of the width of the panel. The edges of the panel are rigidly fastened to the framework. Such a panel, because of its size, has a low critical buckling load that can be increased manyfold by the addition of a few stiffeners at regular intervals along its length. These smaller panels, between stiffeners, then have clamped loaded edges and are simply supported by the stiffeners. It was the purpose of this investigation to develop a method for computing the critical buckling loads of these smaller panels.

It was desired to employ the critical buckling formula (6) of Forest Products Laboratory Report No. 1316,

$$p_{cr} = k_c E_L \frac{h^2}{a^2} \quad (1)$$

and to determine the proper value of k_c for plywood plates clamped at their loaded edges.

The value of this coefficient is dependent upon the construction, dimensions, and elastic properties of the plywood, as well as upon the edge conditions and the direction of the face grain of the plate. In particular, it was desired to determine a curve similar to that shown in figure 103 (Forest Products Laboratory Report No. 1316-G) for simply supported plates of plywood having the direction of its face grain at 45° to its edges, but which would apply to similar plates having their loaded edges clamped instead of simply supported.

Preparation of Specimens

The test plates consisted of three- and five-ply yellow birch and three-ply yellow-poplar plywood made at the Forest Products Laboratory to AN-NN-P-511a specifications and were designed so that the ratios of their estimated critical buckling stresses to their proportional-limit

stresses had values of 0.2, 0.4, or 0.6. They were cut from 2- by 6-foot plywood panels.

The mechanical and elastic properties of the plywood were determined from coupons cut from parts of the large panels adjacent to the test plates and tested in accordance with tentative ASTM Method D805-45T.

The coupons and the test plates were conditioned to constant weight at 75° F. and 64 percent relative humidity.

Description of Test Apparatus

The testing apparatus is shown in figures 150 and 151. The plates were tested in a hydraulic testing machine of 100,000-pound capacity with their loaded edges clamped and their other two edges simply supported.

The clamps consisted of pairs of steel bars 2 by 2 inches in cross section and 30 inches long. The bars were clamped to the plate by means of six 3/8-inch bolts spaced along the centerline of each pair of bars.

The upper clamp was bolted to the head of the testing machine. The lower clamp rested on a 1-inch-thick steel plate 12 inches wide and 30 inches long. This plate was centrally supported by a transverse 1/2-inch-diameter hard-steel roller resting on the bed of the testing machine and was, therefore, free to tilt longitudinally. The two clamps were alined in the same vertical plane. This arrangement allowed good alinement of the specimen and distribution of the load.

The sides of the plate or the edges that were not loaded were simply supported. These supports were 1- by 1-1/2-inch hardwood rails 1/8 inch shorter than the free length of the plate. Dovetail grooves 3/16 inch deep were cut along the centerline of one side of each of these two rails as shown in figure 150. The throat of each groove fitted the edge of the plate snugly to give it simple support.

Two dial indicators reading to 0.001 inch were placed so that they indicated the deflection normal to the surface of the plate. The readings of the indicator at the center of the plate were plotted for the determination of the critical buckling load when the plate was of such proportions that it buckled into a single half wave. The readings of the other dial indicator at a point approximately one-third the length of the plate gave a better indication of the critical buckling load when the width of the plate

was reduced on the subsequent tests, and also when the plate became of such proportions that it buckled into a full wave.

A sample plot of typical lateral-deflection readings and of their critical buckling loads is shown in figure 147.

Preparation of Specimen for Test

The load-bearing edges of the plates were cut straight and parallel to each other. The plates were then inserted in the testing machine and pretested for uniformity of bearing at a low initial load of about 100 pounds. A feeler gage was used to determine the exactness of fit between the bearing surfaces of the testing apparatus and the edges of the plate. If the fit was not satisfactory, the plate was retrimmed. When a satisfactory fit was obtained, the plate was centered, marked, and drilled for the clamping bolts and slotted as shown in figure 150. This was done so that the plate could be inserted and removed from the clamps without completely dismantling them each time. The plate was then clamped in place ready for test as shown in figure 151.

Test Procedure

The plate was loaded at a constant rate of total strain, and readings of the lateral deflection of the plate were plotted at regular increments of load until the critical load was indicated. The load increments chosen were about one-fifteenth of the estimated critical load, and the rate of strain was adjusted so that readings were obtained about every 5 seconds. The plate was then removed from the testing machine, and the width "a" of the plate was symmetrically reduced by increments such that the original width was reduced to one-half in four or five operations. After each reduction the plate was again inserted in the testing machine and its critical buckling load was determined as before.

The critical loads were determined from the points of inflection of the load-lateral-deflection curves as shown by figure 147. This method is described in Forest Products Laboratory Report No. 1525-A.

Explanation of Tables and Figures

Table 30 contains the dimensions, elastic properties, and constants for the plywood test plates according to species and construction. Columns 1 through 4 contain plate designation and dimensions. Columns 5 and 6 contain the effective modulus of elasticity of the plywood in bending, as determined from matched coupons, parallel and perpendicular, respectively, to the direction of the grain of the face plies computed according to the formula

$$E_{fx} \text{ or } E_{fw} = \frac{PL^3}{4wh^3y}$$

where P is the load, L is the span, w is the width, h is the thickness, and y is the deflection in the center, and $L = 48h$ when parallel to face grain and $24h$ when perpendicular to face grain. Column 7 contains the values of the modulus of elasticity of wood in the direction parallel to the grain as computed by the equation $E_L = \frac{20}{21}(E_{fw} + E_{fx})$. Columns 8 and 9 contain values for the particular plywood obtained from the curves of figure 148.

Table 31 contains the data on the effect of the plate width on the buckling loads, and the coordinates for figure 149. Column 1 lists the widths of the plates tested. Column 2 contains the critical loads obtained from the tests. Column 3 contains the corresponding critical stresses. Column 4 shows the values of k_c that are obtained from

$$k_c = \frac{P_{cr}}{E_L \frac{h}{a^2}}$$

which is a variation of equation 1. Columns 5 and 6 contain the values of the coordinates for the plot in figure 149. Column 5 contains the values for the ratio $k_c/(k_c)_\infty$, which was obtained by dividing each of the ratios in column 4, table 31, by the appropriate value in column 9, table 30. Column 6 contains values obtained by dividing each of those in

column 5, table 31, by the product of the appropriate values in columns 4 and 8 of table 30.

In figure 148 the values of the constants $\underline{k_c}$ and $\frac{b'}{a}$ are plotted as functions of the ratio $\frac{E_{fw}}{E_{fw} + E_{fx}}$. The curve for $\underline{k_{c\infty}}$ is reproduced from figure 35, Forest Products Laboratory Report No. 1316-B, and the values for the curve $\frac{b'}{a}$ are from the results of calculations made in connection with the same report.

Figure 149 compares the plot of the test values for the buckling of 45° face-grain plywood in compression, tested with the loaded edges clamped and the others simply supported, to the curve for 45° face-grain plywood tested in compression with all edges simply supported. This curve is obtained from that of figure 103, Forest Products Laboratory Report No. 1316-G by multiplying the values of the abscissas by 1.5.

Figure 147 shows the plots of the lateral-deflection readings of two typical plates and indicates the method of determining the critical buckling load.

Figures 150 and 151 are photographs of the test set-up, showing the method of fitting the test plate into the apparatus and the supported plate ready for test.

Analysis of Results

The coefficient $\underline{k_c}$ for each test plate was computed from the critical buckling data obtained in the investigation by the use of formula 1, i. e.:

$$P_{cr} = k_c E_L \frac{h^2}{a^2}$$

thus

$$k_c = \frac{P_{cr} a^2}{E_L h^2}$$

in which \underline{a} is the width of the plate, \underline{h} the thickness of the plywood, $\underline{p_{cr}}$ the observed critical stress, and $\underline{E_L}$ the modulus of elasticity of the wood parallel to the grain.

The ratios of this observed coefficient $\underline{k_c}$ to the mathematically determined coefficient $\underline{k_{c\infty}}$ for an infinitely long plate are plotted as ordinates in figure 149. The values for $(\underline{k_{c\infty}})$ are obtained from the upper curve in figure 148. This curve was determined by the energy method and is presented in figure 35 of Forest Products Laboratory Report No. 1316-B.

The abscissas in figure 149 are $\frac{\underline{b_c}}{\underline{b'}}$, which is the ratio of the lengths of the plates tested to the lengths of one half wave of similar infinitely long plates. The values for $\underline{b'}$ were obtained from the lower curve in figure 148, in which \underline{a} is the width of the test plate. It was desired to develop a curve for the loaded edges clamped similar to that developed for the simply supported edges, as shown in the upper curve in figure 103, Forest Products Laboratory Report No. 1316-G. Such a curve is drawn in figure 149 to fit the plotted points. From a comparison of these curves it was found that the values of the abscissas for the clamped condition are about 1.5 times those for the simply supported condition. The curve in figure 149 was drawn according to this assumption.

The plot of the test points conforms approximately to the contour of the curve within the range of the data. Some scatter is to be expected due to the nonuniformity of the material and to inaccuracies in the experimental determinations of the critical stresses.

Notation

The major symbols used in this report are:

- \underline{a} = the length of the loaded sides of a plywood plate, for compressive loads.
- \underline{b} = the length of the unloaded sides of a plywood plate, for compressive loads, with all edges simply supported.
- $\underline{b_c}$ = the length of the unloaded sides of a plywood plate, for compressive loads, with loaded edges clamped.

b' = the half-wave length of a buckled surface in the case of an infinitely long plate.

h = the thickness of the plywood.

E_{fx} = effective modulus of elasticity of the plywood in bending measured perpendicular to the grain direction of the face plies.

E_{fw} = effective modulus of elasticity of the plywood in bending measured parallel to the grain direction of the face plies.

E_L = modulus of elasticity of wood in the direction parallel to the grain
$$\frac{20}{21} (\underline{E_{fw}} + \underline{E_{fx}})$$

k_c = coefficient in formula,
$$p_{cr} = k_c E_L \frac{h^2}{a^2}$$

$k_{c\infty}$ = value of the coefficient $\underline{k_c}$ for an infinitely long plate.

P_{cr} = compressive load at which buckling occurs.

p_{cr} = compressive stress at which buckling occurs.

P = load in static-bending tests.

L = span in static-bending tests.

y = deflection in static-bending tests.

w = width in static-bending tests.

Table 30. -- Description and pertinent properties of the plates tested

Plate No.	Thick- ness h	Length b _c	Width a	E _{fw}	E _{fx}	E _L	$\frac{b'}{a}$	$(k_c)_\infty$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	<u>Inch</u>	<u>Inches</u>	<u>Inches</u>	<u>1,000</u> <u>p. s. i.</u>	<u>1,000</u> <u>p. s. i.</u>	<u>1,000</u> <u>p. s. i.</u>		

Yellow Birch -- Three 1/16-Inch Plies

3 x J1	: 0.175	: 16	: 10	: 2,173	: 187.4	: 2,248	: 0.848	: 0.872
3 x J2	: .178	: 28.06	: 22	: 2,173	: 187.4	: 2,248	: .848	: .872
3 x J2a	: .178	: 12.94	: 10	: 2,173	: 187.4	: 2,248	: .848	: .872

Yellow-Poplar -- Three 1/16-Inch Plies

6 x 1	: .178	: 25.5	: 22	: 1,790	: 120	: 1,819	: .821	: .786
6 x 1a	: .178	: 13.25	: 11	: 1,790	: 120	: 1,819	: .821	: .786
6 x 2	: .178	: 25.59	: 22	: 1,790	: 120	: 1,819	: .821	: .786
6 x 2a	: .178	: 12.96	: 11	: 1,790	: 120	: 1,819	: .821	: .786

Yellow Birch -- Five 1/48-Inch Plies

13 x J1	: .110	: 31.62	: 22	: 1,690	: 525	: 2,109	: .968	: 1.444
13 x J1a	: .110	: 14.4	: 10	: 1,690	: 525	: 2,109	: .968	: 1.444
13 x J2	: .108	: 31.62	: 22	: 1,690	: 525	: 2,109	: .968	: 1.444
13 x J2a	: .108	: 15.06	: 11	: 1,690	: 525	: 2,109	: .968	: 1.444
	:	:	:	:	:	:	:	:

Table 31.--The effect of various widths on the critical buckling loads of plywood plates.

Width a	Critical load P _{cr}	Critical stress P _{cr}	k _c	k _c (k _c) _∞	b _c b'	Width a	Critical load P _{cr}	Critical stress P _{cr}	k _c	k _c (k _c) _∞	b _c b'
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
Inches	Pounds	P.s.i.				Inches	Pounds	P.s.i.			
YELLOW BIRCH -- THREE 1/16-INCH PLYS						YELLOW BIRCH -- FIVE 1/48-INCH PLYS					
Plate No. 3 x J2						Plate No. 13 x J1					
22	650	163	1.108	1.271	1.50	22	290	118	2.237	1.549	1.49
19	750	217	1.100	1.261	1.74	19	275	129	1.824	1.263	1.72
16	850	292	1.050	1.204	2.07	16	310	172	1.725	1.195	2.04
13	1,000	420	.997	1.143	2.55	13	365	248	1.642	1.137	2.52
10	1,200	650	.913	1.047	3.30	11	400	320	1.517	1.051	2.97
Plate No. 3 x J2a						Plate No. 13 x J1a					
10	1,500	812	1.140	1.307	1.53	10	625	548	2.147	1.487	1.49
8	1,600	1,073	.964	1.106	1.91	7-3/4	625	699	1.645	1.139	1.92
6	2,000	1,763	.891	1.022	2.55	5-1/2	800	1,238	1.467	1.016	2.70
5	2,300	2,404	.844	.968	3.06	5	875	1,480	1.450	1.004	2.97
Plate No. 3 x J1						Plate No. 13 x J2					
10	1,300	716	1.040	1.193	1.89	22	300	124	2.439	1.689	1.49
8	1,600	1,092	1.015	1.164	2.36	19	300	143	2.098	1.453	1.72
6	2,000	1,793	.938	1.076	3.15	16	325	184	1.914	1.325	2.04
5	2,700	2,870	1.042	1.195	3.78	13	350	242	1.662	1.151	2.52
YELLOW-POPLAR -- THREE 1/16-INCH PLYS						11	450	366	1.800	1.247	2.97
Plate No. 6 x 1						Plate No. 13 x J2a					
22	550	138	1.159	1.475	1.41	11	550	448	2.203	1.526	1.42
19	575	167	1.046	1.331	1.63	9	550	543	1.788	1.238	1.72
16	650	223	.991	1.261	1.93	7	650	816	1.625	1.125	2.22
13	750	315	.924	1.176	2.39	5-1/2	800	1,261	1.550	1.073	2.84
11	850	420	.882	1.122	2.82						
Plate No. 6 x 1a											
11	1,100	543	1.140	1.450	1.47						
9	1,200	719	1.011	1.286	1.80						
7	1,400	1,066	.906	1.153	2.31						
5-1/2	1,700	1,626	.853	1.085	2.94						
Plate No. 6 x 2											
22	535	134	1.125	1.431	1.42						
19	585	170	1.065	1.355	1.65						
16	650	223	.991	1.261	1.95						
13	775	326	.956	1.216	2.40						
11	850	420	.882	1.122	2.84						
Plate No. 6 x 2a											
11	1,150	568	1.193	1.518	1.44						
9	1,200	719	1.011	1.286	1.75						
7	1,400	1,066	.906	1.153	2.25						
5-1/2	1,650	1,578	.828	1.053	2.87						

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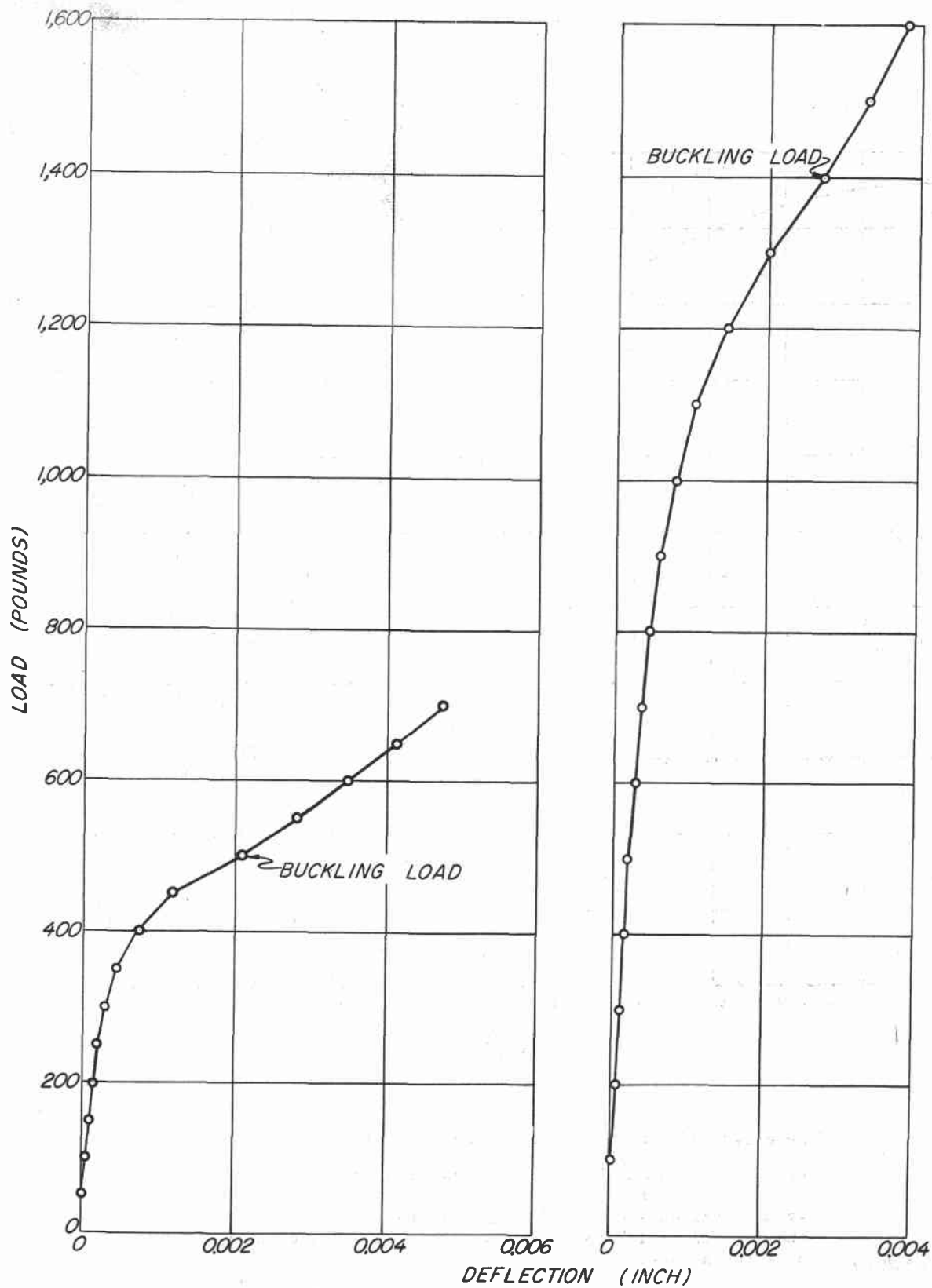


Figure 147.--Plot of lateral deflections of typical test plates
 6 x 2 -- 22 inches wide and 6 x 2a -- 7 inches wide, indicating
 critical buckling load at point of inflection.

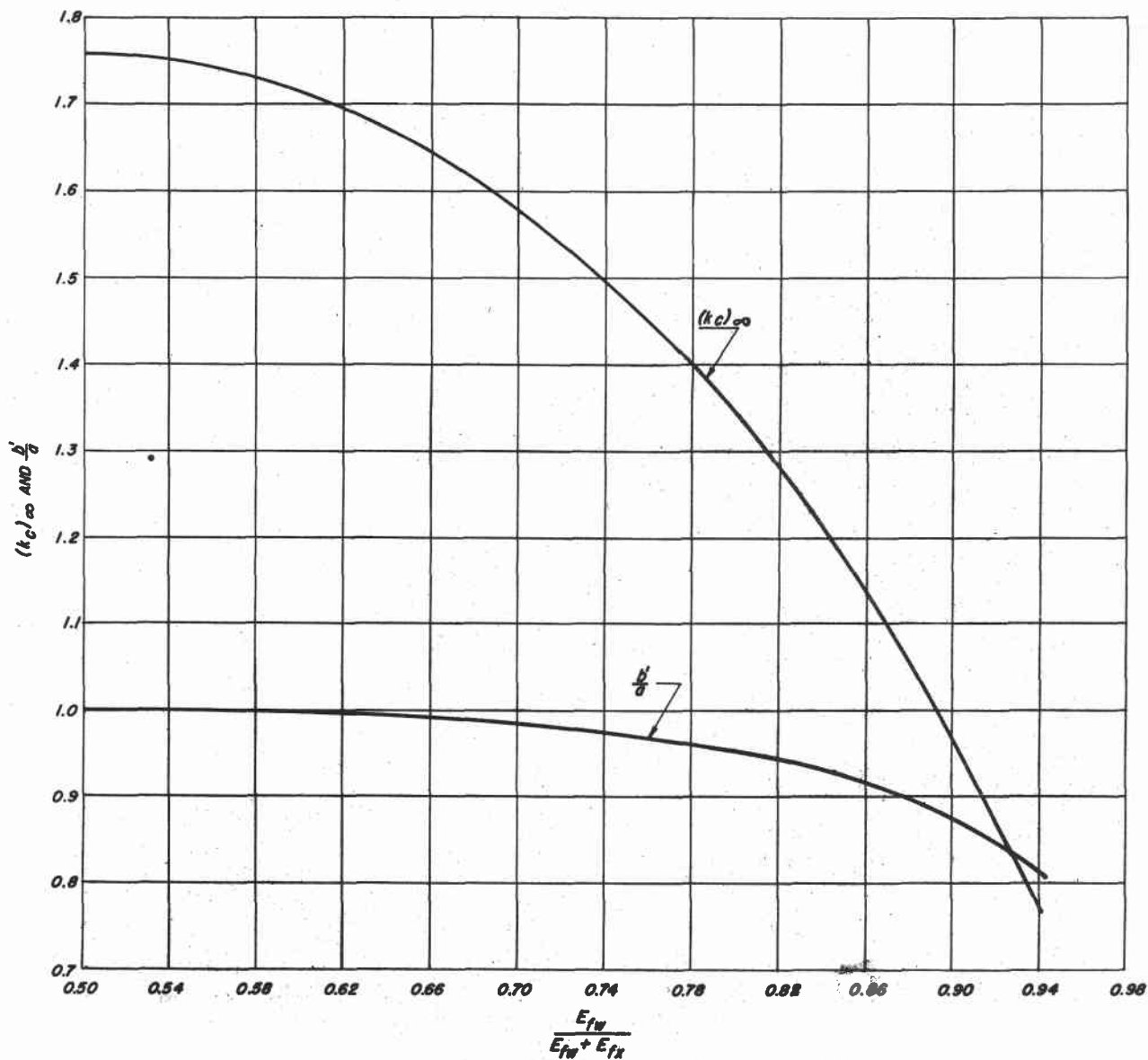


Figure 148.--The constants $k_{c\infty}$ and b'/a , both plotted as functions of the ratio, $\frac{E_{fw}}{E_{fw} + E_{fx}}$. Buckling of infinitely long plates of symmetrical construction, having the direction of the grain of the face plies 45° to the direction of the applied uniform compressive stress.

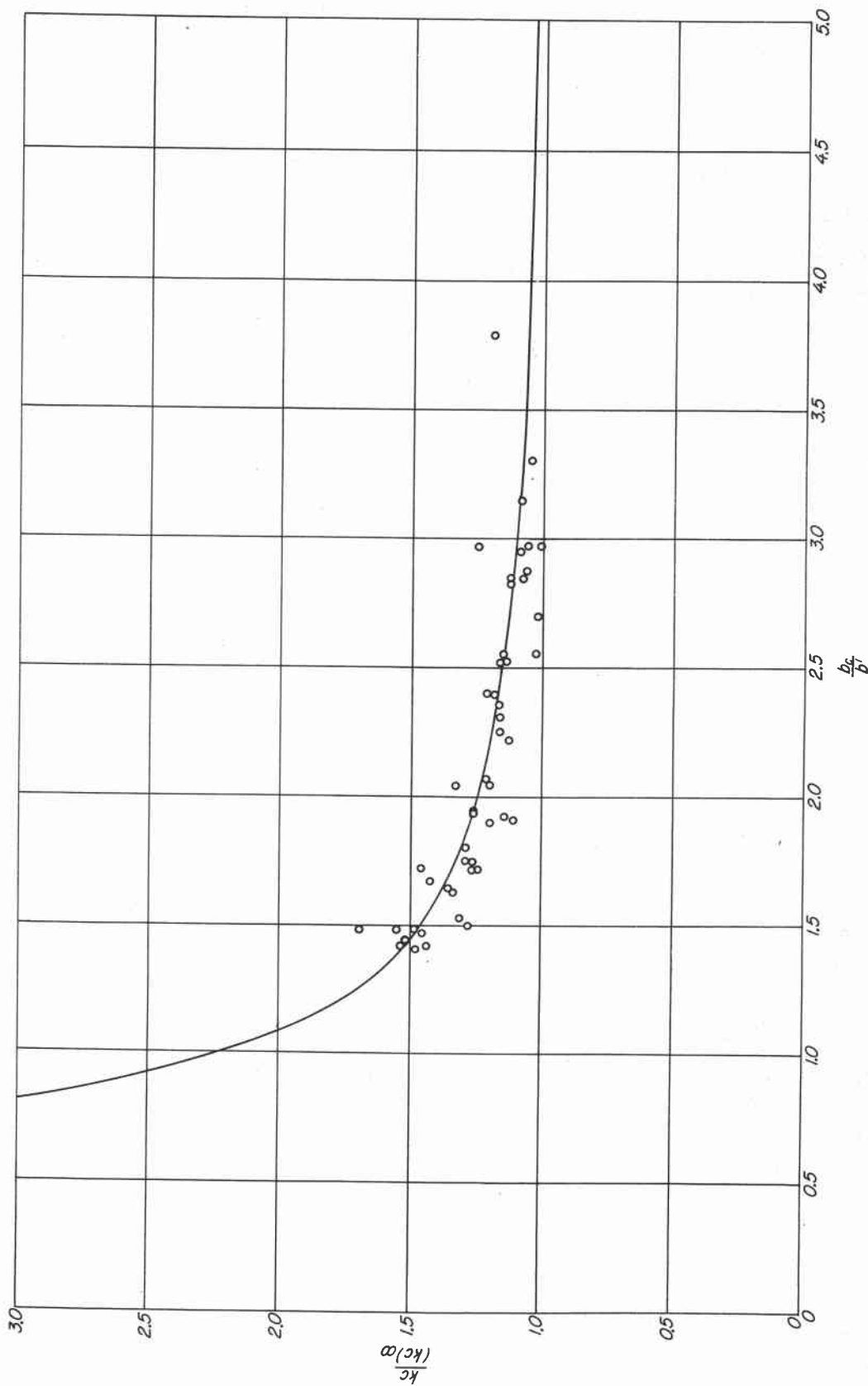


Figure 149.--Suggested curve for the determination of the critical buckling loads of plywood plates subjected to edgewise compression. Direction of face grain of the plywood is 45° to the edge of the plate. Loaded edges of the plate are clamped and the remaining edges are simply supported.

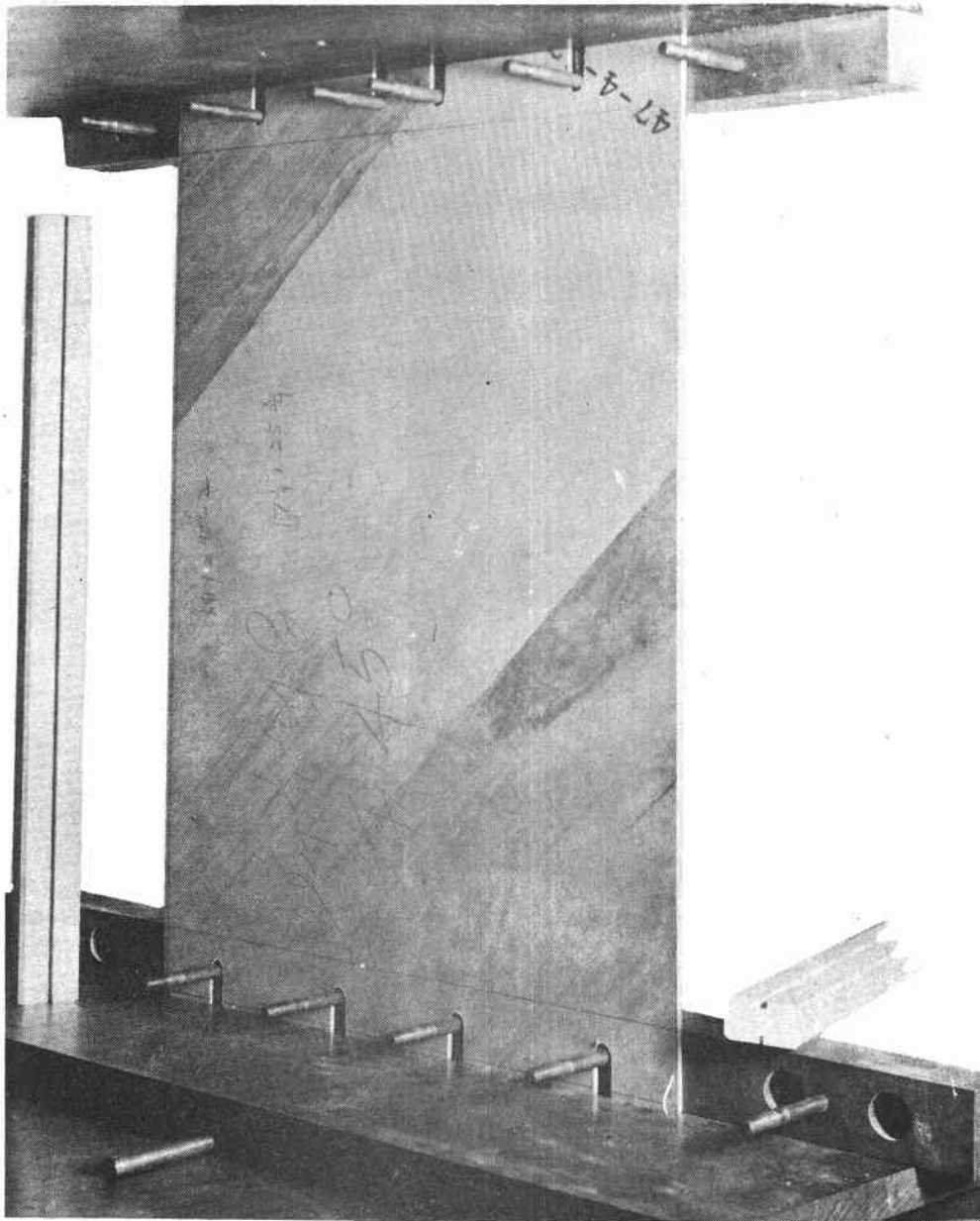


Figure 150.--Test plate in position before clamping. Note uniformity of bearing, both top and bottom, groove in side support, and slots in plate for clamping bolts.

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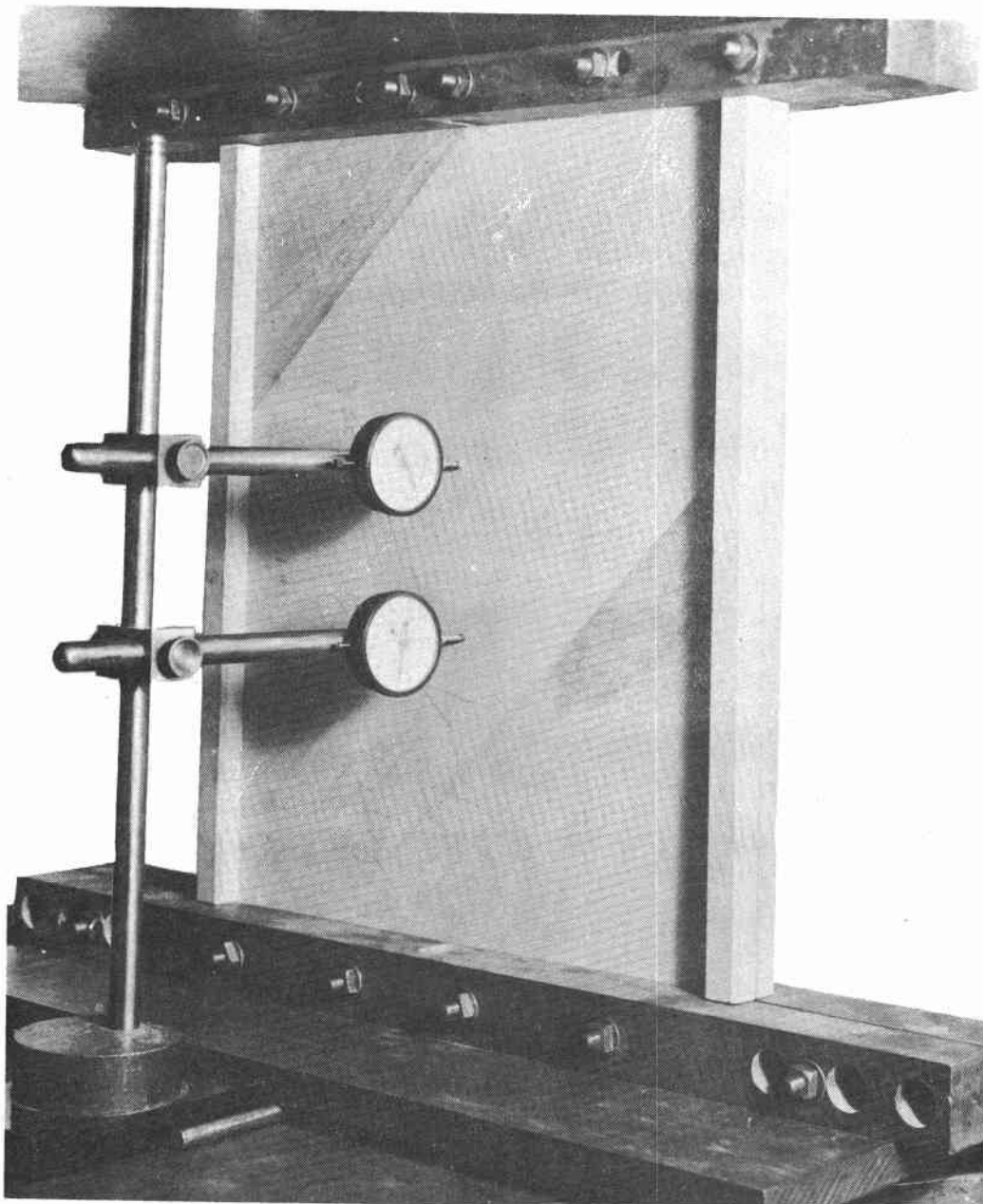


Figure 151.--Plate clamped in position ready for test. Note clamps, side-rail clearance, and transverse roller.

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