AN ABSTRACT OF THE THESIS OF

Zhengkun Zhang for the degree of Doctor of Philosophy in Agricultural and Resource Economics presented on May 4, 1994.

Title: Pricing Strategies, Discount Rates and Fishery Management in the Market for Seafood

Redacted for Privacy

Abstract approved: ______________________
Richard S. Johnston

Three related topics have been explored in this thesis. Those three topics deal with the pricing problems both in the market for access rights to fish and in the market for seafood. The purpose of this study is to contribute some insights into the global picture of fishery management and the market for seafood.

As the consequence of the 200-mile EEZs claims, many coastal nations and distant water fleets have been looking for cooperative fishing arrangements and the market for access rights is emerging. By using a linear control model, the study has demonstrated that the maximum present value of net revenue derived from the resource is negatively related with the discount rate under the sole owner assumption. This finding provides some help both in analyzing the coastal nations' and distant water fleets' competitive abilities and in examining how prices are determined in the market for access.

Most seafood at the retail level is sold to consumers through supermarkets that are multiproduct sellers and have some monopoly power. This study has explored the

pricing strategy of a multiproduct monopolist, examined the difference in the comparative statics as between the single and multiproduct monopolist, and provided a conceptual explanation for a phenomenon in the seafood industry: purchase restrictions imposed by seafood processors on fishermen.

Tying selling is a specific pricing strategy used by many multiproduct sellers. In the literature on tying sales, one issue that has received little attention is tying sales as a reaction to price controls. This study has demonstrated (a) how a competitively supplied good can be used as a tying good and how it works under price controls, (b) how tying sales arrangements under price controls can be used as a price discrimination tool to increase the monopolist's profits, in addition to be used in evasion of price controls, and (c) how the pure bundling pricing may dominate mixed bundling pricing in most circumstances under price controls.

Pricing Strategies, Discount Rates and Fishery Management in the Market for Seafood

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Completed May 4, 1994

Commencement June 1994

APPROVED:

Redacted for Privacy

Professor of Agricultural and Resource Economics in charge of major

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Date thesis presented May 4, 1994

Typed by Zhengkun Zhang

ACKNOWLEDGEMENTS

I have had a wonderful time as a graduate student at the Department of Agricultural and Resource Economics, Oregon State University. Many individuals have assisted me in many ways to the development of this dissertation. I would like to express my gratitude to a number of special individuals.

My advisor and graduate committee chair, Professor Richard Johnston, has provided guidance, moral and financial support over last five years; his contribution to this research has been pervasive. Without his continuous encouragement and support, I would not have persevered to the completion of this dissertation. His combination of honesty, scholarship and helpfulness will always be the ideal that I gauge my own efforts against.

Professor Patricia Lindsey has generously shared her time and wisdom; her valuable comments and suggestions have been very helpful in improving this research.

I am also very grateful to Professor James Cornelius, Professor James Nielsen, Professor Chi-Chur Chao and Professor Marshall English, graduate committee members, for their time and help.

Finally, this dissertation is dedicated to my parents, Yiyu Zhang and Lanzi Yang, and to my wife Crystal; their love and support has been unequivocal.

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CHAPTER 1
INTRODUCTION

In the world's fishery sectors, resource management, domestic seafood markets and international trade are a very complicated system. Fishery management decisions (such as how and how many fishery resources will be harvested, etc.) affect seafood production and supply, which influence the seafood market and trade. In turn, market demand, market structure, prices and pricing behavior in the market channels - both domestic and foreign - feed back to influence decision-making at the fishery resource management level.

For example, in order to protect the fishery resources from overexploitation or destruction, public managers of U.S. fisheries impose seasonal closures in certain water areas. Because of the closures, the total catches are reduced, at least in the short run, and then the supplies to the seafood markets are reduced. Consequently, the total output of domestic fishery products decreases and the domestic market prices go up. This often means that imports will increase because of the high prices in the domestic market.

The simple example above tells us how the management of fishery resources can affect the seafood market. On the other hand, what happens in the seafood market also influences fishery management decisions. For instance, if the demand for seafood in the U.S. market increases, the prices will go up, given other things unchanged. The high prices will in turn motivate the fishermen to catch more fish by increasing their fishing effort. This increased effort can cause overexploitation of an unmanaged fishery resource and put the resource in jeopardy. Thus, fishery resource managers may impose regulations (such as quotas, fishing licenses, taxes, etc.) to protect the fishery resources. One of the goals of such management is to maximize the net present value of the stream of outputs, both present and future, from the fisheries.

Figure 1.1 compendiously shows the interdependency among fishery management, the domestic market and foreign markets for seafood. Domestic markets are influenced not only by domestic fishery management decisions but also by our trade policy and foreign markets. Conversely, fishery management decisions are influenced not only by domestic markets but also by trade policy and foreign markets.

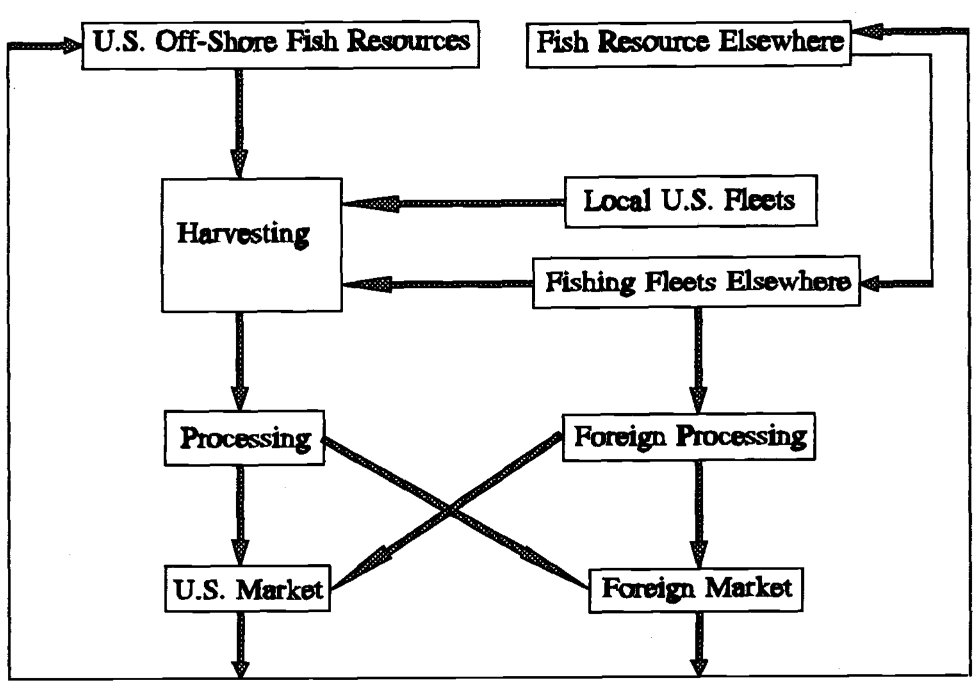

Figure 1.1 A diagram of the interdependency among fishery management and domestic and foreign markets for seafood.

Throughout most of their history, the world's ocean fisheries have been characterized by their open access nature. With very weak property rights in the ocean's resources, coastal nations had little opportunity to manage the fisheries of the adjacent waters. However, things have changed dramatically in the last fifteen to twenty years. Coastal nations have claimed 200-mile Exclusive Economic Zones (EEZs) one after another. As a consequence of the claims, the coastal nations control the fishery resources in their 200-mile EEZs, and can keep any other nation's fishing fleets from accessing and fishing in their EEZs. Those 200-mile EEZs contain more than 95% of the world's conventional marine fishery resources (Royce 1987). How to efficiently and effectively use the fishery resources is a big management issue facing and challenging both the decision-makers and the economists.

Table 1.1 Some statistics of world fisheries (FAO 1990)

Year	World Nominal Catches (Unit: 1000 mt.)	World Trade Value (Unit: 1 million US$)
1981	74,627	32,707
1982	76,817	32,496
1983	77,526	33,035
1984	83,925	33,419
1985	86,378	35,968
1986	92,829	47,293
1987	94,399	58,716
1988	99,062	67,662
1989	100,333	68,592
1990	97,246	75,839

The world's production, consumption and trade of seafood also have increased in recent years (Table 1.1). Since 1981, the world nominal catches have increased 30% and the world trade value has increased 132%. Many issues are waiting for exploration. For example, what factors determine the prices of and demand for seafood? What are the market structures and economic performance of the fishery sectors? What are the opportunities in the domestic and the international markets for U.S. fisheries and aquacultural products? These and many other issues need to be studied.

Much research has been done in the area of the fishery resource management and the market for seafood. However, our knowledge and understanding are still very limited, especially compared to the agriculture sector (Johnston 1992). I choose three related topics as my thesis and hope that my research will extend our knowledge and understanding of fishery resources management and the market for seafood.

The three topics in my thesis are as follows:

(1) The maximum present value, social discount rate and fishery management. This topic addresses some problems related to the management of a fishery (which usually focuses on the "Harvesting" and the "Fish resources" boxes of Figure 1.1).

(2) A comparative statics analysis of the pricing strategy of a multiproduct monopolist. This topic addresses some problems related to the U.S. seafood market (see, especially, the "Processing" and the "Market" boxes of Figure 1.1).

(3) The economics of tying sales under price controls. This topic addresses some problems related to the foreign market (Figure 1.1), although the research on that topic also has implications for the study of various "commodity bundling" strategies in the U.S. market.

In the next three sections, I elaborate my motivations, objectives for choosing the three topics respectively, and how they are related to each other.

1.1 The First Topic: The Maximum Present Value, Social Discount Rate and Fishery Management

As a consequence of the EEZ growth, most of the conventional open-access fishing areas no longer exist. Many coastal nations do not have enough fishing and processing capacities to efficiently use their newly-endowed fishery resources. On the other hand, many distant water fishing nations have limited accessible fishing areas, and have excess fishing and processing capacities. Some economists have pointed out that certain cooperative fishing arrangement between coastal nations and distant water fishing nations or fleets could benefit both partners and is also practicable (Munro 1985; Johnston & Wilson 1987; Clarke & Munro 1991). In the past decades, different forms of cooperative fishing arrangements among coastal nations and distant water fleets have been established (Munro 1982; Samples 1985, 1989; Abdullah et al. 1990).

When many coastal nations want to sell the access rights to their resources and many distant water fleets want to buy the access rights, a market for access rights to fish emerges (Queirolo and Johnston 1992). How, then, are prices determined in the market for the access rights to fish? How and by what factors are coastal nations' and distant water fleets' competitive abilities affected in the market for access?

For a sole owner of a fishery resource, its goal should be to efficiently use the resource. Capital theory can be applied to the resource management of fisheries. "The fish population, or biomass, can be viewed as a capital stock in that, like 'conventional' or man-made capital, it is capable of yielding a sustainable consumption flow through time. As with 'conventional' capital, today's consumption decision, by its impact upon the stock level, will have implications for future consumption options" (Clark & Munro 1975, p92). The resource management problem thus is to maximize the present value of net revenue by selecting an optimal consumption flow through time, which in turn implies selecting an optimal stock, or biomass level as a function of time. Given some simplifying assumptions (Clarke & Munro 1990), the goal of management can be expressed as

$$\max \quad PV(p,c,E) = \int_0^{\infty} e^{-\delta t}[px(t) - c]E(t)dt$$

where p is the price of the harvested fish; c is the unit cost of fishing effort; E(t) is the fishing effort; x(t) is the biomass level; δ is the social discount rate. This is a typical optimal control problem, in which the effort E(t) is a control variable, and biomass level x(t) is a state variable.

It is obvious that the maximum present value derived from the resource is the value of the resource to the profit maximizing owner. In the market for access, we would expect that the higher does a coastal nation value its resource, the higher is the price it asks for access rights. On the other hand, the higher does a distant water fleet value the coastal nation's resource, the higher is the price it is willing to pay for access. From the above equation we can expect that the maximum present value depends on the discount rate. Therefore, the discount rates will influence coastal nations' and distant water fleets' competitive abilities in the market for access, given other things equal. In addition, the discount rates will play an important role in determining prices in the market for access.

In capital budgeting theory, the relationship between present value and the discount rate is very straightforward. If the discount rate increases, the present value will decrease because the future inflow of the net revenue is independent on the discount rate. However, the situation is different in an optimal control model. The optimal output is determined by the optimal effort and optimal biomass level. And in turn, the future optimal inflow of the net revenue is determined by the optimal effort and optimal biomass level under the assumption of constant price and cost. Thus, if the social discount rate changes, the future inflow of net revenue shall change, and further the maximum present value will change.

What is the relationship between the maximum present value and the social discount rate? A priori we would expect is to be a negative relationship, as in capital budgeting theory. But, changing the discount rate may, for some growth functions, alter both the date at which one reaches the "steady state" biomass and the biomass level itself. Thus, it is not clear a priori that such a negative relationship (between the discount rate and maximum present value) holds in all cases. This research examines the question.

Why am I interested in the relationship between the social discount rate and maximum present value? There are at least two reasons:

First, it will provide some help in three microeconomic areas: one is in the analysis of the distant water fleets' willingness to pay for access to a coastal nation's resources; one is in the analysis of a distant water fleet's competitive position in the search for access; the last is in the analysis of how prices are determined in the market for access.

Second, it will provide some help in the analysis of the impacts of macroeconomic policies on resource management.

Cooperative fishing arrangements can take different forms such as "fee" fishing, joint ventures, etc. A cooperative fishing arrangement could involve multiple distant water partners who have different social discount rates. Since their social discount rates are different, they may value the resource differently (because the maximum present value is associated with the social discount rate). Therefore, the social discount rate will affect the distant water partners' willingness to pay for access to the waters of coastal nations. For any particular distant water fleet, this may also affect its competitive position in the search for access. Understanding of these issues is very important for fishery management, especially management that involves cooperative fishing arrangements.

Thus, I seek to determine how the social discount rate will affect the present value of net revenue. This present value of net revenue will, in turn, affect resource management decisions in fisheries.

What factors will influence the social discount rate? And how does one determine the value of the social discount rate? These are big issues facing economists and there are no unique solutions yet (Hartman 1990; Lind 1990; Lyon 1990; Scheraga 1990). Some macroeconomic policies can affect the social discount rate and, indirectly, affect resource management decisions. For example, the government fiscal policy and monetary policy will influence the market interest rate and the expectation of the economy's growth, etc., which will, in turn, influence the social discount rate. Therefore, if we know the relationship between the present value of net revenue and the social discount rate, we will be able to evaluate the impacts of macroeconomic policies on fishery management. In other words, analysis of the relationship of the maximum present value and social discount rate provides some help in connecting macroeconomic policies and fishery management together. Thus, this is another motivation for me to study the relationship between maximum present value and the social discount rate.

Economists have noted the effect of social discount rate on the management of fishery resources (Clarke & Munro 1991; Queirolo & Johnston 1988; 1992). However, nobody, as I know at present, has explicitly studied this issue in detail. What is the relationship between the maximum present value of net revenue and the social discount rate? Can we figure out the relationship? Is it a decreasing or increasing relationship? I will provide a detailed analysis of the relationship between these two variables through use of an optimal control model. And I will also analyze the effect of changes in the social discount rate on the optimal biomass level and optimal effort, and its implications for fishery management and the seafood market. The result of the analysis will be applicable to renewable resource management and market relationships in general.

In sum, the above topic addresses the problems related to the resource management of fisheries. Particularly, it explores (a) how prices are determined in the market for access rights to fish, (b) how and by what factors are coastal nations' and distant water fleets' competitive abilities affected, and (c) their implications for fishery management and seafood markets. The results will provide some insights into the resource management of fisheries and some knowledge of the potential of seafood production and supply.

1.2 The Second Topic: A Comparative Statics Analysis of the Pricing Strategy of a Multiproduct Monopolist

Demand theory says that for normal goods the quantity demanded will increase as the price decreases, i.e., a seller can sell more at a lower price. It has been observed in the past that in the Northwest Pacific Coast, sometimes fishermen have wanted to sell more fish to the buyers at lower prices, but the buyers didn't want to take larger quantities of fish even at those lower prices (Hanna 1990). Why is that? Is it because the buyers don't have enough capacity to handle larger quantities of fish or they cannot sell more to the customers even at lower prices? The facts don't support that. This phenomenon cannot easily be explained by conventional arguments.

A review of the literature on seafood markets has discovered that most research treats prices as having been determined in a competitive market (see, for example, Cheng and Capps 1988; Kim et al. 1988; Hermann and Lin 1988; Wellman 1992). However, business practices in the real world do not support that assumption.

The market channels for seafood have changed dramatically in the last twenty years. For example, Tony's Fish Market, a fish store located in Oregon City, and established in 1934, had sold fresh fish and shellfish for decades. However, as supermarket cases filled with fresh fish, customers slowly stopped making the trip to Tony's. In 1962, Tony's cooked 5,000 pounds of crabs weekly; today they boil approximately 500 pounds weekly during the December to February season. In 1985, the owner knew the fish store was gasping for air; supermarkets had stolen too hefty a share of the fresh fish market. Customers wanted convenience. The owner turned its major business to wholesaling of smoked fish to supermarkets and away from selling fresh fish to customers (Trappen 1993).

Now, most seafood at retail level is sold to final customers by supermarkets rather than by the traditional fish stores. Supermarkets are the sellers of many products. They sell not only seafood, but also beef, pork, poultry and other products. In addition, supermarkets have some monopoly power and are not simply price takers (Bliss 1988; Walden 1988; Mulhern and Leone 1991). In order to understand today's seafood market, we have to pay a lot of more attention to the pricing strategies of these multiproduct monopolists.

If several products are sold, it will usually be found either that the demands are interrelated or that the costs of production of the several products are interrelated or that both demands and costs are interrelated. A change of one product in either price or quantity or input price will affect the sales of other interrelated products and the seller's total profit. The price problem and quantity problem faced by multiproduct sellers have been studied by just a few economists (Edgeworth 1925; Hotelling 1932; Coase 1946; Bailey 1954; Salinger 1991). In the past decades, the problems have been almost ignored. There exists a big gap between price theory and real world pricing behavior for the multiproduct problem.

Through examining the multiproduct monopolist's pricing behavior, we may better understand some real world business practices. For example, regarding the phenomenon of the Northwest Pacific Coast fishermen case, one hypothesis would be that under certain market conditions, selling more fish at a lower price would reduce a multiproduct seller's profits because of the substitution effect. By choosing this topic, I attempt to explore the pricing strategy for a multiproduct monopolist, especially a supermarket, and to examine the difference in the comparative statics as between the single and multiproduct monopolist.

Thus, this topic addresses some problems related to the U.S. market as shown in Figure 1.1. The findings of this research will not only help us to understand the pricing behavior in seafood markets, but will also extend our understanding of price theory about multiproduct sellers.

1.3 The Third Topic: The Economics of Tying Sales Under Price Controls

As I said above, I am interested in the pricing strategies of multiproduct sellers because most seafood is sold to final consumers through supermarkets that are multiproduct sellers. A multiproduct seller can have many different price and selling strategies, in which tying sales and bundling are often observed. For example, we go to a restaurant for dinner and are offered different combinations: fried shrimp with beef steak, soup and vegetable salad; or baked fish with ham, french fries and dessert. Those products offered in combination could be substitutes, complements or unrelated.

The consumption of seafood in restaurants is a large portion of the total seafood consumption. If we want to estimate the total demand for seafood, we need to know the demand facing restaurants, and further we need to understand the pricing problem in restaurants, including tying sales and bundling. Thus, tying sales and commodity bundling influence prices and quantities in the seafood market.

Much research on aspects of tying sales and bundling has been done (Bowman 1957; Burstein 1960a, 1960b; Stigler 1968; Adams and Yellen 1976; Schmalensee 1982, 1984; Rao 1984; Tellis 1986; McAfee et al. 1989; Whinston 1990). As revealed by a review of the literature, economists have studied the reasons for tying sales and bundling, such as lowing costs, increasing profits, reputation preservation, etc. However, I found that one important phenomenon has not been well explored, namely, tying sales under price controls. Palfrey (1983, p463) pointed out that "Tied sales may be a convenient way to avoid price controls. A recent example of this occurred during the May, 1979 gasoline shortage, during which some gasoline stations offered gasoline only to customers who also paid for a carwash". Scherer and Ross (1990, p567) also mentioned that tying sales "...may be employed to evade governmental price controls". However, no research, to my knowledge, has further explored this issue so far. I explore this and extend our knowledge of tying sales and bundling.

The term tying sale (or tie-in sale) means that the buyer of the tying product is required to buy one or more tied products from the seller of the tying product. For example, a seller who produces two products, product A and product B, could choose product A as a tying product, and product B as a tied product. If customers buy product A, they are also required to buy product B from the seller. Generally speaking, the tying sale will not only affect the demand for the tying product, but also affect the demand for the tied product.

Now we go to the seafood market. The United States is a large trader in the international seafood market. In 1990, the U.S. imported \$5,573.2 million of seafood and exported \$2,269.8 million of seafood (FAO 1990). It is no doubt that foreign seafood markets and the U.S. seafood market affect each other. For example, suppose that in a two-country world, a foreign country is an exporter and the U.S. is an importer in the international seafood market. If the supply of seafood in the foreign country decreases, then its exports will decrease also. As the consequence of a change in the supply in the foreign country, the price will go up, imports will go down and the consumption will go down in the U.S. seafood market. Thus, what happens in the foreign market can influence the U.S. market. Understanding of foreign markets and their sellers' pricing behavior and strategies would help us to fully understand the U.S. market for seafood. That is why I have an interest in studying the economics of tying sales under price controls even though price controls are seldom seen in the U.S. economy.

Price controls are seen quite often in some foreign countries, especially in the centrally-planned countries. For example, China and the former Soviet Union, two of the largest producers of fish and shellfish, have experienced price regulations for a long time. I have personally experienced the popularity of tying sales arrangements under price controls in China.

Since 1949, China has been a centrally-planned economy. The central government controlled many prices and didn't let them increase even in the face of rising demand (Fung 1987; Guo 1992). As the result of price controls, many commodities were in short supply. Ration coupons, long queues, and tying sales arrangements existed simultaneously everywhere. Tying sales arrangements were used not only by retailers, but also by manufacturers and wholesalers. For example, brand cigarettes were tied with soy sauce, sugar was tied with salt, color TVs were tied with bicycles, shrimp was tied with fish, etc.

In 1979, China started its economic reform. Since then, the government has lifted some price controls gradually. When I came back to China in January 1993, I found out surprisingly that the cases of tying sales arrangements had been reduced in number dramatically. I could hardly see any obvious examples of tying sales in the market. I asked some people if they could give me some examples of tying sales in today's market, but they could not. Some people told me that the cigarette industry is an exception. I asked why that is, and was told that the cigarette industry is still highly monopolized, and the prices are firmly controlled by the government.

Many issues related to the economics of tying sales under price controls are waiting for further exploring. For example, (a) what are the reasons or motivations for tying sales under price controls in different market structures, (b) how does the tying sale work in different circumstances, (c) what are the consequences of tying sales under price controls, and (d) what is the relationship between the theory of tying sales and real business practices? I attempt to answer some of those questions in this paper through studying the economics of tying sales under price controls.

Therefore, this study addresses some problems related to the foreign market as shown in Figure 1.1, and it also helps to increase understanding of the domestic market at the same time. This study also contributes to the theory of price in the case of multiproduct sellers.

1.4 Summarizing

As stated before, fishery resource management and seafood markets are a very complicated system. The knowledge and information are very limited at present. The studies of the three topics in this thesis address some pricing problems related to the resource management of fisheries, the domestic market and foreign markets. Figure 1.2 compendiously shows the relationship among these three topics.

In particular, the first topic explores how prices are determined and how and by what factors coastal nations' and distant water fleets' competitive abilities are influenced in the market for access, through analyzing the relationship between the maximum

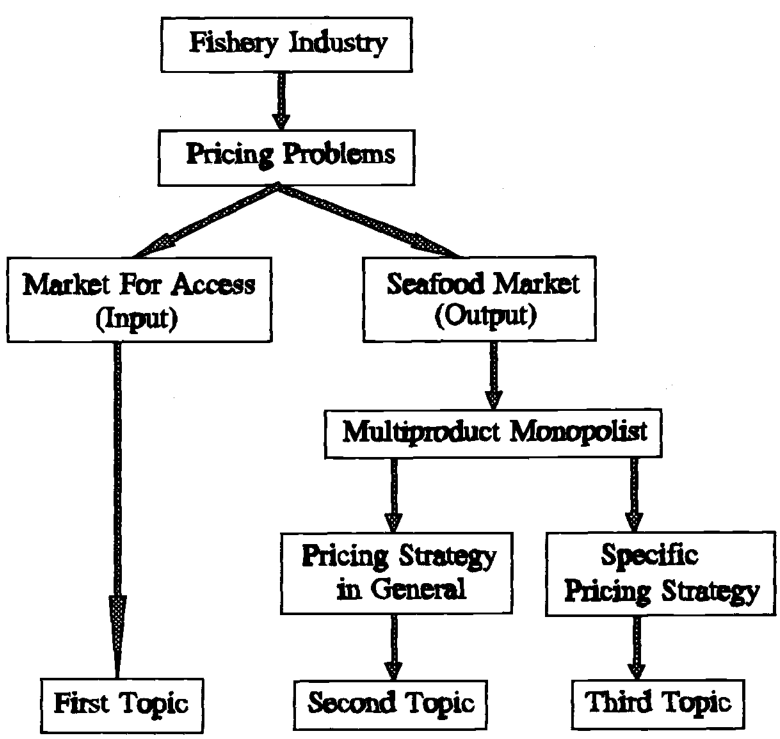

Figure 1.2 The relationship among the three topics.

present value of net revenue and the social discount rate. The fishery resource can be considered as an input in seafood production. Thus, the first topic examines the pricing problems in the input market. The second examines the pricing problems in the seafood market, an output market; specifically, it examines the pricing strategies of a multiproduct monopolist in general. The third topic examines a specific pricing strategy in the seafood market under a specific circumstance, namely, the tying sales strategy under price controls. The findings of these studies will not only help us to understand an important decision of fishery management, to understand this phenomenon in the seafood industry, and to extend our understanding of price determination in the presence of multiproduct sellers, but will also close some gaps between price theory and the pricing problems in real world business practices. Overall, the research contributes some insights into the global picture of fishery resource management and the market for seafood.

CHAPTER 2
THE MAXIMUM PRESENT VALUE, SOCIAL DISCOUNT RATE AND FISHERY MANAGEMENT

2.1 Introduction

Because of the claims for 200-mile exclusive economic zones (EEZs) the property rights for most fishery resources have been assigned to the coastal nations. Open-access resources have become managed resources. Fishery economists have argued, from a social perspective, that the managerial objective can be properly assumed to be maximization of the present value of the net revenue derived from the resources through time (Clark 1990; Clarke and Munro 1991). In calculating the present value of net revenue derived from a given resource, the owner will discount the future revenue by some discount rate. Given other things constant, there must exist some relationship between the maximum present value and the discount rate. Because of the dynamic nature of resource management the relationship is not as straightforward as we may intuitively think. The purpose of this chapter is to explore through a linear control model the relationship between the maximum present value of the net revenue and the social discount rate. Through the analysis I attempt to provide some insights into the management of fishery resources and the market for seafood.

The rest of this chapter is arranged as follows: in section 2.2, I briefly review the literature on fishery management and how the issue has been raised; in section 2.3, I discuss the possibility of using capital budgeting theory and the envelope theorem in exploring the present value - discount rate relationship; in section 2.4, I explore the relationship between the maximum present value and the discount rate within the framework of the Schaefer model; in section 2.5, I discuss the implications for the management of fishery resources and for cooperative fishing arrangements; the summary and conclusions are in section 2.6.

2.2 A Review of the Development of Fishery Management Under Extended Jurisdiction and the Issue Raised

Throughout most of their history, the world's ocean fisheries have been characterized by their open access nature. With very weak property rights in the ocean's resources, coastal nations had little opportunity to manage the fisheries of the adjacent waters. However, things have changed dramatically in the last fifteen to twenty years. Coastal nations have claimed 200-mile Exclusive Economic Zones (EEZs) one after another. As the consequence of these claims, the coastal nations literally control the fishery resources in their 200-mile EEZs, and can keep any other nation's fishing fleets from accessing and fishing in their EEZs (Munro 1989).

There is no doubt that the newly-endowed fishery resources have enormous potential benefits to the coastal nations. For example, the coastal nations can charge fishing "fees" on those foreign fishing fleets in their EEZs, or they can keep all the fishery resources in their EEZs for exclusive use by their own nations' fishing fleets. In these ways, they may increase their benefits from the newly-endowed fishery resources. However, those benefits are potential but not actual. For example, in the "fees" fishing arrangement, to collect the fees, the coastal nations need to monitor the foreign fishing fleets. If the monitoring costs are high, the benefits of "fee" fishing could be totally offset by the monitoring costs. On the other hand, the benefits of allowing only the domestic fleets to fish in the EEZ may be less than those resulting from allowing both domestic and foreign fleets to fish in the EEZ. Thus, the optimal strategy for a coastal nation may be to have foreign fishing fleets participate in the EEZ. How to achieve and maximize the potential benefits from the newly-endowed fishery resource is a challenge for both the economists and the decision-makers in the management of fishery resources.

Suppose that a coastal nation's objective in managing its fisheries within its EEZ is to maximize the net benefits through time in the fisheries. For the sake of simplicity, we can approximately measure these benefits in terms of net revenue. The coastal nation's objective can be expressed mathematically as:

$$\textit{maximize} \qquad R = \int_0^\infty e^{-\delta t} \pi(t)\,dt,$$

where R is the present value of net revenue, $\pi(t)$ is the inflow of net revenue at time t, and δ is the social discount rate.[1]

To achieve the objective, a coastal nation faces basically two issues: one is how to generate $\pi(t)$, the net revenue from its fishery resources within its EEZ; another is how to maximize R, the present value of net revenue through time. Generally, the coastal nation has two alternative strategies for exploiting its fisheries: (a) developing its fishing industry and allowing only domestic fishing fleets to fish within its EEZ; (b)

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[1] As a sole owner of the fisheries within its EEZ, the coastal nation's interest is in a long term rather than a short term arrangement. Here I extend the time to infinity.

negotiating cooperative fishing arrangements with distant-water fishing nations or fleets, and having them participate in the exploitation of fishery resources within its EEZ.

At first, the coastal nation might be inclined to choose strategy (a) and allow only domestic fishing fleets to exploit the fisheries within its EEZ. By doing this, the coastal nation may raise its total catches and increase the total revenue in the fisheries. However, when the opportunity costs and the comparative advantage issues are considered, the conclusion may be the opposite.

We know that most of the coastal nations have not engaged in distant-water fishing, and lack fishing assets, technologies, experience, etc. They are absolutely in a disadvantageous position compared to the distant-water nations or fleets. If the coastal nations want to exploit their fisheries solely through domestic fishing fleets, they need to invest a lot of capital in the fisheries. At the same time, the available capital is very limited in those coastal nations. Thus, it is highly possible that if the coastal nations invest their limited capital in other industries, the benefits may be much larger than those from investing in fisheries. Once the opportunity cost is greater than the benefit, the investment in the fisheries should not be pursued. Hence, when the opportunity costs and the comparative advantage issue are considered, the best strategy for a coastal nation may be to have a distant-water nation or fleet participate in the fisheries within its EEZ.

For different cooperative fishing arrangements, such as "fee" fishing, joint ventures, etc., we can consider them in terms of international trade. The coastal nations either import fishing capacities, technologies and expertise, or export their fishery resources. The distant-water nations or fleets either export their expertise and fishing capacities or import fishery resources. Free trade will benefit both sides as long as comparative advantages exist. This means that cooperative fishing arrangements among coastal nations and distant-water nations or fleets may be the wisest choice. A rich research on the issue of EEZs and distant-water fleets has predicted this trend (see, for example, Munro 1985, 1989; Johnston and Wilson 1987; Queirolo and Johnston 1989; Terry and Queirolo 1989). Cooperative fishing arrangements may be the best choice even for those coastal nations who have engaged in distant-water fishing and have some comparative advantages. Queirolo and Johnston (1989) have shown that Americanization (i.e., toward exclusive domestic utilization of the U.S. EEZ) may not be the best choice for the United States.

Since many coastal nations understand the potential benefits from allowing distant-water nations or fleets to be present in their EEZs, they have established different forms of cooperation with distant-water nations in the past decades. The practices of fishery cooperation have some success stories and some failure stories (see, for example, Munro 1982; Samples 1985, 1989; Abdullah et al. 1990). There are two major problems in realizing the potential benefits from fishery cooperation. One is the problem of optimal exploitation from the coastal nations' perspective. Another is the problem of income distribution. In some cases the coastal nations lack effective and efficient tools to control the distant-water fleets' actions and maintain the exploitation on an optimal path from the coastal nations' viewpoint. In most cases, the fishery resources either are overexploited or underexploited from the coastal nations' perspective. At the same time, the coastal nations have a problem to get a fair share of the income generated from the cooperation. The distant-water fleets have an incentive to cheat on their partner and retain as much of the earnings as possible, and the partner has no efficient way to prevent it.

Economists have completed much research on these issues. The work has provided a rich insight. However, as Queirolo and Johnston (1988, 1992) pointed out, much of the research focuses on the economics of bilateral and multilateral arrangements and the short run. Clarke and Munro (1987, 1991) provided a pioneer work on the issue of optimal long-term cooperative fishery arrangements by using a Principal-Agent model. In their model, a coastal nation, the principal, has a contractual arrangement with a distant-water fleet, the agent. By the arrangement, the agent makes its own decision on the exploitation of the resource within the principal's EEZ, while the principal uses a fishing "fee" or tax to generate revenue and also to indirectly influence the agent's action. Clarke and Munro (1991) have concluded that, when the coastal nation and the distant-water fleet have different discount rates, it will be difficult, if not impossible, to design a contractual arrangement that provides the distant-water fleet with the incentive to exploit the resource at a level that is optimal from the coastal nation's perspective.

Queirolo and Johnston (1988, 1992) have contributed a new thought in addressing the issue of cooperative fishing arrangements under Extended Jurisdiction. They argue that little attention in the literature has been paid to the long run. Although Clarke and Munro (1987, 1991) addressed the long term arrangement in the context of a principal-agent analysis, they still haven't considered the responses from other coastal nations and distant-water fleets in the long run. When many distant-water fleets bid for the access rights and many coastal nations want to sell the access rights, a market for access emerges. "The presence of competing buyers and sellers of access eliminates Pareto relevant externalities and, in so doing, removes the need to design complex contractual, monitoring, and policing arrangements" (Queirolo and Johnston 1992, p3). Thus, "competitive conditions on both sides of the market for access to resources may lead to a consequence of equity and efficiency considerations" (ibid., p1). If the market for access exists suggested by Queirolo and Johnston (1988, 1992), one logical question that we need to ask is that how the price level will be determined. I believe that the price will depend on the valuation of the fishery resources.

Suppose a cooperative fishing arrangement is as follows. The distant-water fleet offers the coastal nation a lump sum payment and obtains sole access rights, for an indefinite period, to the coastal nation's fishery resources. In this arrangement, the distant-water fleet will be interested in maximizing the benefit from the resources through time. The maximum present value to the distant-water fleet is

$$R* = \text{maximize} \quad \int_0^\infty e^{-\delta_d t}\pi(t)dt,$$

where R* is the maximum present value, $\pi(t)$ is the net revenue at time t, δ_d is the distant-water fleet's discount rate. Consequently, the maximum present value depends on the distant-water fleet's discount rate. The maximum present value can be interpreted as the valuation of the resource from the distant-water fleet's perspective.

Let P* be a lump sum payment such that:

$$P^* = \beta . R^*,$$

where β is a constant, and $0<\beta<1$. Thus, if all distant-water fleets face the same conditions but have different discount rates, the distribution of biding price P* (willingness to pay as a lump sum) will depend on the distribution of discount rates. The same is true for coastal nations. The distribution of their asking prices or willingness to accept lump sum payments will depend on their discount rates, if other conditions among them are the same except for the discount rates. The distributions of discount rates will affect distant-water fleets' and coastal nations' competitive abilities in the market for access rights. To see how the discount rates affect those competitive abilities, we need to study the relationship between the maximum present value and the discount rate. In the next section, I address some issues and problems related to the maximum present value and the discount rate.

2.3 The Envelope Theorem, Capital Budgeting and Optimal Control

Before we explore the relationship between the maximum present value and the discount rate as an optimal control problem, let us look at a couple of issues that may, at first, generate confusion.

First, in financial management, capital budgeting also deals with the present value and discount rate (see, for example, Campsey and Brigham 1989). To evaluate a project, we need to calculate the net present value, NPV, which can be expressed as

$$NPV=\sum_{t=1}^{n}\frac{CF_t}{(1+k)^t}-C,$$

where CF_t is the expected net cash flow from the project at period t, n is the project's expected life, k is the discount rate, and C is the project's cost. It is straightforward that NPV decreases when k increases. However, the problem with an optimal control model is much more complicated than that because the optimal net revenue at time t depends on the discount rate (we will see this in the following). Thus, we cannot use knowledge of capital budgeting to examine the relationship between NPV and the discount rate. The appropriate analytical technique is, instead, optimal control.

Second, in static maximization, the envelope theorem provides a very useful tool to analyze the rate of change of the maximum value of the objective function with respect to the parameters (see, for example, Dixit 1990). Suppose that an objective function is

$$F(x,\alpha),$$

where x is a variable, and α is a constant parameter. Maximizing the objective function would yield

$$V^*(\alpha)=F[x^*(\alpha),\alpha].$$

If we want to know the rate of change of the maximum value $V^*(\alpha)$ with respect to the parameter α, we have, by the envelop theorem:

$$\frac{dV*}{d\alpha}=\frac{\partial F}{\partial x}\frac{\partial x}{\partial \alpha}+\frac{\partial F}{\partial \alpha}=\frac{\partial F}{\partial \alpha} \qquad (since \quad \frac{\partial F}{\partial x}=0).$$

Generally, the sign of $\partial F/\partial\alpha$ can be determined, so can the sign of $dV^*/d\alpha$. However, the envelope theorem cannot be applied to the problem with optimal control model. In an optimal control problem, the first term on the right-hand side of the above equation is not zero and its sign generally cannot be determined.

To explore the relationship between the maximum present value and the discount rate, I now define a typical optimal control model for fishery management. Suppose that the owner has definite property rights to the fishery resource. His objective is to maximize the net revenue through time. This objective can be well addressed by an optimal control model:

$$\textit{Maximize:} \qquad R=\int_0^{\infty} e^{-\delta t}[p-C(x(t))]h(t)dt,$$

$$\textit{subject to:} \quad \frac{dx}{dt}=f(x,t,h(t))=F(x)-h(t)$$

$$0 \le t \le \infty$$

$$x(0)=x_0$$

$$0 \le h(t) \le h_{max},$$

where x(t) is the biomass level at time t; p is the price of fish (here I assume that the price p is constant through time); C(x) is the harvest cost, which depends on the biomass level x; h(t) is the harvesting rate at time t; F(x) is the natural growth rate at biomass level x; dx/dt is the rate of change of the biomass level; x_0 is the initial biomass level; h_{max} is the upper-limit of the harvest rate. This is a control problem with h(t) as the control variable, x(t) as the state variable, and dx/dt as the equation of motion. The Hamiltonian function for this problem is

$$H(x, h, y, t)=e^{-\delta t}[p-C(x)]+y(t)f(x, t, h),$$

where y(t) is an unknown function called the costate variable. By the maximum principle, the necessary conditions for the optimal control are as follows:

$$\frac{\partial H}{\partial h}=0 , \quad \frac{dx}{dt}=f(x,\ t,\ h)=\frac{\partial H}{\partial y} \quad \textit{and} \quad \frac{dy}{dt}=-\frac{\partial H}{\partial x} .$$

Suppose that the solutions for this problem are h*, x* and y*. The maximum present value of the objective function can be expressed as (see Intriligator 1971)

$$R*=\int_0^{\infty} e^{-\delta t}\{H(x*,\ h*,\ y*,\ t)+\frac{dy*}{dt}x*\}dt+y*(0)x_0$$

$$=f(x*,\ h*,\ y*,\ t,\ \delta).$$

Since h*, x* and y* all depend on the discount rate δ, the maximum present value R* also depends on δ. In order to figure out the relationship between the maximum present value R* and the discount rate δ, we need to know the sign of the total derivative of the maximum present value with respect to the discount rate, i.e., the sign of

$$\frac{dR*}{d\delta}=\frac{\partial f}{\partial x*}\frac{\partial x*}{\partial \delta}+\frac{\partial f}{\partial h*}\frac{\partial h*}{\partial \delta}+\frac{\partial f}{\partial y*}\frac{\partial y*}{\partial \delta}+\frac{\partial f}{\partial \delta}\ .$$

Generally, the sign cannot be determined in this problem through use of the necessary conditions.

Optimal control theory has been used by economists in many areas, such as exhaustible resource management, renewable resource management, environment economics, economic growth theory, etc. Those problems usually include the discount rate in the objective functions. However, to my best knowledge, no information on the relationship between the maximum value of the objective function and the discount rate has been provided in those studies. For example, in the literature of economic growth theory, researchers have explored the relationship between the optimal consumption per capita and the discount rate, but not the relationship of the maximum value of the objective function to the discount rate (see, for example, Shell 1967; Branson 1989).

In the literature on fishery management, researchers have explored the relationship between the optimal biomass level and the discount rate, but provided no information on the relationship between the maximum present value and the discount rate (see, for example, Clark and Munro 1975; Clark 1985, 1990; Clarke and Munro 1987, 1991). A review of the literature on natural resource economics suggests that here, too, the issue has not explored (see, for example, Mirman and Spulber 1982; Conrad and Clark 1987; Neher 1990).

2.4 The Relationship Between the Maximum Present Value and the Social Discount Rate

As discussed in the above section, it is very difficult, maybe impossible in some cases, to explore the relationship between the maximum present value and the discount rate with a general optimal control model. In this section, I first define a specific and relatively simple optimal control model in fishery management, and then explore the relationship between the maximum present value and the discount rate.

2.4.1 The Basic Model

Let x=x(t) represent the biomass level at time t. Corresponding to each level of biomass, there exists (see Schaefer 1957) a certain natural rate of increase, F(x):

$$\frac{dx}{dt}=F(x). \tag{2.1}$$

It is assumed that:

$$F(x)>0 \quad for \quad 0<x<K, \quad F(0)=F(K)=0, \quad and \quad \frac{d^2F(x)}{dx^2}<0, \tag{2.2}$$

where K denotes the carrying capacity of the environment, i.e.,

$$\lim_{t\to\infty} x(t)=K.$$

When harvesting is introduced, equation (2.1) is altered to

$$\frac{dx}{dt}=F(t)-h(t), \tag{2.3}$$

where $h(t)\geq 0$ and represents the harvest rate. I also assume

$$C_E=cE, \tag{2.4}$$

where C_E is total effort cost, E is effort, and c is a constant. I further assume that

$$h(t)=bE^{\alpha}x^{\beta}, \tag{2.5}$$

where b, α, and β are constants. It is also assumed that $\alpha=\beta=b=1$ for the sake of simplicity. From equation (2.5) we have

$$E=h(t)/x.$$

Substituting this into equation (2.4), we then have

$$\begin{aligned} C_E &= c\frac{h(t)}{x} \\ &= C(x)h(t), \qquad where \quad C(x)=\frac{c}{x}\ . \end{aligned}$$

Therefore, the implications of equations (2.4) and (2.5) are that harvesting costs are linear in harvesting but are a decreasing function of biomass level x.

Given the assumptions of constant price and costs linear in harvesting, the present value of the net revenue derived from the resources can be expressed as

$$R=\int_0^\infty e^{-\delta t}[p-C(x(t))]h(t)dt, \tag{2.6}$$

where δ is the discount rate, p is the price, C(x) is the unit cost of harvesting at biomass x.

As a sole owner of the fishery resource, the objective of fishery management is assumed to be:

$$\begin{aligned} \textit{maximize:} \quad & R=\int_0^\infty e^{-\delta t}[p-C(x(t))]h(t)dt \\ & =\int_0^\infty e^{-\delta t}[px(t)-c]E(t)dt \ , \end{aligned} \tag{2.7}$$

$$\textit{subject to:} \quad \frac{dx}{dt}=f(x,\ E,\ t)=F(x)-h(t)=F(x)-xE(t)$$

$$x(0)=x_0 \ .$$

This is a control problem. E(t) is the control variable, x(t) is the state variable, dx/dt is the equation of motion. Assume that

$$0 \le E(t) \le E_{max},$$

where E_{max} may be viewed as being determined by the fishing industry's capacity to harvest at any point in time.

We can solve this problem by using the maximum principle (see, for example, Intriligator 1971). The Hamiltonian function is:

$$
\begin{aligned}
H &= e^{-\delta t}(px-c)E+y(t)\frac{dx}{dt} \\
&= e^{-\delta t}(px-c)E+y(t)[F(x)-h(t)] \qquad (2.8) \\
&= e^{-\delta t}(px-c)E+y(t)[F(x)-xE] \\
&= [e^{-\delta t}(px-c)-y(t)x]E+y(t)F(x),
\end{aligned}
$$

where y(t) is an unknown function of time t. Because the Hamiltonian function is linear in the control variable E(t), this is a linear control problem. Let

$$s(t)=e^{-\delta t}(px-c)-y(t)x,$$

where s(t) is a switching function. Thus, the Hamiltonian function can be rewritten as

$$H=s(t)E+y(t)F(x)$$

Maximizing H with respect to the control variable, E(t), in this linear control problem, we have

$$E*(t)=\begin{cases} E_{\max} & if \quad s(t)>0 \\ 0 & if \quad s(t)<0 \end{cases}$$

Such a solution is known as a **bang-bang control**.

When the switching function s(t) vanishes, the Hamiltonian becomes independent of E. The most important case (called the singular case) arises when s(t) vanishes identically over some time interval of positive length. The corresponding singular control E(t) can be determined as follows.

Let

$$s(t)=e^{-\delta t}(px-c)-yx=0. \qquad (2.9)$$

From equation (2.9) we then have

$$y=e^{-\delta t}(p-\frac{c}{x}) . \tag{2.10}$$

Taking the derivative of equation (2.10), we have

$$\begin{aligned}\frac{dy}{dt}&=-\delta e^{-\delta t}(p-\frac{c}{x})+e^{-\delta t}(\frac{c}{x^2}\frac{dx}{dt})\\&=-\delta e^{-\delta t}(p-\frac{c}{x})+e^{-\delta t}\frac{c}{x^2}(F-xE).\end{aligned} \tag{2.11}$$

Taking the derivative of equation (2.8), we have

$$\frac{\partial H}{\partial x}=e^{-\delta t}pE+y(\frac{dF}{dx}-E).$$

Substituting equation (2.10) into the above equation, we have

$$\begin{aligned}\frac{\partial H}{\partial x}&=e^{-\delta t}pE+e^{-\delta t}(p-\frac{c}{x})(\frac{dF}{dx}-E)\\&=e^{-\delta t}pE+e^{-\delta t}p\frac{dF}{dx}-e^{-\delta t}pE-e^{-\delta t}\frac{c}{x}\frac{dF}{dx}+e^{-\delta t}\frac{c}{x}E\\&=e^{-\delta t}(p-\frac{c}{x})\frac{dF}{dx}+e^{-\delta t}\frac{c}{x}E.\end{aligned} \tag{2.12}$$

Now, according to the maximum principle, we have

$$\frac{dy}{dt}=-\frac{\partial H}{\partial x} ,$$

i.e., equation (2.11)=-equation (2.12). That is

$$-\delta e^{-\delta t}(p-\frac{c}{x})+e^{-\delta t}\frac{c}{x^2}(F-xE) =-e^{-\delta t}(p-\frac{c}{x})\frac{dF}{dx}-e^{-\delta t}\frac{c}{x}E .$$

Multiplying both sides by $e^{\delta t}$, we then have

$$
\begin{aligned}
-\delta(p-\frac{c}{x})+\frac{c}{x^2}(F-xE) &= -(p-\frac{c}{x})\frac{dF}{dx}-\frac{c}{x}E \\
-\delta(p-\frac{c}{x})+\frac{c}{x^2}F-\frac{c}{x}E &= -(p-\frac{c}{x})\frac{dF}{dx}-\frac{c}{x}E \\
-\delta(p-\frac{c}{x})+\frac{c}{x^2}F &= -(p-\frac{c}{x})\frac{dF}{dx} \\
\delta(p-\frac{c}{x}) &= \frac{c}{x^2}F+(p-\frac{c}{x})\frac{dF}{dx} \ .
\end{aligned}
\tag{2.13}
$$

It is assumed that

$$F(x)=rx(1-\frac{x}{K}) \ . \tag{2.14}$$

Taking the derivative of equation (2.14), we then have

$$\frac{dF}{dx}=r-\frac{2r}{K}x \ ,$$

and, after substituting the above two equations into equation (2.13),

$$
\begin{aligned}
\delta(p-\frac{c}{x}) &= \frac{c}{x^2}rx(1-\frac{x}{K})+(p-\frac{c}{x})(r-\frac{2rx}{K}) \\
&= \frac{cr}{x}(1-\frac{x}{K})+(p-\frac{c}{x})(r-\frac{2rx}{K}) \ .
\end{aligned}
$$

Next divide both sides by c/x,

$$\delta(\frac{p}{c}x-1)= r(1-\frac{x}{K})+(\frac{p}{c}x-1)(r-\frac{2r}{K}x)$$

$$\delta\frac{p}{c}x-\delta= r-\frac{r}{K}x+\frac{pr}{c}x-\frac{2rp}{cK}x^2-r+\frac{2r}{K}x \tag{2.15}$$

$$\frac{2rp}{cK}x^2+(\frac{\delta p}{c}-\frac{r}{K}-\frac{pr}{c})x-\delta=0 \ .$$

By using the quadratic formula, from equation (2.15) we have

$$X=\frac{-(\frac{\delta p}{c}-\frac{r}{K}-\frac{pr}{c})+\sqrt{(\frac{\delta p}{c}-\frac{r}{K}-\frac{pr}{c})^2+4\frac{2rp}{cK}\delta}}{2\frac{2rp}{cK}} \tag{2.16}$$

$$=\frac{1}{4}\{K(1-\frac{\delta}{r})+\frac{c}{p}+[(k(1-\frac{\delta}{r})+\frac{c}{p})^2+\frac{8cK}{pr}\delta]^{\frac{1}{2}}\} \ .$$

Therefore, X is the optimal biomass.[2] It is independent of time t and of the initial biomass level x_0. Let us write X as

$$X=X(\delta, p, c, r, K)$$

It is obvious that the optimal biomass level X depends on the discount rate δ.

Because X is independent of time t, we then have

$$\frac{dX}{dt}=0,$$

i.e.,

[2] Capital X is used to represent the optimal biomass level in the rest of this chapter.

$$\frac{dX}{dt}=F(X)-XE=0.$$

Therefore, the corresponding optimal control variable E(t) is

$$E*(t)=\frac{F(X)}{X}$$

When the initial biomass level x_0 is not the same as X, the optimal approach to the steady state biomass X is the most rapid or "Bang-Bang" one. That is

$$E*(t)=\begin{cases} E_{max} & if \;\; x(t)>X \\ \frac{F(X)}{X} & if \;\; x(t)=X \\ 0 & if \;\; x(t)<X \end{cases} \tag{2.17}$$

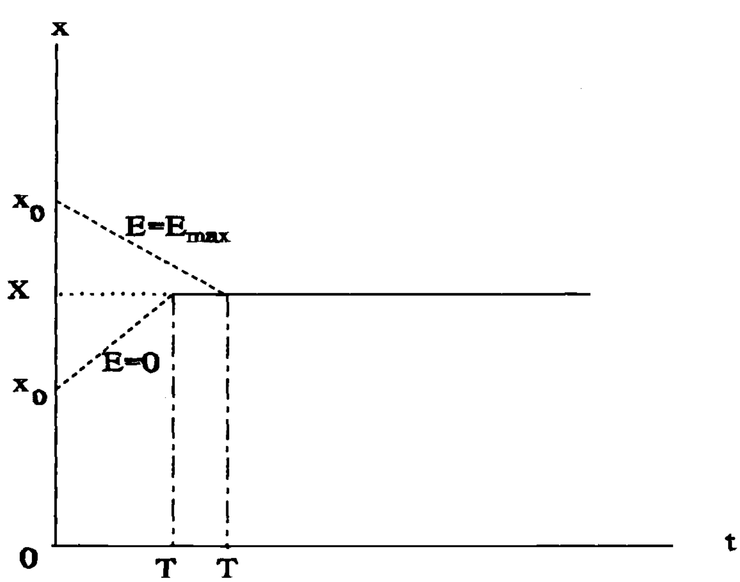

Figure 2.1 The optimal population trajectory x=x(t).

Figure 2.1 shows the relationship among the optimal effort, the optimal biomass, the initial biomass and the time. When the initial biomass level x_0 is less than the optimal level X, the best strategy is to have no harvesting (E=0) and let the biomass level reach X as soon as possible. On the other hand, when the initial biomass level x_0 is greater than the optimal level X, the best strategy is to harvest the resource at the maximum effort ($E=E_{max}$) and let the biomass level reach X as soon as possible. When the biomass is at its optimal level, the best strategy is to harvest the resources at the effort level that keeps the biomass level stable, i.e., to keep the harvesting rate equal to the natural growth rate [h(t)=F(x)].

2.4.2 The Relationship Between the Optimal Biomass X and the Discount Rate δ

In order to explore the relationship between the maximum present value and the discount rate, I now first analyze the relationship between the optimal biomass X and the discount rate δ. It would be much easier for us to start from equation (2.15) than from equation (2.16). From equation (2.15) we have

$$\frac{2rp}{ck}X^2+(\frac{\delta p}{c}-\frac{r}{K}-\frac{pr}{c})X-\delta= 0\ ,$$

i.e.,

$$\frac{2rp}{cK}X^2+(-\frac{r}{K}-\frac{pr}{c})X+\frac{p}{c}X\delta-\delta= 0\ ,$$

i.e.,

$$\frac{2rp}{cK}X^2-(\frac{r}{K}+\frac{pr}{c})X=(1-\frac{p}{c}X)\delta\ .$$

Therefore, we have

$$\delta=\frac{\frac{2rp}{cK}X^2-(\frac{r}{K}+\frac{pr}{c})X}{1-\frac{p}{c}X}$$

$$=\frac{2prX^2-(cr+Kpr)X}{cK-KpX}\ .$$

Take the total differential on both sides with respect to δ and X:

$$d\delta=\frac{[4prXdX-(cr+Kpr)dX](cK-KpX)-[2prX^2-(cr+Kpr)X](-Kp)dX}{(cK-KpX)^2}$$

$$=\frac{[4prX-(cr+Kpr)](cK-KpX)dX+[2prX^2-(cr+Kpr)X]KpdX}{(cK-KpX)^2}$$

$$=\frac{(4prX-cr-Kpr)(cK-KpX)+[2prX^2-(cr+Kpr)X]Kp}{(cK-KpX)^2}dX\ .$$

So, we have

$$\frac{dX}{d\delta}=\frac{(cK-KpX)^2}{(4prX-cr-Kpr)(cK-KpX)+Kp[2prX^2-(cr+Kpr)X]}$$

$$=\frac{(cK-KpX)^2}{4cKprX-4Kp^2rX^2-c^2Kr+cKprX-cK^2pr+K^2p^2rX+2Kp^2rX^2-cKprX-K^2p^2rX}$$

$$=\frac{(cK-KpX)^2}{4cKprX-2Kp^2rX^2-c^2Kr-cK^2pr}$$

$$=\frac{(c-pX)^2K^2}{rK(4cpX-2p^2X^2-c^2-cKp)}\ ,$$

that is,

$$\frac{dX}{d\delta}=\frac{K(c-pX)^2}{2r(2cpX-p^2X^2-\frac{1}{2}c^2-\frac{1}{2}ckp)}$$

$$=\frac{K(c-pX)^2}{2r(2cpX-p^2X^2-c^2+c^2-\frac{1}{2}c^2-\frac{1}{2}cKp)} \qquad (2.18)$$

$$=\frac{K(c-pX)^2}{2r[-(pX-c)^2-\frac{1}{2}c(Kp-c)]}\ .$$

Assume that the net revenue at time t is positive; otherwise, the owner won't want to harvest the resource. That is

$$pX-c>0.$$

Because K>X, we must have

$$Kp-c>0.$$

Therefore, from equation (2.18) we have

$$\frac{dX}{d\delta}<0\ . \qquad (2.19)$$

Equation (2.19) means that the optimal biomass level X decreases as the discount rate δ increases. Thus, if the owner of the fishery resource has a relatively high discount rate, he or she[3] will be more interested in increasing the current net revenue than he would be with a lower rate. Consequently, he will increase the harvesting rate and reduce the optimal biomass level X. An owner with a lower discount rate will be more interested in reducing current net revenue and increasing future net revenue.

[3] For the rest of the paper, the reader is asked to accept use of the male pronoun as shorthand for "he or she," "him or her".

Consequently, he would decrease the harvesting rate, and the optimal biomass level X will increase.

In cooperative fishing arrangements, if the partners have different discount rates, the distant-water partner's fishing behavior will not be optimal from the coastal nation's perspective. If the coastal nation's discount rate is higher than the partner's, the coastal nation will see that the resource is underexploited. Conversely, the coastal nation will see that the resource is overexploited. Certainly, this conclusion and analysis base on the assumptions employed.

2.4.3 The Relationship Between the Maximum Present Value and Social Discount Rate

For different initial biomass levels, the optimal approaches to reach the optimal biomass level are different. Consequently, the maximum present values corresponding to each optimal approach are different. Depending on the initial biomass level, the analysis can be completed in three cases: the first is $x(0)=x_0=X$; the second is $x(0)=x_0<X$; and the third is $x(0)=x_0>X$.

$\underline{x(0) = x_0 = X}$

When the initial biomass level is the same as the optimal biomass, the owner can harvest the resources at $E^*(t)=F(X)/X$ from the beginning to infinite time. The maximum present value of the net return is

$$
\begin{aligned}
R &= \int_0^\infty e^{-\delta t}[pX(t)-c]E*(t)dt \\
&= \int_0^\infty e^{-\delta t}[pX-c]\frac{F(X)}{X}dt \\
&= \frac{1}{\delta}(pX-c)\frac{F(X)}{X} \ .
\end{aligned}
\tag{2.20}
$$

Substituting equation (2.14) into equation (2.20), we have

$$
R = \frac{r}{\delta}(pX-c)(1-\frac{X}{K}) \ . \tag{2.21}
$$

Taking the derivative with respect to δ:

$$
\frac{dR}{d\delta} = \frac{\partial R}{\partial \delta} + \frac{\partial R}{\partial X}\frac{dX}{d\delta} \ . \tag{2.22}
$$

If we can determine the sign of equation (2.22), we then know the relationship between the maximum present value and the discount rate in this case. Consider the first term on the right-hand side of equation (2.22). Taking the partial derivative of equation (2.21), we have

$$
\frac{\partial R}{\partial \delta} = -\frac{r}{\delta^2}(pX-c)(1-\frac{X}{K})<0 \qquad (since \quad pX-c>0) \ . \tag{2.23}
$$

The meaning of equation (2.23) is very intuitive. If we hold constant all parameters but the discount rate δ, then, the present value R will decline as the discount rate δ increases because the future return contributes less to R than before. I would label this the "pure discount rate effect".

Now consider the second term on the right-hand side of equation (2.22). From equation (2.21) we obtain

$$\frac{\partial R}{\partial X}\frac{dX}{d\delta}=\frac{r}{\delta}(p-\frac{2p}{K}X+\frac{c}{K})\frac{dX}{d\delta}\ . \tag{2.24}$$

The sign of equation (2.24) is to be determined. Suppose that

$$p-\frac{2p}{K}X+\frac{c}{K}>0\ ,$$

we then have

$$pK-2pX+c>0$$

i.e.,

$$X<\frac{1}{2p}(pK+c)=\frac{1}{2}(K+\frac{c}{p})\ .$$

So, if

$$X<\frac{1}{2}(K+\frac{c}{p})\ ,$$

we then have

$$p-\frac{2p}{K}X+\frac{c}{K}>0\ .$$

We know from equation (2.16) that

$$X=\frac{1}{4}\{K(1-\frac{\delta}{r})+\frac{c}{p}+[(k(1-\frac{\delta}{r})+\frac{c}{p})^2+\frac{8cK}{pr}\delta]^{\frac{1}{2}}\}\ .$$

If δ=0, we then have

$$X=\frac{1}{4}\{K+\frac{c}{p}+[(K+\frac{c}{p})^2+0]^{\frac{1}{2}}\}$$

$$=\frac{1}{2}(K+\frac{c}{p}) \ .$$

Thus, when $\delta>0$, then

$$X<\frac{1}{2}(K+\frac{c}{p}) \qquad (since \ \ \frac{dX}{d\delta}<0) \ . \tag{2.25}$$

Therefore, we have

$$p-\frac{2p}{K}X+\frac{c}{K}>0 \ . \tag{2.26}$$

Substituting equation (2.26) into equation (2.24), we get

$$\frac{\partial R}{\partial X}\frac{dX}{d\delta}=\frac{r}{\delta}(p-\frac{2p}{K}X+\frac{c}{K})\frac{dX}{d\delta}<0 \ . \tag{2.27}$$

The meaning of equation (2.27) may be explained as follows. When the discount rate δ increases, the owner is more interested in the short term return, and will increase the current harvesting rate. Consequently, the optimal biomass level X declines as the result of higher harvesting rate. I assume in the model that the unit cost of harvesting, C, equals c/x. When the optimal biomass level X declines, the unit cost of harvesting increases. So, the net revenue (p-c/x) per unit has decreased when X declines.

Therefore, the maximum present value of net revenue R decreases as the result of increased unit cost of harvesting. I label this the "cost effect".[4]

We now substitute equations (2.23) and (2.27) into equation (2.22) and obtain

$$\frac{dR}{d\delta}<0\ .$$

We can summarize as follows: if the initial biomass, $x_0=X$, the optimal biomass, then the maximum present value of net revenue decreases when the discount rate increases. The total effect of changes in the discount rate δ on the maximum present value of net revenue R can be decomposed two effects: the "pure discount rate effect", and the "cost effect".

---

[4] In order to better understand the "cost effect", let us discuss it from a different perspective. From equation (2.20) we know

$$R=\frac{1}{\delta}(p-\frac{c}{X})F(X)\ .$$

So, taking the partial derivative with respect to δ, we have

$$\frac{\partial R}{\partial X}\frac{dX}{d\delta}=\frac{1}{\delta}(p-\frac{c}{X})\frac{dF(X)}{dX}\frac{dX}{d\delta}+\frac{1}{\delta}F(X)\frac{c}{X^2}\frac{dX}{d\delta}$$

$$=\{(p-\frac{c}{X})\frac{dF(X)}{dX}+F(X)\frac{c}{X^2}\}\frac{1}{\delta}\frac{dX}{d\delta}\ .$$

F(X) is the natural growth rate at biomass level X, and would be the same as the harvesting rate when X is stable. The effect of dF(X)/dX on the maximum present value can be called the "harvesting rate effect" which is a part of the "cost effect". dF(X)/dX could be either positive or negative in the Schaefer model. When it is positive, a reduction in X would cause F(X) to decrease, which, in turn, causes the harvesting rate to decrease. Consequently, the maximum present value would be reduced. Thus, in this case, the "harvesting rate effect" enhances the "cost effect". When dF(X)/dX is negative, a reduction in X would cause F(X) to increase, in turn, cause the harvesting rate to increase. Consequently, the maximum present value would be increased through the "harvesting rate effect". However, the total "cost effect" is negative and the maximum present value is reduced as shown by equation (2.27).

<u>$x(0) = x_0 < X$</u>

When $x(0)=x_0<X$, the fishery resource is overexploited in the initial period (t=0). Let T(X) denote the time required for the biomass to climb to the optimal level X from its initial level x_0 in the absence of any harvesting. Then the maximum present value of net revenue derived from the resource is

$$R=\int_0^{\infty} e^{-\delta t}[px(t)-c]E*(t)dt$$

$$=\int_0^{T(X)} e^{-\delta t}[px(t)-c]E*(t)dt+\int_{T(X)}^{\infty} e^{-\delta t}[pX(t)-c]E*(t)dt \ .$$

Since E*(t)=0 when 0<t<T(X) and E*(t)=F(X)/X when t ≥T(X), we have

$$R=\int_{T(X)}^{\infty} e^{-\delta t}[pX-c]\frac{F(X)}{X}dt$$

$$=\frac{e^{-\delta T(X)}}{\delta}(pX-c)\frac{F(X)}{X} \tag{2.28}$$

$$=\frac{re^{-\delta T(X)}}{\delta}(pX-c)(1-\frac{X}{K}) \ .$$

Taking the derivative of equation (2.28) with respect to δ, we have

$$\frac{dR}{d\delta}=\frac{\partial R}{\partial\delta}+\frac{\partial R}{\partial X}\frac{dX}{d\delta}+\frac{\partial R}{\partial T}\frac{dT}{d\delta} \ . \tag{2.29}$$

In order to examine the relationship between the maximum present value and the discount rate, we need to determine the sign of equation (2.29). Consider the first term on the right-hand side of equation (2.29). Taking the partial derivative of equation (2.28) with respect to δ, we have

$$\frac{\partial R}{\partial \delta} = -\frac{re^{-\delta T}}{\delta^2}(pX-c)(1-\frac{X}{K}) + \frac{re^{-\delta T}}{\delta}(-T)(pX-c)(1-\frac{X}{K})$$

$$= -(\frac{1}{\delta}+T)\frac{re^{-\delta T}}{\delta}(pX-c)(1-\frac{X}{K}) < 0 \ . \tag{2.30}$$

Equation (2.30) can be interpreted as the "pure discount rate effect" as before, and it is negative. Next, consider the second term on the right-hand side of equation (2.29). From equation (2.28) we have

$$\frac{\partial R}{\partial X}\frac{dX}{d\delta} = \frac{re^{-\delta T}}{\delta}(p - \frac{2p}{K}X + \frac{c}{K})\frac{dX}{d\delta} < 0 \ . \tag{2.31}$$

Equation (2.31) can be interpreted as the "cost effect" on R. It is also negative.

Now, consider the last term on the right-hand side of equation (2.29). We know that the harvesting rate h(t)=0 when x(t)<X. Thus,

$$\frac{dx}{dt} = F(x(t)) - h(t) = F(x(t)).$$

By definition of x(t) we have

$$x(T(X)) = X.$$

Taking the derivative with respect to δ on both sides simultaneously, we have

$$\frac{dx}{dT}\frac{dT}{d\delta} = \frac{dX}{d\delta} \ .$$

Therefore, we obtain

$$\frac{dT}{d\delta}=\frac{\frac{dX}{d\delta}}{\frac{dx}{dT}}=\frac{\frac{dX}{d\delta}}{F(X)} .$$

Then from equation (2.28) we have

$$\frac{\partial R}{\partial T}\frac{dT}{d\delta}=\frac{1}{\delta}re^{-\delta T}(pX-c)(1-\frac{X}{K})(-\delta\frac{dT}{d\delta})$$

$$=-re^{-\delta T}(pX-c)(1-\frac{X}{K})\frac{\frac{dX}{d\delta}}{F(X)} \tag{2.32}$$

$$=-e^{-\delta T}(p-\frac{c}{X})\frac{dX}{d\delta}>0 .$$

The time T is that required for the biomass x to climb to the optimal biomass level X from the initial biomass level x_0. T is a function of both X and x_0, that is,

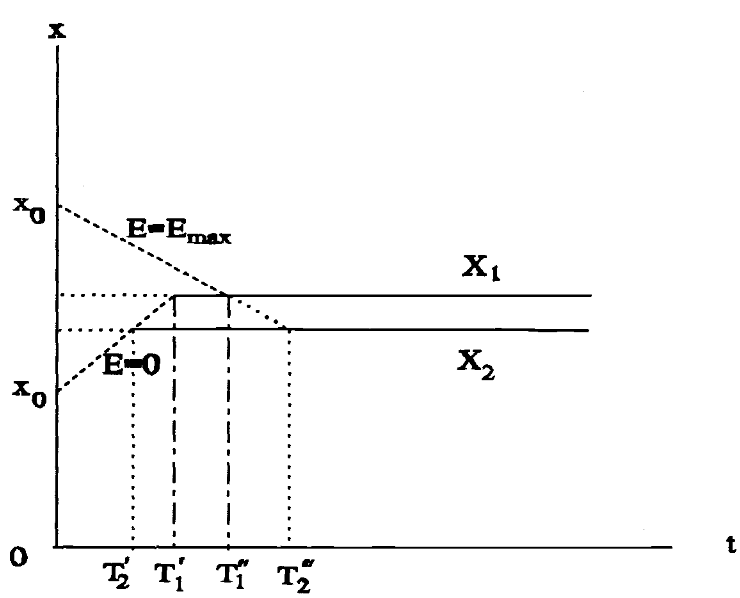

Figure 2.2 The times required for the initial biomass to climb to the optimal biomass. As the discount rate increases, the optimal biomass changes from X_1 to X_2.

$T=f(X, x_0)$. When the discount rate δ increases, the optimal biomass level X declines, and, thus, time T decreases. Figure 2 shows the relationship between the time T and the discount rate. Suppose the discount rate is δ_1 and the optimal biomass level is X_1 and the time required for the initial biomass to climb to the optimal biomass level is T_1'. When the discount rate increases to δ_2, the optimal biomass level drops to X_2, and the time required for the initial biomass to climb to the optimal biomass level is reduced to T_2'. Because of the decreased time T, the owner can start fishing earlier than before, so that the maximum present value of the net revenue R increases. This kind of effect on R can be interpreted as the "time effect".

Finally, consider the second term and the third term together on the right-hand side of equation (2.29). From equations (2.31) and (2.32) we have

$$\begin{aligned}
&\frac{\partial R}{\partial X}\frac{dX}{d\delta}+\frac{\partial R}{\partial T}\frac{dT}{d\delta} \\
&= \frac{re^{-\delta T}}{\delta}\left(p-\frac{2p}{K}X+\frac{c}{K}\right)\frac{dX}{d\delta}-e^{-\delta T}\left(p-\frac{c}{X}\right)\frac{dX}{d\delta} \qquad (2.33) \\
&= e^{-\delta T}\frac{dX}{d\delta}\left[\left(p-\frac{2p}{K}X+\frac{c}{K}\right)\frac{r}{\delta}-\left(p-\frac{c}{X}\right)\right] .
\end{aligned}$$

Let

$$\left(p-\frac{2p}{K}X+\frac{c}{K}\right)\frac{r}{\delta}-\left(p-\frac{c}{X}\right)=A \ . \qquad (2.34)$$

Multiply both sides by δKX:

$$\delta KX(p-\frac{2p}{K}X+\frac{c}{K})\frac{r}{\delta}-\delta KX(p-\frac{c}{X})=A*\delta KX$$

$$rKpX-2prX^2+crX-\delta KpX+\delta cK=A*\delta KX$$

$$-2prX^2+(cr+Kpr-\delta Kp)X+\delta cK=A*\delta KX\ ,$$

and divide both sides by cK, then

$$-\frac{2pr}{cK}X^2+(cr+Kpr-\delta Kp)X\frac{1}{cK}+\delta=A*\frac{\delta X}{c}$$

$$-\frac{2pr}{cK}X^2+(\frac{r}{K}+\frac{pr}{c}-\frac{p}{c}\delta)X+\delta=A*\frac{\delta X}{c} \tag{2.35}$$

$$\frac{2pr}{cK}X^2+(\frac{\delta p}{c}-\frac{r}{K}-\frac{pr}{c})X-\delta=-A*\frac{\delta X}{c}\ .$$

From equation (2.15) we know

$$\frac{2pr}{cK}X^2+(\frac{\delta p}{c}-\frac{r}{K}-\frac{pr}{c})X-\delta=0\ .$$

Substituting this into equation (2.35), we then have

$$-A*\frac{\delta X}{c}=0\ .$$

Thus,

$$A=0 \qquad \text{(since } \delta,\ X,\ c\neq 0).$$

This means that equation (2.33)=0, that is

$$\frac{\partial R}{\partial X}\frac{dX}{d\delta}+\frac{\partial R}{\partial T}\frac{dT}{d\delta}=0\ . \tag{2.36}$$

This is interesting. The "cost effect" and "time effect" actually cancel each other out. Although an increase in discount rate causes a reduction in the optimal biomass level, X, it actually has no effect on the maximum present value of the net revenue, R.

Now substituting equations (2.30) and (2.36) into equation (2.29), we have

$$\begin{aligned}\frac{dR}{d\delta} &= \frac{\partial R}{\partial \delta}+\frac{\partial R}{\partial X}\frac{dX}{d\delta}+\frac{\partial R}{\partial T}\frac{dT}{d\delta}\\ &=\frac{\partial R}{\partial \delta}<0 \ .\end{aligned} \tag{2.37}$$

Therefore, a partial conclusion can be made now. When the initial biomass ($x(0)=x_0$) is less than the optimal biomass X, i.e., the fishery resource is overexploited at the initial time, then the maximum present value of net revenue decreases as the discount rate increases under the assumptions employed. The larger is the discount rate, the smaller is the maximum present value of net revenue.

When the discount rate increases, the optimal biomass is reduced and the unit profit margin (p-c/x) is reduced due to the increased unit harvesting cost (c/x). Consequently, the "cost effect" causes a reduction in the maximum present value. On the other hand, while the optimal biomass is reduced because of the increased discount rate, the time required for the biomass to climb from the initial level to the optimal level is reduced and the owner can start harvesting earlier than before. Therefore, the "time effect" causes an increase in the maximum present value. The owner reacts to an increase in the discount rate such that the loss due to the "cost effect" equals the gain due to the "time effect". The reduction in the maximum present value is not because of either the increased harvesting cost or the reduced biomass. It is simply because the higher discount rate has the owner value the net revenue derived from the resource less.

$\underline{x(0) = x_0 > X}$

When $x(0)=x_0>X$, the fishery resource is underexploited at the initial time. In this case, the maximum present value of the net revenue is

$$
\begin{aligned}
R &= \int_0^{\infty} e^{-\delta t}[px(t)-c]E*(t)dt \\
&= \int_0^{T(X)} e^{-\delta t}[px(t)-c]E_{max}dt + \int_{T(X)}^{\infty} e^{-\delta t}[pX-c]\frac{F(X)}{X}dt \\
&= \int_0^{T(X)} e^{-\delta t}[px(t)-c]E_{max}dt + \int_{T(X)}^{\infty} e^{-\delta t}[pX-c]E(X)dt \\
&= R_1 + R_2 \ ,
\end{aligned} \tag{2.38}
$$

where

$$
\begin{aligned}
R_1 &= \int_0^{T(X)} e^{-\delta t}[px(t)-c]E_{max}dt \\
R_2 &= \int_{T(X)}^{\infty} e^{-\delta t}[pX-c]E(X)dt \ .
\end{aligned} \tag{2.39}
$$

Taking the derivative of equation (2.38), we have

$$\frac{dR}{d\delta}=\frac{dR_1}{d\delta}+\frac{dR_2}{d\delta}$$
$$=\frac{\partial R_1}{\partial \delta}+\frac{\partial R_1}{\partial x}\frac{dx}{d\delta}+\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial \delta}+\frac{\partial R_2}{\partial X}\frac{dX}{d\delta}+\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}\ . \tag{2.40}$$

If we know the sign of equation (2.40), we then know the relationship of the maximum present value, R, and the discount rate, δ. Thus, we need to determine the sign of equation (2.40). Once again we examine the terms on the right-hand side of equation (2.40) one at a time. Consider the first term. Taking the partial derivative of equation (2.39) we have

$$\frac{\partial R_1}{\partial \delta}=\int_0^{T(X)} -te^{-\delta t}[px(t)-c]E_{max}dt$$
$$=-\int_0^{T(X)} te^{-\delta t}[px(t)-c]E_{max}dt\ .$$

Since $te^{-\delta t}[px(t)-c]E_{max}>0$ when $0<t<T$, we then have

$$\frac{\partial R_1}{\partial \delta}=-\int_0^{T(X)} te^{-\delta t}[px(t)-c]E_{max}dt<0\ . \tag{2.41}$$

This is the "pure discount rate effect" which, as before, is negative.

Turn next to the second term in equation (2.40). Since the biomass x is independent of δ, i.e., $dx/d\delta=0$, we have

$$\frac{\partial R_1}{\partial x}\frac{dx}{d\delta}=0. \tag{2.42}$$

Now consider the third and last terms in equation (2.40). When the initial biomass $x_0>X$, we then have

$$\frac{dx}{dt}=F(x)-h(t)$$

$$=F(x)-xE_{max}<0$$

and

$$x(T(X))=X$$

$$\frac{dx}{dT}\frac{dT}{d\delta}=\frac{dX}{d\delta} .$$

Therefore,

$$\frac{dT}{d\delta}=\frac{\frac{dX}{d\delta}}{\frac{dx}{dt}}=\frac{\frac{dX}{d\delta}}{F(X)-XE_{max}}=\frac{1}{X[\frac{F(X)}{X}-E_{max}]}\frac{dX}{d\delta}$$

$$=\frac{1}{X[E(X)-E_{max}]}\frac{dX}{d\delta} .$$

Since $dX/d\delta<0$ and $[E(X)-E_{max}]<0$, we then have

$$\frac{dT}{d\delta}=\frac{1}{X[E(X)-E_{max}]}\frac{dX}{d\delta}>0 . \tag{2.43}$$

When the discount rate δ increases, the optimal biomass level X decreases, and in turn, the time T increases. Figure 2 shows the relationship between the time T and the discount rate. Suppose the discount rate is δ_1 and the optimal biomass level is X_1 and the time required for the initial biomass to reach the optimal biomass level is T_1''. When the discount rate increases to δ_2, the optimal biomass level drops to X_2, and the time required for the initial biomass to reach the optimal biomass level increases to T_2''.

From equation (2.39) we have

$$\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}=e^{-\delta T}(pX-c)E_{\max}\frac{dT}{d\delta}>0\ . \tag{2.44}$$

So, the "time effect" on R_1 is positive. Again, from equation (2.39)

$$R_2=\int_{T(X)}^{\infty} e^{-\delta t}(pX-c)E(X)dt$$

$$=\frac{e^{-\delta T}}{\delta}(pX-c)E(X)\ .$$

So,

$$\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}=-e^{-\delta T}(pX-c)E(X)\frac{dT}{d\delta}<0\ . \tag{2.45}$$

Equation (2.45) means that the "time effect" on R_2 is negative.

From equations (2.44) and (2.45) we have

$$\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}$$

$$= e^{-\delta T}(pX-c)E_{\max}\frac{dT}{d\delta}-e^{-\delta T}(pX-c)E(X)\frac{dT}{d\delta}$$

$$=e^{-\delta T}(pX-c)[E_{\max}-E(X)]\frac{dT}{d\delta}\ .$$

Substituting equation (2.43) into it, we have

$$\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}$$

$$=e^{-\delta T}(pX-c)[E_{max}-E(X)]\frac{1}{X[E(X)-E_{max}]}\frac{dX}{d\delta} \tag{2.46}$$

$$=-e^{-\delta T}(pX-c)\frac{1}{X}\frac{dX}{d\delta}>0 \qquad (since\ \frac{dX}{d\delta}<0)\ .$$

Equation (2.46) is the "time effect" on the total present value of return R. It is positive.

The fourth term in equation (2.40) is next. Since R_2 is the same as R in equation (2.28), from equation (2.30) we have

$$\begin{aligned}\frac{\partial R_2}{\partial \delta} &= -\frac{re^{-\delta T}}{\delta^2}(pX-c)(1-\frac{X}{K})+\frac{re^{-\delta T}}{\delta}(-T)(pX-c)(1-\frac{X}{K})\\ &=-(\frac{1}{\delta}+T)\frac{re^{-\delta T}}{\delta}(pX-c)(1-\frac{X}{K})<0\ .\end{aligned} \tag{2.47}$$

This is the "pure discount rate effect" on R_2 and it is negative.

Finally, we turn to the fifth term in equation (2.40). From equation (2.31) we have

$$\frac{\partial R_2}{\partial X}\frac{dX}{d\delta}=\frac{re^{-\delta T}}{\delta}(p-\frac{2p}{K}X+\frac{c}{K})\frac{dX}{d\delta}<0\ . \tag{2.48}$$

The "cost effect" on R_2 is negative. Therefore, from equations (2.46) and (2.48) we have

$$\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial X}\frac{dX}{d\delta}$$

$$=-e^{-\delta T}(pX-c)\frac{1}{X}\frac{dX}{d\delta}+\frac{e^{-\delta T}}{\delta}(pr-\frac{2pr}{K}X+\frac{cr}{K})\frac{dX}{d\delta}$$

$$=e^{-\delta T}\frac{dX}{d\delta}\{\frac{1}{\delta}(pr-\frac{2pr}{K}X+\frac{cr}{K})-(pX-c)\frac{1}{X}\} \tag{2.49}$$

$$=e^{-\delta T}\frac{dX}{d\delta}*B \ ,$$

where

$$B=\frac{1}{\delta}(pr-\frac{2pr}{K}X+\frac{cr}{K})-(pX-c)\frac{1}{X} \ .$$

We then have

$$B*\delta X=X(pr-\frac{2pr}{K}X+\frac{cr}{K})-\delta(pX-c)$$

$$=(prX-\frac{2pr}{K}X^2+\frac{cr}{K}X-\delta pX+c\delta)$$

and further we have

$$-B*\frac{\delta X}{c}=\frac{2pr}{cK}X^2-(\frac{\delta p}{c}-\frac{pr}{c}-\frac{r}{K})X-\delta$$

$$=0 \qquad \textit{(from equation (2.15))} \ .$$

Since δ, $X\neq 0$, we then must have

B=0.

Therefore, from equation (2.49) we have

$$\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial X}\frac{dX}{d\delta}=0 \ . \tag{2.50}$$

This is very interesting too. The "time effect" and "cost effect" actually cancel each other out. Though an increase in the discount rate causes a reduction in the optimal biomass level X, it actually has no effect on the maximum present value of the net revenue, R.

Now substituting equations (2.41), (2.42), (2.47) and (2.50) into equation (2.40), we have

$$\frac{dR}{d\delta}=\frac{dR_1}{d\delta}+\frac{dR_2}{d\delta}$$

$$=\frac{\partial R_1}{\partial \delta}+\frac{\partial R_1}{\partial x}\frac{dx}{d\delta}+\frac{\partial R_1}{\partial T}\frac{dT}{d\delta}+\frac{\partial R_2}{\partial \delta}+\frac{\partial R_2}{\partial X}\frac{dX}{d\delta}+\frac{\partial R_2}{\partial T}\frac{dT}{d\delta}$$

$$=\frac{\partial R_1}{\partial \delta}+\frac{\partial R_2}{\partial \delta}<0 \ .$$

In sum, when the initial biomass ($x(0)=x_0$) is greater than the optimal biomass X, i.e., when the fishery resource is underexploited at the initial time, then the maximum present value of net revenue decreases as the discount rate increases under the assumptions employed. The larger is the discount rate, the smaller is the maximum present value of net revenue.

When the discount rate increases, the optimal biomass is reduced and the unit profit margin (p-c/x) falls due to the increased unit harvesting cost (c/x). Consequently, the "cost effect" causes a reduction in the maximum present value. On the other hand, while the optimal biomass is reduced because of the increased discount rate, the time required for the biomass to reach the optimal level from the initial level has increased and the owner can start harvesting longer at the maximum effort E_{max} than before. Therefore, the "time effect" causes an increase in the maximum present value. The owner reacts to an increase in the discount rate in a way that has the loss due to the "cost effect" equal the gain due to the "time effect". The reduction in the maximum present value is not because of, as we might think, either the increased harvesting cost or the reduced biomass. It is simply because the higher discount rate has the owner value the total net revenue derived from the resources less.

2.4.4 Conclusions for This Section

(1) I have demonstrated that the sole owner's maximum present value of the net revenue has a negative relationship with his discount rate given other things constant, no matter what the initial biomass level is, under the assumptions employed.

(2) When the initial biomass level x_0 is actually at the optimal biomass level X, both the "pure discount rate effect" and the "cost effect" cause the maximum present value R to decline as the discount rate δ increases.

(3) When the initial biomass level x_0 is not the same as the optimal biomass level X, that is, either greater or less than the optimal biomass level X, only the "pure discount rate effect" causes the maximum present value R to decline as the discount rate δ increases.

(4) The "time effect" is canceled out by the "cost effect" in the case of the initial biomass level x_0 not equal to the optimal biomass level X. Since time T is a function

of X and x_0, i.e., $T=f(X,x_0)$, the initial biomass level x_0 and optimal biomass level X have no effect on the present value of return R when we analyze the relationship between the present value of return R and the discount rate δ.

2.5 Implications for Fishery Management and Cooperative Fishing Arrangements

In the above section, I have demonstrated that the maximum present value of net revenue derived from the resource is negatively related to the discount rate under the sole owner assumption. This finding can be applied to address some issues in the management of fishery resources.

The maximum present value actually represents the potential value of the fishery resource. The larger the maximum present value, the larger the value of the fishery resource. Since the maximum present value is negatively related to the discount rate, the value of the fishery resource is also negatively related to the discount rate. Suppose that in a cooperative fishing arrangement, the coastal nation grants the access rights of the fishery resource completely to a distant-water fleet for an indefinite period, in exchange for a lump sum payment. In that case, we expect that the distant-water fleet would manage and exploit the resource in a way that is consistent with the interests of a sole owner. The value of the fishery resources to the distant-water fleet is the maximum present value. Therefore, when both have the same discount rate, how much both the coastal nation and the distant-water fleet value the resource depends on the maximum present value, which, in turn, depends on the discount rate.

Suppose that there is a market for access. One coastal nation comes to the market and auctions the access rights of its fishery resources to many bidders, the distant-water fleets. The highest price that each bidder could bid is the value of the fishery resource, i.e., the maximum present value, to him. The final winner would be the one who has the lowest discount rate among the bidders, with other things equal. In the process of competing for the access rights, the coastal nation obtains the best offer, and the most competitive (i.e., with the lowest discount rate) distant-water fleet gets the access rights. Similarly, suppose a distant-water fleet wants to buy the access rights, and many coastal nations come to offer such rights. The final winner would be the coastal nation that has the highest discount rate among the coastal nations, with other things equal. Thus, the discount rate will affect coastal nations' and the distant-water fleets' competitive abilities in cooperative fishing arrangements. The lower the discount rate, the higher the competitive ability of a distant-water fleet; the higher the discount rate, the higher the competitive ability of a coastal nation.

Through competition for access rights, many problems (for example asymmetric information, monitoring, cheating, etc.) associated with cooperative fishing arrangements can be avoided. The winner of the distant-water fleets gives its best offer, and will behave in a fashion which is optimal from the coastal nation's perspective. At the same time, the coastal nation doesn't need to monitor or police the distant fleet.

In conclusion, competition for access rights avoids many difficulties associated with cooperative fishing arrangements. The discount rates will affect coastal nations' and distant-water fleets' competitive abilities in the market for access rights. Further study of the discount rate and the factors which influence the discount rate will provide us with rich insights into the management of fishery resources and the market for seafood.

2.6 Summary and Conclusions

In this chapter, I have used an optimal control model to explore a sole owner's exploiting behavior and to attempt to provide some insights into the management of fishery resources and cooperative fishing arrangements. When a sole owner of the fishery resource wants to maximize the present value of the net revenue derived from the resource through time, the maximum present value of the net revenue is negatively related to the owner's discount rate.

When using this finding in a competitive market for access rights, we may uncover some insights into the management of fishery resources and cooperative fishing arrangements. For example, given other things equal, the discount rate will affect both the coastal nations' and distant-water nations' (or fleets') competitive abilities in the market for access rights.

2.6.1 The Limits

The analysis is based on the Schaefer model with its assumptions of constant price and infinite time period. Under those assumptions I am basically dealing with a linear control problem, which simplifies the analysis. The Schaefer model and the assumptions are very restrictive, however. In the real world, things are much more complicated than are assumed in this chapter. For example, the price is unlikely to be constant, but rather, likely to be determined by demand and supply. Thus, price may be influenced by events in any particular fishery. In addition, the total harvesting cost is not likely to be linear in harvesting rate, etc. Therefore, the analysis in this chapter has certain limits. When we apply this analysis to the management of particular fishery resources and cooperative fishing arrangements, we should not overlook these limits.

When we relax those assumptions, the problem that we deal with may be a nonlinear control problem. Therefore, to explore an explicit relationship between the maximum present value and discount rate would be very complicated, if it is not impossible. I discuss this briefly in the next section.

2.6.2 Some Suggestions for Future Research

Our knowledge of fishery resource management and the market for seafood is relatively limited at present. Many unknown things need to be explored in the future. However, I would suggest some future research which is closely related to the research in this chapter.

(1) The finding in this chapter needs to be generalized. In the analysis of this chapter, I use a specific fishery model, i.e., Schaefer model. Though the Schaefer model is very popular among fishery economists, it has been criticized by some economists. Other fishery models are also used among fishery economists (see, for example, Pella and Tomlinson 1969; Shepherd 1982). Thus, we first need to study the issue of the maximum present value and the discount rate by using other fishery models. Further,

we need to study this issue by using a more general fishery model, and find out under what conditions the finding will hold.

(2) If the finding in this chapter can be applied to other fishery models, it needs to be tested by using empirical data. The finding in this chapter suggests that in bidding for access rights, the higher is the bidder's discount rate, the lower the bidding price is, with other things equal among the bidders. If we can collect data on the offers, "fees" or prices of cooperative fishing arrangements, we should be able to test the hypothesis.

(3) It is appropriate to apply the finding in this chapter to other research on fishery management and the market for seafood. I have concluded that the level of the discount rate will affect coastal nations' and distant-water fleets' competitive abilities in the market for access rights. How will a change in those factors that influence the discount rate eventually affect the fishery management and the market for seafood? For example, how will macroeconomic policies influence social discount rates, and in turn influence fishery management and the market for seafood?

(4) Finally, the hypothesis that the discount rates may be different among coastal nations and among distant-water fleets needs to be studied both theoretically and empirically. Are the discount rates really different? Why and how are they different? This kind of knowledge will be important and useful to further research on the management of fishery resources and the market for seafood.

CHAPTER 3
A COMPARATIVE STATICS ANALYSIS OF THE PRICING STRATEGY OF A MULTIPRODUCT MONOPOLIST

3.1 Introduction

Some interesting phenomena in the seafood industry have been observed in recent years:

(1) Researchers have found that some price elasticities of demand for seafood are positive. For example, Kim et al. (1988) reported positive price elasticities of demand for some seafood in Japan; Wellman (1992) reported positive price elasticities of demand for some seafood in the U.S.; and Lambert (1991) reported positive price elasticities of demand for chicken and turkey in the U.S.

(2) In recent years, the prices of seafood to final consumers have increased and the fish prices to the fishermen have decreased. " Producer prices of seafood over the same period (1982 to 1992) rose to 154", Johnson and Dore (1993, p48) reported, "...Only at the ex-vessel level have seafood prices been held down. ...it appears that fishermen have not kept pace with inflation, while the distributors and retailers of seafood have managed to increase their margins."

(3) In the Northwest Pacific coast, sometimes the purchasers don't want to take larger quantities of fish from the fishermen even at lower prices. Actually, the purchasers want fishermen to limit their total catches to certain maximum amounts. Hanna (1990) did a survey and found that such limits do exist.

How to explain the phenomena in the seafood industry is a challenge to the economists and to economic theory. Some economists have attempted to explore those phenomena. For example, Johnston and Larson (1992) provide an explanation for the phenomenon of positive price elasticity of demand at the consumer level.

As a matter of fact, most seafood at the retail level in the U.S. is sold to final consumers through supermarkets (Trappen 1993). "Supermarkets dominated the retail food market", Senauer et al. (1991, p299) reported, compared to other food stores, "although the number of supermarkets is lowest, their sales are highest, reflecting consumer acceptance of the wide variety of products, service, and convenient one-stop shopping that they offer." The supermarkets are multiproduct sellers for which one of the following usually holds: (1) the costs of production of the several products are interrelated, (2) the demands are interrelated or (3) both costs and demands are interrelated. For profit maximization, the monopolist has to set optimal prices to maximize total profits instead of setting prices to maximize each individual product's profit. Corresponding to any changes in demands and costs, a multi-product monopolist's response will be much more complicated than a single-product monopolist. Through examining the multi-product monopolist's pricing behavior, we may better understand some real world business practices.

The purpose of this chapter, then, is to explore the pricing strategy for a multiproduct monopolist, especially a supermarket, and to examine the difference in the comparative statics as between the single and multiproduct monopolist. In addition, I hope that I can provide a conceptual explanation for some observed phenomena in the seafood industry.

3.2 A Review of the Literature on Multiproduct Pricing

If several products are sold, it will usually be found either that the costs of production of the several products are interrelated or that the demands are interrelated or that both costs and demands are interrelated. In the case of a profit maximization monopolist, the firm has to set optimal prices to maximize the total profits instead of setting the prices to maximize the individual product's profit. Tirole (1989, p70) shows that the prices maximizing the individual product's profit may be either too high or too low from the point of view of maximizing total profits.

The literature on marketing has found that in supermarkets the cross-price elasticities of demand among food items are non-zero and the demands are interrelated. The supermarkets' managers do consider the demand interdependency when they set their pricing strategies (see, for example, Little and Shapiro 1980; Reibstein and Gatignon 1984; Tellis 1986; Mulhern and Leone 1991). However, from the point of view of economics, little research has been done to date on the issues related to the multiproduct monopolists. Indeed, "The multi-product problem has been largely ignored" (Spence 1980, p821).

Because of either the interrelated demands or interrelated costs or both, a change in tax, cost, or demand will cause all the prices and quantities to change. However, this change is much more complicated than that in the single-product case. For example, in the single-product monopolist case, when a tax per unit of output is imposed on the product, the price always goes up and the quantity goes down (see, for example Henderson and Quandt 1980, p187). By contrast, in the multiproduct monopolist case, when a tax per unit of output is imposed on one product, the changes in prices and quantities may be very different under different conditions.

Edgeworth (1925) did a comparative static analysis of a multiproduct monopolist and showed that a tax on one of two substitute products sold by a monopolist can result in the reduction of both prices. This result is known as Edgeworth's paradox of taxation. Hotelling (1932) argued that the paradox might occur in practice. He produced a quite plausible discrete example concerning different classes of railway service in which the paradox arises. Coase (1946) and Bailey (1954) provide excellent graphical presentations of multiproduct pricing and why the paradox can arise.

However, since this early work the comparative static analysis of multiproduct monopolists is "a point that seems to have been largely forgotten" (Salinger 1991, p545). There exists a large gap between the price theory of multiproduct monopolists and the pricing practices in real business.

Nevertheless, some research has been done on the comparative static analysis of multiproduct pricing through different oligopoly models (see, for example, Bulow et al. 1985; Dixit 1986; Anderson and Fischer 1989; Gaudet and Salant 1991). They explore the pricing behavior of either multiproduct or multimarket sellers through the comparative statics analysis that considers the rivals' responses. Although this literature provides rich insights into oligopoly pricing, it provided few insights into the pricing behavior of multiproduct monopolists.

3.3 A Comparative Statics Analysis of a Multiproduct Monopolist in Conventional Models

In this section I explore how a multiproduct monopolist prices its products and how it responses to changes in parameters. I first consider the case of a single-product monopolist and then the case of a multiproduct monopolist in conventional models. By conventional models, I mean that the demand functions are continuous, differentiable and strictly quasi-convex.

3.3.1 A Single-Product Monopolist

Assume that a monopolist produces one product with constant marginal cost, c. The demand for the product facing the monopolist is

$$x=x(p),$$

where p is the price and x is the quantity demanded at price p. I further assume that

$$\frac{dx}{dp}<0 \quad and \quad \frac{d^2x}{dp^2}\leq 0.$$

That is, the demand function is strictly quasi-convex.

The monopolist's profit function is

$$\pi=(p-c)x.$$

To maximize the profit, the first order condition is

$$\frac{d\pi}{dp}=(p-c)\frac{dx}{dp}+x=0 \tag{3.1}$$

The second order condition is

$$\frac{d^2\pi}{dp^2}=(p-c)\frac{d^2x}{dp^2}+\frac{dx}{dp}+\frac{dx}{dp}$$
$$=2\frac{dx}{dp}+(p-c)\frac{d^2x}{dp^2}<0 \tag{3.2}$$

Taking the total differential of equation (3.1), we have

$$\frac{d^2\pi}{dp^2}dp+\frac{d^2\pi}{dpdc}dc=0 \tag{3.3}$$

From equation (3.1) we have

$$\frac{d^2\pi}{dpdc}=-\frac{dq}{dp}.$$

Substituting this into equation (3.3), we have

$$\frac{dp*}{dc}=-\frac{\frac{d^2\pi}{dpdc}}{\frac{d^2\pi}{dp^2}}=\frac{\frac{dq}{dp}}{\frac{d^2\pi}{dp^2}}>0 \tag{3.4}$$

From equation (3.1) we know that for a given demand function, the optimal price p* is a function of the constant marginal cost, c, i.e.,

p*=p*(c).

Substituting p* into the demand function, then

x*=x*(p*(c)).

Taking the derivative with respect to c, we have

$$\frac{dx*}{dc}=\frac{dx*}{dp*}\frac{dp*}{dc}<0, \tag{3.5}$$

since dx*/dp*<0 and dp*/dc>0 from equation (3.4). Therefore, the price will decrease and the quantity will increase as the marginal cost decreases in this model.

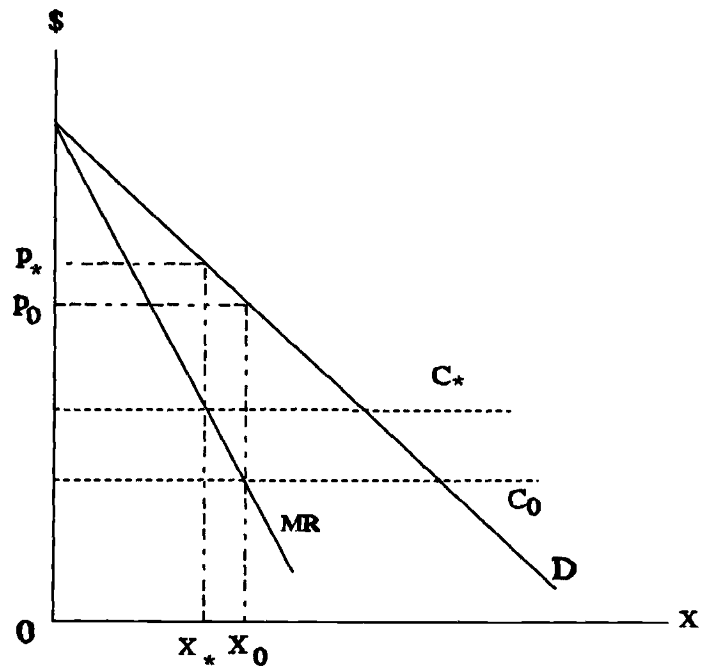

Figure 3.1. A comparative statics analysis of a single-product monopolist.

Suppose the monopolist is a fish-seller who buys fresh fish from the fishermen and sells it to the final consumers. The marginal cost for the seller can be considered as the price which he pays to the fishermen if we ignore other costs. When the input price decreases, the seller will reduce its product's price and sell more to the consumers. In turn, the seller will buy more raw fresh fish from the fishermen when the fishermen's price goes down. This conclusion can be well demonstrated by Figure 3.1. When marginal cost changes from c_* to c_0, the price decreases from p_* to p_0 and the quantity demanded increases from x_* to x_0.

3.3.2 A Multiproduct Monopolist

When a monopolist sells more than one product, the pricing strategy is much more complicated than for a single-product monopolist. Consequently, the comparative static analysis is also much more complicated than that for the one-product monopolist. In the following, I first explore a comparative static analysis of the prices for a multiproduct monopolist and then explore a comparative static analysis of the outputs.

3.3.2.1 A Comparative Static Analysis of the Prices

Suppose a monopolist produces two products, x and y, at constant marginal costs, c_x and c_y, respectively. The demands for x and y are

$$x=x(p_x, p_y) \quad \text{and} \quad y=y(p_x, p_y)$$

respectively, where p_x is the price for product x and p_y is the price for product y. We further assume that

$$\frac{\partial x}{\partial p_x}<0, \quad \frac{\partial y}{\partial p_y}<0, \quad \frac{\partial x}{\partial p_y}>0 \quad and \quad \frac{\partial y}{\partial p_x}>0 \ .$$

Thus, the demands for product x and product y are interrelated and the products are substitutes. The total profit for the monopolist is

$$\pi=(p_x-c_x)x+(p_y-c_y)y.$$

To maximize profit, the first order conditions are

$$\frac{\partial\pi}{\partial p_x}=x+(p_x-c_x)\frac{\partial x}{\partial p_x}+(p_y-c_y)\frac{\partial y}{\partial p_x}=0 \tag{3.6}$$

and

$$\frac{\partial\pi}{\partial p_y}=y+(p_x-c_x)\frac{\partial x}{\partial p_y}+(p_y-c_y)\frac{\partial y}{\partial p_y}=0 \tag{3.7}$$

The second order conditions are

$$\frac{\partial^2\pi}{\partial p_x^2}<0, \quad and \quad \frac{\partial^2\pi}{\partial p_x^2}\cdot\frac{\partial^2\pi}{\partial p_y^2}-\left(\frac{\partial^2\pi}{\partial p_x\partial p_y}\right)^2>0, \tag{3.8}$$

or the determinant

$$D=\begin{vmatrix} \dfrac{\partial^2\pi}{\partial p_x^2} & \dfrac{\partial^2\pi}{\partial p_x\partial p_y} \\ \dfrac{\partial^2\pi}{\partial p_x\partial p_y} & \dfrac{\partial^2\pi}{\partial p_y^2} \end{vmatrix} \tag{3.9}$$

is negative definite.

Taking the total differential of equations (3.6) and (3.7), we have

$$\frac{\partial^2\pi}{\partial p_x^2}dp_x+\frac{\partial^2\pi}{\partial p_x\partial p_y}dp_y+\frac{\partial^2\pi}{\partial p_x\partial c_x}dc_x+\frac{\partial^2\pi}{\partial p_x\partial c_y}dc_y=0 \tag{3.10}$$

and

$$\frac{\partial^2\pi}{\partial p_y\partial p_x}dp_x+\frac{\partial^2\pi}{\partial p_y^2}dp_y+\frac{\partial^2\pi}{\partial p_y\partial c_x}dc_x+\frac{\partial^2\pi}{\partial p_y\partial c_y}dc_y=0 \tag{3.11}$$

Rearranging equations (3.10) and (3.11), we then have

$$\frac{\partial^2\pi}{\partial p_x^2}dp_x+\frac{\partial^2\pi}{\partial p_x\partial p_y}dp_y=-\frac{\partial^2\pi}{\partial p_x\partial c_x}dc_x-\frac{\partial^2\pi}{\partial p_x\partial c_y}dc_y \tag{3.12}$$

and

$$\frac{\partial^2\pi}{\partial p_y\partial p_x}dp_x+\frac{\partial^2\pi}{\partial p_y^2}dp_y=-\frac{\partial^2\pi}{\partial p_y\partial c_x}dc_x-\frac{\partial^2\pi}{\partial p_y\partial c_y}dc_y \tag{3.13}$$

Equations (3.12) and (3.13) can be expressed as

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial p_x^2} & \frac{\partial^2\pi}{\partial p_x\partial p_y} \\ \frac{\partial^2\pi}{\partial p_x\partial p_y} & \frac{\partial^2\pi}{\partial p_y^2} \end{pmatrix}\begin{pmatrix} dp_x \\ dp_y \end{pmatrix}=\begin{pmatrix} -\frac{\partial^2\pi}{\partial p_x\partial c_x}dc_x-\frac{\partial^2\pi}{\partial p_x\partial c_y}dc_y \\ -\frac{\partial^2\pi}{\partial p_y\partial c_x}dc_x-\frac{\partial^2\pi}{\partial p_y\partial c_y}dc_y \end{pmatrix} \tag{3.14}$$

From equations (3.6) and (3.7) we have

$$\frac{\partial^2\pi}{\partial p_x\partial c_x}=-\frac{\partial x}{\partial p_x} \quad and \quad \frac{\partial^2\pi}{\partial p_y\partial c_x}=-\frac{\partial x}{\partial p_y}\ .$$

Substituting them into equation (3.14) and letting dc_y=o, we have

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial p_x^2} & \frac{\partial^2\pi}{\partial p_x\partial p_y} \\ \frac{\partial^2\pi}{\partial p_x\partial p_y} & \frac{\partial^2\pi}{\partial p_y^2} \end{pmatrix}\begin{pmatrix} dp_x \\ dp_y \end{pmatrix}=\begin{pmatrix} \frac{\partial x}{\partial p_x}dc_x \\ \frac{\partial x}{\partial p_y}dc_x \end{pmatrix}$$

According to Cramer's rule, we have

$$\frac{dp_x}{dc_x}=\frac{1}{D}\begin{vmatrix} \frac{\partial x}{\partial p_x} & \frac{\partial^2\pi}{\partial p_x \partial p_y} \\ \frac{\partial x}{\partial p_y} & \frac{\partial^2\pi}{\partial p_y^2} \end{vmatrix}=\frac{1}{D}\{\frac{\partial x}{\partial p_x}\cdot\frac{\partial^2\pi}{\partial p_y^2}-\frac{\partial x}{\partial p_y}\cdot\frac{\partial^2\pi}{\partial p_x \partial p_y}\} \tag{3.15}$$

and

$$\frac{dp_y}{dc_x}=\frac{1}{D}\begin{vmatrix} \frac{\partial^2\pi}{\partial p_x^2} & \frac{\partial x}{\partial p_x} \\ \frac{\partial^2\pi}{\partial p_x \partial p_y} & \frac{\partial x}{\partial p_y} \end{vmatrix}=\frac{1}{D}\{\frac{\partial^2\pi}{\partial p_x^2}\cdot\frac{\partial x}{\partial p_y}-\frac{\partial x}{\partial p_x}\cdot\frac{\partial^2\pi}{\partial p_x \partial p_y}\} \tag{3.16}$$

where

$$D=\begin{vmatrix} \frac{\partial^2\pi}{\partial p_x^2} & \frac{\partial^2\pi}{\partial p_x \partial p_y} \\ \frac{\partial^2\pi}{\partial p_x \partial p_y} & \frac{\partial^2\pi}{\partial p_y^2} \end{vmatrix}>0$$

The signs of equations (3.15) and (3.16) cannot be determined according to the second order conditions and the assumptions made on the demand functions.

From equations (3.15) and (3.16), we can have following conclusions:

(1) *if* $\frac{\partial^2\pi}{\partial p_x \partial p_y}<0$, *then* $\frac{dp_x}{dc_x}>0$ *and* $\frac{dp_y}{dc_x}<0$; (3.17)

$$(2) \quad if \quad \frac{\partial^2\pi}{\partial p_x \partial p_y}>0 \quad and \quad \{\frac{\partial x}{\partial p_x}\cdot\frac{\partial^2\pi}{\partial p_y^2}-\frac{\partial x}{\partial p_y}\cdot\frac{\partial^2\pi}{\partial p_x \partial p_y}\}>0,$$

$$i.e., \quad 0<\frac{\partial^2\pi}{\partial p_x \partial p_y}<\frac{\frac{\partial x}{\partial p_x}}{\frac{\partial x}{\partial p_y}}\cdot\frac{\partial^2\pi}{\partial p_y^2}, \quad then \quad \frac{dp_x}{dc_x}>0 \; ; \tag{3.18}$$

$$(3) \quad if \quad \frac{\partial^2\pi}{\partial p_x \partial p_y}>\frac{\frac{\partial x}{\partial p_x}\frac{\partial^2\pi}{\partial p_y^2}}{\frac{\partial x}{\partial p_y}}, \quad then \quad \frac{dp_x}{dc_x}<0 \; ; \tag{3.19}$$

$$(4) \quad if \quad \frac{\partial^2\pi}{\partial p_x \partial p_y}=\frac{\frac{\partial x}{\partial p_x}\frac{\partial^2\pi}{\partial p_y^2}}{\frac{\partial x}{\partial p_y}}, \quad then \quad \frac{dp_x}{dc_x}=0 \; ; \tag{3.20}$$

From equation (3.16),

$$(5) \quad if \quad \frac{\partial^2\pi}{\partial p_x \partial p_y}>0 \quad and \quad \{\frac{\partial^2\pi}{\partial p_x^2}\cdot\frac{\partial x}{\partial p_y}-\frac{\partial x}{\partial p_x}\cdot\frac{\partial^2\pi}{\partial p_x \partial p_y}\}>0,$$

$$i.e., \quad \frac{\partial^2\pi}{\partial p_x \partial p_y}>\frac{\frac{\partial^2\pi}{\partial p_x^2}\cdot\frac{\partial x}{\partial p_y}}{\frac{\partial x}{\partial p_x}}, \quad then \quad \frac{dp_y}{dc_x}>0 \; ; \tag{3.21}$$

$$\text{(6)} \quad \text{if} \quad \frac{\partial^2\pi}{\partial p_x \partial p_y} = \frac{\dfrac{\partial^2\pi}{\partial p_x^2} \cdot \dfrac{\partial x}{\partial p_y}}{\dfrac{\partial x}{\partial p_x}} \;, \quad \text{then} \quad \frac{dp_y}{dc_x} = 0 \;; \qquad (3.22)$$

For the sake of convenience, the signs of dp_x/dc_x and dp_y/dc_x and the corresponding conditions can be summarized as follows:

$$\frac{dp_x}{dc_x} \begin{cases} >0 & \text{if} \quad \dfrac{\partial^2\pi}{\partial p_x \partial p_y} < \dfrac{\dfrac{\partial x}{\partial p_x} \dfrac{\partial^2\pi}{\partial p_y^2}}{\dfrac{\partial x}{\partial p_y}} \\ =0 & \text{if} \quad \dfrac{\partial^2\pi}{\partial p_x \partial p_y} = \dfrac{\dfrac{\partial x}{\partial p_x} \dfrac{\partial^2\pi}{\partial p_y^2}}{\dfrac{\partial x}{\partial p_y}} \\ <0 & \text{if} \quad \dfrac{\partial^2\pi}{\partial p_x \partial p_y} > \dfrac{\dfrac{\partial x}{\partial p_x} \dfrac{\partial^2\pi}{\partial p_y^2}}{\dfrac{\partial x}{\partial p_y}} \end{cases} \qquad (3.23)$$

and

$$\frac{dp_y}{dc_x}\begin{cases} >0 & if \quad \dfrac{\partial^2\pi}{\partial p_x \partial p_y} > \dfrac{\dfrac{\partial x}{\partial p_y}\dfrac{\partial^2\pi}{\partial p_x^2}}{\dfrac{\partial x}{\partial p_x}} \\ =0 & if \quad \dfrac{\partial^2\pi}{\partial p_x \partial p_y} = \dfrac{\dfrac{\partial x}{\partial p_y}\dfrac{\partial^2\pi}{\partial p_x^2}}{\dfrac{\partial x}{\partial p_x}} \\ <0 & if \quad \dfrac{\partial^2\pi}{\partial p_x \partial p_y} < \dfrac{\dfrac{\partial x}{\partial p_y}\dfrac{\partial^2\pi}{\partial p_x^2}}{\dfrac{\partial x}{\partial p_x}} \end{cases} \tag{3.24}$$

From equations (3.23) and (3.24) we can see that when one product's marginal cost changes, the profit maximizing monopolist may increase, decrease or not change the corresponding product's price. The other product's price also could be increased, be decreased or be unchanged. The phenomenon here is much different from the single-product monopolist. Suppose the multiproduct monopolist is a seller of fish and meat (beef, pork, poultry). Because fish and meat are substitutes in consumption the demands for fish and meat are interrelated. When the fish's input price goes down, the monopolist may increase or reduce or not change the fish's price to its customers. On the other hand, the fish's price may change when the meat's marginal cost changes, though there is no change in the fish's cost or input price.

To understand the phenomena created by either equation (3.23) or equation (3.24), we need to consider the two equations together. Let

$$A = \frac{\dfrac{\partial x}{\partial p_x}\dfrac{\partial^2 \pi}{\partial p_y^2}}{\dfrac{\partial x}{\partial p_y}} \quad and \quad B = \frac{\dfrac{\partial x}{\partial p_y}\dfrac{\partial^2 \pi}{\partial p_x^2}}{\dfrac{\partial x}{\partial p_x}} \tag{3.25}$$

According to the assumptions on the demand functions, the first order conditions and the second order conditions, we have

$$A>0, \; B>0$$

and

$$A*B = \frac{\dfrac{\partial x}{\partial p_x}\dfrac{\partial^2 \pi}{\partial p_y^2}}{\dfrac{\partial x}{\partial p_y}} * \frac{\dfrac{\partial x}{\partial p_y}\dfrac{\partial^2 \pi}{\partial p_x^2}}{\dfrac{\partial x}{\partial p_x}}$$

$$= \frac{\partial^2 \pi}{\partial p_y^2}\frac{\partial^2 \pi}{\partial p_x^2} > \left(\frac{\partial^2 \pi}{\partial p_x \partial p_y}\right)^2$$

Combining (3.23) and (3.24), we have following conclusions:

$$(a) \quad if \quad \frac{\partial^2 \pi}{\partial p_x \partial p_y} < 0 \quad or \quad \frac{\partial^2 \pi}{\partial p_x \partial p_y} < \min(A, B), \quad then \quad \frac{dp_x}{dc_x} > 0 \quad and \quad \frac{dp_y}{dc_x} < 0. \tag{3.26}$$

$$(b) \quad if \quad A>B \quad and \quad B < \frac{\partial^2 \pi}{\partial p_x \partial p_y} < A, \quad then \quad \frac{dp_x}{dc_x} > 0 \quad and \quad \frac{dp_y}{dc_x} > 0. \tag{3.27}$$

$$(c) \quad if \quad \frac{\partial^2 \pi}{\partial p_x \partial p_y} = B < A, \quad then \quad \frac{dp_x}{dc_x} > 0 \quad and \quad \frac{dp_y}{dc_x} = 0. \tag{3.28}$$

$$(d) \quad if \quad A<B \quad and \quad A < \frac{\partial^2 \pi}{\partial p_x \partial p_y} < B, \quad then \quad \frac{dp_x}{dc_x} < 0 \quad and \quad \frac{dp_y}{dc_x} < 0. \tag{3.29}$$

$$(e) \quad if \quad \frac{\partial^2\pi}{\partial p_x \partial p_y}=A<B, \quad then \quad \frac{dp_x}{dc_x}=0 \quad and \quad \frac{dp_y}{dc_x}<0. \tag{3.30}$$

Thus, when $dp_x/dc_x>0$, dp_y/dc_x could be less than, greater than or equal to zero, depending on conditions. However, when $dp_x/dc_x\leq 0$, dp_y/dc_x must be negative. This may be explained as follows. For instance, when c_x decreases and p_x increases or does not change, the profit margin, (p_x-c_x), of product x increases. It is obvious that the monopolist would like to sell more of product x to maximize profits. However, if the price p_y is reduced, the customers may buy more of the product y and less of the product x. In order to induce customers to buy more of product x, the price for product y has to be increased when the price of product x increases.

On the other hand, when $dp_y/dc_x\geq 0$, dp_x/dc_x is unambiguously positive. This may be explained as follows. For instance, when c_x increases and p_y increases or does not change, the profit margin, (p_y-c_y), of product y increases or does not change, while the profit margin, (p_x-c_x), of product x decreases. It is obvious that the monopolist would like to sell less of product x and more of product y to maximize profits. However, if the price p_y is increased, customers may buy more of product x and less of product y. In order to prevent customers from doing that, the price for product x has to be increased when both c_x and p_y increase.

In conclusion (d), when the marginal cost c_x increases, both prices are reduced. This is a kind of Edgeworth Paradox (Hotelling 1932). Why does the monopolist reduce both prices, instead of increasing them, when one of the marginal costs increases?

To make the Edgeworth Paradox intuitively understandable, let us use a figure to illustrate the phenomenon in conclusion (d). Each of the first order conditions (3.6)

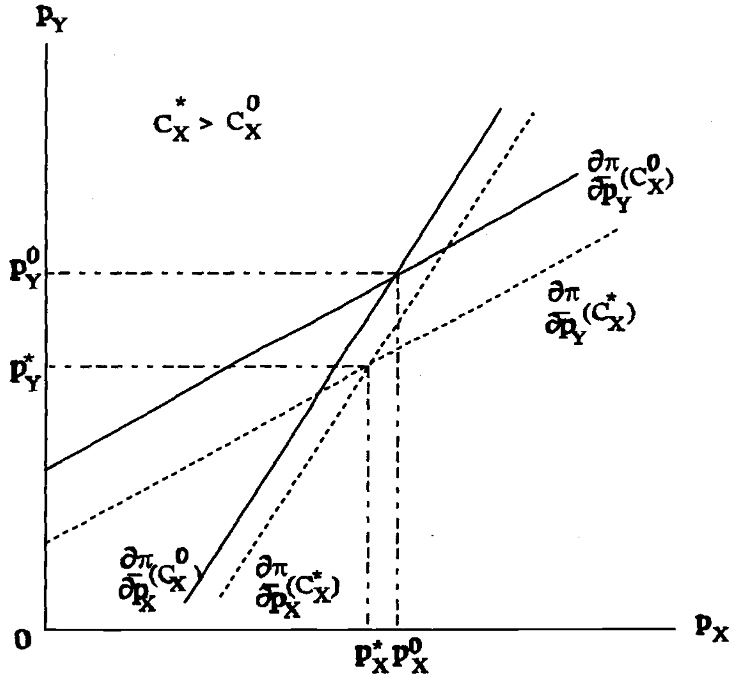

Figure 3.2. A comparative statics analysis of a multiproduct monopolist.

and (3.7) actually determines a unique relationship between p_x and p_y. This relationship can be expressed as a curve in the p_x-p_y plane as in Figure 3.2. Let $\partial\pi/\partial p_x$ represent the relationship obtained from equation (3.6) between p_x and p_y, and let $\partial\pi/\partial p_y$ represent the relationship obtained from equation (3.7) between p_x and p_y. Figure 3.2 shows a case in which an increase in the marginal cost c_x causes both prices to drop.[5]

First, let us see how the slopes of the curves in Figure 3.2 are determined and in what directions an increase in c_x will shift the curves. Taking the partial differential of equation (3.6) with respect to p_x and p_y, we have

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[5] The relationship between the two prices may be non-linear. For simplicity, I use linear curves here.

$$\frac{\partial^2\pi}{\partial p_x^2}\partial p_x+\frac{\partial^2\pi}{\partial p_x\partial p_y}\partial p_y=0 \tag{3.31}$$

From equation (3.31) we have

$$slope\ of\ \frac{\partial\pi}{\partial p_x}=\frac{\partial p_y}{\partial p_x}=-\frac{\frac{\partial^2\pi}{\partial p_x^2}}{\frac{\partial^2\pi}{\partial p_x\partial p_y}}\begin{cases}>0 & if\ \ \frac{\partial^2\pi}{\partial p_x\partial p_y}>0\\ <0 & if\ \ \frac{\partial^2\pi}{\partial p_x\partial p_y}<0\end{cases} \tag{3.32}$$

Taking the partial differential of equation (3.7) with respect to p_x and p_y, we have

$$\frac{\partial^2\pi}{\partial p_y\partial p_x}\partial p_x+\frac{\partial^2\pi}{\partial p_y^2}\partial p_y=0 \tag{3.33}$$

From equation (3.33) we have

$$slope\ of\ \frac{\partial\pi}{\partial p_y}=\frac{\partial p_y}{\partial p_x}=-\frac{\frac{\partial^2\pi}{\partial p_x\partial p_y}}{\frac{\partial^2\pi}{\partial p_y^2}}\begin{cases}>0 & if\ \ \frac{\partial^2\pi}{\partial p_x\partial p_y}>0\\ <0 & if\ \ \frac{\partial^2\pi}{\partial p_x\partial p_y}<0\end{cases} \tag{3.34}$$

Thus, when $\partial^2\pi/\partial p_x\partial p_y>0$, both curves of $\partial\pi/\partial p_x$ and $\partial\pi/\partial p_y$ are upward-sloping.

From equations (3.32) and (3.34) we have

$$\frac{slope\ of\ \frac{\partial\pi}{\partial p_x}}{slope\ of\ \frac{\partial\pi}{\partial p_y}}=\frac{-\frac{\partial^2\pi}{\partial p_x^2}/\frac{\partial^2\pi}{\partial p_x\partial p_y}}{-\frac{\partial^2\pi}{\partial p_x\partial p_y}/\frac{\partial^2\pi}{\partial p_y^2}}=\frac{\frac{\partial^2\pi}{\partial p_x^2}\frac{\partial^2\pi}{\partial p_y^2}}{(\frac{\partial^2\pi}{\partial p_x\partial p_y})^2}>1 \tag{3.35}$$

according to the second order condition. Thus the slope of $\partial\pi/\partial p_x$ is greater than that of $\partial\pi/\partial p_y$ when they are both positive (see Figure 3.2).

Taking the partial differential of equation (3.6) with respect to p_x and c_x, we have

$$\frac{\partial^2\pi}{\partial p_x^2}\partial p_x+\frac{\partial^2\pi}{\partial p_x\partial c_x}\partial c_x=0$$

From the above equation we then obtain

$$\frac{\partial p_x}{\partial c_x}=-\frac{\dfrac{\partial^2\pi}{\partial p_x\partial c_x}}{\dfrac{\partial^2\pi}{\partial p_x^2}}=\frac{\dfrac{\partial x}{\partial p_x}}{\dfrac{\partial^2\pi}{\partial p_x^2}}>0 \tag{3.36}$$

because

$$\frac{\partial x}{\partial p_x}<0 \quad and \quad \frac{\partial^2\pi}{\partial p_x^2}<0$$

Hence, an increase in c_x will cause the line of $\partial\pi/\partial p_x$ to shift to the right, given price p_y.

Taking the partial differential of equation (3.7) with respect to p_x and c_x, we have

$$\frac{\partial^2\pi}{\partial p_y^2}\partial p_y+\frac{\partial^2\pi}{\partial p_y\partial c_x}\partial c_x=0$$

From the above equation we then obtain

$$\frac{\partial p_y}{\partial c_x}=-\frac{\frac{\partial^2\pi}{\partial p_y \partial c_x}}{\frac{\partial^2\pi}{\partial p_y^2}}=\frac{\frac{\partial x}{\partial p_y}}{\frac{\partial^2\pi}{\partial p_y^2}}<0 \tag{3.37}$$

because

$$\frac{\partial x}{\partial p_y}>0 \quad and \quad \frac{\partial^2\pi}{\partial p_y^2}<0.$$

Thus, an increase in c_x will cause line $\partial\pi/\partial p_y$ to shift to the right, given price p_x.

We now look at Figure 3.2. The two curves have positive slopes and the slope of $\partial\pi/\partial p_x$ is greater than that of $\partial\pi/\partial p_y$. At the initial constant marginal cost c_x^0, the optimal prices are p_x^0 and p_y^0 respectively. When the marginal cost increases from c_x^0 to c_x^*, the curve $\partial\pi/\partial p_x$ shifts to the right. Thus, for any given price p_y, the price p_x increases. This is how a single-product monopolist will respond in order to maximize profits when the marginal cost increases. However, things have been changed in the case of a multiproduct monopolist. Because the demands are interrelated, the monopolist has to consider the impact of changing one price on the others. In the case of Figure 3.2, the curve $\partial\pi/\partial p_y$ also shifts to the right when the marginal cost increases from c_x^0 to c_x^*. The optimal prices are now p_x^* and p_y^* respectively at c_x^*. Both prices are reduced.

3.3.2.2 Generalizing to an N-Product Monopolist

We now generalize the analysis to the case of an n-product monopolist to see the monopolist's pricing behavior. Suppose a monopolist produces n products: $x_1, x_2, ..., x_n$.

The demand function facing the monopolist for product i is

$$x_i = x_i(p_1, p_2, \ldots, p_n), \; i=1, 2, \ldots, n.$$

It is also assumed that the marginal costs are constant: $c_1, c_2, \ldots, c_n$. The total profit for the monopolist is

$$\pi = \sum_{i=1}^{n} (p_i - c_i) x_i \tag{3.38}$$

The first order conditions for profit maximization are

$$\frac{\partial \pi}{\partial p_i} = x_i + \sum_{j=1}^{n} (p_j - c_j) \frac{\partial x_j}{\partial p_i} = 0 \tag{3.39}$$

where $i=1, 2, \ldots, n$.

The second order conditions are that

$$D = \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_1^2} & \frac{\partial^2 \pi}{\partial p_1 \partial p_2} & \cdots & \frac{\partial^2 \pi}{\partial p_1 \partial p_n} \\ \frac{\partial^2 \pi}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi}{\partial p_2^2} & \cdots & \frac{\partial^2 \pi}{\partial p_2 \partial p_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi}{\partial p_n \partial p_1} & \frac{\partial^2 \pi}{\partial p_n \partial p_2} & \cdots & \frac{\partial^2 \pi}{\partial p_n^2} \end{vmatrix} \tag{3.40}$$

is negative definite, i.e.,

$$\frac{\partial^2\pi}{\partial p_1^2}<0, \quad \begin{vmatrix} \frac{\partial^2\pi}{\partial p_1^2} & \frac{\partial^2\pi}{\partial p_1 \partial p_2} \\ \frac{\partial^2\pi}{\partial p_2 \partial p_1} & \frac{\partial^2\pi}{\partial p_2^2} \end{vmatrix}>0, \ \ldots,$$

$$(-1)^n \begin{vmatrix} \frac{\partial^2\pi}{\partial p_1^2} & \frac{\partial^2\pi}{\partial p_1 \partial p_2} & \cdots & \frac{\partial^2\pi}{\partial p_1 \partial p_n} \\ \frac{\partial^2\pi}{\partial p_2 \partial p_1} & \frac{\partial^2\pi}{\partial p_2^2} & \cdots & \frac{\partial^2\pi}{\partial p_2 \partial p_n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \frac{\partial^2\pi}{\partial p_n \partial p_1} & \frac{\partial^2\pi}{\partial p_n \partial p_2} & \cdots & \frac{\partial^2\pi}{\partial p_n^2} \end{vmatrix}>0$$

Taking the total differential of equation (3.39), we obtain

$$\sum_{j=1}^{n} \frac{\partial^2\pi}{\partial p_i \partial p_j} dp_j + \sum_{j=1}^{n} \frac{\partial^2\pi}{\partial p_i \partial c_j} dc_j = 0 \quad (i=1,2,\ldots,n). \tag{3.41}$$

From equation (3.39) we have

$$\frac{\partial^2\pi}{\partial p_i \partial c_j} = \frac{\partial x_j}{\partial p_i}, \qquad i,\ j=1,2,\ldots,n.$$

Thus, equation (3.41) can be rewritten as

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial p_1^2} & \frac{\partial^2\pi}{\partial p_1\partial p_2} & \cdot\cdot & \frac{\partial^2\pi}{\partial p_1\partial p_n} \\ \frac{\partial^2\pi}{\partial p_2\partial p_1} & \frac{\partial^2\pi}{\partial p_2^2} & \cdot\cdot & \frac{\partial^2\pi}{\partial p_2\partial p_n} \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \frac{\partial^2\pi}{\partial p_n\partial p_1} & \frac{\partial^2\pi}{\partial p_n\partial p_2} & \cdot\cdot & \frac{\partial^2\pi}{\partial p_n^2} \end{pmatrix} \begin{pmatrix} dp_1 \\ dp_2 \\ \cdot \\ \cdot \\ dp_n \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \frac{\partial x_j}{\partial p_1} dc_j \\ \sum_{j=1}^{n} \frac{\partial x_j}{\partial p_2} dc_j \\ \cdot \\ \cdot \\ \sum_{j=1}^{n} \frac{\partial x_j}{\partial p_n} dc_j \end{pmatrix} \tag{3.42}$$

Letting $dc_i=0$, $i=2, 3, ..., n$, from equation (3.42) we have

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial p_1^2} & \frac{\partial^2\pi}{\partial p_1\partial p_2} & \cdot\cdot & \frac{\partial^2\pi}{\partial p_1\partial p_n} \\ \frac{\partial^2\pi}{\partial p_2\partial p_1} & \frac{\partial^2\pi}{\partial p_2^2} & \cdot\cdot & \frac{\partial^2\pi}{\partial p_2\partial p_n} \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \frac{\partial^2\pi}{\partial p_n\partial p_1} & \frac{\partial^2\pi}{\partial p_n\partial p_2} & \cdot\cdot & \frac{\partial^2\pi}{\partial p_n^2} \end{pmatrix} \begin{pmatrix} \frac{dp_1}{dc_1} \\ \frac{dp_2}{dc_1} \\ \cdot \\ \cdot \\ \frac{dp_n}{dc_1} \end{pmatrix} = \begin{pmatrix} \frac{\partial x_1}{\partial p_1} \\ \frac{\partial x_1}{\partial p_2} \\ \cdot \\ \cdot \\ \frac{\partial x_1}{\partial p_n} \end{pmatrix} \tag{3.43}$$

Let

$$D_i=\begin{vmatrix} \frac{\partial^2\pi}{\partial p_1^2} & \frac{\partial^2\pi}{\partial p_1\partial p_2} & \cdots & \frac{\partial^2\pi}{\partial p_1\partial p_{i-1}} & \frac{\partial x_1}{\partial p_1} & \cdots & \frac{\partial^2\pi}{\partial p_1\partial p_n} \\ \frac{\partial^2\pi}{\partial p_2\partial p_1} & \frac{\partial^2\pi}{\partial p_2^2} & \cdots & \cdot & \frac{\partial x_1}{\partial p_2} & \cdots & \frac{\partial^2\pi}{\partial p_2\partial p_n} \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \frac{\partial^2\pi}{\partial p_n\partial p_1} & \frac{\partial^2\pi}{\partial p_n\partial p_2} & \cdots & \cdot & \frac{\partial x_1}{\partial p_n} & \cdots & \frac{\partial^2\pi}{\partial p_n^2} \end{vmatrix} \tag{3.44}$$

According to Cramer's rule, we have

$$\frac{dp_i}{dc_1}=\frac{D_i}{D}, \qquad i=1,2,\ldots,n. \tag{3.45}$$

The signs of $dp_i/dc_1=D_i/D$ are undetermined by the second order conditions. It is possible for dp_1/dc_1 to be negative or to be equal to zero. That is, when the marginal cost increases the price for that corresponding product may not change or may decrease.

3.3.2.3 A Comparative Static Analysis of the Quantities for a Multiproduct Monopolist

I have analyzed what could happen to the prices when the marginal cost changes. What may happen to the quantities when the marginal cost changes? We now take the quantities as independent variables and the inverse demand functions for product x and product y are expressed as

$$p_x=p_x(x, y) \quad \text{and} \quad p_y=p_y(x, y)$$

respectively, where p_x is the price for product x and p_y is the price for product y. We further assume that

$$\frac{\partial p_x}{\partial x}<0, \quad \frac{\partial p_y}{\partial y}<0, \quad \frac{\partial p_x}{\partial y}<0, \quad and \quad \frac{\partial p_y}{\partial x}<0.$$

Thus, the demands for product x and product y are interrelated and the products are substitutes. The total profit for the monopolist is

$$\pi=(p_x-c_x)x+(p_y-c_y)y.$$

To maximize the profit, the first order conditions are

$$\frac{\partial \pi}{\partial x}=p_x-c_x+x\frac{\partial p_x}{\partial x}+y\frac{\partial p_y}{\partial x}=0 \tag{3.46}$$

and

$$\frac{\partial \pi}{\partial y}=p_y-c_y+x\frac{\partial p_x}{\partial y}+y\frac{\partial p_y}{\partial y}=0 \tag{3.47}$$

The second order conditions for maximizing profit are

$$\frac{\partial^2 \pi}{\partial x^2}<0, \quad \frac{\partial^2 \pi}{\partial y^2}<0 \quad and \quad \frac{\partial^2 \pi}{\partial x^2}\frac{\partial^2 \pi}{\partial y^2}-(\frac{\partial^2 \pi}{\partial x \partial y})^2 > 0,$$

or the determinant

$$D=\begin{vmatrix} \frac{\partial^2 \pi}{\partial x^2} & \frac{\partial^2 \pi}{\partial x \partial y} \\ \frac{\partial^2 \pi}{\partial x \partial y} & \frac{\partial^2 \pi}{\partial y^2} \end{vmatrix} \tag{3.48}$$

is negative definite.

Taking the total differential of equations (3.46) and (3.47), we have

$$\frac{\partial^2\pi}{\partial x^2}dx+\frac{\partial^2\pi}{\partial x\partial y}dy+\frac{\partial^2\pi}{\partial x\partial c_x}dc_x+\frac{\partial^2\pi}{\partial x\partial c_y}dc_y=0 \tag{3.49}$$

and

$$\frac{\partial^2\pi}{\partial y\partial x}dx+\frac{\partial^2\pi}{\partial y^2}dy+\frac{\partial^2\pi}{\partial y\partial c_x}dc_x+\frac{\partial^2\pi}{\partial y\partial c_y}dc_y=0 \tag{3.50}$$

Rearranging equations (3.49) and (3.50), we then obtain

$$\frac{\partial^2\pi}{\partial x^2}dx+\frac{\partial^2\pi}{\partial x\partial y}dy=-\frac{\partial^2\pi}{\partial x\partial c_x}dc_x-\frac{\partial^2\pi}{\partial x\partial c_y}dc_y \tag{3.51}$$

and

$$\frac{\partial^2\pi}{\partial y\partial x}dx+\frac{\partial^2\pi}{\partial y^2}dy=-\frac{\partial^2\pi}{\partial y\partial c_x}dc_x-\frac{\partial^2\pi}{\partial y\partial c_y}dc_y \tag{3.52}$$

Equations (3.51) and (3.52) can be expressed as

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x^2} & \frac{\partial^2\pi}{\partial x\partial y} \\ \frac{\partial^2\pi}{\partial x\partial y} & \frac{\partial^2\pi}{\partial y^2} \end{pmatrix}\begin{pmatrix} dx \\ dy \end{pmatrix}=\begin{pmatrix} -\frac{\partial^2\pi}{\partial x\partial c_x}dc_x-\frac{\partial^2\pi}{\partial x\partial c_y}dc_y \\ -\frac{\partial^2\pi}{\partial y\partial c_x}dc_x-\frac{\partial^2\pi}{\partial y\partial c_y}dc_y \end{pmatrix} \tag{3.53}$$

From equations (3.46) and (3.47) we have

$$\frac{\partial^2\pi}{\partial x\partial c_x}=-1 \quad and \quad \frac{\partial^2\pi}{\partial y\partial c_x}=0\ . \tag{3.54}$$

Substituting equation (3.54) into equation (3.53) and letting dc_y=o, we have

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x^2} & \frac{\partial^2\pi}{\partial x\partial y} \\ \frac{\partial^2\pi}{\partial x\partial y} & \frac{\partial^2\pi}{\partial y^2} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} dc_x \\ 0 \end{pmatrix}$$

According to Cramer's rule, we have

$$\frac{dx}{dc_x} = \frac{1}{D} \begin{vmatrix} 1 & \frac{\partial^2\pi}{\partial x\partial y} \\ 0 & \frac{\partial^2\pi}{\partial y^2} \end{vmatrix} = \frac{1}{D} \cdot \frac{\partial^2\pi}{\partial y^2} \tag{3.55}$$

and

$$\frac{dy}{dc_x} = \frac{1}{D} \begin{vmatrix} \frac{\partial^2\pi}{\partial x^2} & 1 \\ \frac{\partial^2\pi}{\partial x\partial y} & 0 \end{vmatrix} = \frac{-1}{D} \frac{\partial^2\pi}{\partial x\partial y} \tag{3.56}$$

where

$$D = \begin{vmatrix} \frac{\partial^2\pi}{\partial x^2} & \frac{\partial^2\pi}{\partial x\partial y} \\ \frac{\partial^2\pi}{\partial x\partial y} & \frac{\partial^2\pi}{\partial y^2} \end{vmatrix} > 0$$

According to the second order conditions for profit maximization, we have from equation (3.55)

$$\frac{dx}{dc_x} < 0. \tag{3.57}$$

The sign of dy/dc_x is undetermined by the second order conditions. However, it can be stated as follows:

$$\frac{dy}{dc_x}\begin{cases} >0 & if \quad \frac{\partial^2\pi}{\partial x\partial y}<0 \\ =0 & if \quad \frac{\partial^2\pi}{\partial x\partial y}=0 \\ <0 & if \quad \frac{\partial^2\pi}{\partial x\partial y}>0 \end{cases} \tag{3.58}$$

Although we may have some conclusions according to equations (3.57) and (3.58), let us first extend our analysis to the case of an n-product monopolist.

3.3.3 Generalizing to an N-Product Monopolist

Suppose a monopolist produces n products: x_1, x_2, ..., x_n. The inverse demand function facing the monopolist for product i is

$$p_i=p_i(x_1, x_2, ..., x_n), \ i=1, 2, ..., n.$$

Assume that the marginal costs are constant: c_1, c_2, ..., c_n. Thus the total profit for the monopolist is

$$\pi=\sum_{i=1}^{n}(p_i-c_i)x_i$$

The first order conditions for profit maximization are

$$\frac{\partial\pi}{\partial x_i}=(p_i-c_i)+x_i\frac{\partial p_i}{\partial x_i}+\sum_{j\neq i}x_j\frac{\partial p_j}{\partial x_i}=0 \tag{3.59}$$

where i, j=1, 2, ..., n. The second order conditions are that

$$D=\begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & \cdots & \frac{\partial^2\pi}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdots & \frac{\partial^2\pi}{\partial x_2\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2\pi}{\partial x_n\partial x_1} & \frac{\partial^2\pi}{\partial x_n\partial x_2} & \cdots & \frac{\partial^2\pi}{\partial x_n^2} \end{vmatrix} \tag{3.60}$$

is negative definite, i.e.,

$$\frac{\partial^2\pi}{\partial x_1^2}<0, \quad \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} \end{vmatrix}>0, \ \ldots,$$

$$(-1)^n \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & \cdots & \frac{\partial^2\pi}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdots & \frac{\partial^2\pi}{\partial x_2\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2\pi}{\partial x_n\partial x_1} & \frac{\partial^2\pi}{\partial x_n\partial x_2} & \cdots & \frac{\partial^2\pi}{\partial x_n^2} \end{vmatrix}>0$$

Taking the total differential of equation (3.59), we have

$$\sum_{j=1}^{n} \frac{\partial^2\pi}{\partial x_i\partial x_j}dx_j + \frac{\partial^2\pi}{\partial x_i\partial c_i}dc_i=0 \quad (i=1,2,\ldots,n). \tag{3.61}$$

Since

$$\frac{\partial^2\pi}{\partial x_i \partial c_i} = -1 \ , \quad i=1,\ 2,\ ...,\ n,$$

equation (3.61) can be rewritten as

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1 \partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1 \partial x_n} \\ \frac{\partial^2\pi}{\partial x_2 \partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_2 \partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n \partial x_1} & \frac{\partial^2\pi}{\partial x_n \partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{pmatrix} = \begin{pmatrix} dc_1 \\ dc_2 \\ \cdot \\ \cdot \\ dc_n \end{pmatrix} \tag{3.62}$$

Letting $dc_i=0$, $i=2, 3, ..., n$, from equation (3.62) we have

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1 \partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1 \partial x_n} \\ \frac{\partial^2\pi}{\partial x_2 \partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_2 \partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n \partial x_1} & \frac{\partial^2\pi}{\partial x_n \partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{pmatrix} \begin{pmatrix} \frac{dx_1}{dc_1} \\ \frac{dx_2}{dc_1} \\ \cdot \\ \cdot \\ \frac{dx_n}{dc_1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \tag{3.63}$$

Let

$$D_i = \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1 \partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1 \partial x_{i-1}} & 1 & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1 \partial x_n} \\ \frac{\partial^2\pi}{\partial x_2 \partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\ \cdot & \cdot & 0 & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_2 \partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n \partial x_1} & \frac{\partial^2\pi}{\partial x_n \partial x_2} & \cdot\ \cdot & \cdot & 0 & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{vmatrix} \tag{3.64}$$

According to Cramer's rule, we have

$$\frac{dx_i}{dc_1} = \frac{D_i}{D}, \qquad i=1,2,\ldots,n. \tag{3.65}$$

The signs of $dx_i/dc_1=D_i/D$ are undetermined by the second order conditions except $dx_1/dc_1=D_1/D$. From the second order conditions, we know if $D>0$, then $D_1<0$. Therefore,

$$\frac{dx_1}{dc_1} = \frac{D_1}{D} < 0 .$$

So far, I have explored a comparative static analysis of prices and of quantities by using conventional models. When a product's marginal cost increases for a multiproduct monopolist, the price for the corresponding product may increase, decrease or stay the same. However, the sales of the corresponding product always go down as the marginal cost for that product increases. On the other hand, when the marginal cost decreases, the monopolist always increases the sales of the corresponding product. The sales of the other products may increase, decrease or stay the same.

3.4 Input Demand of a Multiproduct Monopolist in Conventional Models

A multiproduct monopolist's pricing behavior has been analyzed when demands are interrelated. How will a multiproduct monopolist's demands for inputs be influenced when the demands for the outputs are interrelated? For example, will the quantity demanded of an input increase when the price for that input decreases under the multiproduct case? I examine those issues next. I start with the case of one-product and then extend the analysis to the multiproduct case.

3.4.1 One Output with Two Inputs and Unconstrained Production Capacity

Assume a monopolist produces one product, y, with two inputs, x_1 and x_2. The production function is

$$y=y(x_1, x_2).$$

The prices for x_1 and x_2 are given as w_1 and w_2 respectively. The demand facing the monopolist for product y is

$$p=p(y),$$

where p is the price for product y. The profit function is

$$\pi=p(y)y(x_1, x_2)-w_1x_1-w_2x_2.$$

The first order conditions for profit maximization are

$$\frac{\partial \pi}{\partial x_1}=\frac{\partial p}{\partial y}\frac{\partial y}{\partial x_1}y(x_1, x_2)+p\frac{\partial y}{\partial x_1}-w_1=0 \tag{3.66}$$

and

$$\frac{\partial\pi}{\partial x_2}=\frac{\partial p}{\partial y}\frac{\partial y}{\partial x_2}y(x_1,\ x_2)+p\frac{\partial y}{\partial x_2}-w_2=0 \tag{3.67}$$

The second order conditions are

$$\frac{\partial^2\pi}{\partial x_1^2}<0,\quad \frac{\partial^2\pi}{\partial x_2^2}<0,\quad \textit{and}\quad \frac{\partial^2\pi}{\partial x_1^2}\frac{\partial^2\pi}{\partial x_2^2}-(\frac{\partial^2\pi}{\partial x_1\partial x_2})^2>0\ . \tag{3.68}$$

Taking the total differential of equations (3.66) and (3.67), we have

$$\frac{\partial^2\pi}{\partial x_1^2}dx_1+\frac{\partial^2\pi}{\partial x_1\partial x_2}dx_2+\frac{\partial^2\pi}{\partial x_1\partial w_1}dw_1=0 \tag{3.69}$$

and

$$\frac{\partial^2\pi}{\partial x_2\partial x_1}dx_1+\frac{\partial^2\pi}{\partial x_2^2}dx_2+\frac{\partial^2\pi}{\partial x_2\partial w_2}dw_2=0 \tag{3.70}$$

From equations (3.66) and (3.67) we have

$$\frac{\partial^2\pi}{\partial x_1\partial w_1}=\frac{\partial^2\pi}{\partial x_2\partial w_2}=-1\ .$$

Substitute it into equations (3.69) and (3.70), and rewrite them as

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} \\ \frac{\partial^2\pi}{\partial x_1\partial x_2} & \frac{\partial^2\pi}{\partial x_2^2} \end{pmatrix}\begin{pmatrix} dx_1 \\ dx_2 \end{pmatrix}=\begin{pmatrix} dw_1 \\ dw_2 \end{pmatrix} \tag{3.71}$$

Letting $dw_2=0$, we then have

$$\frac{dx_1}{dw_1}=\frac{1}{D}\begin{vmatrix}1 & \frac{\partial^2\pi}{\partial x_1\partial x_2}\\ 0 & \frac{\partial^2\pi}{\partial x_2^2}\end{vmatrix}=\frac{1}{D}\cdot\frac{\partial^2\pi}{\partial x_2^2}<0 \tag{3.72}$$

because, according to the second order conditions,

$$D=\begin{vmatrix}\frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2}\\ \frac{\partial^2\pi}{\partial x_1\partial x_2} & \frac{\partial^2\pi}{\partial x_2^2}\end{vmatrix}>0 \quad and \quad \frac{\partial^2\pi}{\partial x^2}<0.$$

We also have

$$\frac{dx_2}{dw_1}=\frac{1}{D}\begin{vmatrix}\frac{\partial^2\pi}{\partial x_1^2} & 1\\ \frac{\partial^2\pi}{\partial x_1\partial x_2} & 0\end{vmatrix}=-\frac{1}{D}\cdot\frac{\partial^2\pi}{\partial x_1\partial x_2} \tag{3.73}$$

The sign of dx_2/dw_1 is undetermined according to the second order conditions. However, it can be stated as follows:

$$\frac{dx_2}{dw_1}\begin{cases}>0 & if \quad \frac{\partial^2\pi}{\partial x_1\partial x_2}<0\\ =0 & if \quad \frac{\partial^2\pi}{\partial x_1\partial x_2}=0\\ <0 & if \quad \frac{\partial^2\pi}{\partial x_1\partial x_2}>0\end{cases} \tag{3.74}$$

In this case, the monopolist's demand for an input is always negative according to equation (3.72). That is, when an input's price goes up, the demand for that input always goes down. On the other hand, when an input's price goes down, the demand for that input will be expected to increase.

3.4.2 Generalizing to the M-Output-N-Input Case

Assume that a monopolist produces m products by using n inputs, that is

$$y_i,\ x_j,\ i=1,\ 2,\ \ldots,\ m \text{ and } j=1,\ 2,\ \ldots,\ n.$$

The production functions are

$$y_i=y_i(x_1,\ x_2,\ \ldots,\ x_n),\ i=1,\ 2,\ \ldots,\ m.$$

The inverse demand functions facing the monopolist are

$$p_i=p_i(y_1,\ y_2,\ \ldots,\ y_m),\ i=1,\ 2,\ \ldots,\ m.$$

The prices for the inputs are given as w_1, w_2, ..., w_n. The profit for the monopolist is

$$\pi=\sum_{i=1}^{m}p_iy_i - \sum_{j=1}^{n}w_jx_j$$

The first order conditions for profit maximization are

$$\frac{\partial\pi}{\partial x_i}=\sum_{j=1}^{m}p_j\frac{\partial y_j}{\partial x_i} + \sum_{j=1}^{m}\sum_{k=1}^{m}y_j\frac{\partial p_j}{\partial y_k}\frac{\partial y_k}{\partial x_i}-w_i=0 \qquad i=1,\ 2,\ \ldots,\ n. \tag{3.75}$$

The second order conditions are

$$\frac{\partial^2\pi}{\partial x_1^2}<0,\quad \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} \end{vmatrix}>0,\ \ldots,$$

$$(-1)^n \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & \cdots & \frac{\partial^2\pi}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdots & \frac{\partial^2\pi}{\partial x_2\partial x_n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \frac{\partial^2\pi}{\partial x_n\partial x_1} & \frac{\partial^2\pi}{\partial x_n\partial x_2} & \cdots & \frac{\partial^2\pi}{\partial x_n^2} \end{vmatrix}>0$$

Taking the total differential of equations (3.75), we have

$$\sum_{j=1}^{n}\frac{\partial^2\pi}{\partial x_i\partial x_j}dx_j+\frac{\partial^2\pi}{\partial x_i\partial w_i}dw_i=0 \qquad i=1,2,\ldots,n \tag{3.76}$$

From equations (3.75) we know

$$\frac{\partial^2\pi}{\partial x_i\partial w_i}=-1\ .$$

Therefore, equations (3.76) can be written as

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_2\partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n\partial x_1} & \frac{\partial^2\pi}{\partial x_n\partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ \cdot \\ \cdot \\ dx_n \end{pmatrix} = \begin{pmatrix} dw_1 \\ dw_2 \\ \cdot \\ \cdot \\ dw_n \end{pmatrix} \tag{3.77}$$

Letting $dw_i=0$, $i=2, 3, ..., n$, we then have

$$\begin{pmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_2\partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n\partial x_1} & \frac{\partial^2\pi}{\partial x_n\partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{pmatrix} \begin{pmatrix} \frac{dx_1}{dw_1} \\ \frac{dx_2}{dw_1} \\ \cdot \\ \cdot \\ \frac{dx_n}{dw_1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \tag{3.78}$$

Let

$$D_i = \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1\partial x_{i-1}} & 1 & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\ \cdot & \cdot & 0 & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_2\partial x_n} \\ \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot\ \cdot\ \cdot & & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot\ \cdot\ \cdot & & \cdot \\ \frac{\partial^2\pi}{\partial x_n\partial x_1} & \frac{\partial^2\pi}{\partial x_n\partial x_2} & \cdot\ \cdot & \cdot & 0 & \cdot\ \cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{vmatrix} \tag{3.79}$$

and

$$D=\begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1 \partial x_2} & \cdot\cdot & \frac{\partial^2\pi}{\partial x_1 \partial x_n} \\ \frac{\partial^2\pi}{\partial x_2 \partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & \cdot\cdot & \frac{\partial^2\pi}{\partial x_2 \partial x_n} \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n \partial x_1} & \frac{\partial^2\pi}{\partial x_n \partial x_2} & \cdot\cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{vmatrix} \tag{3.80}$$

Therefore,

$$\frac{dx_1}{dw_1}=\frac{D_1}{D}=\frac{1}{D}\begin{pmatrix} \frac{\partial^2\pi}{\partial x_2^2} & \frac{\partial^2\pi}{\partial x_2 \partial x_3} & \cdot\cdot & \frac{\partial^2\pi}{\partial x_2 \partial x_n} \\ \frac{\partial^2\pi}{\partial x_3 \partial x_2} & \frac{\partial^2\pi}{\partial x_3^2} & \cdot\cdot & \frac{\partial^2\pi}{\partial x_3 \partial x_n} \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \frac{\partial^2\pi}{\partial x_n \partial x_2} & \cdot & \cdot\cdot & \frac{\partial^2\pi}{\partial x_n^2} \end{pmatrix}<0, \tag{3.81}$$

according to the second order conditions.

Thus, the demand for the input is downward sloping in its own price. That is, when an input's price goes up, the quantity demanded for that input always goes down. On the other hand, when an input's price goes down, the quantity demanded for that input will increase. The signs of dx_i/dw_1 are indeterminate (i=2, 3, ..., n).

3.4.3 Constrained Demand for Inputs

When the production capacity is constrained, how will a multiproduct monopolist's demands for inputs be influenced? Suppose the monopolist's production capacity is constrained as

$$x_1 + x_2 +, \ldots, + x_n \leq Z.$$

The monopolist's objective is to

$$\begin{aligned} &\textit{Maximize}: \quad \pi=\sum_{i=1}^{m} p_i y_i - \sum_{j=1}^{n} w_j x_j \\ &\textit{Subject to}: \quad \sum_{i=1}^{n} x_i \leq Z \end{aligned} \tag{3.82}$$

The Lagrange function is

$$L=\sum_{i=1}^{m} p_i y_i - \sum_{j=1}^{n} w_j x_j + \lambda(Z-\sum_{i=1}^{n} x_i) \tag{3.83}$$

The first order conditions are[6]

$$\begin{aligned} &\frac{\partial L}{\partial x_i}=\sum_{j=1}^{m} p_j \frac{\partial y_j}{\partial x_i} + \sum_{j=1}^{m} \sum_{k=1}^{m} y_j \frac{\partial p_j}{\partial y_k} \frac{\partial y_k}{\partial x_i} - w_i - \lambda = 0 \qquad i=1,\ 2,\ \ldots,\ n. \\ &\frac{\partial L}{\partial \lambda}=Z-\sum_{i=1}^{n} x_i = 0\ . \end{aligned} \tag{3.84}$$

The second order conditions are

[6] For simplicity, I assume that the constraint is binding and do not consider corner solutions.

$$\begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \frac{\partial^2\pi}{\partial x_1\partial x_2} & -1 \\ \frac{\partial^2\pi}{\partial x_2\partial x_1} & \frac{\partial^2\pi}{\partial x_2^2} & -1 \\ -1 & -1 & 0 \end{vmatrix}>0, \ \ldots, \ (-1)^i \begin{vmatrix} \frac{\partial^2\pi}{\partial x_1^2} & \ldots & \frac{\partial^2\pi}{\partial x_1\partial x_i} & -1 \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial^2\pi}{\partial x_i\partial x_1} & \ldots & \frac{\partial^2\pi}{\partial x_i^2} & -1 \\ -1 & \ldots & -1 & 0 \end{vmatrix}>0$$

where i=3, 4, ..., n.

Taking the total differential of equations (3.84), we have

$$\begin{aligned} &\sum_{j=1}^{n}\frac{\partial^2 L}{\partial x_i\partial x_j}dx_j+\frac{\partial^2 L}{\partial x_i\partial w_i}dw_i+\frac{\partial^2 L}{\partial x_i\partial\lambda}d\lambda=0 \qquad i=1,\ 2,\ \ldots,\ n \\ &\sum_{j=1}^{n}\frac{\partial^2 L}{\partial\lambda\partial x_j}dx_j+\frac{\partial^2 L}{\partial\lambda^2}d\lambda+\frac{\partial^2 L}{\partial\lambda\partial Z}dZ=0 \end{aligned} \tag{3.85}$$

From equations (3.84) we know

$$\begin{aligned} &\frac{\partial^2 L}{\partial x_i\partial w_i}=-1, \quad \frac{\partial^2 L}{\partial x_i\partial\lambda}=-1, \quad \frac{\partial^2 L}{\partial\lambda\partial x_i}=-1, \qquad i=1,\ 2,\ \ldots,\ n \\ &\frac{\partial^2 L}{\partial\lambda^2}=0, \quad and \quad \frac{\partial^2 L}{\partial\lambda\partial Z}=1\ . \end{aligned} \tag{3.86}$$

Letting dw_i=0, i=2, 3, ..., n and dZ=0, equations (3.85) then can be written as

$$\begin{pmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \cdot\cdot & \frac{\partial^2 L}{\partial x_1 \partial x_n} & -1 \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \cdot\cdot & \frac{\partial^2 L}{\partial x_2 \partial x_n} & -1 \\ \cdot & \cdot & \cdot\cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\cdot & \cdot & -1 \\ \frac{\partial^2 L}{\partial x_n \partial x_1} & \frac{\partial^2 L}{\partial x_n \partial x_2} & \cdot\cdot & \frac{\partial^2 L}{\partial x_n^2} & -1 \\ -1 & -1 & \cdot\cdot & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{dx_1}{dw_1} \\ \frac{dx_2}{dw_1} \\ \cdot \\ \cdot \\ \frac{dx_n}{dw_1} \\ \frac{d\lambda}{dw_1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \tag{3.87}$$

According to the second order conditions, we then have

$$\frac{dx_1}{dw_1} = \frac{1}{H} \begin{pmatrix} \frac{\partial^2 L}{\partial x_2^2} & \cdot\cdot & \frac{\partial^2 L}{\partial x_2 \partial x_n} & -1 \\ \cdot & \cdot\cdot & \cdot & \cdot \\ \cdot & \cdot\cdot & \cdot & -1 \\ \frac{\partial^2 L}{\partial x_n \partial x_2} & \cdot\cdot & \frac{\partial^2 L}{\partial x_n^2} & -1 \\ -1 & \cdot\cdot & -1 & 0 \end{pmatrix} < 0, \tag{3.88}$$

where

$$H=\begin{pmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \cdot\ \cdot & \frac{\partial^2 L}{\partial x_1 \partial x_n} & -1 \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \cdot\ \cdot & \frac{\partial^2 L}{\partial x_2 \partial x_n} & -1 \\ \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot \\ \frac{\partial^2 L}{\partial x_n \partial x_1} & \frac{\partial^2 L}{\partial x_n \partial x_2} & \cdot\ \cdot & \frac{\partial^2 L}{\partial x_n^2} & -1 \\ -1 & -1 & \cdot\ \cdot & -1 & 0 \end{pmatrix} \tag{3.89}$$

From the above analysis we can conclude that a multiproduct monopolist's demand for input in the conventional models is always downward sloping in own price even with a capacity constraint. That is, when an input's price goes up, the quantity demanded for that input always goes down. On the other hand, when an input's price goes down, the quantity demanded for that input will increase.

3.5 Pricing Strategy and Input Demand of a Multiproduct Monopolist in a Discrete Choice Model

In the conventional models, the input quantity demanded and the input price always move in the opposite directions. Thus, they fail to explain why the buyers don't want to buy more fish from the fishermen even when the fishermen are willing to lower their prices.

In conventional models, it is assumed that (1) the demand function facing the monopolist is continuous and continuously differentiable, and (2) the second order conditions are met. In the real world, consumers' preferences for food items may not be well characterized by continuous functions. For example, a consumer may purchase a finite amount of meat per time period and his preference is characterized by local satiation. The utility of any quantities of the meat beyond the finite amount to the consumer is zero because of the high cost of at-home storage, perishability, and the fact that the consumer can return to the store (or to another store) during a subsequent shopping period (Johnston and Larson 1992). When the satiation phenomenon is considered in conventional models, the second order conditions may not be met.

On the other hand, for a multiproduct seller like a supermarket, the demands are usually interrelated through shopping complementarity, consumption complementarity and consumption substitution (Balderston 1956; Reibstein and Gatignon 1984; Bliss 1988; Mulhern and Leone 1991; Klemperer 1992). When a supermarket reduces one product's price, it expects more customers will come to its store. Once customers arrive at the store, they will purchase not only the product whose price is reduced but also other products. Sales of all the other products, no matter whether they are complements or substitutes in consumption, are likely to increase. However, when the product's price is reduced further, the customers in the store may increase their purchases of the reduced price product and reduce their purchases of substitute products. Therefore, if two products are substitutes in consumption, the demand facing the supermarket for one product may increase first and then decrease as the price of the other product decreases, depending on whether or not the effect of shopping complementarity dominates the effect of consumption substitution. When this phenomenon is considered in conventional models, the second order conditions may not be met.

To close the gap between pricing theory and real world business practices, a theoretical model must be able to better capture not only the multiproduct sellers' pricing behavior and characteristics, especially the supermarkets', but also the consumers' shopping behavior. In this section, I use a discrete choice model to address this issue and attempt to provide an explanation for some observed phenomena in the seafood industry.

3.5.1 The Model and Assumptions

A supermarket usually sells many different products. Among those products, some are independent of each other in consumption, some are substitutes and some are complements. When a customer makes decision on which store to go to, he will consider several factors such as shopping costs (including time cost, transportation cost, etc.), convenience, varieties, prices, etc. (Engel 1990; Wilkie 1990; Senauer et al. 1991). Once a customer is attracted to the store by the availability of a specific product or by a sale price for a specific product, he will usually purchase more than one product and try to complete his shopping in one store; otherwise, his shopping costs would increase. "Consumers often prefer to concentrate their purchases with a single supplier. It is more convenient to do one's grocery shopping in a single supermarket than to visit several." (Klemperer 1992, p740) I call this phenomenon shopping complementarity. A customer may conduct his main shopping once a week and generally operates within a budget constraint or quantity constraint (explained below) for such shopping (Johnston and Larson 1992). Although there may be more than one supermarket in a given area (e.g., a city or a town), each supermarket has some monopoly power because of the customers' shopping costs. That is, "once consumers are inside a specific supermarket, travel and time costs between supermarkets give each store a monopoly position." (Walden 1988, p54)

With the above assumptions regarding shopping behavior and supermarket characteristics, we can use the following model to explore a supermarket's pricing strategies.

Suppose a monopolist sells three goods: y_1, y_2 and y_3. The prices of those three goods are p_1, p_2, and p_3 respectively, while the corresponding marginal costs are constant at c_1, c_2 and c_3. The customers are represented by three persons, A, B and C, who have different preferences.

Good y_1 is consumption-independent of good y_2, and good y_3. Good y_2 and good y_3 are substitutes in consumption. Each person's preferences are represented by his reservation prices for those three goods.

Person i's reservation prices (i = A, B, C) for the three goods are:

V_{i1}^1: the reservation price for the first unit of good y_1.

V_{i1}^2: the reservation price for the second unit of good y_1. Assume that $V_{i1}^1 > V_{i1}^2$, and the reservation price for the third unit of good y_1 is zero. Hence, if $V_{i1}^1 \geq p_1$, person i will buy one unit of y_1. If $V_{i1}^2 \geq p_1$, person i will buy two units of y_1. The person will buy no more than two units of good y_1 so long as $p_1 \geq 0$.[7]

[7] The assumption that only two units have positive reservation prices is made for simplicity and is not critical to the analysis. What is critical is that, per time period, preferences are characterized by local satiation. The justifications for this include the high cost of at-home storage, perishability, and the assumption that the customer can

V_{i2}^1: the reservation price for the first unit of good y_2.

V_{i2}^2: the reservation price for the second unit of good y_2. Assume that $V_{i2}^1 > V_{i2}^2$, and the reservation price for the third unit of good y_2 is zero. The person will buy no more than two units of good y_2.

V_{i3}^1: the reservation price for the first unit of good y_3.

V_{i3}^2: the reservation price for the second unit of good y_3. Assume that $V_{i3}^1 > V_{i3}^2$, and the reservation price for the third unit of good y_3 is zero. The person will buy no more than two units of good y_3.

Further assume that each person will buy no more than two units of good y_2 plus good y_3. That is, a customer wants no more than two units of either good y_2 or good y_3, or both. For example, if a person purchases two units of y_2, he will not purchase any more of either y_2 or y_3. If he purchases one unit of y_2 and one unit of y_3, he will not want any more of either y_2 or y_3. What is the rationale for this assumption? Suppose that y_3 is seafood and y_2 is a non-seafood meat (for example, beef, poultry, pork). A person can eat only a certain amount of both y_2 and y_3 in a certain period. If the person wants to eat more y_2, he reduces his consumption of y_3.[8]

When customer i makes purchasing decisions, he tries to maximize his consumer's surplus and has the following choices:

(1) If $V_{i1}^1 \geq p_1$, buy one unit of y_1. If $V_{i1}^2 \geq p_1$, then buy two units of y_1. If $V_{i1}^1 < p_1$, buy none of y_1.

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return to the store (or to another store) during a subsequent shopping period.

[8] For further discussion see Johnston and Larson (1992). They provide a detailed discussion on this issue.

(2) If $0 \leq (V^2_{i2}\text{-}p_2) > (V^1_{i3}\text{-}p_3)$, buy two units of y_2 and none of y_3.

(3) If $(V^2_{i2}\text{-}p_2) < (V^1_{i3}\text{-}p_3) \geq 0$, and $(V^2_{i3}\text{-}p_3) < (V^1_{i2}\text{-}p_2) \geq 0$, buy one unit of y_2 and one unit of y_3.

(4) If $0 \leq (V^2_{i3}\text{-}p_3) > (V^1_{i2}\text{-}p_2)$, buy two units of y_3 and none of y_2.

(5) If $(V^1_{i2}\text{-}p_2) \geq 0$, $(V^2_{i2}\text{-}p_2) < 0$, and $(V^1_{i3}\text{-}p_3) < 0$, buy one unit of y_2 and none of y_3.

(6) If $(V^1_{i3}\text{-}p_3) \geq 0$, $(V^2_{i3}\text{-}p_3) < 0$, $(V^1_{i2}\text{-}p_2) < 0$, buy one unit of y_3 and none of y_2.

(7) If $(V^1_{i2}\text{-}p_2) < 0$ and $(V^1_{i3}\text{-}p_3) < 0$, buy neither of y_3 and y_2.

We also assume that if the seller sells all three goods, all three customers will come to shop in his store. However, if the seller sells only goods y_1 and y_2, customer A will not come while customers B and C will come anyhow. The rationale for this assumption can be explained as follows.

Suppose the shopping costs (including time cost, transportation cost, etc., but not the goods' own costs) for all three customers are lower when shopping at this store than at other stores. Customer A values good y_3 relatively highly and expects to enjoy some consumer surplus if he purchases good y_3. Ceteris paribus, he would go to this store but not to the others.[9] However, when the store doesn't sell good y_3, he would go to one of the other stores because he believes that the gains from purchasing three goods in another store would dominate the losses derived from higher shopping costs. On the other hand, customers B and C will come anyhow to the store for shopping either

[9] It is assumed that consumers have full information on prices, varieties, etc. about each store before they make decision on which store to go for their shopping. To get this information in the real world, consumers have to do some researching work. This researching procedure is ignored for the analytic simplicity.

because they don't want good y_3 or because they value it relatively low and the gains from purchasing three goods in another store are not likely to dominate the losses derived from the associated higher shopping costs. Generally, similar stories might be told for the other goods. For example, when good y_1 is not sold, some customers may go to other stores instead of shopping at the store. For analytical simplicity, we consider only what happens when good y_3 is sold or not sold.

Under the above assumptions the demands for goods y_1, y_2 and y_3 are interrelated. When the price of good y_3 changes, the demands for goods y_1 and y_2 will change too. Therefore, when the seller makes pricing decisions, he needs to consider not only the effect of shopping complementarity but also the effect of consumption complementarity and substitution.[10]

3.5.2 A Numerical Example

We now look at a numerical example to see how the seller makes an optimal decision. Table 3.1 shows the three customers' reservation prices for the three goods and their costs for the seller.

First, suppose the seller sells only two goods: y_1 and y_2. Customer A will not come to the store because good y_3 is not available; customers B and C will shop at this store. Because the demands for y_1 and y_2 are independent in consumption, we can set an optimal price for each good separately. Table 3.2 shows that the maximum profit for

[10] Consumption complementarity is not considered further in the model because shopping complementarity should include that effect.

Table 3.1 Customer's reservation prices for y_1, y_2 and y_3.

Customer	y_1		y_2		y_3	
	1st	2nd	1st	2nd	1st	2nd
A	100	80	80	60	85	81
B	110	90	80	65	70	60
C	90	70	70	60	30	20
Cost	c_1=60		c_2=42		c_3=70	

Table 3.2 The maximum profits and optimal price for good y_1 in the absence of customer A.

Customer	p_1=110	p_1^*=90	p_1=70
B	1	2	2
C	0	1	2
Profit π_1	50	3x30=90*	4x10=40

good y_1 is π_1^*=90 at price p_1^*=90. Customer B buys two units of good y_1 and customer C buys one unit.

For good y_2, the optimal price is p_2^*=60 and the maximum profit is π_2^*=72. Customers B and C both buy two units of y_2 (see Table 3.3).

Therefore, when only y_1 and y_2 are sold, the optimal prices are p_1^*=90 and p_2^*=60. The maximum profit is

$$\pi^*=\pi_1^*+\pi_2^*=90+72=162. \qquad (3.90)$$

Table 3.3 The maximum profit and optimal price for good y_2 in the absence of customer A.

Customer	p_2=80	p_2=70	p_2=65	p_2^*=60
B	1	1	2	2
C	0	1	1	2
Profit π_2	38	2x28=56	3x23=69	4x18=72*

Suppose now that all three goods are offered for sale. All three customers will come to shop if the price for good y_3 is equal to or less than customer A's reservation price for the first unit of y_3. Because the demand for good y_1 is independent of y_2 and y_3 in consumption, we can determine an optimal price for y_1 first. From Table 3.4, we can see that the optimal price p_1^{**}=90, and the profit

$$\pi_1^{**}=120. \tag{3.91}$$

Customers A and C both buy one unit of y_1 and B buys two units.

Table 3.4 The maximum profit and the optimal price for good y_1.

Customer	p_1=110	p_1=100	p_1^{**}=90	p_1=80	p_1=70
A	0	1	1	2	2
B	1	1	2	2	2
C	0	0	1	1	2
Profit π_1	50	2x40=80	4x30=120**	5x20=40	6x10=60

In setting the prices for y_2 and y_3, the seller has to consider the substitution between those two goods. The maximum price that the seller can charge for good y_3 is

Table 3.5 The maximum profits and optimal prices for y_2 and y_3.

Customer	$p_2=80$ $p_3=85$		$p_2=70$ $p_3=85$		$p_2^*=65$ $p_3^*=85$		$p_2=60$ $p_3=85$	
	y_2	y_3	y_2	y_3	y_2	y_3	y_2	y_3
A	1	1	1	1	1	1	1	1
B	1	0	1	0	2	0	2	0
C	0	0	1	0	1	0	2	0
Total Profit	76+15=91		84+15=99		92+15=107*		90+15=105	

85; otherwise, customer A will not come. If $p_3=85$, the profit margin per unit from selling y_3 is (85-70)=15.

In the set of price choices (80, 70, 65, 60) for y_2, the profit margin per unit of y_2 is always greater than 15. Given the number of customers, an increase in sales of y_3 will cause a drop in sales of y_2. Thus, the optimal pricing strategy for good y_3 would be one that attracts as many potential customers as possible and sells as few units of y_3 as possible. In this numerical example, the optimal price for y_3 would be $p_3=85$ and only one unit of y_3 would be sold. Table 3.5 shows the combinations of p_2 and p_3 and their corresponding profits. The optimal prices are $p_2^{**}=65$ and $p_3^{**}=85$. The maximum profit is

$$\pi_{2,3}^{**}=107. \tag{3.92}$$

Thus, the optimal prices for the three-good seller are $p_1^{**}=90$, $p_2^{**}=65$ and $p_3^{**}=85$. From equations (3.91) and (3.92), the maximum profit is

$$\pi^{**}=\pi_1^{**}+\pi_{2,3}^{**}=227, \tag{3.93}$$

which is greater than π^* (=162) in equation (3.90).

In sum, by introducing the third good y_3, the seller attracts more customers. Consequently, the demands for y_1 and y_2 are increased because of the shopping complementarity effect. The total profits are increased. Table 3.6 shows the differences between the two-good case and three-good case. By introducing the third good y_3, the profits from selling y_1 and y_2 are both increased, as are total profits.

Table 3.6 The differences between the two-good and three-good case.

		y_1	y_2	y_3	Total Profit
Two-good case	price	90	60		162
	profit	90	72		
Three-good case	price	90	65	85	227
	profit	120	92	15	

3.5.3 Discussion

From the above numerical example, we see that although the profit margin of the third good is small and less than that of its substitute, the monopolist's total profits are increased by adding the third good on the shelf. The seller is able to use the third good to attract more customers to the store and increase the sales of the other goods because of the shopping complementary effect. The major contributions to the increase in total profits come from the increased sales of goods y_1 and y_2 not the sales of good y_3. From this fact we can conclude

(1) A multiproduct seller may charge one good below its cost as long as the gains from the increased sales of the other goods dominate the losses from the sales of

the underpriced good. This is the "loss leader" case.[11] For example, when c_3 increases from 70 to 90 in the numerical example, the price would be less than the cost if the seller still sells good y_3 at price p_3=85. If the seller increases good y_3's price to avoid losing money from selling good y_3, consumer A would not come to the store, consequently, the total sales of other goods would be reduced. Thus, in this case the best strategy is to keep the price p_3=85, even though p_3 is less than the cost.

(2) A multiproduct seller's input demand may be perfectly inelastic in some price ranges. Again in our numerical example, when the marginal cost c_3 increases or decreases within certain ranges, the seller would not change price p_3. If we let c_3 be the input price, it would mean that the seller's demand for the input is perfectly inelastic. The rationale for this is as follows. Good y_3's main contribution to the seller's total profits is through the increased sales of the other goods, not through profits from its own sales. When price p_3 increases, the sales of the other goods decrease through the effect of shopping complementarity, so that the total profits are reduced. On the other hand, when price p_3 is reduced, the sales of the other goods go down through the substitution effect, so that the total profits are reduced. In the presence of these two offsetting influences, there are circumstances under which the seller's demand for an input may be perfectly price-inelastic.

(3) When a good is sold through multiproduct sellers, the input price and output price may move in opposite directions, i.e., when the input price goes down, the output price goes up. For instance, when consumer demand for good y_3 goes up, the seller may

[11] "loss leader" (or "loss leading") means that the shop sells one or more items at less than cost. (Tellis 1986, Bliss 1988)

continue to set a price such that the same amount of y_3 will be purchased because the gains from selling more y_3 may be less than the losses in the reduced sales of the substitute goods. Thus, when the demand for y_3 goes up, the price will go up while the quantity remains the same. Because the store's supply of y_3 is inelastic, its demand for y_3's input is inelastic. Thus, if the supply of the input increases, the price of that input may go down and the quantity demanded remains the same. Consequently, while the input price goes down, the output price goes up.

We are now in a position to explain some phenomena observed in the seafood industry. In recent years, fish prices to the fishermen have decreased and the seafood prices to the final consumers have increased (Johnson and Dore 1993). In the Pacific Northwest it has been observed that, in the face of expanded availability of fish (through favorable oceanographic or biological conditions, for example) processors are often unwilling to take greater quantities of fish from the fishermen even at lower prices. On the contrary, the processors often set quantity limits for the fishermen, a phenomenon known in the industry as placing the fishermen "on limits" (Hanna 1990). This phenomenon cannot easily be explained by conventional arguments. However, one possible explanation follows from the above analysis, as discussed next.

Most seafood at the retail level is sold through supermarkets. The consumption of seafood is relatively small and the storage cost is relatively high compared to other meats (Senauer et al. 1991). Consequently, the profit from selling seafood – at least some seafood – is relatively small compared to other meats. Through offering seafood for sale, supermarkets hope they will meet some customers' demands for seafood and can attract more customers to their stores. They expect the total profits to increase no matter how much of the increase can be attributed directly to the sales of seafood. When a customer in the store purchases more seafood, he will reduce the purchase of other meat. Because the profit margin for seafood is less than for its substitutes such as beef, pork and poultry, the optimal pricing strategy would be to set the price for seafood at the level that attracts as many customers to the store as possible while inducing the customers, once in the store, to buy as little seafood as possible.[12] By adopting this pricing strategy, the supermarkets may want to sell only a certain amount of seafood. Therefore, the supply of seafood to the final consumers is inelastic. In other words, when price increases, the quantity supplied may stay the same. Consequently, when consumers' demands for seafood increase, the price goes up. On the other hand, the supermarkets' demands for fish may be perfectly inelastic, i.e., the quantity of fish demanded by supermarkets stays the same when fish price changes. Thus, the processors don't want to take more fish from the fishermen because their buyers, the supermarkets, don't want more no matter what the price is. Thus, with a perfectly inelastic derived demand for fish, the fish price goes down when fishermen increase their supply; this, then, represents one possible explanation for the fishermen being placed "on limits."[13]

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[12] This gives seafood some attributes of a "loss leader" (see Thompson and Formby, pp. 388-9). The difference between the present analysis and that of loss-leader pricing, however, is that the latter generally pertains to the phenomenon of setting a low (perhaps even below-cost) price on a relatively popular item in order to attract additional customers to the multi-product seller's store. In such a model, reduced input costs can be expected to lead to lower prices of the loss leader, a phenomenon that does not necessarily emerge within the present framework because of the partitioning of consumer groups according to their preferences and because of the assumption of local satiation.

[13] Other possible explanations include (1) limited storage capacity and (2) established fishermen-processor relationships that prevent price declines, to name two. While these may explain the "on limits" phenomenon in the short run they are less satisfactory long

In conclusion, I believe I have in this section developed a model that extends the current literature on multi-product pricing and that provides a possible explanation for processor behavior in the seafood industry.

3.6 Summary and Conclusions

In real world business practices, many firms produce or sell more than one product. Our knowledge of single-product monopolists is not enough to understand some issues in the real business world. The pricing behavior and comparative static analysis of a multi-product monopolist are complicated. In this chapter, I have used some conventional models and a discrete choice model to explore the pricing behavior and comparative statics of a multi-product monopolist. The conclusions and suggestions for further research are discussed next.

3.6.1 The Main Findings

The main findings from the analysis of the comparative statics for multiproduct monopolists in the conventional models can be summarized as follows:

(1) For a single-product monopolist, when the marginal cost decreases, the price always decreases and the output increases.

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run explanations. The current model looks for the explanation further down the market channel and, in the process, offers an extension of the current literature on multi-product pricing in general.

(2) For a multiproduct monopolist, when a product's marginal cost decreases, the sales of the corresponding product always increase. However, the sales of the other products can either increase or decrease or remain the same.

(3) For a two-product monopolist, when a product's marginal cost decreases, the corresponding product's price can either increase or decrease or remain the same, depending on conditions. The other product's price can also either increase or decrease or remain the same. However, when the first product's price increases, the second product's price cannot decrease. In this chapter, I have also identified the conditions under which the Edgeworth paradox may occur.

(4) Combining (2) and (3), a product's quantity and price may both move in the same direction, corresponding to a change in its marginal cost. That is, when the price goes up, the quantity demanded may also increase.

(5) For both a single-product monopolist and a multiproduct monopolist, the input demand and the input price always move in opposite directions in conventional models. That is, when an input price goes down, the quantity demanded for that input will increase. A binding capacity constraint won't change this conclusion. However, a change in one input's price can cause the quantity demanded for other inputs to either increase or decrease, depending on conditions.

From (5) we can see that conventional models fail to address the phenomenon in which the purchasers don't want to take more fish from the fishermen even at lower prices. It may be because most seafood is sold to final consumers through supermarkets and the conventional models do not adequately capture the supermarkets' special characteristics, pricing behavior and the consumers' shopping behavior. By using a discrete choice model, it has been shown that:

(1) the input price and output price may move in opposite directions, i.e., while the input price goes down the output price goes up;

(2) the seller's demand for an input can be perfectly price-inelastic in some range, even if the customers' demand for the output is price-elastic.

3.6.2 The Limits and Suggestions for Further Research

This chapter attempts to explore the pricing behavior and comparative statics for multiproduct monopolists and to provide an explanation for at least one phenomenon in the seafood industry. There exist some limits in the analysis and some further research needs to be done.

In the analysis it is assumed that marginal costs are constant and the costs are not interrelated, i.e., the productions are independent. Relaxation of the assumptions will bring the model closer to real world business practices, though it may make the analysis much more complicated.

The major weakness of the discrete choice model is its discontinuity. Although it well addresses the supermarket pricing behavior and consumers' shopping behavior, the discrete choice model needs to be developed into a conceptual and continuous model. When I used the findings from the discrete choice model to explain some phenomena in the seafood industry, I made an assumption that the profit margin of seafood in supermarkets is less than that of other substitutes (for example, beef, pork, poultry). This assumption waits empirical testing.

CHAPTER 4
THE ECONOMICS OF TYING SALES UNDER PRICE CONTROLS

4.1 Introduction

In today's food markets, more and more producers are producing multiple products (Senauer et al. 1991). These products may be substitutes, complements, or independent in consumption. For a multiproduct producer or seller, the pricing strategies are much more complicated than those for a producer or seller of a single product. Among those pricing behaviors and pricing strategies, tying selling is the strategy used by many sellers and observed in many business practices (see, for example, Adams and Yellen 1976; Tirole 1989).

A tying sale (or tie-in sale) is one that either conditions the sale of one good (the tying good) on the sale of another (the tied good), or conditions the lease of a machine on the use of supplies or services furnished by the lessor. For example, a seller of copy machines sells copy machines to the buyers on the condition that the buyers also purchase the special ink (toner) from him. A conceptually related practice is commodity bundling, under which the seller insists that the buyer take a package of products, bundled together and offered at a single price per bundle. For example, a restaurant offers some combination dinners as opposed to (or in addition to) "à la carte" selections. Commodity bundling can be considered as a special form of tying sales.

There exists a considerable body of literature that explores the economics of tying sales. Most of the literature explains tying sales as an extension of market power, economies of joint sales or joint production, metering and price discrimination, etc. One issue that has received little attention is that of tying sales as a reaction to price controls. Since Bowman (1957) pointed out that tying sales can be used to evade the price regulation, nobody, to the best of my knowledge, has gone further to study the economics of tying sales under price controls.

Skousen (1977) pointed out the popularity of tying sales under price controls. For instance, in the period of World War II, many prices were regulated by the U.S. government. "During 1945, a New York meat retailer purchased $4200 worth of stock in a corporation called the United Meat Co., Inc., in order to obtain some meat at ceiling prices". During the Korean War price controls, "a common tying arrangement involved car "trade-ins." In many cases, a car dealer would not sell a new car until he received a used one at a price far below its resale value" (Skousen 1977, p184). I have also personally experienced the popularity of tying sales arrangements under price controls in China.

Since 1949, China has been a centrally-planned economy. The central government controlled many prices and didn't let them increase even in the face of rising demand (Fung 1987; Guo 1992). As the result of price controls, many commodities were in short supply. Ration coupons, long queues, and tying sales arrangements existed simultaneously everywhere. Tying sales arrangements were used not only by retailers, but also by manufactures and wholesalers. For example, brand name cigarettes were tied with soy sauce, sugar was tied with salt, color TVs were tied with bicycles, shrimp was tied with fish, etc.

In the 1970s the price of matches was regulated at two fens per box (two fens were about 0.005 U.S. dollars at that time) and hadn't changed for several years. The supply of matches was short. Whenever I wanted to buy a box of matches for my family, the seller would require me to buy a notebook or a bottle of ink or pencils, sometimes even a box of cigarettes from him. Otherwise, the seller would not sell any matches to me. Even though I didn't need those items or I knew that his prices of those items were higher than the market prices, I still had to buy the matches from him because literally every seller of matches was using tying sales arrangements and I had no way to avoid them.

In 1979, China started its economic reform. Since then, the government has lifted some price controls gradually. When I came back to China in January 1993, I found to my surprise that the prevalence of tying sales arrangements had been dramatically reduced. I could hardly find evidence of tying sales in the market. I asked some people if they could give me some examples of tying sales in today's market, but they could not. Some people told me that the cigarette industry is an exception. I asked why that is, and was told that the cigarette industry is still highly monopolized, and the prices are firmly controlled by the government.

In this chapter, I study some issues related to the economics of tying sales under price controls: (a) what are the reasons or motivations for tying sales under price controls in different market structures, (b) how does the tying sale work in different circumstances, (c) what are the consequences of tying sales under price controls, and (d) what is the relationship between the theory of tying sales and real business practices.

The rest of this chapter is arranged as follows: in section 4.2, I briefly review the literature on tying sales and price controls; in section 4.3, I study a competitive-tying model under price controls; in section 4.4, I study a monopoly-tying model under price controls; the summary and conclusions are in section 4.5.

4.2 A Review of the Literature on Tying Sales and Price Controls

Because of the popularity of tying sales in business practices, much research has been done on the topic. Economists have tried to explore the reasons or motivations for tying sales. Since tying sales include so many different cases with different circumstances, there is no general theory which can explain all tying sales arrangements. Different explanations exist simultaneously and can explain different cases of the tying sales arrangements. Generally the reasons for tying sales can be divided into eight categories:

(1) Extension of market power.

In tying sales, the monopolistic seller of a tying good can extend its monopoly power to the tied good, a second market. This explanation is referred to as the "leverage theory of tying": "tying provides a mechanism whereby a firm with monopoly power in one market can use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market" (Whinston 1990, p837).

The leverage theory of tying has been criticized by many economists in the past thirty years. The critics point out that a monopolist cannot extend its monopoly power in one market to a second market by tying sales (Director and Levi 1956; Bowman 1957; Stigler 1968; Posner 1976). A typical criticism can be illustrated as follows. Suppose that a firm is a monopolist of good x that a consumer values at level v_x and that costs

c_x to produce. The consumer also purchases another competitively supplied good y that he values at v_y and that can be produced at a unit cost of c_y. Because of the competitive assumption for good y, v_y must be equal to c_y. Now, the monopolist could require the consumer to purchase good y from it if he wants good x, but what will the firm gain? The consumer will only purchase such a bundle if its price is no larger than (v_x+v_y), and so the monopolist can do no better than earning (v_x-c_x), the level earned from selling good x independently. In short, there is only one monopoly profit that can be extracted. Thus, the critics contend, if a monopolist does use tying, the motivation cannot be leverage.

However, in recent years, some economists have defended the leverage theory of tying sales in new models. The critics of leverage theory assumed that the tied good market has a competitive, constant returns-to-scale structure. With this assumption, the use of leverage to affect the market structure of the tied good market is impossible. However, if the tied good market is imperfectly competitive, tying may extend a monopolist's power to the tied good's market. Whinston (1990, p855) pointed out that "once one allows for scale economies and strategic interaction, tying can make continued operation by a monopolist's tied market rival unprofitable by leading to the foreclosure of tied good sales". In cases where the rivals' entry or exit decisions are unaffected by tying, a monopolist's tying decision can influence the behavior of rival tied good producers and increase profits. Carbajo et al. (1990) showed that a monopolist can increase its profits through tying because it causes rivals to act less aggressively. Thus, tying can extend a monopolist's power to the tied good market, a second market.

(2) Economies of joint sales or joint production.

The economies realized through joint sales or joint production is an important straightforward reason for tying sales in some circumstances (Bowman 1957; Singer 1968; Kenney and Klein 1983; Scherer and Ross 1990). For example, automobile engines and chassis are sold as a unit primarily because doing so dramatically reduces transactions and transportation costs. The negotiation costs, administrative costs and shipping costs may all be reduced because of the tying sales.

(3) The protection of good will.

Both Singer (1968) and Palmer (1984) pointed out that the protection of good will may be the reason for tying sales in some business practices. For example, the producer of a technically complex machine may tie to control the quality of raw materials used with its machine so that its reputation is not sullied by breakdowns caused through the use of faulty supplies. In the Pick case, General Motors insisted that its dealers sell and install only those parts authorized by the company.[14] It was argued that substandard parts installed in a General Motors car by a General Motors dealer would have a deleterious effect on the company's good will because users would not ordinarily associate the improper functioning of their automobiles with the use of non-General Motors parts.

(4) Metering and price discrimination.

One of the most popular explanations for the existence of tying sales is the use of the tied good as a metering device to facilitate price discrimination. Suppose that one

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[14] Pick Mfg. Co. v. General Motors Corp., 80 F.2d 641 (7th Cir. 1935).

copying machine user makes 4,000 copies per week while other makes 8,000 copies per week. It would be difficult for a producer selling only copying machines to price its machines in such a way as to extract more profits from the more intensive user. (It is assumed that the producer cannot know in advance the intensity with which the buyers will use the machines). But if the machine producer can tie the purchase of special ink (toner) to the purchase of his machine, and if it can price the supplies (ink or toner) so as to realize a supra-normal profit margin on them, it will be able to extract additional profits from the higher-volume user. In this sort of tying, the sale of supplies serves as a substitute for placing a meter on the machine itself and billing the consumer (who leases the machine rather than buying it) on the basis of metered usage.

Aaron Director (1956) was the first one who developed the argument of price discrimination as a reason for tying sales. Since then, many papers have been published on the issue of price discrimination with tying sales (see, for example, Bowman 1957; Burstein 1960a, 1960b). Stigler (1968) and Adams and Yellen (1976) provided an alternative technique for price discrimination with commodity bundling, where bundling is viewed as a specific form of tying sales. Later, Schmalensee (1982, 1984) and McAfee et al. (1989) further studied the case of bundling and suggested some conditions for the profitability of bundling.

(5) The allocation of risk.

Burstein (1960b) and Liebowitz (1983) have suggested that tying sales may be used as a risk allocation mechanism.

Suppose that a patentee sells machines embodying the patent. But potential purchasers are concerned lest they incur large fixed costs. The patentee can ease this concern by leasing or renting them the machines, hence reducing their fixed costs. The concern can be eased even further if the rental rate for the machine is set below costs and the difference is covered by charging the lessee a higher price for a tied good (the tied good is an input in production: for instance, the tying good is a salt brine mixing machine and the tied good is the salt). In this way, if the lessee's business prospers, the patentee will share the benefits, and if the lessee's business does poorly, the patentee will share in the losses. Because the benefits are approximately equal to losses across all the lessees, the patentee will be assuming very little additional risk while reducing the amount of risk faced by each of the lessees.

(6) Cheating on a cartel.

A cartel normally sets a floor price at the level that is higher than the competitively determined price and requests all its members to charge their products at least as high as the floor price. The profit for each member of the cartel should be larger than that under competition. However, a member of the cartel can increase his profit by charging a price lower than the floor price without causing other members to cut their prices. Hence, for profit maximization, a member of the cartel has a potential to cheat. Tying sales can be used to cheat on a cartel.

In a study of the International Salt Case, Peterman (1979) provided an example and detailed analysis. Suppose that a salt producer is a member of a salt-selling cartel. The producer would like to increase his profits by selling more salt and would be tempted to cut the price, but he fears retaliatory price-cutting by other members of the cartel. Still, he might be able to increase his market share by reducing the rental fee for his patented machine to mix salt and water and by concurrently requiring the lessees to purchase all of their salt requirements from him at the cartel price. This tying sale will effectively reduce the price of the salt to the users without necessarily setting off a price war within the cartel. In this case, the seller prefers the floor-price control to competition; at the same time, he tries to cheat.

(7) Policing a cartel.

As said above, the members of a cartel have a potential to cheat. The cartel needs to find a way to prevent its members from cheating. Tying sales can be used to police cartels. Cummings and Ruhter (1979) provided a good example in their paper on "The Northern Pacific Case". The Northern Pacific railroad leased land to its clients subject to the tying condition that the lessee ship with Northern Pacific or allow it the opportunity to match a lower freight rate offered by a competitor. Since the Interstate Commerce Commission set freight rates above the competitive level, this tying sale gave Northern Pacific the opportunity to determine whether any of its competitors were violating the rate regulations and enabled Northern Pacific, along with the Interstate Commerce Commission, to enforce the cartel-type freight rates.

The above two cases (6 & 7) deal with floor-price controls in a cartel. Tying sales can be used either to evade the floor-price control or to enforce the floor-price control. Next we consider the case of ceiling-price controls.

(8) Evasion of ceiling-price controls.

In the case of ceiling-price controls, a seller cannot charge for his product more than a ceiling price set by the government. Bowman (1957) first pointed out that tying sales may be used as an evasion of price regulation and provided an example of such a tying sale under price regulation.

In Bowman's example, two products are used together in fixed proportions, such as one nut and one bolt. From the buyer's point of view the two together might as well be a single product. The price of the combination is the only matter of interest. A monopolist who sells one part of such a combination while the other part is sold competitively could extract as much monopoly revenue from the sale of one part as from the sale of both. For example, if the price of bolts were set by a monopolist and the price of nuts were set by competition, tying the sale of nuts to the sale of bolts would not increase the monopoly profit. However, if the government were to place a ceiling on the price of bolts and leave the price of nuts uncontrolled, a tying arrangement would become beneficial. By this means, the monopolist could increase the nut price to maintain the profit-maximizing price for the bolt-nut combination. Thus, tying sales can be used to get around price controls and increase monopoly profits.

It can be seen that Bowman's analysis is based on the assumptions: (a) two products are used in fixed proportion, (b) one good is produced only by a monopolist and another one is produced competitively, and (c) one price is regulated and another one is free. If the two goods are used not in fixed proportion, the two goods are both supplied competitively or monopolistically, and both prices are regulated, will tying sales still be profitable? Bowman didn't provide further analysis.

After Bowman's analysis (1957), in all the literature of tying sales, economists argued that tying sales could be used to get around price controls (Singer 1968; Liebowitz 1983; Palmer 1984; Scherer and Ross 1990). For example, Palmer (1984, p679) pointed out that "Another motivation for tying sales is to evade price controls. Thus, if the price of gasoline is set at less than the market-clearing price, consumers may be unsubtly encouraged to purchase fan belts, tune-ups, etc. at comparatively high prices along with their gasoline". Scherer and Ross (1990, p567) also stated that "tying contracts may be employed to evade governmental price control - for example, when a firm supplying some commodity such as natural gas or telephone service whose price is regulated requires consumers to buy from it fixtures and attachments whose prices are not effectively controlled. Then the firm will set the fixture prices above competitive levels, capturing profits that regulation prevented it from retaining in its core monopolized business". However, no further research has been done on the issue of tying sales under price regulations.[15]

In addition, a review of literature on price regulations uncovered very little mention of the issue of tying sales under price regulations. Most of the literature on price regulations focuses on issues such as shortages, parallel or black markets, rent seeking, disequilibrium, welfare, etc., but not tying sales (see, for example, Galbraith 1952; Buchanan et al. 1980; Fung 1987; Deacon and Sonstelie 1989; Jonung 1990; Guo 1992). Skousen (1977) did mention that "tying agreements constitute one of the chief methods used to evade controls", but he also didn't provide further analysis.

In sum, the literature on tying sales and price controls suggests that tying sales can be used to evade price controls, but has not provided rigorous analysis. At the same time, the literature also hasn't provided alternative reasons for tying sales under price

[15] I am concerned with the government's price regulations, not the cartel's price controls. I will focus on the issue of tying sales under ceiling-price controls in the rest of the paper.

controls. It is my interest to study the issue of tying sales under price controls, to identify the reasons for tying sales and to determine how they work under price controls.

4.3 A Competitive-Tying Model Under Price Controls

A tying sale includes at least two goods: one is called the tying good and one is called the tied good. Those goods can both be produced competitively or by a monopolist. It is also possible that one good is produced competitively and the other good is produced by a monopolist. How the goods are produced will affect the analysis of tying sales under price controls. In this section, I first consider the phenomenon in which both goods in a tying sale are produced competitively, which I call a competitive-tying model under price controls. In the next section, I consider the phenomenon in which at least one good in a tying sale is produced only by a monopolist, which I will call a monopoly-tying model under price controls.

4.3.1 Demands Are Independent

When a consumer makes a decision on what goods and how much of each good to purchase in a multiple goods market, he will consider all the prices and the benefits to him among the goods, and of course his income. Thus a consumer's demand for a good normally depends not only on the good's own price but also on the other goods' prices. In that case, we say that the demands are interrelated. If the demand for each good depends only on its own price but not on others', we say that the demands are independent. For the sake of simplicity, I first consider the case of independent demands, and later consider the case of interrelated demands.

Assume that in a competitive economy there are two goods: good 1 and good 2. All the consumers have the same preferences. A representative consumer's demand functions for good 1 and good 2 are

$$x_1=x_1(p_1) \text{ and } x_2=x_2(p_2).$$

I also assume that $dx_i/dp_i<0$, $i=1, 2$ and $dx_i/dp_j=0$, $i\neq j$. Hence, the demands for good 1 and good 2 are independent. For simplicity I also assume that the demands are also independent of the consumer's income. In other words, there is no income effect on the demands. Therefore, the consumer's surplus under the demand curve will be the true measurement of the consumer's welfare.[16]

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[16] The assumptions of independent demands and no income effect are very strict. However, these properties of the demand functions can be derived from a specific utility function. For example, suppose a representative consumer's utility function is as follow:

$$U(x_1, x_2, y)=u_1(x_1)+u_2(x_2)+y.$$

Assume $u_i'(x_i)>0$, $u_i''(x_i)<0$, $i=1,2$. Thus, $u_i(x_i)$ are strictly quasi-concave and the demand curves are well-defined. A representative consumer maximizes his utility subject to his budget constrain:

$$\begin{aligned} &Max: && U(x_1,x_2,y)=u_1(x_1)+u_2(x_2)+y \\ &subject\ \ to: && p_1x_1+p_2x_2+y=m \end{aligned}$$

where p_i=price of per unit good x_i, $i=1,2$; m=income; the price of good y=1. The first order conditions are

$$\frac{dU_i}{dx_i}=\frac{du_i}{dx_i}=p_i \qquad i=1,2$$

Then from the first conditions we have demand functions

$$x_i=x_i(p_i) \qquad i=1,2$$

Hence, the demand for good x_i is a function of its own price, p_i. The demands for good 1 and good 2 are independent of each other and independent of the income, m.

The market demands for each good are the aggregations of the individual consumers' demands. Since I assume that the consumers' preferences are identical and the demands for good 1 and good 2 are independent of consumers' income, the market demands for each good can be expressed as follows:[17]

$$X_i = X_i(p_i) = Nx_i(p_i), \quad i=1,\ 2,$$

where X_i is the market demand for good i, and N is the total number of consumers. By the assumptions, $X_i(p_i)$ is well-defined demand curve. Since $dx_i/dp_i<0$, then $dX_i/dp_i<0$ (i=1, 2), i.e., the market demand is a downward-sloping curve.

I further assume that the total demand and supply curves of X_i are linear, as shown in Figure 4.1 and Figure 4.2. The linearity assumption is just for drawing simplicity but is not critical for the analysis. Its relaxation will not affect the results.

In Figure 4.1, p_{1E} is the free competitive equilibrium price at which the total demand for good 1 is equal to the total supply at quantity X_{1E}. In Figure 4.2, p_{2E} is the free competitive equilibrium price at which the supply and demand for good 2 are equal at X_{2E}. In both cases, the markets are "cleared".

Now, in the market for good 1, suppose for some reason, that the government imposes a ceiling price p_{1C} which is below the free competitive equilibrium price p_{1E}. As the price is artificially reduced, the consumers would want to buy more; on the other hand, the producers would reduce their supply. Consequently, the quantity demanded would be greater than the quantity supplied. Therefore, the market is not in equilibrium

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[17] I use small x to represent individual's demand, and use capital X to represent the market demand in this chapter.

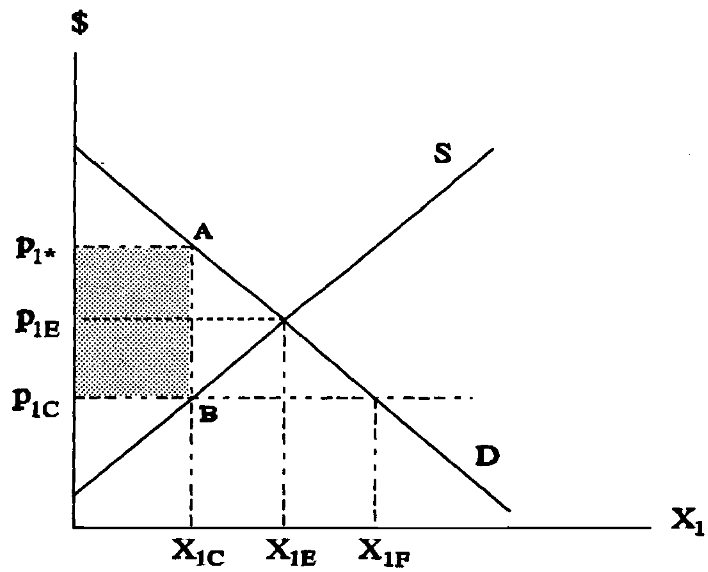

Figure 4.1. Market demand for and supply of good 1.

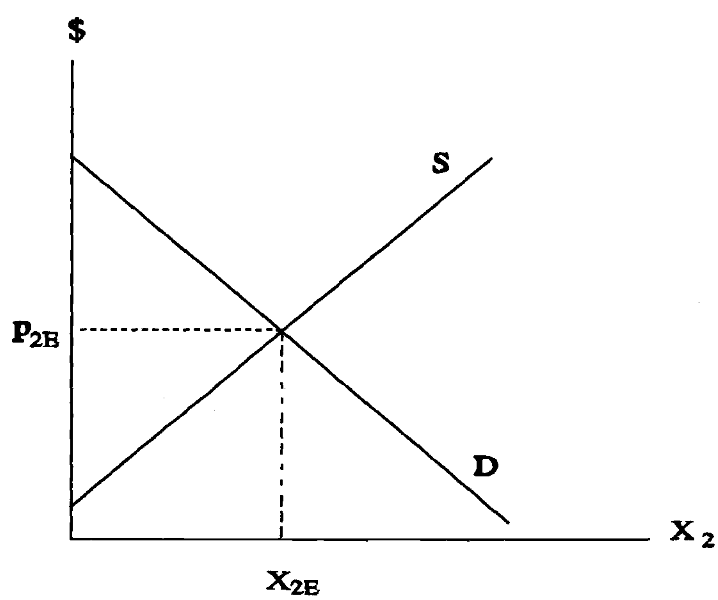

Figure 4.2. Market demand for and supply of good 2.

at the ceiling price. In Figure 4.1, at ceiling price p_{1C}, the quantity demanded of good 1 is X_{1F} and the supply is X_{1C}. There exists an excess demand equal to $(X_{1F}-X_{1C})$.

For the amount X_{1C} of good 1, the uncontrolled equilibrium price would be p_{1*} which is higher than the ceiling price p_{1C}. Price controls would generate a shortage and disequilibrium in most cases (Mohammad and Whalley 1984; Davis and Charemza 1989; Deacon and Sonstelie 1989). For analytic convenience, we may call the area $p_{1*}ABp_{1C}$ in Figure 4.1 "contrived surplus".[18] The property right of the contrived surplus is generally not assigned. I will discuss this more later.

Facing the shortage and disequilibrium problems, sellers usually have three ways to sell their goods: (1) on a first-come-first-serve basis, (2) through rationing, and (3) through a privilege plus first-come-first-serve arrangement.[19]

Under the privilege plus first-come-first-serve arrangement, those people who are privileged (e.g., through membership in particular organizations) can buy whatever they want. The rest of the people have to buy the good on a first-come-first-serve basis,

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[18] Some researchers also call the area "rent" (see for example, Deacon and Sonstelie 1989; Mohammad and Whalley 1982). Fung (1987) calls this "contrived surplus". Precisely, it is neither a "rent" nor a "contrived surplus". It is not rent because it is a part of consumer surplus and producer surplus in the absence of price control. It is not a "contrived surplus" because it exists in the absence of the price control. It is a surplus whose property right is generally not assigned under the price control. For analytic convenience, I may call it "contrived surplus" in the analysis.

[19] The sellers also could sell their goods in a black market. When price controls are imposed, black markets are likely to occur. Many issues of black markets are very important to the economy and the producers and the consumers under price controls. Much research has been done (Michaely 1954; Browning and Culbertson 1974; Mukherji et al. 1980; Bevan et al. 1989). However, I don't consider it as a formal selling channel here because the black market is illegal.

which causes queues. Felder (1993) and Fung (1987) have provided detailed analyses of this case, which has been particularly relevant in centrally-planned economies.

Under rationing, the people who have ration coupons can buy quantity of the good up to the limit of the coupons. Ration coupons have been used quite often when the government controls prices in times of war or shortages caused by natural disaster. Much research has been done on this phenomenon (see, for example, Tobin 1955; Pollak 1969; Neary 1980).

Under the first-come-first-serve arrangement, people who want to buy the good have to come earlier than others in order to make sure that they can buy the good. People who don't want to wait will have no chance to buy any amount of the good. (unless they pay someone to stand in line for them. I will discuss this more later.) Controlled prices and the accompanying shortages force some consumers out of the market and cause queues simultaneously. This case is important for the present study because I believe that shortages and queues can motivate the sellers to use a tying sales strategy to increase their profits.

Again, consider Figure 4.1. The ceiling price creates a contrived surplus which is equal to the Area ($p_{1*}ABp_{1C}$). Under the ration coupon or privilege cases, the people who have the ration coupons or the privilege buy the good and capture the contrived surplus. Under the first-come-first-serve case, the people who buy the good capture the contrived surplus. In the last case, however, people who buy the good have to be in the queue, in which some waiting costs occur. So the real price paid by a buyer is the ceiling price plus waiting costs. In equilibrium, the real price to the buyers would be

p_{1*} at which the demand for good 1 is equal to X_{1C}. The contrived surplus created by the ceiling price is totally dissipated by waiting costs.[20]

In the market for good 2, I assume that there is no price control or that the ceiling price is higher than the free competitive equilibrium price.

Since it is assumed that the consumers are identical, each consumer will buy the same amount of good 1 and of good 2. If the total supply of good 1 is X_{1C} at the ceiling price p_{1C}, the supply of good 1 to a representative consumer would be $x_{1C}= X_{1C}/N$. Figure 4.3 shows the demand for and supply of good 1 for a representative consumer. For that consumer, the supply of good 1 is fixed at x_{1C} because of the ceiling price. The supply of good 2 to a consumer is infinite at price p_{2E} because of free competition (Figure 4.4).

Now consider a competitive seller who sells both good 1 and good 2. The seller will know that the ceiling price for good 1 is lower than the free competitive market price and queues occur, by observing the market transactions. I assume that the seller knows the representative consumer's demands for good 1 and good 2 such as in Figure 4.3 and Figure 4.4. The consumer purchases amount x_{1C} of good 1 at ceiling price p_{1C} plus waiting costs, i.e., the total cost is x_{1C} times p_{1*} (Figure 4.3), so that the real price

[20] p_{1*} represents the consumers' willingness to pay for good 1 at the supply of X_{1C}. Instead of waiting by himself in the queue, a consumer may be willing to pay someone else to stand in line for him. Because all consumers are the same, competition among them will fore each one to pay $(p_{1*}-p_{1C})$. The real price for a consumer is p_{1*}. Thus, the value of $(p_{1*}-p_{1C})$ represents either a consumer's waiting cost or a consumer's payment to someone else standing in line for him. In real world most consumers may stand in line by themselves rather than pay someone else to stand in line for them. Therefore, for the sake of simplicity, I will assume that all the consumers who want to buy the price-controlled good will stand in line by themselves, and their real cost for the controlled good includes the ceiling price and waiting cost.

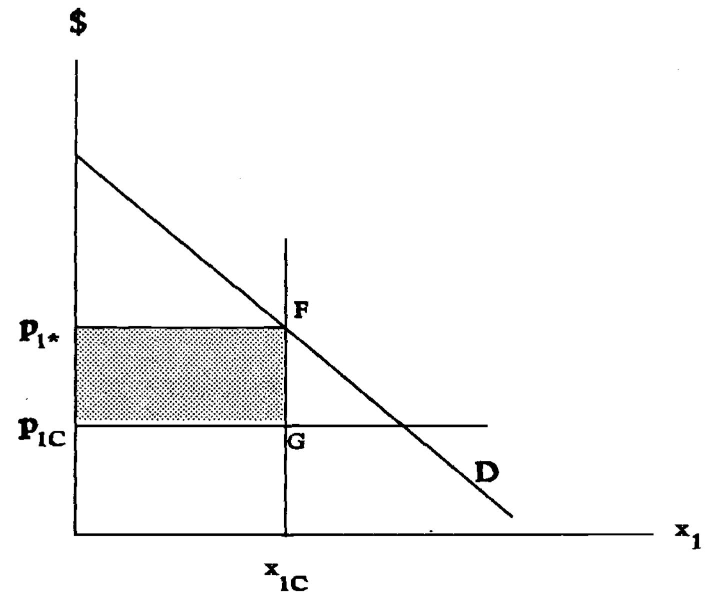

Figure 4.3. Demand for and supply of good 1 for a consumer.

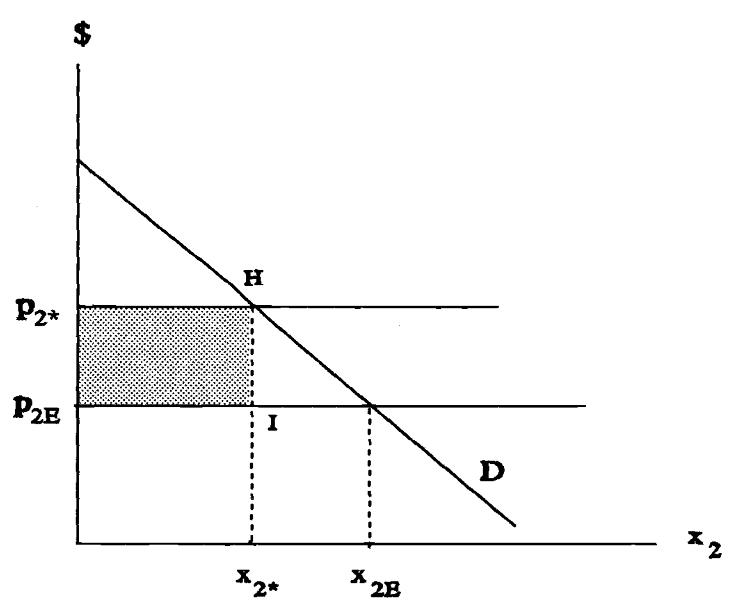

Figure 4.4. Demand for and supply of good 2 for a consumer.

of good 1 to the consumer is p_{1*}. Once the seller realizes that good 1 is in short supply and the consumer pays much more than the ceiling price to obtain the good, he will try to extract some of the contrived surplus that is dissipated to the waiting costs. The tying sales would be a good pricing strategy to achieve his objective. Let us analyze how the tying sales arrangements work.

Suppose that the seller raises the price of good 2 from p_{2E} to p_{2*} in Figure 4.4. Now instead of selling x_{1C} at ceiling price p_{1C} on a first-come-first-serve basis, the seller decides to tie the two goods together. He wants to sell good 1 at the ceiling price only to those who also buy amount x_{2*} of good 2 at price p_{2*} from him. So that in the presence of tying sales, a consumer faces two supply curves for good 2 as in Figure 4.4. One is provided by the seller of tying sales at price p_{2*}, and one is provided by the competitive market at price p_{2E}. If the price is at p_{2*}, the consumer will buy amount x_{2*} of good 2. If the consumer decides to take the package, he could buy good 1 right away and have no waiting cost. If the consumer doesn't like the package deal, he can buy good 2 at the competitive market and buy good 1 in the queue. Would the consumer accept the offer?

Under the tying sales arrangement, a consumer's purchase can be completed in two steps. First, the consumer purchases amount x_{1C} of good 1 at ceiling price p_{1C} without any waiting cost, and at the same time, purchases amount x_{2*} of good 2 at price p_{2*}. Second, he purchases amount $(x_{2E}-x_{2*})$ of good 2 at price p_{2E} from the competitive market. By taking the package of tying sales, the consumer saves all the waiting costs (Area $p_{1*}FGp_{1C}$) for good 1, and pays more money (Area $p_{2*}HIp_{2E}$) for good 2, and consumes the same amount of good 1 and good 2 as that in the absence of the tying sales arrangement. If the price p_{2*} and amount x_{2*} are such that

$$(p_{2*}-p_{2E})x_{2*} \leq (p_{1*}-p_{1C})x_{1C},$$

i.e., the Area ($p_{1*}FGp_{1C}$) in Figure 4.3 is less than or equal to the Area ($p_{2*}HIp_{2E}$) in Figure 4.4, the consumer would spend less or the same amount of money and consume the same amount of both good 1 and good 2 as in the absence of tying sales. The consumer's surplus has not declined. Thus, the consumer will accept the tying sales arrangement. On the other side, the seller makes a profit of Area ($p_{1*}FGp_{1C}$) through the tying sales arrangement. Under tying sales, the real price that the consumer pays is still p_{1*}, and the real price received by the seller goes up from p_{1C} to p_{1*}. The former waiting costs become the seller's profits.

Readers may ask two questions. One is can the seller of good 1 use a pricing strategy involving a two-part tariff or "fee" to extract the contrived surplus? Yes, he can do that theoretically. However, any two-part tariff or "fee" can be easily considered as directly raising the price of the controlled good. The authority of the price controls will not tolerate any direct raising of the price of the controlled good. Thus, I would assume that a two-part tariff or "fee" is illegal under the price controls.

Another question is, instead of only the contrived surplus, why doesn't the seller extract all the consumer's surplus by using tying sales arrangements? If the seller extracts all the consumer's surplus by tying sales, he will make the consumer worse off than that without tying sales. In a competitive market, the consumer will go to other sellers. So, for a seller in a competitive market, the best strategy would be to extract all the contrived surplus and leave the consumer as well off as without the tying sale.

4.3.2 Equilibrium Analysis

In the above analysis, the seller makes a positive profit through tying sales. This profit comes from the contrived surplus created by the ceiling price (see Figure 4.1). The positive profit with tying sales will influence the market for good 1 in three aspects: (1) motivating the rest of the sellers of good 1 to use the tying sales strategy, (2) encouraging the producers who are already in the market at the ceiling price to produce more, and (3) attracting some new entrants.

Suppose that the demand and supply of good 1 for a representative producer are as in Figure 4.5. The demand facing a competitive producer is a horizontal line at the market price. The producer's supply is upward-sloping. At the ceiling price p_{1C}, he would produce amount x_{1C} of good 1. If the market price were p_{1*}, he would increase his production to x_{1*}. Since the real price of good 1 to a producer under the tying sales arrangement goes up from p_{1C} to p_{1*}, the producer will increase his production of good 1.

At the same time, those producers who lose money at the ceiling price would now want to enter the market because they can capture some of the contrived surplus by tying sales arrangements. Suppose that a seller of good 1 loses money at the ceiling price, therefore, is not in the market after the ceiling price is imposed. Now this seller would enter the market as long as the contrived surplus that he captures through a tying sales arrangement is greater than or equal to the loss of his selling good 1 at the ceiling price.

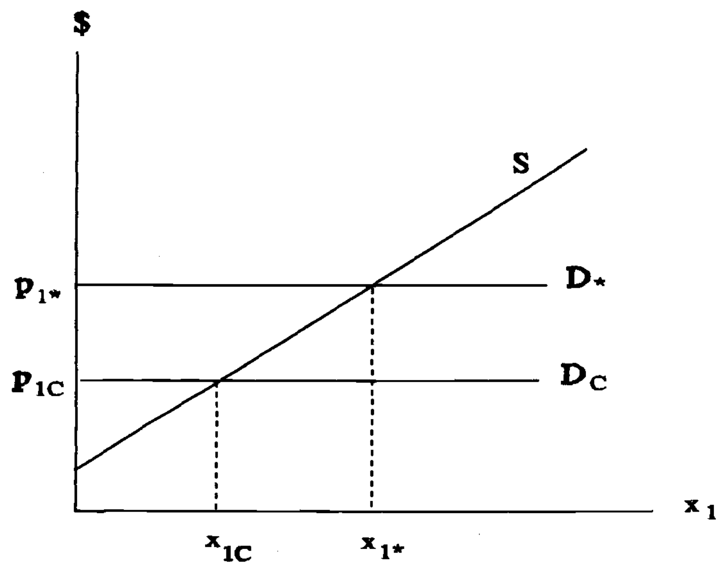

Figure 4.5. Demand for and supply of good 1 for a competitive producer.

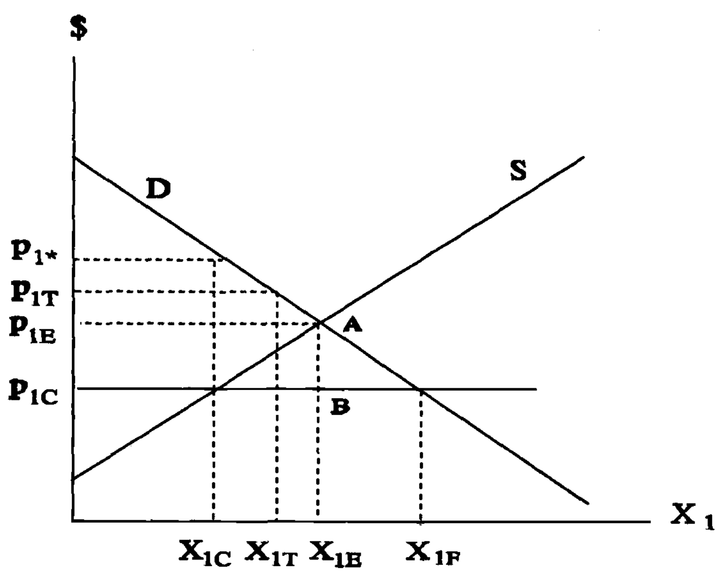

Figure 4.6. Market demand for and supply of good 1.

Therefore, the positive profits through the tying sales arrangements not only motivate the producers to produce more, but also attract some new entrants. Consequently, the market supply of good 1 will increase along the supply curve in Figure 4.6,[21] though the ceiling price is still the same.

While the supply of good 1 increases along the supply curve, the consumers' demand price decreases along the demand curve. For instance, when the supplied quantity of good 1 increases from X_{1C} to X_{1T}, the demand price decreases from p_{1*} to p_{1T}. I have pointed out that for any quantity of good 1, the real price level facing a consumer is not the ceiling price because of the waiting costs: for each quantity supplied the real price to the consumer is determined along the demand curve . This price level is also the real price to a seller who uses the pricing strategy of tying sales. As long as the demand price is greater than the supply price for each quantity, the producer will be able to make a positive profit through a tying sale arrangement. While the positive profits exist, the quantity supplied of good 1 will keep increasing. It can be expected that equilibrium will be achieved at the point (p_{1E}, X_{1E}) in Figure 4.6.

At (p_{1E}, X_{1E}), the Area ($p_{1E}ABp_{1C}$) is the total contrived surplus captured by the producers through tying sales arrangements. At that quantity level of good 1, the real price to a producer is p_{1E} which is equal to the supply price. For any quantity level beyond X_{1E}, the real price to a producer will be less than the supply price. Thus, the point (p_{1E}, X_{1E}) is indeed an equilibrium point. We may have already noted that the equilibrium levels of price and quantity are the same as those in the absence of price

[21] Figure 4.6 is the same as Figure 4.1. I redraw it here just for convenience.

controls. Therefore, the producers have fully gotten around the price controls. The real price level and the quantity sold are the same both to the consumers and to the producers as those in the absence of price controls.

However, we should note some things that are different. Under the price control, the market demand for good 1 is X_{1F} at the ceiling price $\bar{p_{1C}}$. Even though the quantity supplied increases from X_{1C} to X_{1E} through tying sales arrangements, there still exists an excess demand (X_{1F} - X_{1E}) for good 1. It is the excess demand that forces the consumers to accept the tying sales arrangements, in order to avoid the queues and associated costs.

Therefore, in a price controlled competitive market, the strategy of tying sales eventually increases the total quantity supplied and reduces the real price paid by the consumers. Although the market equilibrium under tying sales arrangements is the same as that in the absence of price controls, there exists an excess demand under price controls, i.e., the quantity demanded at the ceiling price level is still greater than the quantity supplied.

4.3.3 Demands Are Interrelated

In the above analysis, I assumed that the demands are independent. What happens in one market has no influence on the other market. However, in actual business practices, tying sales arrangements include not only independent goods, but also substitute or complementary goods. When substitute or complementary goods are involved in tying sales arrangements, the demands for the tying good and tied good are interrelated by definition. We need to analyze how the interrelated demands will affect the tying sales strategy in the competitive model.

Suppose that a representative consumer has demand functions for good 1 and good 2 as follows:

$$x_1 = x_1(p_1, p_2) \tag{4.1}$$

$$x_2 = x_2(p_1, p_2) \tag{4.2}$$

where p_i is the price of good i (i=1, 2). It is also assumed that

$$dx_i/dp_i<0 \quad \text{and} \quad dx_i/dp_j\neq 0 \ (i, j=1, 2) \tag{4.3}$$

i.e., the demand for each good is downward sloping in own price and depends on both goods' prices. When either price changes, demand for both goods will be influenced. Thus, the demands for good 1 and good 2 are interrelated. If $dx_i/dp_j > 0$, we say that the two goods are substitutes in consumption. If $dx_i/dp_j < 0$, we say that the two goods are complements in consumption.

In addition, I assume that the market demand for X_1 is the aggregation of N identical consumers' demands given by equation (4.1). The market demands for X_2 are the aggregation of N identical consumers' demands given by equation (4.2). Therefore, we have

$$X_1 = Nx_1(p_1, p_2) = X_1(p_1, p_2) \tag{4.4}$$

$$X_2 = Nx_2(p_1, p_2) = X_2(p_1, p_2). \tag{4.5}$$

From equation (4.3) we have

$$dX_i/dp_i<0 \quad \text{and} \quad dX_i/dp_j\neq 0, \ i, j=1, 2. \tag{4.6}$$

Hence, the market demand for each good depends on both goods' prices. When either price changes, both will be influenced. The market demands for good 1 and good 2 are

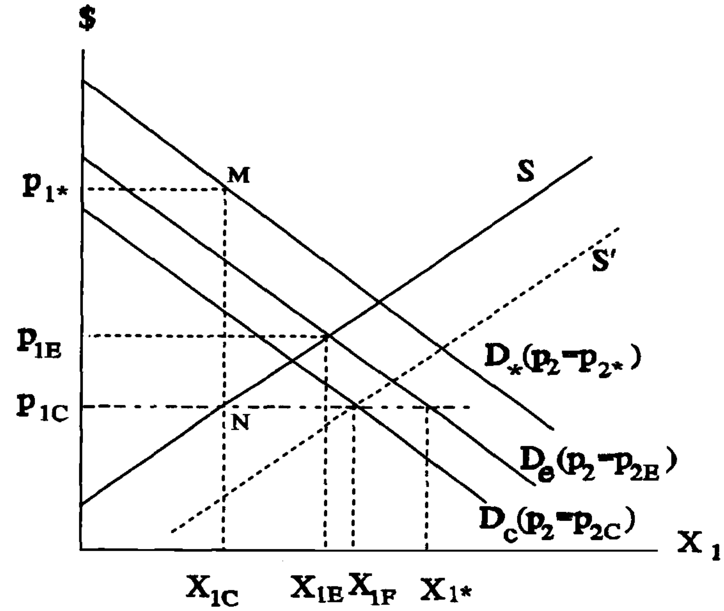

Figure 4.7. Market demand for and supply of good 1 when demands are interrelated.

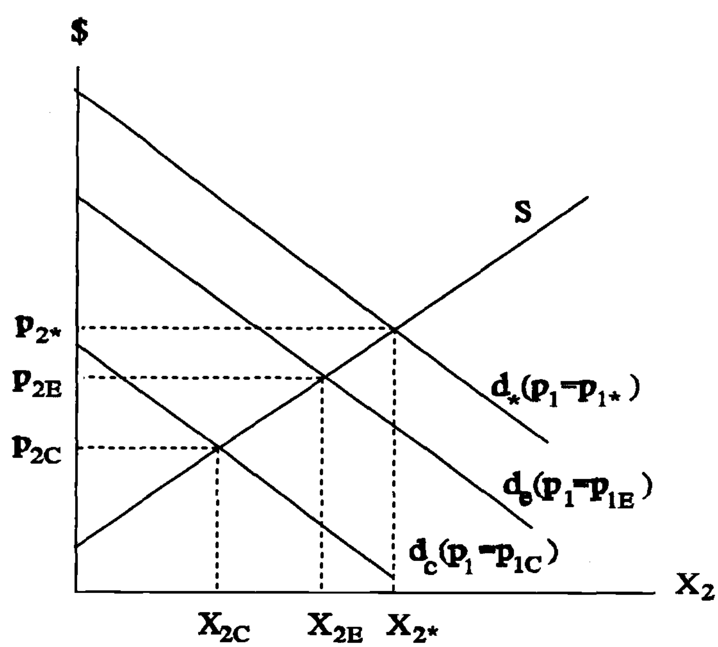

Figure 4.8. Market demand for and supply of good 2 when demands are interrelated.

interrelated. If $dX_i/dp_j > 0$, we say that the two goods are substitutes in consumption. If $dX_i/dp_j < 0$, we say that the two goods are complements in consumption. In the following analysis, I assume that the goods are substitutes. The main findings will be applicable to the case of complementary demands.

For any given price of good i, we can express the demand for good j as a function of its own price. Thus, each market demand curve in Figure 4.7 and Figure 4.8 corresponds to a given price level of the other good. For example, the curve $D_e(p_2=p_{2E})$ in Figure 4.7 is drawn given $p_2=p_{2E}$. If the price p_2 changes, the demand curve for good 1 will change. Suppose that in the absence of price controls, both markets are cleared at (p_{1E}, X_{1E}) and (p_{2E}, X_{2E}) (see Figure 4.7 and Figure 4.8).

Now, in the market for good 1, suppose that the supply curve shifts to the right from S to S'. Given $p_2=p_{2E}$, the equilibrium price in the market for good 1 will go down along the demand curve $D_e(p_2=p_{2E})$. Since the price of good 1 goes down, the quantity of good 2 demanded will drop given any price of good 2; that is, the demand curve for good 2 will shift to the left. When the demand curve for good 2 shifts to the left, the equilibrium price of good 2 goes down, in turn, the demand curve for good 1 also shifts to the left. Consequently, an increase in supply of good 1 causes both demands for good 1 and good 2 shift to the left, and the new equilibria are

$$D_c(p_2=p_{2C}),\ (p_{1C}, X_{1F}),\ d_c(p_1=p_{1C}) \text{ and } (p_{2C}, X_{2C})$$

for good 1 and good 2 respectively. However, the way things work under ceiling prices is much different.

Instead of a change in supply of good 1, suppose that for whatever reason, the government imposes a ceiling price p_{1C} which is below the free competitive equilibrium price p_{1E} (Figure 4.7).

While the market price for good 1 goes down from p_{1E} to the ceiling price p_{1C}, the demand for good 2 shifts to the left from $d_e(p_1=p_{1E})$ to $d_c(p_1=p_{1C})$. The market price for good 2 goes down to p_{2C}. At the same time, the demand for good 1 shifts to the left from $D_e(p_2=p_{2E})$ to $D_c(p_2=p_{2C})$. Because the price p_{1C} is artificially imposed by the government and not determined by the market, the supply of good 1 is fixed at X_{1C}, and the demand for good 1 is larger than the supply at the ceiling price p_{1C}. On a first-come-first-serve basis, the consumers have to be in a queue to purchase good 1. The real unit cost of good 1 to a consumer, then, is the ceiling price p_{1C} plus some waiting costs. Because the real price of good 1 is larger than the ceiling price p_{1C}, the demand curve $d_c(p_1=p_{1C})$ for good 2 will shift to the right and the market price of good 2 will increase; in turn, the demand curve $D_c(p_2=p_{2C})$ for good 1 will shift to the right. Suppose that $D_*(p_2=p_{2*})$ and $d_*(p_1=p_{1*})$ are the demand curves for good 1 and good 2 respectively at the new equilibrium prices (p_{1*}, p_{2*}), where p_{1*}, equal to the ceiling price p_{1C} plus waiting costs, is the real price of good 1 to the consumers.

The area $(p_{1*}MNp_{1C})$ in Figure 4.7 is the contrived surplus created by the price control and dissipated to the waiting costs. When the producers of good 1 adopt the strategy of tying sales, they turn the contrived surplus into their profits. As I have pointed out in the section on equilibrium analysis, the positive profits under the tying sales arrangements will not only motivate the producers to produce more, but also attract some new entrants. Consequently, the market supply of good 1 will increase along the supply curve, though the ceiling price is still unchanged. While the supply of good 1 increases, the real price p_{1*} to the consumers decreases. When p_{1*} decreases, the demand for good 2 will shift to the left again and the market price p_{2*} will decrease; in turn, the demand for good 1 will also shift to the left. We can expect that the two markets will eventually arrive at the initial equilibrium (p_{1E}, p_{2E}).

Thus, the producers have fully gotten around the price control. The real price level and the quantity level are the same both to the consumers and the producers as those in the absence of price controls. The price control fails. The conclusions at which I arrived in the case of independent demands are also applied to the case of interrelated market demands.

In the above analysis, I assume that the goods are substitutes in consumption. The analysis is applicable to the case where the goods are complements. It can be expected that the final equilibrium would be the same as that in the case of substitute goods, although the process of reaching the final equilibrium is different.

At this point, I would like to have a brief summary for the competitive-tying model. Price control in a competitive market is accompanied by a shortage. The shortage creates a contrived surplus which is equal to the area between the ceiling price and the market-clearing price. The contrived surplus could be totally dissipated to waiting costs or could be captured by the consumers who have ration coupons or a purchasing privilege. In order to capture the contrived surplus, the producers are motivated to use tying sales arrangements. Under the tying sales arrangements, the tied goods in demand can be independent, substitutes or complements of the tying good. In most cases, the major restriction is that the prices of the tied goods must be allowed to increase legally.

In the analysis, I assume that (a) a representative producer produces both good 1 and good 2, and (b) a representative consumer consumes both good 1 and good 2. Actually these two assumptions are not critical to the competitive-tying model. If the producer produces only good 1, he can always buy any amount of good 2 from the free competitive market at the market price. This means that the choice of the tied goods is very flexible. For example, if the tying good is shrimp, the producer of shrimp can choose salt, sugar or cigarettes as a tied good as long as those goods are available in the competitive markets.

On the consumer side, if we assume that a buyer of a competitive good can always resell the good at the market price and has no transaction cost, we then can relax the assumption of consuming both goods. In that case, even if a consumer does not consume good 2, the tied good, he may still take the tying package. All he needs to do is to resell the tied good on the market and keep the tying good. Skousen (1977, p184) told a very interesting story of tying arrangements under the Nazi controls:

> "A peasant was arrested and put on trial for having repeatedly sold his old dog together with a pig. When a private buyer of pig came to him, a sale was staged according to the official rules. The buyer would ask the peasant:"How much is the pig?" The cunning peasant would answer:"I cannot ask you for more than the official price. But how much will you pay for my dog which I also want to sell?" Then the peasant and the buyer of the pig would no longer discuss the price of the pig, but only the price of the dog. They would come to an understanding about the price of the dog, and when an agreement was reached, the buyer got the pig too. The price for the pig was quite correct, strictly according to the rules, but the buyer had paid a high price for the dog. Afterward, the

> buyer, wanting to get rid of the useless dog, released him, and he ran back to his old master for whom he was indeed a treasure."

The above story in some degree supports my analysis. Whether the producer produces both the tying good and the tied good is not critical in the competitive-tying model. And whether the consumer consumes both the tying good and the tied good is also not critical. The essential assumption is that the ceiling price causes a shortage, and the shortage creates a contrived surplus. In order to capture the contrived surplus, sellers use the tying sales strategy. By doing that, the producer increases his profit beyond that associated with sales of the single good at the controlled price.

In the mainstream of tying sales economics, the tying good is always a monopoly good. No literature has shown that the tying good could be competitively supplied. Economists believe that if a seller wants to use a competitively supplied good as a tying good, the consumers will just go away because the good is freely available in the market. "A competitive supplier, selling at the prevailing price and attempting to impose a tying upon a buyer, would merely be displaced by a seller who did not" (Bowman 1957, p20). I have shown in the above analysis that a competitively supplied good could be used as a tying good in a tying sales arrangement under price controls, and how it works.

4.4 A Monopoly-Tying Model Under Price Controls

In the above analysis, I assumed that in the tying sales arrangements, the tying good and tied good are both competitively supplied. In the real business world, many goods are supplied, instead, by firms that do not take prices as given. In this section, I want to explore what are the motivations for a monopolist to use the strategy of tying sales under price controls, and how it works. Because the tying sales arrangement includes at least one good that is monopoly-supplied, I call the model a monopoly-tying model under price controls.

In the literature on tying sales, tying and bundling are the two terms that are frequently used. Under a tying arrangement, the purchaser of some good agrees as a condition of purchase (or lease) to buy from the seller supplies of some other commodity. Under a bundling practice, the seller insists that the buyer take a package of products, bundled together and offered at a single price per bundle. Thus, bundling can be considered as a special case of tying sales. I may use these two terms interchangeably in the following analysis, though the cases described are closer to bundling.

I first consider the case of a single-good monopolist under price controls, and later consider a two-good monopolist. By saying a single-good monopolist, I mean that the monopolist produces only one monopoly good, though he may be a multiproduct producer. A two-good monopolist, however, produces two monopoly goods.

4.4.1 A Single-Product Monopolist Under Price Controls

To analyze the tying sales strategy for a single-product monopolist, I make the following assumptions:

(a) The producer sells two goods: the producer is the only seller of good 1, but sells good 2 in a competitive market.

(b) Marginal production costs for good 1 and good 2 are constant: c_1, c_2.

(c) Good 1 is indivisible; i.e., it is bought or sold only in discrete units.

(d) Tastes are different across consumers.

(e) Each consumer purchases only one unit of good 1 if the price is less than or equal to his reservation price. The marginal utility of the second unit of good 1 is zero.

(f) Demand for good 1 is a downward-sloping curve as shown in Figure 4.9.

(g) Consumers are uniformly distributed along a negatively-sloped demand curve.

(h) Demand for good 1 is $p_1=p_1(X_1)$, where X_1 is in discrete units only.

Under the above assumptions, the total profit for the monopolist is

$$\pi = p_1(X_1)X_1 - c_1X_1.$$ [22]

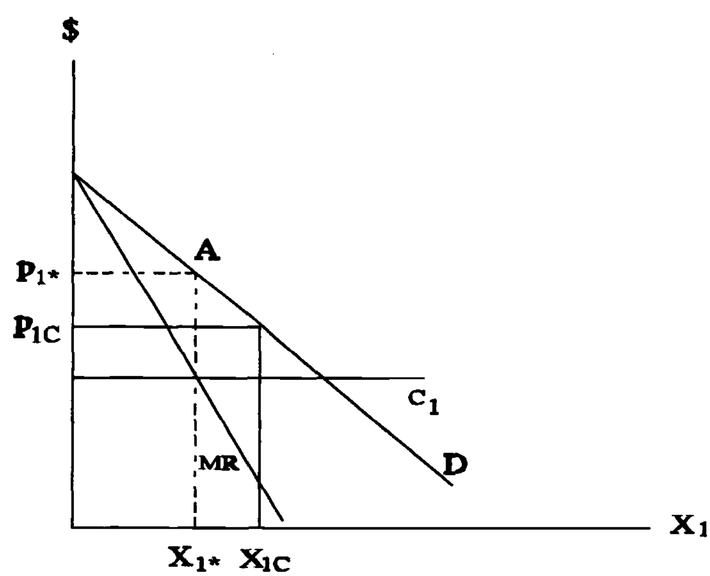

Figure 4.9. Market demand for a monopolist of good 1.

[22] The profit from selling good 2 is zero because of the competition assumption. Thus, π is obtained only from selling good 1.

I also assume that first degree price discrimination is impossible. The monopolist must set a uniform price for all the consumers. The first order condition is

$$\frac{d\pi}{dX_1}=p_1(X_1)+\frac{dp_1(X_1)}{dX_1}X_1-c_1=o$$

That is

$$p_{1*}(X_{1*})+\frac{dp_{1*}(X_{1*})}{dX_{1*}}X_{1*}=c_1$$

Assume that the second order condition is satisfied. Hence, in order to maximize his profit, the monopolist would set the price p_{1*} at the level where the marginal revenue equals the marginal cost (Figure 4.9). The consumers consume amount X_{1*} of good 1 at price p_{1*}. The price p_{1*} can be interpreted as the marginal consumer's reservation price for good 1.

Now, suppose that the government considers the monopolist's price to be too high, and wants to reduce some of the monopolist's market power. To do so, the government imposes a ceiling price p_{1C} on the monopoly market. It is assumed that the ceiling price p_{1C} is less than the monopolist's price and greater than the monopolist's marginal cost. That is

$$c_1<p_{1C}<p_{1*}.$$

Under the price control, the marginal revenue for the monopolist is constant and equal to the ceiling price p_{1C} when the production is less than or equal to X_{1C} (Figure 4.9). The marginal revenue is less than the marginal cost when the production is larger than X_{1C}. The monopolist would supply amount X_{1C} of good 1 to the market. The monopolist's profit is

$$\pi=(p_{1C}-c_1)X_{1C}$$

which must be less than that in the absence of the price control. If there were no price control, the monopolist would increase price to p_{1*} in maximizing profit. However, he cannot raise the price of good 1 to maximize profit under the price control.

Therefore, price control has reduced the monopoly price and indirectly increased the supply of good 1. The controlled market is cleared, i.e., the quantity demanded equals the quantity supplied. The shortage and queues that are observed in a controlled competitive market don't occur here. However, the monopolist may be able to use a tying sales arrangement to get around the price control and maximize profits. Let us see how it works.

It is assumed that good 2 is competitively supplied. Let the market price for good 2 be p_{2E}. The monopolist sells good 1 at price p_{1C} and good 2 at p_{2E}. The total profit is less than that in the absence of the price control. Looking for maximum profits, the monopolist decides to use the strategy of tying sales to get around the price control and maximize his profits. Now, the monopolist requires each buyer of good 1 to buy one unit of good 2 from him at price p_{2*}. Otherwise, he will not sell good 1 alone. The price p_{2*} for the one unit of good 2 will be greater than the market price p_{2E}. In other words, the monopolist bundles one unit of good 1 and one unit of good 2 together and sells them as a package at a price $p_b=(p_{1C}+p_{2*})$ which is greater than the sum $(p_{1C}+p_{2E})$.[23]

[23] It is also possible for the monopolist to bundle one unit of good 1 and two, three or more units of good 2 together. For analytical convenience, I let the monopolist bundle together one unit of each good.

Thus, under the tying sales arrangement consumers are forced to make a decision regarding whether or not to take the package at the price $p_b=(p_{1C}+p_{2*})$. Suppose that consumer i has reservation price R_i for good 1. He will obtain amount (R_i-p_{1C}) of consumer surplus if he purchases one unit of good 1. He will lose amount $(p_{2*}-p_{2E})$ of consumer surplus if he purchases one unit of good 2 from the monopolist rather than from the competitive market. The net benefit for the consumer to take the package is

$$(R_i-p_{ic})-(p_{2*}-p_{2e}).$$

Therefore, if

$$(R_i-p_{ic})-(p_{2*}-p_{2e})\geq 0,$$

consumer i will buy the package. Otherwise, he will not buy good 1, and instead, just buy good 2 from the competitive market at price p_{2E}.

How does the monopolist set an optimal price p_{2*} that can provide the maximum profit? I have shown that the price p_{1*} is the price at which the monopolist has maximum profits in the absence of price controls. The optimal price for a package of one unit good 1 plus one unit good 2 is $p_b=(p_{1*}+p_{2E})$ in the absence of price controls. If the price for a package under the price control is set at the level $p_b=(p_{1*}+p_{2E})$, the profit should be maximized. Therefore, the optimal price p_{2*} would be such that:

$$p_{1*} + p_{2E} = p_{1C} + p_{2*}.$$

Rearranging above equation, we obtain

$$p_{2*} = (p_{1*}-p_{1C}) + p_{2E}.$$

With this arrangement of price p_{2*}, the price for a package of one unit of good 1 plus one unit of good 2 would be $p_b=(p_{1C}+p_{2*})$ and the monopolist doesn't violate the ceiling price for good 1. However, this price arrangement is the same as $(p_{1*}+p_{2E})$. Thus, the real price to a consumer for good 1 is the monopoly price p_{1*}. Tying sales just help the monopolist to get around the price control and obtain a maximum profit. From Figure 4.9, under the tying sales arrangement, the consumers to the right of A will no longer purchase good 1 because their reservation prices are less than p_{1*}, while those to the left of A with reservation prices greater than p_{1*} will purchase good 1. The total quantity of good 1 demanded will be X_{1*}.

Hence, in this single-product monopolist tying model, the monopolist cannot raise the price directly to maximize his profit because of the price regulation. In order to maximize the profit, he uses tying sales to get around the price control and obtain maximum profit. With tying sales arrangements, the consumers and the monopolist are as well off as they are in the absence of the price control. The price control, therefore, totally fails.

4.4.2 A Two-Good Monopolist Under Price Controls

In the above analyses, I have addressed some problems related to the price control and tying sales by using both the competitive-tying model and the single-product monopolist tying model. The producers are motivated by profit maximizing to try to get around the price control and increase their profits by using the strategy of tying sales. Although the objective of using tying sales to avoid price controls has been mentioned in the literature for a long time, I provide the first detailed analysis of the phenomenon. As I have pointed out in the literature review, many economists argue that the objective of using tying sales under price controls is to get around the price controls. However, no one has explored other motivations for use of tying sales in the presence of price controls. In fact, tying sales can also be used to increase the producers' profits without trying to get around the price controls. This is illustrated in the case where a seller is the monopolistic supplier of two goods, both of which are subject to price ceilings. I address this issue next.

The analysis proceeds under the following assumptions:

(1) A monopolist produces two goods: good 1 and good 2, expressed as x_1 and x_2 respectively. Both goods are supplied only by the monopolist.

(2) The unit costs for good 1 and good 2 are constant: c_1 and c_2.

(3) The consumers' preferences are different across consumers, as reflected in different reservation prices. The marginal utility of a second unit of each good is zero for all the consumers. Each consumer, then, will buy no more than one unit of each good.

(4) The demands for good 1 and good 2 are independent, i.e., the demand for one good is independent of the other good's price and consumption. Further, the consumer's reservation price for one good won't be influenced by consuming the other good or by the other good's price.

(5) Every consumer is completely described by a pair of reservation prices (R_1, R_2), where R_1 represents the consumer's reservation price for good 1 and R_2 represents the consumer's reservation price for good 2.

(6) The prices for good 1 and good 2 are both regulated through ceiling prices p_{1c} and p_{2c}, and they are both binding. In another word, both p_{1c} and p_{2c} are less than the monopolistic prices.

(7) It is impossible for consumers to resell the goods.

(8) Each consumer's reservation price (R_B) for a bundle containing one unit of each good is equal to the sum of his reservation prices (R_1, R_2) for the two individual goods (i.e., $R_B=R_1+R_2$).

Thus, I call this model a monopoly-tying model with a two-good monopolist under price controls. Under the above assumptions, the tying sales arrangement and commodity bundling are the same. Thus, I may use the term of bundling instead of tying in the following analysis. Generally, the monopolist has four options to sell his two goods:

(a) Sell the two goods separately at the prices p_1 and p_2 equal to the ceiling prices.

(b) Sell the two goods in a package containing one unit of each good at a bundling price p_B less than or equal to the sum of the ceiling prices.

(c) Combine strategies (a) and (b) by offering each good separately and a package of both, at a set of prices (p_1, p_2, p_B).

(d) Sell one good alone at p_1(or p_2) and sell the other only in a package containing one unit of each good at a bundling price p_B.

Let us call these strategies as separate components pricing, pure bundling pricing, mixed bundling pricing and quasi-mixed bundling pricing respectively. The present analysis differs from the earlier analyses in that adoption of the bundling strategy (tying strategy) is motivated by price controls, this time in the markets for two goods.

4.4.2.1 *The Strategy of Pure Bundling Pricing vs. the Strategy of Separate Components Pricing Under Price Controls*

Figure 4.10 is a two-dimension reservation prices space in which consumers' populations are distributed. Every consumer is completely described by a pair of reservation prices (R_1, R_2) in the reservation prices space, where R_1 represents the consumer's reservation price for good 1 and R_2 represents the consumer's reservation price for good 2.

Suppose that the prices for both goods are regulated at p_{1C} and p_{2C} respectively in Figure 4.10. The monopolist cannot raise either price. He cannot tie the two goods together and sell the bundle at a price which is greater than the sum of ceiling prices p_{1C}

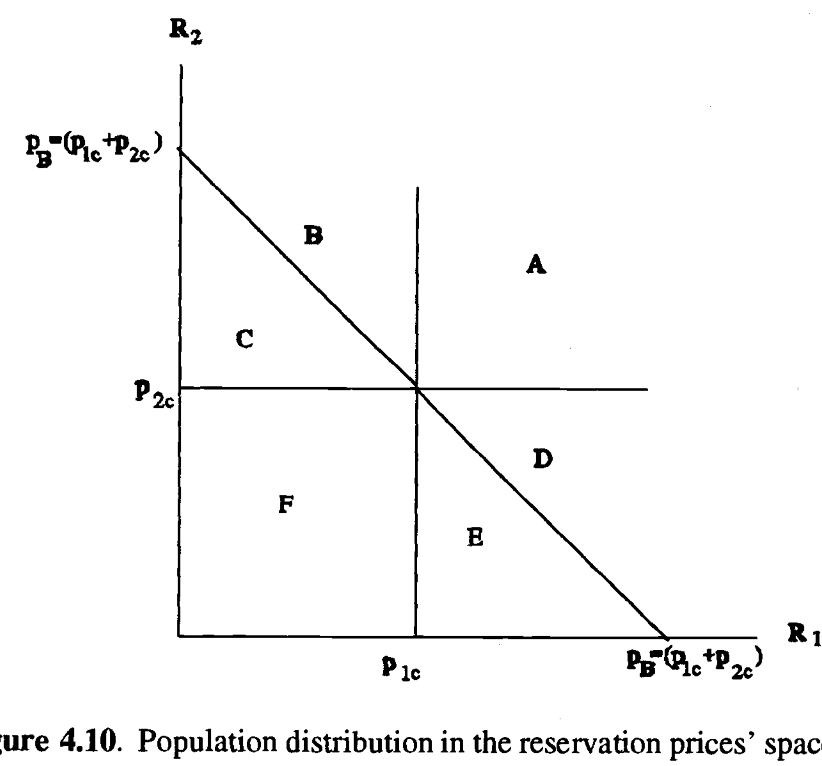

Figure 4.10. Population distribution in the reservation prices' space under price controls.

and p_{2C}. In the earlier analysis, only one price is regulated and the other price is allowed to increase, so that the producer can raise the tied good's price and get around the price controls through a tying sales arrangement. In this case the monopolist has no way to avoid the price control. Does this mean a tying sales strategy is not profitable?

The monopolist may still be interested in using a tying sales arrangement (or bundling), even though he has no way to get around the price controls. The objective of the tying sales arrangement is to increase the monopolist's total profits, within the limitations imposed by the price controls. Let us see the process.

Under the price controls, the monopolist can always choose the strategy of separate components pricing as long as the prices that he charges for good 1 and good 2 are no higher than the ceiling prices. If the monopolist does choose the strategy of separate components pricing by setting the prices for good 1 and good 2 at p_{1C} and p_{2C} respectively, the population is sorted into four groups in Figure 4.10:

Group (B+C): the individuals with reservation price for good 2 equal to or greater than p_{2C} and with reservation price for good 1 less than p_{1C}.

Group A: the individuals with reservation price for good 1 equal to or greater than p_{1C} and with reservation price for good 2 equal to or greater than p_{2C}.

Group F: the individuals with reservation price for good 1 less than p_{1C} and with reservation price for good 2 less than p_{2C}.

Group (D+E): the individuals with reservation price for good 1 equal to or greater than p_{1C}, and with reservation price for good 2 less than p_{2C}.

Hence, under this separate components pricing strategy, the persons in Group (B+C) purchase good 2 only, in Group A purchase both goods, in Group F purchase neither good, and in Group (D+E) purchase good 1 only.

Under the price controls, the monopolist can also use the strategy of pure bundling pricing. Let the monopolist tie one unit of good 1 with one unit of good 2 and sell them as a bundle at price $p_B=p_{1C}+p_{2C}$. This doesn't violate the price control, and may increase the profitability. This is demonstrated next.

Under the pure bundling pricing strategy, the population is sorted into two groups (see Figure 4.10):

Group 1 (A+B+D): individuals with reservation price ($R_B=R_1+R_2$) for the bundle equal to or greater than the bundle's price ($p_B=p_{1C}+p_{2C}$).

Group 2 (C+F+E): individuals with reservation price ($R_B=R_1+R_2$) for the bundle less than the bundle's price ($p_B=p_{1C}+p_{2C}$).

Thus, under the strategy of pure bundling pricing, the persons in Areas A, B, and D purchase the bundle. The rest purchase nothing.

The monopolist may also adopt the strategy of mixed bundling pricing. It will be considered later. I first want to show that the strategy of tying sales arrangements (or bundling) could dominate the strategy of separate components pricing. After that, I will compare the profitability between the strategy of pure bundling pricing and the strategy of mixed bundling pricing.

We now check the differences between the strategy of separate components pricing and the strategy of pure bundling pricing. In Figure 4.10, those persons, who in Area A purchase both goods, and who in Area F purchase neither good, are indifferent as between the two strategies. However, the persons in Area B, who purchase only good 2 under separate components pricing, purchase both goods under pure bundling pricing. The persons in Area C, who purchase only good 2 under separate components pricing, purchase neither good under pure bundling pricing. The persons in Area D, who purchase only good 1 under separate components pricing, purchase both goods under pure bundling pricing. The persons in Area E, who purchase only good 1 under separate components pricing, purchase neither good under pure bundling pricing.

Let A, B, C, D and E represent the populations in their corresponding areas. Under separate components pricing, the total profit to the monopolist is

$$\pi_s=(p_{1C}-c_1)(A+D+E)+(p_{2C}-c_2)(A+B+C).$$

Under pure bundling pricing, the total profit is

$$\pi_B=(p_{1C}+p_{2C}-c_1-c_2)(A+B+D).$$

The difference of the profits between these two pricing strategies is

$$\pi_B-\pi_s=(p_{1C}+p_{2C}-c_1-c_2)(A+B+D)-[(p_{1C}-c_1)(A+D+E)+(p_{2C}-c_2)(A+B+C)].$$

By rearranging the terms on the right-hand side of the above equation, we have

$$\pi_B-\pi_s=(p_{1C}-c_1)(B-E)+(p_{2C}-c_2)(D-C).$$

If $\pi_B-\pi_s>0$, the pure bundling pricing strategy will dominate the strategy of separate components pricing and increase the monopolist's total profit.

Under what conditions, will the sign of $(\pi_b-\pi_s)$ be positive?

(i) If $(B-E)>0$ and $(D-C)>0$, then $\pi_B-\pi_s>0$ (It is assumed that $p_{ic}-c_i>0$, $i=1,2$). Hence, when the population in Area B is larger than that in Area E and the population in Area D is larger than that in Area C, the strategy of pure bundling pricing dominates the strategy of separate components pricing.

From Figure 4.10, we can see that the those in Areas B and C have reservation prices R_1 less than p_{1C}, and R_2 larger than p_{2C}; those in Areas D and E have reservation prices R_1 larger than p_{1C}, and R_2 less than p_{2C}; those in Areas B and D have reservation price (R_1+R_2) larger than p_B; and those in Areas C and E have reservation price (R_1+R_2) less than p_B. From these facts, we can conclude that when (1) there exist some persons in the population with negatively related valuations of the two goods,[24] and (2) in those persons with negatively related valuations of the two goods, more of them have reservation prices whose sum (R_1+R_2) is larger than $(p_{1C}+p_{2C})$, the sum of the single ceiling prices, the strategy of pure bundling pricing is likely to dominate the strategy of separate components pricing.

On the other hand, when (1) there exist some persons in the population with negatively related valuations of the two goods, and (2) in those persons with negatively related valuations of the two goods, more of them have reservation prices whose sum (R_1+R_2) is less than $(p_{1C}+p_{2C})$, the sum of the single ceiling prices, the strategy of separate components pricing is likely to dominate the strategy of pure bundling pricing.

(ii) For (B-E) and (D-C), if one of them is positive, and another one is negative, the sign of $(\pi_B-\pi_s)$ will not only depend on the magnitudes of $(p_{1C}-c_1)$ and $(p_{2C}-c_2)$, but also depend on the magnitudes of (B-E) and (D-C).

Through the above analysis, we can conclude that when both prices are regulated by ceiling prices which are less than the free monopoly prices, the monopolist could

[24] "negatively related valuations" means that the higher is the valuation (willingness to pay) for one good, the lower is the valuation (willingness to pay) for the other good.

adopt a tying sales strategy to increase his profit.[25] Generally, the profitability of tying sales will depend on (1) the distribution of consumers in the reservation price space, (2) the levels of the ceiling prices, and (3) the levels of costs.

In the analysis, I have used the sum of the ceiling prices as the price for a bundle containing one unit of each good. However, an optimal price for the bundle may not necessarily be the sum of the ceiling prices. It could be less than the sum of the ceiling prices in some circumstances. I will demonstrate that in the next section through a numerical example.

Let us use the model to explain a couple of tying sales' examples under price controls. In the 1980's, in Beijing, China, the prices for rice and wheat flour were firmly controlled by the government through ceiling prices. It was very difficult to find a grain shop that sold rice and wheat flour separately to customers. Most grain shops required that each customer purchase equal volumes of rice and wheat flour at their respective ceiling prices. This effectively meant that the grain shops bundled one unit of rice and one unit of wheat flour together and sold them at a price equal to the sum of the ceiling prices. What was the rationale for the grain shops to do so?[26]

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[25] In the analysis, it has demonstrated that a tying sales arrangement could be a dominate pricing strategy under price controls in some circumstances. In the absence of price controls, the dominate pricing strategy could be either a simple monopoly pricing or a tying sales arrangement. In other words, when either a simple monopoly pricing or a tying sales arrangement is the dominate pricing strategy in the absence of price controls, a tying sales arrangement would be a dominate pricing strategy under price controls in come circumstances.

[26] The grain market was highly monopolized in China in that time. All the retail grain shops were controlled by a couple of state-owned companies. Thus, the grain shop can be assumed as a monopolist in this analysis.

In Beijing, the population is composed of roughly two groups: southerners and northerners. Rice is mainly produced in south China and wheat flour is mainly produced in north China. Southerners are used to eating rice and prefer rice products to wheat flour products. Conversely, northerners are used to eating wheat flour products and prefer wheat flour to rice. Therefore, the southerners value rice relatively highly and the northerners value wheat flour relatively highly, but all might be willing to pay roughly the same amount for a bundle of one unit of each good. In the absence of tying sales, most southerners would be likely to buy rice only and most northerners would be likely to buy wheat flour only. Through tying sales arrangements, the grain shops forced both southerners and northerners to buy both rice and wheat flour and captured more consumer surplus, thus, increasing their profits. That is, the strategy of tying sales pricing brought more profits to the sellers than the strategy of separate components pricing.

Figure 4.11 shows the rationale for the grain shops to use a pure bundling pricing strategy under price controls. R_r and R_w represent the reservation prices for rice and wheat flour respectively. p_r and p_w, which are roughly equal to each other, represent the ceiling prices for rice and wheat flour respectively. The sellers' costs for rice and wheat flour are roughly equal. Because the southerners prefer rice to wheat flour and northerners prefer wheat flour to rice, most southerners are expected to be distributed in Areas B and C in the reservation prices' space. Conversely, most northerners are expected to be distributed in Areas D and E. Because all the people are willing to pay roughly the same amount for a bundle of one unit rice and one unit wheat flour, most populations are likely to distribute in Areas A, B and D. Through a pure bundling

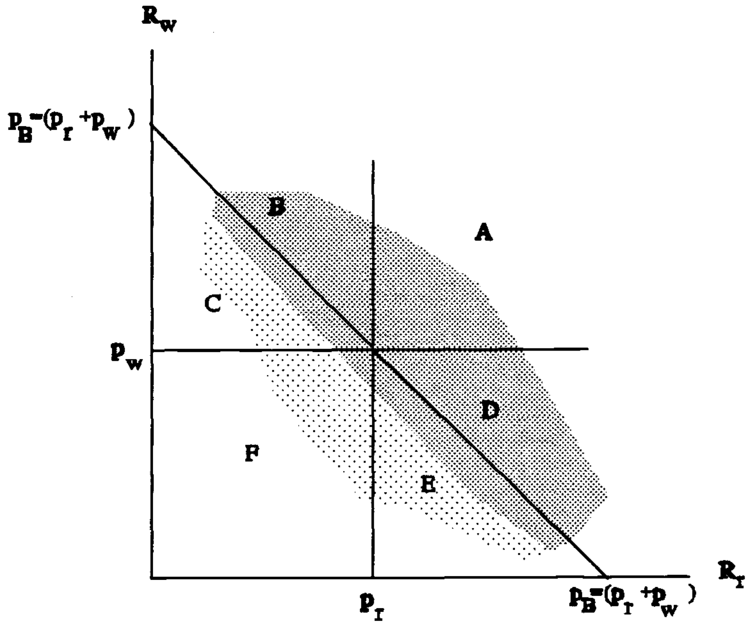

Figure 4.11. Population distribution in the reservation prices' space for rice and wheat flour.

pricing strategy at price $p_B=(p_r+p_w)$, the seller captures more consumer surplus from the people in Areas B and D and loses some profits from the people in Areas C and E. It is obvious that the population in (B) is larger than in (E) and the population in (D) is larger than in (C). Therefore, the gains dominate the losses and total profits are increased through tying sales arrangements.

Another example of tying sales under price controls is in the selling of cigarettes in China. The cigarettes can be divided into two kinds: one is brand name (normally high quality) and the other is generic (normally low quality). Both prices were regulated through ceiling prices until recently. It was observed that most sellers sold their cigarettes through a quasi-mixed bundling (or tying) arrangement: the generic could be bought alone at the ceiling price, the brand name couldn't be bought alone at the ceiling price, and any one who wanted to buy the brand name had to buy a bundle of one unit of each good at a price equal to the sum of the ceiling prices. What is the rationale for a seller to practice this pricing scheme? It can be explained by the tying sales model.

The smokers in China can be divided into three groups according to their family income per capita:

Group (a): the richer (urban people with high salaries and/or the people with average salaries and few dependents),

Group (b): the middle class (urban people with average salaries and/or average number of dependents) and

Group (c): the poor (farmers or urban people with low salaries and/or larger number of dependents).

Group (a) usually value the brand name cigarettes (high quality) relatively highly and value the generic cigarettes (low quality) relatively lowly and smoke the brand name cigarettes. Group (b) usually value the two kinds of cigarettes at a medium level and smoke both kinds. Group (c) usually value the generic relatively highly and value the brand name relatively lowly and smoke the generic cigarettes. Group (c)'s willingness to pay for a bundle of one unit of each good is likely much less than group (a)'s. Thus, pure bundling is not likely to dominate a separate components pricing strategy. However, most in Group (a) have a willingness to pay for the brand name that is much higher than the ceiling price. In the absence of price controls, the seller would be able to increase profits through increasing the brand name's price. Under price controls, he couldn't do so. Given these assumptions, the quasi-mixed bundling pricing strategy was able to capture more consumer surplus from Group (a) without losing any profit from Group (c); therefore, the total profits were increased.

This situation can be illustrated graphically in Figure 4.12. R_H and R_L represent the reservation prices for the brand name and generic cigarettes respectively.[27] p_H and p_L represent the ceiling prices for the brand name and generic cigarettes respectively. p_H is much higher than p_L. Because the richer consumers value the brand name relatively highly and value the generic relatively lowly, and the poor value the generic relatively highly and value the brand name relatively lowly, most people in Group (a)

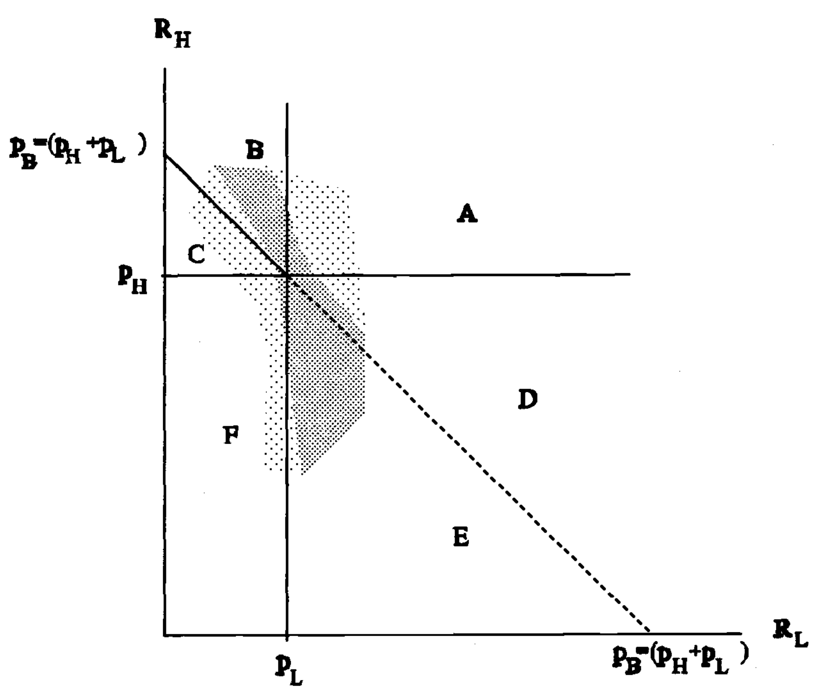

Figure 4.12. Population distribution in the reservation prices' space for the brand name and generic cigarettes.

[27] It is assumed that each consumer purchases a finite number of cigarettes per time period. Thus, the assumptions made in section 4.4.2 are also applicable to this case.

are likely to appear in Areas B and C, while, most people in Group (c) are likely to appear in Areas D and E. Because Group (c)'s willingness to pay for a bundle of one unit of each good is likely to be much less than group (a)'s, populations in Group (a) are densely distributed in Areas B and populations in Group (c) are densely distributed in Area E. Through a pure bundling pricing strategy at price $p_B=(p_H+p_L)$, the seller could capture more consumer surplus from the people in Areas B and D and lose some profits from the people in Areas C and E. Because the population in (E) is larger than in (B), the pure bundling pricing strategy would lose all the customers in Area E and not be likely to increase the seller's profits.

However, the seller can use a quasi-mixed bundling pricing strategy to increase his profits. Suppose that the seller sells (1) the generic alone at price p_L, (2) a bundle of one unit of each good at price $p_B=(p_H+p_L)$ and (3) does not sell the brand name alone. Under this pricing strategy, the people in Area B buy a bundle and in Area C buy neither. Thus, the seller captures more consumer surplus from the people in Area B and loses some profits from the people in Area C.

Let π_m and π_s represent the profits for quasi-mixed bundling pricing and simple components pricing respectively. Let c_H and c_L represent the constant marginal costs for the brand name and generic respectively. Let B and C represent the population in their corresponding areas. The profit difference between the two pricing strategies is

$$\pi_m-\pi_s=B(p_B-c_H-c_L)-(B+C)(p_H-c_H)$$
$$=B(p_L-c_L)-C(p_H-c_H).$$

Because the population in Area B is much larger than in Area C, the quasi-mixed bundling pricing strategy is likely to dominate the simple components pricing strategy when profit margins for both goods are roughly the same. That is one possible explanation of why the cigarette sellers in China were using the tying sales arrangements under price controls.

4.4.2.2 *The Profitability of a Mixed Bundling Pricing Strategy Under Price Controls*

So far, the analysis has focused on the profitability of the strategy of pure bundling pricing compared to the strategy of separate components pricing. In the following, I discuss the profitability of the strategy of mixed bundling pricing.

The strategy of mixed bundling pricing is always at least as profitable as the strategy of pure bundling pricing in the absence of price controls (Adams and Yellen 1976). The reason can be illustrated as follows:

In Figure 4.13, under pure bundling with price p_B, the population is sorted into two groups. The persons in group ($R_2p_B'p_BR_1$) with reservation price $R_B=R_1+R_2\geq p_B$ purchase the bundle, and the rest of the persons purchase nothing. Under mixed bundling, the monopolist sets price p_B for a bundle, and prices p_1 and p_2 for the individual goods. Assume $p_B<p_1+p_2$.[28] The population is sorted into four groups:

Group I (R_2p_2BA): $R_2>p_2$, $R_1<p_B-p_2$.[29]

Group II (ABCD): $R_B=R_1+R_2>p_B$, $R_1>p_B-p_2$, $R_2>p_B-p_1$.

Group III (DCp_1R_1): $R_1>p_1$, $R_2<p_B-p_1$.

[28] When $p_B=p_1+p_2$, it makes no sense for mixed bundling pricing because there is no difference between simple components pricing and mixed bundling pricing.

[29] $p_B-p_2=(p_B'p_2)=p_2B$. It is also noted that $p_B-p_1=(p_Bp_1)=p_1C$.

Group IV (p_2BCp_1): $R_1<p_1$, $R_2<p_2$, $R_B<p_B$.

For a person in Group I, if he purchases a bundle, he obtains consumer surplus $S_B=R_1+R_2-p_B$; If he purchases only good 2, he can obtain consumer surplus $S_2=(R_2-p_2)$. Thus, if $S_2>S_B$, the person will purchase only good 2. We have

$$S_2-S_B=(R_2-p_2)-(R_1+R_2-p_B)$$
$$=(p_B-p_2)-R_1.$$

If $R_1<(p_B-p_2)$, then $(S_2-S_B)>0$, and the person will purchase only good 2. Because $R_1<(p_B-p_2)$ for individuals in Group I, those persons will purchase only good 2. This principle can be applied to other groups. Therefore, we can conclude that persons in Group I purchase only good 2, in Group II purchase both goods, in Group III purchase

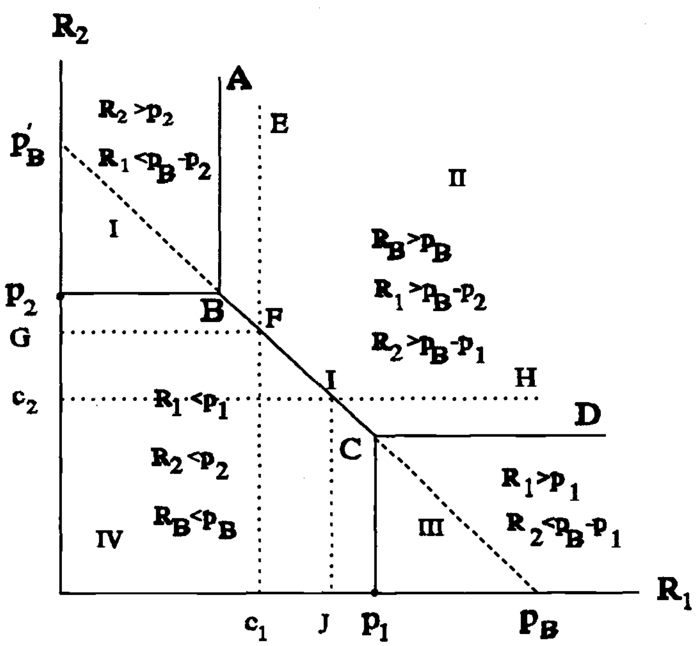

Figure 4.13. Population distribution in the reservation prices' space and mixed bundling pricing.

only good 1, in Group IV purchase neither. The persons in Group I and Group III, who purchase the bundle under pure bundling, purchase only one good under mixed bundling. If the profits from selling only one good to those persons are larger than those from selling two goods to them, the mixed bundling must be better than the pure bundling.

The profit of selling good 2 to a person in Group I is

$$\pi_2=p_2-c_2.$$

The profit of selling a bundle to a person in Group I is

$$\pi_B=p_B-c_1-c_2.$$

Thus, if

$$p_2>(p_B-c_1),$$

then

$$\begin{aligned}\pi_2-\pi_B&=(p_2-c_2)-(p_B-c_1-c_2)\\&=p_2-(p_B-c_1) > 0.\end{aligned}$$

We can easily derive the similar condition for the persons in Group III. The profit of selling good 1 to a person in Group III is

$$\pi_1=p_1-c_1.$$

The profit of selling a bundle to a person in Group III is

$$\pi_B=p_B-c_1-c_2.$$

Thus, if

$$p_1>(p_B-c_2),$$

then

$$\begin{aligned}\pi_1-\pi_B&=(p_1-c_1)-(p_B-c_1-c_2)\\&=p_1-(p_B-c_2) > 0.\end{aligned}$$

Therefore, mixed bundling with a bundle price p_B and single-good prices $p_1 \geq p_B - c_2$ and $p_2 \geq p_B - c_1$ always yields profits at least as high as pure bundling with price p_B. In other words, when there are any persons in Areas ($EFGR_2$) and ($HIJR_1$) in Figure 4.13, mixed bundling might dominate pure bundling. However, this conclusion is no longer valid under price controls.

If the optimal pure bundling price is the sum of the ceiling prices, there is no sense to using mixed bundling because the single-good prices cannot be greater than the ceiling prices. No person will buy the bundle if the single-good prices are less than the ceiling prices and the bundle price is equal to the sum of the ceiling prices.

Mixed bundling is possible when the optimal pure bundling price is less than the sum of the ceiling prices. If the monopolist can set the single-good prices $p_1 \geq p_B - c_2$ and $p_2 \geq p_B - c_1$, mixed bundling will dominate the pure bundling. If $p_1 < p_B - c_2$ and $p_2 < p_B - c_1$, mixed bundling may not be as profitable as pure bundling. Further, if $p_1 << p_B - c_2$ and $p_2 << p_B - c_1$, pure bundling will dominate mixed bundling. Under price controls, the single-good prices cannot be greater than the ceiling prices. That is, if $p_{1C} > p_B - c_2$ and $p_{2C} > p_B - c_1$, the mixed bundling will dominate pure bundling. Otherwise, pure bundling may dominate mixed bundling.

Let us look at the likelihood of $p_{1C} > p_B - c_2$ and $p_{2C} > p_B - c_1$. Let $p_{ic} = y + c_i$, where $i=1,2$, and y is the profit margin per unit good i. Let $p_B - c_j = z + c_1 + c_2 - c_j$, where $j=1,2$ and z is the profit margin per bundle. If

$$p_i > (p_B - c_j), \text{ where } i, j=1, 2 \text{ and } i \neq j,$$

that is

$$y + c_i > z + c_1 + c_2 - c_j$$

we then have

$$y>z.$$

This means that when the profit margin per unit single-good is larger than the profit margin per bundle, the mixed bundling will dominate the pure bundling. Under price controls, the profit margin per single-good is limited. Thus, the profitability of mixed bundling under price controls is very weak because of the fact that the prices for single-good cannot be greater than the ceiling prices. Generally, pure bundling may be more profitable than the mixed bundling under price controls.

4.4.2.3 The Profitability of the Quasi-mixed Bundling Pricing Strategy Under Price Controls

The example of cigarettes selling under price controls in China has illustrated the point that a quasi-mixed bundling pricing strategy might be the optimal strategy under some conditions. Those conditions can be summarized as when (a) many consumers are willing to pay roughly the ceiling price for good 1 and to pay less than the ceiling price for good 2, and (b) many consumers are willing to pay much more than the ceiling price for good 2 and to pay less than the ceiling price for good 1, a quasi-mixed bundling pricing strategy is likely to be most profitable.

4.4.3 A Numerical Example

I have graphically shown that under certain circumstances, bundling pricing may dominate a separate components pricing when both prices are regulated. I now present a numerical example, and hope that it will make the findings more understandable.

Suppose that consumers are distributed in reservation price space as in Figure 4.14.[30] There are fourteen consumers (A, B, C, D, E, F, G, H, I, J, K, L, M, N). Each consumer has a pair of reservation prices (R_1, R_2). A consumer's reservation price for a bundle including one unit of each good is the sum of his single reservation prices, i.e.,

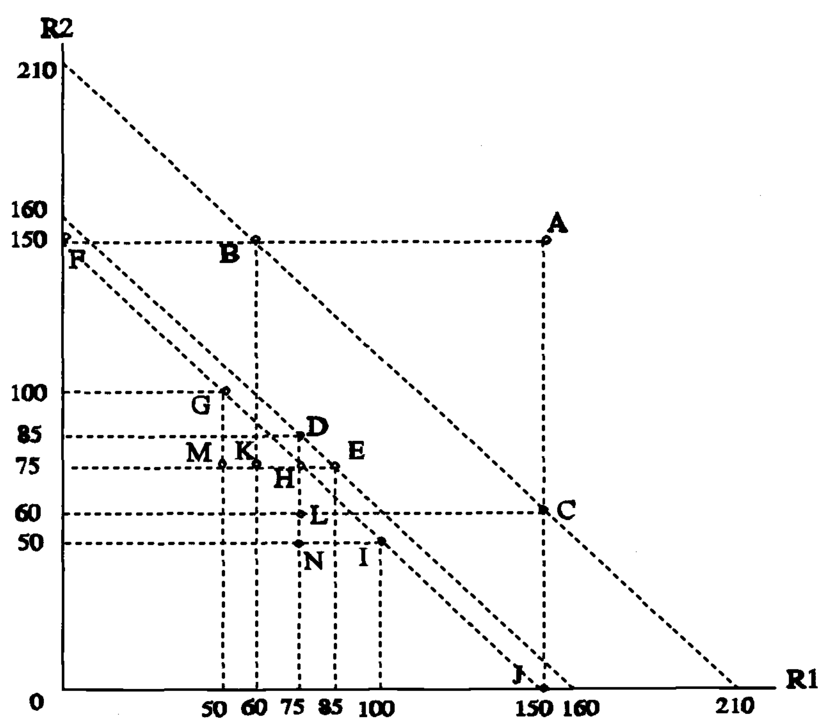

Figure 4.14. Population distribution in the reservation prices' space.

[30] For the analytic convenience, the consumers are symmetrically distributed in the reservation prices space. It is not crucial to the analysis.

$R_B=R_1+R_2$. All the assumptions made at the beginning of the last section are also applied here. The reservation prices for each consumer are as follows:

A(150, 150), B(60, 150), C(150, 60), D(75, 85), E(85, 75), F(0, 150), G(50, 100), H(75, 75), I(100, 50), J(150, 0), K(60, 75), L(75, 60), M(50, 75), N(75, 50).

In the absence of any price regulation, among the three pricing strategies, the separate components pricing strategy provides the largest profits (510) (see Table 4.1).[31] Therefore, if there is no price control, the monopolist would adopt the separate components pricing strategy and set prices for good 1 and good 2 at $p_1^*=150$ and $p_2^*=150$ respectively. In this case, the consumers (C, J) purchase only good 1, (B, F) purchase only good 2, A purchases both goods, and the rest of the consumers purchase neither (Figure 4.14).

Table 4.1 Maximum profits and optimal prices for each pricing strategy in the absence of price controls.

Strategy	p_1^*	p_2^*	p_B^*	Profits	Costs
Separate Components Pricing	150	150		510*	$c_1=65$
Pure Bundling Pricing			210	240	$c_2=65$
Mixed Bundling Pricing	150	150	160	320	

We can see that the monopolist's prices are much higher than its costs and many consumers couldn't afford it. Therefore, suppose the government decides to intervene

[31] Appendix has detailed calculations and shows how this result is obtained.

in the monopoly markets and imposes ceiling prices p_{1C}=75 and p_{2C}=75 for good 1 and good 2 respectively.

In the presence of the price controls, what is the best pricing strategy for the monopolist? In Table 4.2,[32] if the monopolist adopts the separate components pricing strategy, the maximum profit that he can obtain is 180 at the prices $p_1^*=p_{1C}=75$, and $p_2^*=p_{2C}=75$. On the other hand, if the monopolist adopts the pure bundling price strategy, the maximum profit that he can obtain is 200 at the price $p_B^*=150=(p_{2C}+p_{1C})$. If the monopolist adopts the mixed bundling pricing strategy, the profit that he can get is 120 at the prices $p_1^*=p_2^*=75$ and $p_B^*=135$. In this case, the pure bundling pricing strategy is the best. A monopolist whose objective is to maximize profit will adopt the pure bundling price strategy.

Table 4.2 Maximum profits and the optimal prices for each pricing strategy under price controls: $p_{1C}=p_{2C}=75$.

Strategy	p_1^*	p_2^*	p_B^*	Profits	Costs
Separate Components Pricing	75	75		180	c_1=65
Pure Bundling Pricing			150	200*	
Mixed Bundling Pricing	75	75	135	120	c_2=65

With separate components pricing at the prices $p_1^*=p_{1C}=75$, and $p_2^*=p_{2C}=75$, all consumers purchase what they want (Figure 4.14). Consumers (B, F, G, K, M) purchase only good 2. Consumers (C, I, J, L, N) purchase only good 1. Consumers (A, D, E, H)

[32] Appendix has detailed calculations and shows how the result is obtained.

purchase both goods. With pure bundling pricing at the price $p_B^*=150=(p_{2C}+p_{1C})$, consumers (K, L, M, N) are forced out of the market and purchase nothing. The rest of the consumers purchase the bundle. Consumers (F, G, I, J), who enjoy some consumers' surplus under the separate components pricing, now have no consumers' surplus under the pure bundling pricing. The consumers' surplus for (B, C) is also reduced.

Thus, a monopolist who uses no tying sales arrangement in the absence of price controls may want to use tying sales arrangements in the presence of price controls. The objective of using the tying sales arrangements is to increase profits by extracting more consumers' surplus, not by getting around the price controls.

If the ceiling prices are changed to $p_{1C}=p_{2C}=84$, the pure bundling pricing strategy still dominates the separate components pricing and the mixed bundling pricing strategies (see Table 4.3).[33] The optimal price for a bundle is $p_B^*=150$ which is less than the sum of the ceiling prices ($p_1+p_{1C}=168$). From this example we can understand that the optimal price for a bundle is not necessarily equal to the sum of the ceiling prices. It could be less than the sum of the ceiling prices.

Table 4.3 Maximum profits and optimal prices for each pricing strategy under price controls: $p_{1C}=p_{2C}=84$.

Strategy	p_1^*	p_2^*	p_B^*	Profits	Costs
Separate Components Pricing	84	84		190	$c_1=65$
Pure Bundling Pricing			150	200*	
Mixed Bundling Pricing	84	84	150	194	$c_2=65$

[33] Appendix has detailed calculations and shows how the result is obtained.

If the ceiling prices are p_{1C}=84 and p_{2C}=84, and the costs are increased to c_1=c_2=71, then the separate components price strategy dominates the pure bundling pricing strategy and the mixed bundling pricing strategy (Table 4.4).[34] In this case, the monopolist has no interest in tying. He sets the prices for each good at the ceiling price level.

Table 4.4 Maximum profits and optimal prices for each pricing strategy under price controls: p_{1C}=p_{2C}=84.

Strategy	p_1^*	p_2^*	p_B^*	Profits	Costs
Separate Components Pricing	84	84		130*	c_1=71
Pure Bundling Pricing			160	90	
Mixed Bundling Pricing	84	84	150	100	c_2=71

In none of the above cases is the mixed bundling pricing the dominant strategy. Yet, as I said earlier, in the absence of price controls, the mixed bundling is always at least as profitable as the pure bundling. However, the profitability of the mixed bundling is limited under price controls because the prices cannot be greater than the ceiling prices. When the ceiling prices increase, the mixed bundling may be more profitable, and may dominate other strategies in some circumstances. For example, when the ceiling prices increase to p_{1C}=p_{2C}=100, and the costs are c_1=c_2=65, the mixed bundling is the dominative strategy (see Table 4.5).[35]

[34] Appendix has detailed calculations and shows how the result is obtained.

[35] Appendix has detailed calculations and shows how the result is obtained.

Table 4.5 Maximum profits and optimal prices for each pricing strategy under price controls: $p_{1C}=p_{2C}=100$.

Strategy	p_1^*	p_2^*	p_B^*	Profits	Costs
Separate Components Pricing	100	100		280	$c_1=65$
Pure Bundling Pricing			200	210	
Pure Bundling Pricing	100	100	160	300*	$c_2=65$

In sum, a monopolist may have no interest in using a tying sales arrangement in the absence of price controls. However, when both prices are regulated by ceiling prices that are less than the free monopoly prices, the monopolist may have an interest in using tying sales arrangements to increase his profits. The profitability of tying sales will depend on the distribution of consumers in the reservation prices' space, the levels of the ceiling prices, and the levels of costs. Under some circumstances, tying sales may be more profitable than separate components pricing. When the tying sale strategy dominates the separate components pricing, the optimal prices may be less than the ceiling prices.

In the absence of price controls, mixed bundling is always at least as profitable as the pure bundling. This is not true in the presence of price controls. Pure bundling may dominate mixed bundling. It generally depends on the distribution of consumers in the reservation prices' space, the levels of ceiling prices, and on the levels of costs.

4.5 Summary and Conclusions

That tying sales arrangements can be used to evade price controls was pointed out in the literature on tying sales a long time ago. However, no rigorous analysis of the issue has been done so for. On the other hand, no other explanation has been provided for tying sales arrangements under price controls. In this chapter, I attempt to provide detailed analysis of some issues of the economics of tying sales under price controls.

4.5.1 The Main Findings

In this chapter, I have studied two models: one is the competitive-tying model under price controls, and the other is the monopoly-tying model under price controls. These two models address the economics of tying sales under price controls in different market structures.

In a competitive market, if a ceiling price less than the free market equilibrium price is imposed, the quantity that the producers want to produce is less than that in the absence of the price control. At the same time, the demand is greater than that in the absence of the price control. Thus, the supply of the good will be short under the price control. This shortage creates a contrived surplus. The contrived surplus could be captured by consumers or producers or be dissipated to waiting costs.

On a first-come-first-serve basis, queues occur, and the real price to the consumers is the ceiling price plus some waiting costs. In market equilibrium, the contrived surplus is totally dissipated to waiting costs. In this case, a competitive producer could use tying sales arrangements to capture some of the contrived surplus, therefore, to increase his profit. Hence, contrived surplus seeking is the motivation for a competitive producer to use the strategy of tying sales under price control. Some people may consider the motivation in this case is maximizing profits. However, in a competitive market, a competitor is assumed to make no profit but make even. So that I prefer to identify the motivation for a competitor to using tying sales as a contrived surplus seeking than maximizing profits. In fact, the competitors of adopting tying sales strategy make no profits at the final market equilibrium as I pointed out in the equilibrium analysis.

Price controls in a competitive market cause short supply and force consumers to consume less and pay a higher real price than that in the absence of price controls. The tying sales arrangements under the price controls indirectly increase the market supply and lower the real price that consumers pay for the controlled good.

In the mainstream of the literature on tying sales economics, the tying good is always a monopoly good. No literature has shown that the tying good could be competitively supplied. In this chapter, I have shown how a competitively supplied good can be used as a tying good and how it works under price controls. Basically, through tying sales arrangements, the competitive sellers capture the contrived surplus dissipated to consumers' waiting costs and leave the consumers as well off as they were in the absence of any controls.

In a monopoly market, if a ceiling price less than the monopoly price is imposed, the supply of the monopoly good may be increased and the consumers purchase more and pay less than they do in the absence of the price control. On the other hand, the monopolist's profit is less than it would be in the absence of the price control. Those may be exactly what the authority of the price controls looks for.

Searching for a maximum profit, the monopolist would use the strategy of tying sales to get around the price control and maximize his profit. Thus, profit maximization is the motivation for a monopolist to use the strategy of tying sales under price controls. In the analyses, I have shown how the monopolist may get around the price control and maximize his profit through tying sales arrangements.

In this chapter, I have explored the argument that tying sales arrangements under price controls could be used as a price discrimination tool to increase the monopolist's profits, in addition to be used in evasion of price controls. In the analysis, I used a two-good monopolist model. When two markets are both monopolized by a single monopolist, and ceiling prices for both markets are imposed simultaneously, the monopolist could use the strategy of tying sales to increase his total profits even without using tying to get around the price controls. In this case, the tying strategy is a tool of price discrimination. The monopolist sorts the consumers into different groups through the tying sales arrangements, and extracts more consumers' surplus. The profitability of tying sales arrangements will depend on the consumers' distribution in the reservation price space, on the ceiling price levels and on the cost levels.

The mixed bundling pricing strategy is always at least as profitable as the pure bundling pricing strategy in the absence of price controls. I find that this conclusion is no longer valid under price controls. Conversely, the pure bundling pricing strategy may dominate the mixed bundling pricing strategy in most circumstances under price controls. Under some circumstances, the quasi-mixed bundling pricing strategy might be most profitable. Hence, the pure bundling pricing strategy and quasi-mixed bundling pricing strategy are expected to be used under price controls more often than the mixed bundling pricing strategy.

4.5.2 The Limits and Some Suggestions for Future Research

This study is principally conceptual in nature and little empirical evidence is provided of tying sales under price controls. In the two-good monopolist model, the analysis rests on some restrictive assumptions such as discrete demand for all consumers, independence in consumption, etc.

In further research, following needs to be explored:

(1) In the competitive-tying model, the equilibrium analysis found that the supplied quantity and the real price to the consumers and to the producers through tying sales arrangements under price controls are the same as they are in the absence of price controls. Do actual business practices under price controls support this finding? Those need to be explored.

(2) Price controls are used quite a bit in today's world, especially in centrally-planned economies. I personally experienced the popularity of tying sales arrangements in China. Do the business practices in other countries support and show the same popularity under price controls? If not, why and what might be the reasons?

(3) In this chapter, I found that even under price controls, tying sales arrangements could be used as a tool to sort consumers into different groups and extract more of the consumers' surplus. This is an alternative to use of tying arrangements to

get around the price controls. A next step is to generalize: under what (a) price control and (b) customer distribution circumstances do tying sales make sense when both prices are regulated. Resulting hypotheses can be tested via examples of business responses to price controls.

(4) I pointed out that a pure bundling pricing strategy and quasi-mixed bundling pricing strategy may generally dominate the mixed bundling pricing strategy under price controls. Do the business practices support this?

In other words, the research reported here has generated several hypotheses that await empirical testing:

(1) Under price-ceiling controls in a competitive market, the real unit cost to consumers for a controlled good is much higher than the ceiling price. Because consumers are willing to pay more than the ceiling price for the controlled good, the sellers can use tying sales arrangements to go around the price controls and increase their profits.

(2) Under price-ceiling controls in either a competitive market or a monopoly market, tying sales arrangements might bring the market in equilibrium at which the real price and output level are the same as in the absence of price controls.

(3) Under price-ceiling controls in a monopoly market, when consumers' preferences are different tying sales arrangements can be used as a discrimination tool to sort consumers into different groups and help the monopolist to exploit more of the consumer surplus and increase his profits. When some consumers' preferences for two goods are negatively related and most of them are willing to pay roughly the same amount for a bundle of one unit of each good, a pure bundling pricing strategy is likely

to be used. When (a) many consumers are willing to pay roughly the ceiling price for good 1 and to pay less than the ceiling price for good 2, and (b) many consumers are willing to pay much more than the ceiling price for one good 2 and to pay less than the ceiling price for good 1, a quasi-mixed bundling pricing strategy is likely to be used.

CHAPTER 5
CONCLUSIONS

Three related topics have been explored in this thesis. Those three topics deal with pricing problems both in the market for access (input market) and in the market for seafood (output market). The purpose of this study was to provide some help (1) to understand an important decision of fishery management, (2) to understand some phenomena in the seafood industry, (3) to extend our understanding of price determination in the presence of multiproduct sellers, (4) to close some gaps between price theory and the pricing problems in real world business practices, and (5) eventually, to contribute some insights into the global picture of fishery resource management and the market for seafood.

In chapter 1, I elaborated my motivations, the objectives of this study and the relationships among the three topics in this thesis.

Chapter 2 explored the first topic, "the maximum present value, social discount rate and fishery management". In this chapter, I used an optimal control model to explore a resource owner's exploiting behavior. I demonstrated that when a sole owner of the fishery resource wants to maximize the present value of the net revenue derived from the resource through time, the maximum present value is negatively related to the owner's discount rate, i.e., the maximum present value decreases as the owner's discount rate increases.

When using this finding in a competitive market for access rights, we may uncover some insights into the management of fishery resources and cooperative fishing arrangements. For example, given other things equal, the discount rate will affect both the coastal nations' and distant-water nations' (or fleets') competitive abilities in the market for access rights. In addition, this finding may also provide some help in examining how prices are determined in the market for access rights.

The analysis was based on the Schaefer model with its assumptions of constant price and infinite time period. Under those assumptions I was basically dealing with a linear control problem, which simplifies the analysis. The Schaefer model and the assumptions are very restrictive, however. In the real world, things are much more complicated than are assumed in this chapter. For example, the price is unlikely to be constant, but rather, likely to be determined by demand and supply. Thus, price may be influenced by events in any particular fishery. In addition, the total harvesting cost is not likely to be linear in harvesting rate. Therefore, the analysis in chapter 2 has certain limits. When we apply this analysis to the management of particular fishery resources and cooperative fishing arrangements, we should not overlook these limits.

Chapter 3 explored the second topic, "a comparative statics analysis of the pricing strategy of a multiproduct monopolist". This study attempted to explore the pricing behavior and comparative statics for multiproduct monopolists and to provide an explanation for at least one phenomenon in the seafood industry, specifically, purchase restrictions imposed on fishermen by seafood processors. Two kinds of models were used in the analysis: one is the conventional model; the other is the discrete choice model.

The main findings in the conventional models can be summarized as follows:

(1) For a single-product monopolist, when the marginal cost decreases, the price always decreases and the output increases.

(2) For a multiproduct monopolist, when a product's marginal cost decreases, the sales of that product always increase. However, the sales of the other products can either increase or decrease or remain the same.

(3) For a two-product monopolist, when a product's marginal cost decreases, that product's price can either increase or decrease or remain the same, depending on conditions. The other product's price can also either increase or decrease or remain the same. However, when the first product's price increases, the second product's price cannot decrease. In this chapter, I also identified the conditions under which the Edgeworth paradox may occur.

(4) Combining (2) and (3), a product's quantity demanded and price may both move in the same direction, corresponding to a change in its marginal cost. That is, when the price goes up, the quantity demanded may also increase.

(5) For both a single-product monopolist and a multiproduct monopolist, the quantity demanded of an input and that input's price always move in opposite directions in the conventional models. That is, when an input price goes down, the quantity demanded for that input will increase. A binding capacity constraint won't change this conclusion. However, a change in one input's price can cause the quantity demanded for other inputs to either increase or decrease, depending on conditions.

From (5) we can see that the conventional models fail to address the phenomenon in which the purchasers don't want to take more fish from the fishermen even at lower prices. An alternative explanation may lie in the fact that most seafood is sold to final consumers through supermarkets and the conventional models do not adequately capture the supermarkets' special characteristics, pricing behavior and the consumers' shopping behavior.

By using a discrete choice model, it was demonstrated that

(1) The input price and the output price may move in opposite directions; i.e., while the input price goes down, the output price may go up;

(2) The seller's demand for an input can be perfectly price-inelastic in some range, even if the customers' demand for the output is price-elastic.

There exist some limits in the analysis of chapter 3 and some further research needs to be done. In the analysis, it was assumed that the marginal costs are constant and the costs are not interrelated, i.e., the production relationships are independent. Relaxation of the assumptions will bring the model closer to real world business practices, though it may make the analysis much more complicated.

The major weakness of the discrete choice model is its discontinuity. Although it well addresses the supermarket pricing behavior and consumers' shopping behavior, the discrete choice model needs to be developed into a conceptual and continuous model. When I used the findings in the discrete choice model to explain some phenomena in the seafood industry, I made an assumption that the profit margin of seafood in supermarkets is less than those of other substitutes (for example, beef, pork, poultry). This assumption waits empirical testing.

The third topic, "the economics of tying sales under price controls", was studied in chapter 4. This study attempted to provide detailed analysis of some issues of the economics of tying sales under price controls. Two models were used in the analysis: one is the competitive-tying model under price controls, and the other is the monopoly-tying model under price controls. These two models addressed the economics of tying sales under price controls in different market structures.

Price controls in a competitive market cause short supply and force consumers to consume less and pay a higher real price than that in the absence of price controls. A competitive producer can use tying sales arrangements to capture some of the contrived surplus, therefore, to increase his profits. The tying sales arrangements under the price controls indirectly increase the quantity supplied and lower the real price that consumers pay for the controlled good.

In addition, in the mainstream of the literature on tying sales economics, the tying good is always a monopoly good. No literature has shown that the tying good could be competitively supplied. In this chapter, I demonstrated how a competitively supplied good can be used as a tying good and how it works under price controls.

In a monopoly market, if a ceiling price less than the monopoly price is imposed, the quantity supplied of the monopoly good may increase and the consumers purchase more and pay less than they do in the absence of the price control. On the other hand, the monopolist's profit is less than it would be in the absence of the price control. In the analysis, I showed how the monopolist may get around the price control and maximize his profits through tying sales arrangements.

In addition, this chapter explored the argument that tying sales arrangements under price controls may be used as a price discrimination tool to increase the monopolist's profits. In the analysis, I used a two-good monopolist model. When two markets are both monopolized by a single monopolist, and ceiling prices for both markets are imposed simultaneously, the monopolist can use the strategy of tying sales to increase his total profits even without using tying to get around the price controls. In this case, the tying strategy is a tool of price discrimination. The monopolist sorts the consumers into different groups through the tying sales arrangements, and extracts more consumers' surplus. The profitability of tying sales arrangements will depend on the consumers' distribution in the reservation price space, on the ceiling price levels and on the cost levels.

The mixed bundling pricing strategy is always at least as profitable as the pure bundling pricing strategy in the absence of price controls. I found that this conclusion is no longer valid under price controls. On the contrary, the pure bundling pricing strategy may dominate the mixed bundling pricing strategy in most circumstances under price controls. Under some circumstances, the quasi-mixed bundling pricing strategy might be most profitable. Hence, the pure bundling pricing strategy and quasi-mixed bundling pricing strategy are expected to be used under price controls more often than the mixed bundling pricing strategy.

The study in chapter 4 was principally conceptual in nature and little empirical evidence was provided of tying sales under price controls. In the two-good monopolist model, the analysis rested on some restrictive assumptions such as discrete demand for all consumers, independence in consumption. Future researchers may wish to relax these assumptions as they develop statistical models to examine the relationship between the presence of price controls and the use of tying arrangements in actual markets.

As stated before, the fishery resource management and seafood markets are a very complicated system. The knowledge and information are very limited at present. Hopefully, this thesis contributed some insights into the global picture of fishery resource management and the market for seafood.

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APPENDIX

APPENDIX
THE PROFITABILITY ANALYSES FOR DIFFERENT PRICING STRATEGIES UNDER PRICE CONTROLS

In this appendix, I analyze the profitability for each pricing strategy. Assume that the customers' populations are distributed in the reservation prices' space as in Figure 4.14, and the costs for good 1 and good 2 are constant.

When we analyze the separate components pricing and the mixed bundling pricing, we need to set a price level for each good. Because the distribution of the population in the reservation space is symmetric, and the costs are the same for the two goods, we can expect that for any possible optimal pair of prices, the two prices must be the same.[36] For example, if the prices (p_1, p_2) are the optimal prices for the separate components pricing strategy, p_1 must be equal to p_2. If the prices (p_1, p_2, p_B) are the optimal prices for the strategy of mixed bundling pricing, p_1 must be equal to p_2. Thus, when I calculate the profitability for various price levels, I always give the same value to p_1 and p_2.

For each pricing strategy, I obtain different profit values by changing the price level from high to low. In doing so, I obtain the maximum profit for each pricing strategy. Comparing those maximum profits among the three strategies, the optimal pricing strategy is found.

In calculating the profit for the mixed pricing, when a customer with one reservation price less than the cost, and when (R_1, R_2) is located at the intersection of the

[36] The symmetry assumption is for the sake of simplicity.

bundling line and the single-price line for the second good, he would be indifferent between purchasing the bundle and purchasing the single good. However, the seller gets less profit if the buyer he purchases the bundle. For example, when the single price is 150, and the bundle price is 210 in Figure 4.14, customer B with $R_1=60$ which is less than the unit cost, would be indifferent between the bundle and the single good 2. If he purchases the bundle, the profit to the seller is 80. If he purchases only good 2, the profit to the seller is 85. Thus, it is to the seller's benefit to not let him purchase the bundle. In this case, the seller can reduce the single-price a very little, and let the customer purchase only one good. So, I assume that those customers like B purchase only one good in our calculations.

A.1 In the absence of price controls and $c_1=c_2=65$

Figure A.1 shows how each price level is determined for the separate components pricing strategy. Table A.1 shows the purchasers and the profits to the seller corresponding to each price level by using the separate components pricing strategy in the absence of price controls. The maximum profit is 510 at prices (150, 150).

Table A.1 Separate components pricing and its profits in the absence of price controls.

Prices (p_1,p_2)	Purchasers			Profits
	Good 1	Good 2	Both Goods	
150,150	C,J	B,F	A	510*
100,100	C,J,I	B,F,G	A	280
85,85	C,J,I,E	B,F,G,D	A	200
75,75	C,J,I,L,N	B,F,G,K,M	A,D,E,H	180

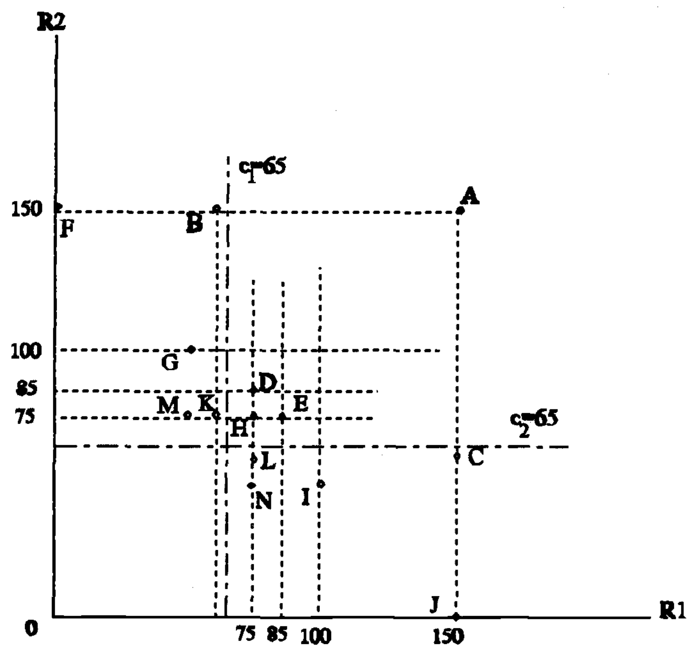

Figure A.1. Population distribution and separate Components Pricing in the absence of price controls.

Figure A.2 shows how each price level is determined for the pure bundling pricing strategy. Table A.2 shows the purchasers and the profits to the seller corresponding to each price level by using the pure bundling pricing strategy in the absence of price controls. The maximum profit is 240 at price p_B=210.

Table A.2 Pure Bundling Pricing and Its Profits.

Price/Bundle	Purchasers	Profits
300	A	170
210	A,B,C	240*
160	A,B,C,D,E	150
150	A,B,C,D,E,F,G,H,I,J	200
135	A,B,C,D,E,F,G,H,I,J,K,L	60
125	A,B,C,D,E,F,G,H,I,J,K,L,M,N	-70

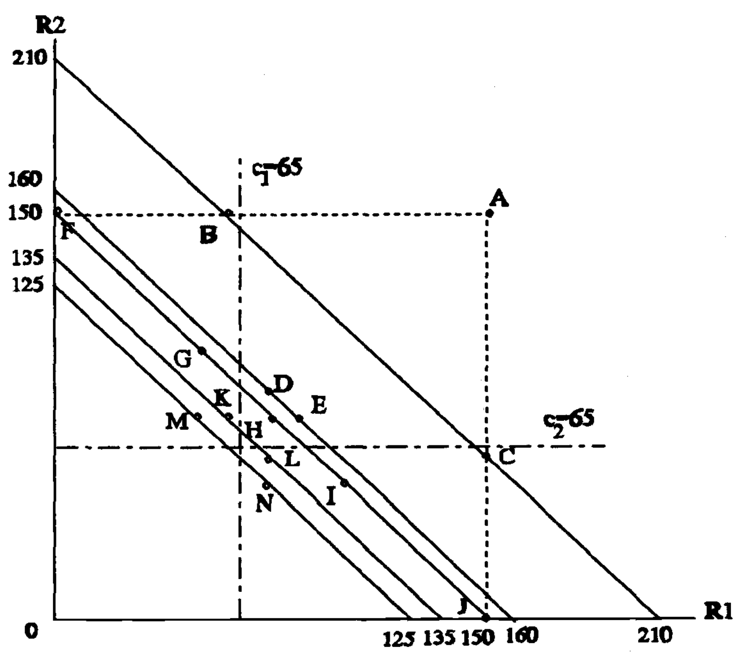

Figure A.2. Population distribution and the strategy of pure bundling pricing in the absence of price controls.

Table A.3 Mixed Bundling Pricing and Its Profits.

Prices (p_1, p_2, p_B)	Purchasers			Profits
	Good 1	Good 2	Bundle	
150,150,210	C,J	B,F	A	420*
150,150,160	J	F	A,B,C,D,E	320
100,100,160	C,J,I	B,F,G	A,D,E,	300
100,100,150	J,I	F,G	A,B,C,D,E,H	260
100,100,135	J	F	A,B,C,D,E,G,H,I,K,L	120
75,75,135	C,J,I,L,N	B,F,G,K,M	A,D,E,H	120

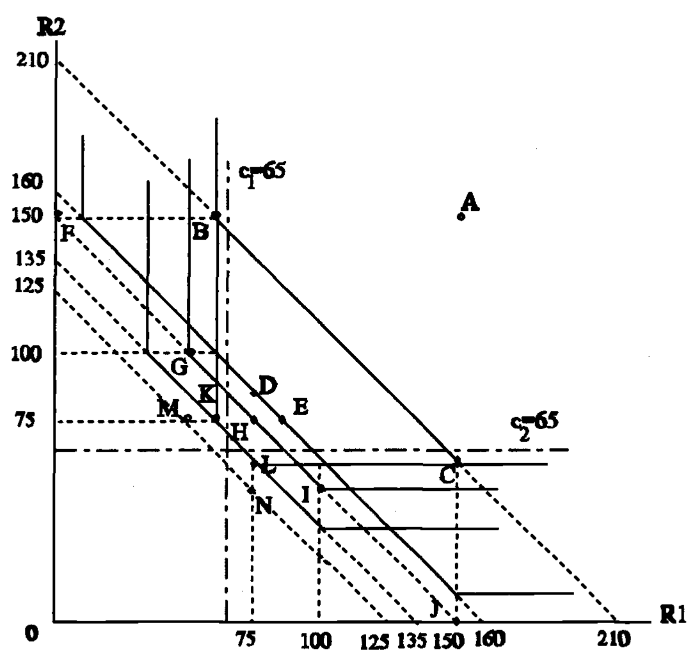

Figure A.3. Population distribution and the strategy of mixed bundling pricing in the absence of price controls.

Figure A.3 shows how each price level is determined for the mixed bundling pricing strategy. Table A.3 shows the purchasers and the profits to the seller

corresponding to each price level by using the mixed bundling pricing strategy in the absence of price controls. The maximum profit is 420 at prices (150, 150, 210).

From the above three tables A.1, A.2 & A.3, we can see that 510 is the largest profit value obtained by using the separate components pricing strategy. Thus, the separate components pricing strategy is the optimal strategy in the absence of price controls. The maximum profit is 510 at prices ($p_1=p_2=150$).

A.2 Under price controls: ceiling prices, $p_{1c}=p_{2c}=75$; $c_1=c_2=65$

In the presence of price controls, the price for each good cannot be greater than the ceiling price, the price for a bundle including one unit of each good cannot be greater than the sum of the ceiling prices.

Figure A.4 shows how the price level is determined for the separate components pricing strategy. Table A.4 shows the purchasers and the profits to the seller corresponding to the price level by using the separate components pricing strategy under price controls. In this case, only one price level is available. The profit is 180 at the ceiling prices (75, 75).

Table A.4 Separate Components Pricing and Its Profits

Prices (p_1,p_2)	Purchasers			Profits
	Good 1	Good 2	Both Goods	
75,75	C,J,I,L,N	B,F,G,K,M	A,D,E,H	180*

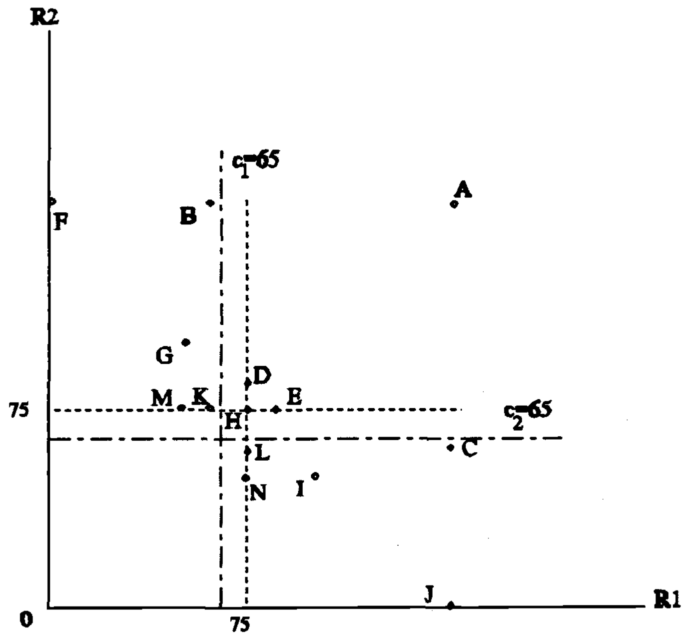

Figure A.4. Population distribution and the strategy of separate components pricing under price controls (p_{1c}=p_{2c}=75, C_1=C_2=65).

Table A.5 Pure Bundling Pricing and Its Profits Under Price Controls

Price/Bundle	Purchasers	Profits
150	A,B,C,D,E,F,G,H,I,J	200*
135	A,B,C,D,E,F,G,H,I,J,K,L	60
125	A,B,C,D,E,F,G,H,I,J,K,L,M,N	-70

Figure A.5 shows how each price level is determined for the pure bundling pricing strategy. Table A.5 shows the purchasers and the profits to the seller corresponding to each price level by using the pure bundling pricing strategy under price

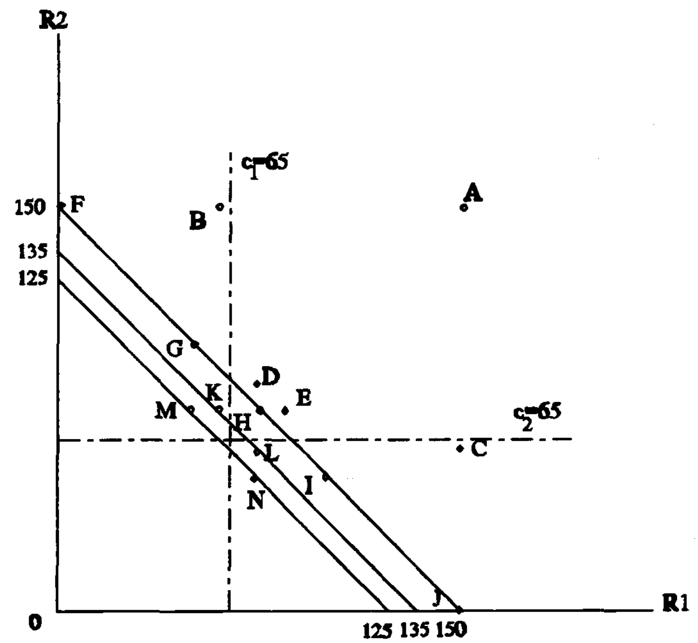

Figure A.5. Population distribution and the strategy of pure bundling pricing under price controls (p_{1c}=p_{2c}=75, C_1=C_2=65).

Table A.6 Mixed Bundling Pricing and Its Profits.

Prices (p_1,p_2,p_B)	Purchasers			Profits
	Good 1	Good 2	Bundle	
75,75,135	C,J,I,L,N	B,F,G,K,M	A,D,E,H	120*
75,75,125	J,I,N	F,G,M	A,B,C,D,E,H,K,L	20

controls. The maximum profit is 200 at price p_B=150 which is equal to the sum of the ceiling prices.

Figure A.6 shows how each price level is determined for the mixed bundling pricing strategy. Table A.6 shows the purchasers and the profits to the seller

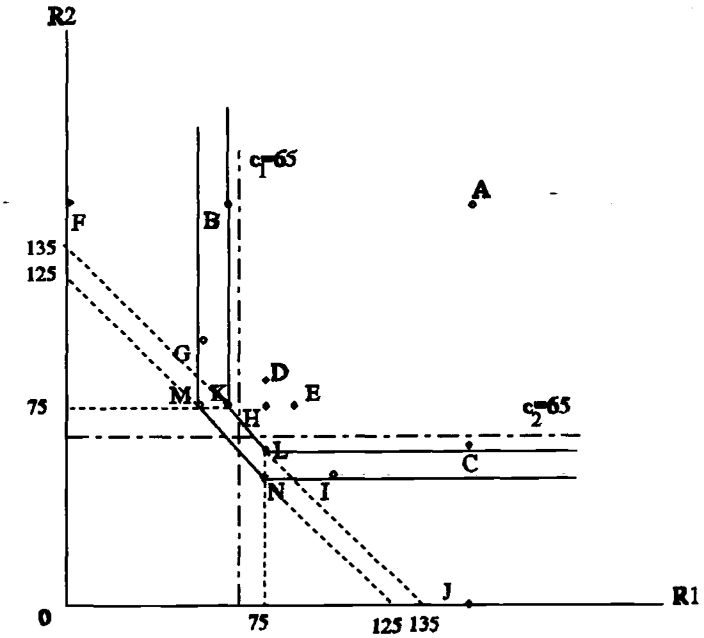

Figure A.6. Population distribution and the strategy of mixed bundling pricing under price controls (p_{1c}=p_{2c}=75, C_1=C_2=65).

corresponding to each price level by using the mixed bundling pricing strategy under price controls. The maximum profit is 120 at prices (75, 75, 135).

From the above three tables A.4, A.5 & A.6, we can see that 200 is the largest profit value obtained by using the pure bundling pricing strategy. Thus, the pure bundling pricing strategy is the optimal strategy under price controls. The maximum profit is 200 at prices (p_B=150). 1.3 Under price controls: ceiling prices: p_{1c}=84 and p_{2c}=84; c_1=65 and c_2=65.

A.3 Under price controls: ceiling prices, $p_{1c}=p_{2c}=84$; $c_1=c_2=65$

In this case, I change the ceiling prices to $p_{1c}=84$ and $p_{2c}=84$, and see how a change in the ceiling prices will influence the profitability of the each pricing strategy.

Figure A.7 shows how each price level is determined for the separate components pricing strategy. Table A.7 shows the purchasers and the profits to the seller corresponding to each price level by using the separate components pricing strategy under price controls. In this case, only two price levels are available. The maximum profit is 190 at the ceiling prices (84, 84).

Table A.7 Separate Components Pricing and Its Profits

Prices (p_1,p_2)	Purchasers			Profits
	Good 1	Good 2	Both Goods	
84,84	C,J,I,E	B,F,G,D	A,	190*
75,75	C,J,I,L,N	B,F,G,K,M	A,D,E,H	180

Figure A.8 shows how each price level is determined for the pure bundling pricing strategy. Table A.8 shows the purchasers and the profits to the seller corresponding to each price level by using the pure bundling pricing strategy under price controls. The maximum profit is 200 at price $p_B=150$ which is less than the sum of the ceiling prices.

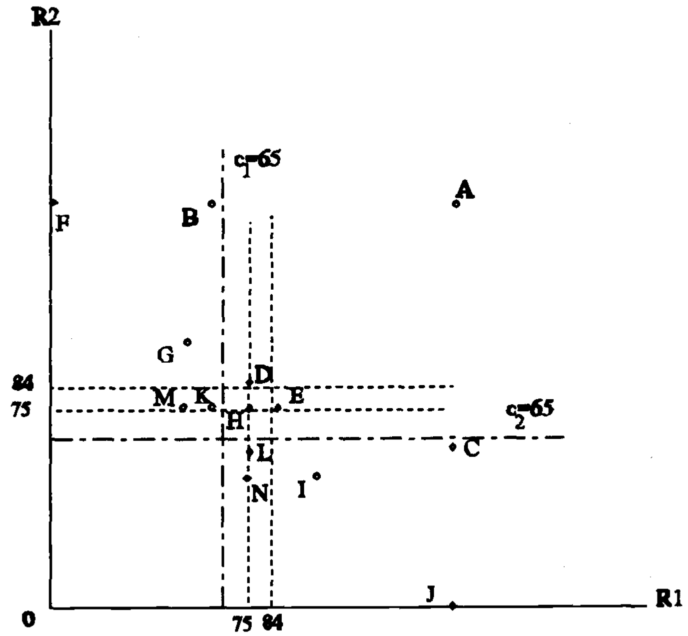

Figure A.7. Population distribution and the strategy of separate components pricing under price controls ($p_{1c}=p_{2c}=84$, $C_1=C_2=65$).

Table A.8 Pure Bundling Pricing and Its Profits.

Price/Bundle	Purchasers	Profits
168	A,B,C,	114
160	A,B,C,D,E	150
150	A,B,C,D,E,F,G,H,I,J	200*
135	A,B,C,D,E,F,G,H,I,J,K,L	60
125	A,B,C,D,E,F,G,H,I,J,K,L,M,N	-70

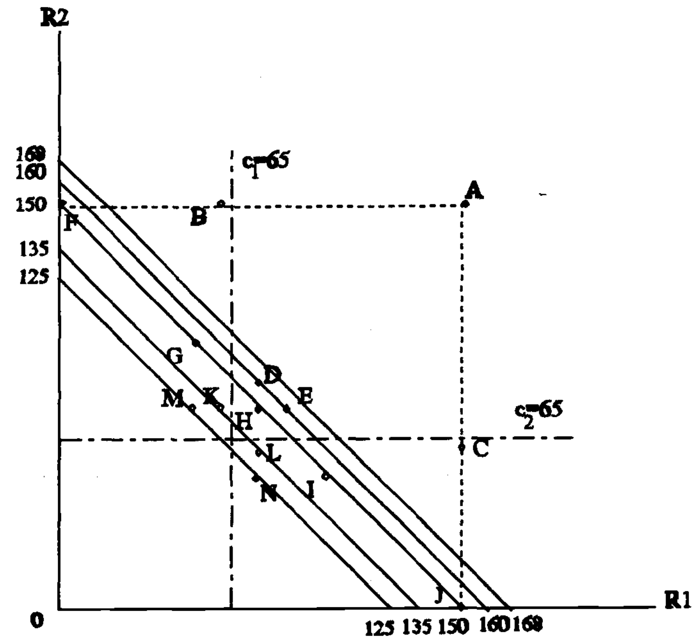

Figure A.8. Population distribution and the strategy of pure bundling pricing under price controls (p_{1c}=p_{2c}=84, C_1=C_2=65).

Table A.9 Mixed Bundling Pricing and Its Profits.

Prices (p_1,p_2.p_B)	Purchasers			Profits
	Good 1	Good 2	Bundle	
84,84,150	C,J,I	B,F,G	A,D,E,H	194*
84,84,135	J,I	F,G	A,B,C,D,E,H,K,L	116
75,75,135	C,J,I,L,N	B,F,G,K,M	A,D,E,H	120
84,84,125	J	F	A,B,C,D,E,G,H,I,K,L,M,N	-22
75,75,125	J,I,N	F,G,M	A,B,C,D,E,H,K,L	20

Figure A.9 shows how each price level is determined for the mixed bundling pricing strategy. Table A.9 shows the purchasers and the profits to the seller

corresponding to each price level by using the mixed bundling pricing strategy under price controls. The maximum profit is 194 at prices (84, 84, 150).

From the above three tables A.7, A.8 & A.9, we can see that 194 is the largest profit value obtained by using the mixed bundling pricing strategy. Thus, the mixed bundling pricing strategy is the optimal strategy under price controls in this case. The maximum profit is 194 at prices (84, 84, 150).

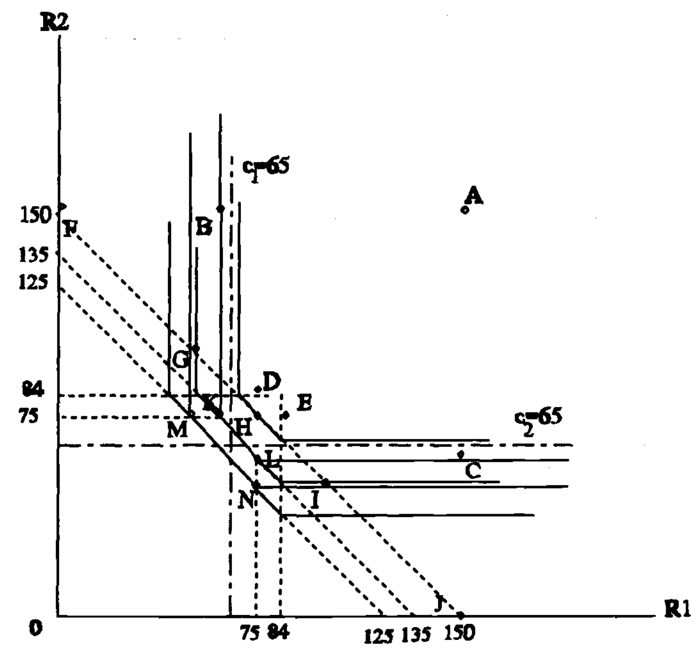

Figure A.9. Population distribution and the strategy of mixed bundling pricing under price controls (p_{1c}=p_{2c}=84, C_1=C_2=65).

A.4 Under price controls: ceiling prices, $p_{1c}=p_{2c}=84$; $c_1=c_2=71$

In this case, I change the costs to $c_1=71$ and $c_2=71$, and see how a change in the costs will influence the profitability of the each pricing strategy.

Figure A.10 shows how each price level is determined for the separate components pricing strategy. Table A.10 shows the purchasers and the profits to the seller corresponding to each price level by using the separate components pricing strategy under price controls. In this case, only two price levels are available. The maximum profit is 130 at the ceiling prices (84, 84).

Table A.10 Separate Components Pricing and Its Profits

Prices (p_1,p_2)	Purchasers			Profits
	Good 1	Good 2	Both Goods	
84,84	C,J,I,E	B,F,G,D	A	130*
75,75	C,J,I,L,N	B,F,G,K,M	A,D,E,H	72

Figure A.11 shows how each price level is determined for the pure bundling pricing strategy. Table A.11 shows the purchasers and the profits to the seller corresponding to each price level by using the pure bundling pricing strategy under price controls. The maximum profit is 90 at price $p_B=160$ which is less than the sum of the ceiling prices.

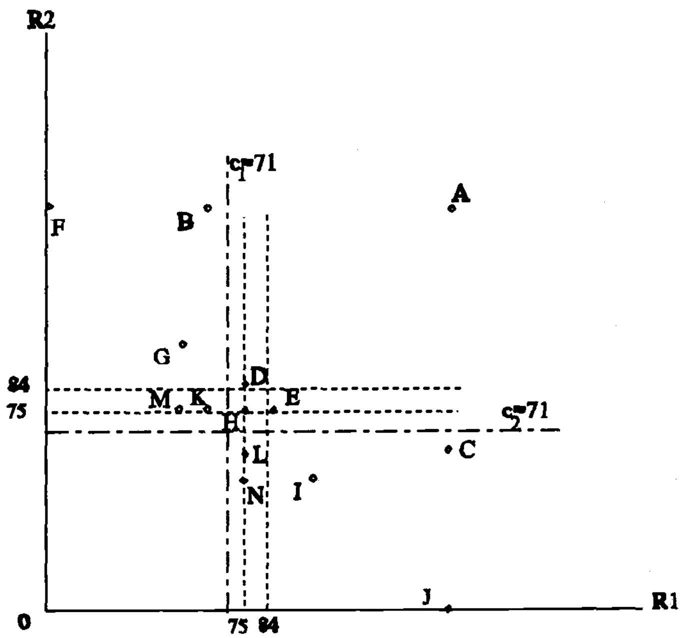

Figure A.10. Population distribution and the strategy of separate components pricing under price controls ($p_{1c}=p_{2c}=84$, $C_1=C_2=71$).

Table A.11 Pure Bundling Pricing and Its Profits.

Price/Bundle	Purchasers	Profits
168	A,B,C,	78
160	A,B,C,D,E	90*
150	A,B,C,D,E,F,G,H,I,J	80
135	A,B,C,D,E,F,G,H,I,J,K,L	-84

Figure A.12 shows how each price level is determined for the mixed bundling pricing strategy. Table A.12 shows the purchasers and the profits to the seller

corresponding to each price level by using the mixed bundling pricing strategy under price controls. The maximum profit is 110 at prices (84, 84, 150).

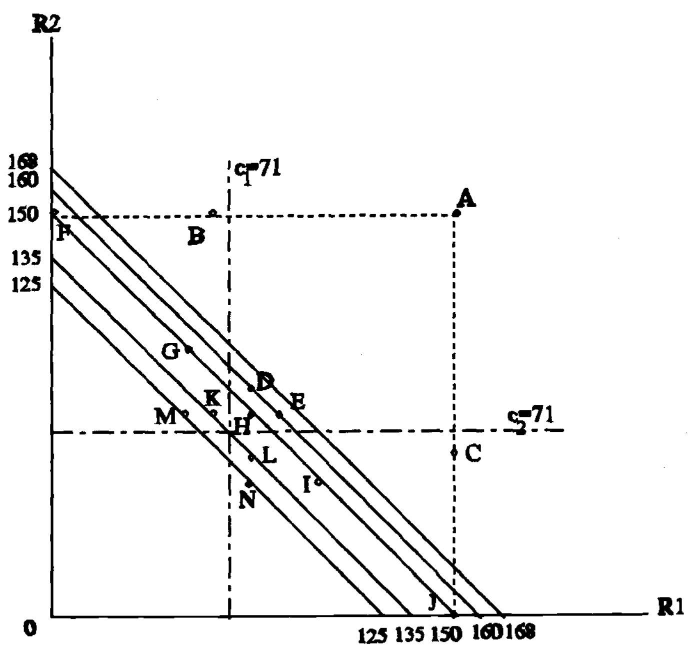

Figure A.11. Population distribution and the strategy of pure bundling pricing under price controls ($p_{1c}=p_{2c}=84$, $C_1=C_2=71$).

Table A.12 Mixed Bundling Pricing and Its Profits.

Prices (p_1,p_2,p_B)	Purchasers			Profits
	Good 1	Good 2	Bundle	
84,84,150	C,J,I	B,F,G	A,D,E,H	110*
84,84,135	J,I	F,G	A,B,C,D,E,H,K,L	-4
75,75,135	C,J,I,L,N	B,F,G,K,M	A,D,E,H	12
84,84,125	J	F	A,B,C,D,E,G,H,I,K,L,M,N	-178
75,75,125	J,I,N	F,G,M	A,B,C,D,E,H,K,L	-112

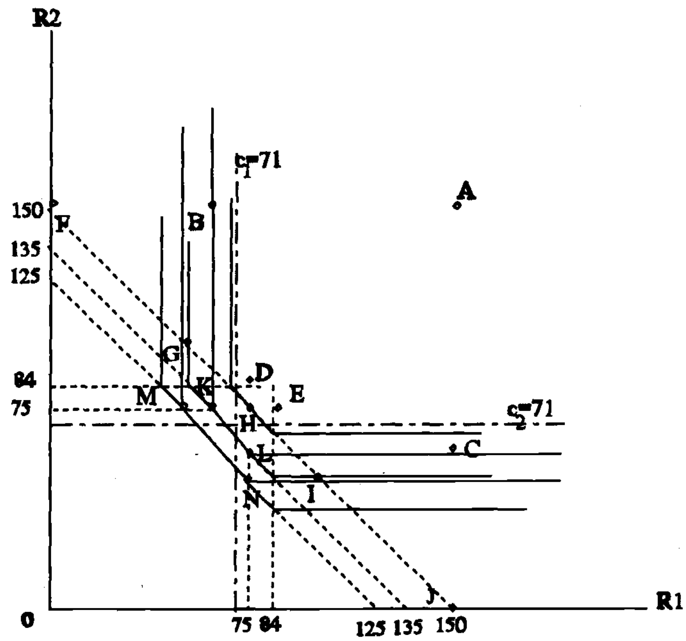

Figure A.12. Population distribution and the strategy of mixed bundling pricing under price controls (p_{1c}=p_{2c}=84, C_1=C_2=71).

From the above three tables A.10, A.11 & A.12, we can see that 130 is the largest profit value obtained by using the separate components pricing strategy. Thus, the separate components pricing strategy is the optimal strategy under price controls in this case. The maximum profit is 130 at prices (84, 84).

A.5. Under price controls: ceiling prices, p_{1c}=p_{2c}=100; c_1=c_2=65

In this case, I change the ceiling prices to p_{1c}=100 and p_{2c}=100, and see how a change in the ceiling prices will influence the profitability of the each pricing strategy.

Figure A.13 shows how each price level is determined for the separate components pricing strategy. Table A.13 shows the purchasers and the profits to the seller corresponding to each price level by using the separate components pricing strategy under price controls. The maximum profit is 280 at the ceiling prices (100, 100).

Table A.13 Separate Components Pricing and Its Profits

Prices (p_1,p_2)	Purchasers			Profits
	Good 1	Good 2	Both Goods	
100,100	C,J,I	B,F,G	A	280*
85,85	C,J,I,E,	B,F,G,D	A	200
75,75	C,J,I,L,N	B,F,G,K,M	A,D,E,H	180

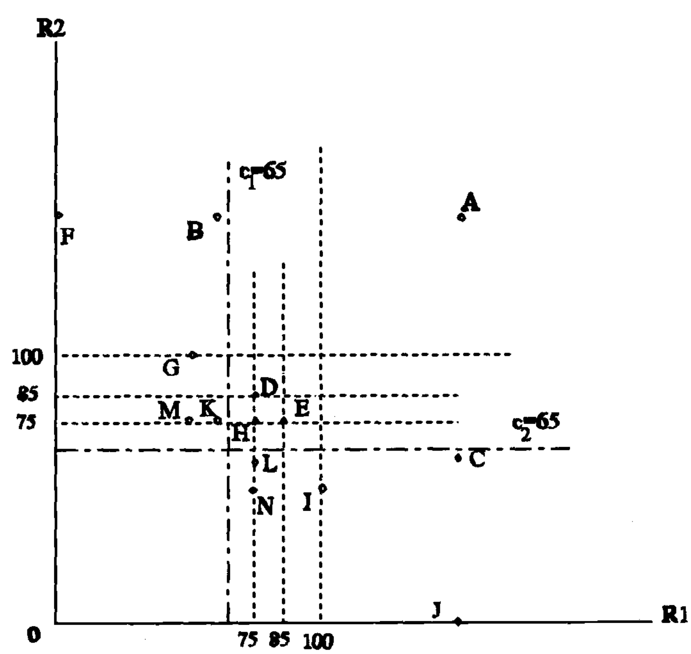

Figure A.13. Population distribution and the strategy of separate components pricing under price controls (p_{1c}=p_{2c}=100, C_1=C_2=65).

Figure A.14 shows how each price level is determined for the pure bundling pricing strategy. Table A.14 shows the purchasers and the profits to the seller corresponding to each price level by using the pure bundling pricing strategy under price controls. The maximum profit is 210 at price p_B=200 which is equal to the sum of the ceiling prices.

Table A.14 Pure Bundling Pricing and Its Profits.

Price/Bundle	Purchasers	Profits
200	A,B,C,	210*
160	A,B,C,D,E	150
150	A,B,C,D,E,F,G,H,I,J	200
135	A,B,C,D,E,F,G,H,I,J,K,L	60
125	A,B,C,D,E,F,G,H,I,J,K,L,M,N	-70

Figure A.15 shows how each price level is determined for the mixed bundling pricing strategy. Table A.15 shows the purchasers and the profits to the seller corresponding to each price level by using the mixed bundling pricing strategy under price controls. The maximum profit is 300 at prices (100, 100, 160).

From the above three tables A.13, A.14 & A.15, we can see that 300 is the largest profit value obtained by using the mixed bundling pricing strategy. Thus, the mixed bundling pricing strategy is the optimal strategy under price controls in this case. The maximum profit is 300 at prices (100, 100, 160).

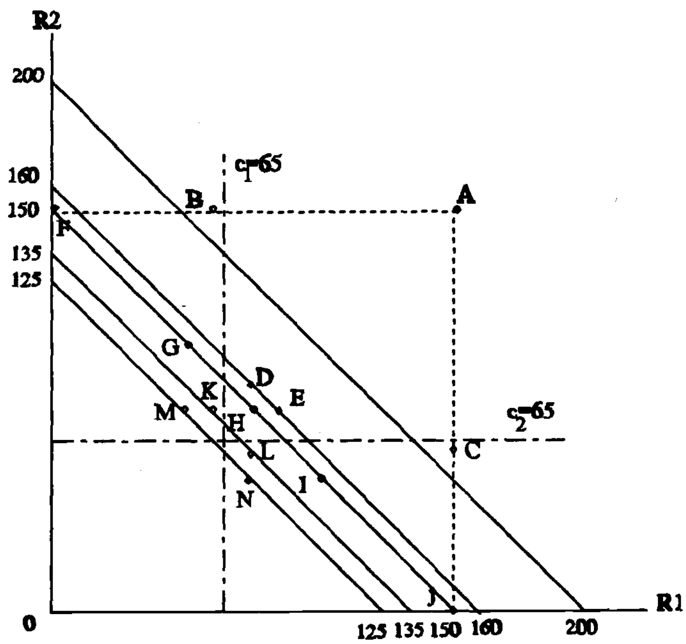

Figure A.14. Population distribution and the strategy of pure bundling pricing under price controls ($p_{1c}=p_{2c}=100$, $C_1=C_2=65$).

Table A.15 Mixed Bundling Pricing and Its Profits.

Prices (p_1,p_2,p_B)	Purchasers			Profits
	Good 1	Good 2	Bundle	
100,100,160	C,J,I	B,F,G	A,D,E	300*
100,100,150	J,I	F,G	A,B,C,D,E,H	280
85,85,150	C,J,I	B,F,G	A,D,E,H	200
100,100,135	J	F	A,B,C,D,E,G,H,I,K,L	120
85,85,135	J,I	F,G	A,B,C,D,E,H,K,L	90
75,75,135	C,J,I,L,N	B,F,G,K,M	A,D,E,H	120

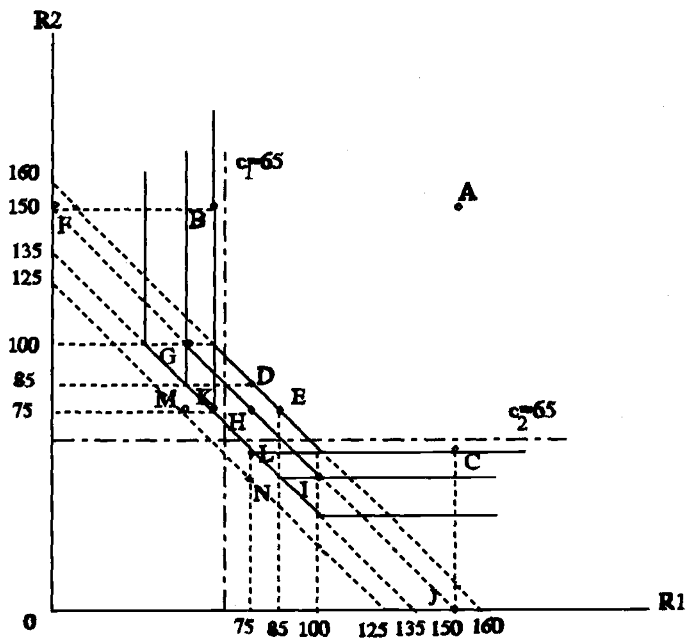

Figure A.15. Population distribution and the strategy of mixed bundling under price controls ($p_{1c}=p_{2c}=100$, $C_1=C_2=65$).

In summary, tying sales arrangements under price controls can be used as a discriminate tool to sort customers into different groups and help the monopolist to explore more of the consumer surplus and increase his profit. The profitability of tying sales arrangements will depend on the consumers' population distribution in the reservation price space, on the ceiling price levels and on the cost levels.

The mixed bundling pricing strategy is always at least as profitable as the pure bundling pricing strategy in the absence of price controls. This conclusion is no longer valid under price controls. Conversely, the pure bundling pricing strategy may dominate the mixed bundling pricing strategy in most circumstances under price controls.