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for Controlling Process Mean

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In production processes, there are two types of variations that affect production quality -- variations produced by chance causes and variations produced by assignable causes. One of the main instruments in quality control used to control quality by distinguishing between variations produced by chance causes and a real process change is the control chart. Each type of control chart has advantages and disadvantages in a specified situation. For example, some control charts fail to detect small shifts, while the others are ineffective to detect large shifts in process mean.

In this study, three types of control charts, namely, $\overline{X}$, cumulative sum, and geometric moving average control charts were compared on an economic basis. A simulation model was developed to simulate the control chart functions in a typical production process. The simulation

was executed in BASIC on an IBM PC/XT. Before comparison, each control chart was matched so that all the control charts have the same characteristics when the process operates in-control for a certain period of time. The effects of the type of control chart, sample size, sampling interval, and the magnitude of shift in process mean on profit per hour were observed and analyzed using Analysis of Variance (ANOVA).

The results show that, in general, the cumulative sum control chart has advantage over the other two types of control charts when shift of small magnitude of about 0.5σ is present. $\bar{X}$-control chart is ineffective to detect small shifts; however, its effectiveness increases sharply as the magnitude of shift increases to values of 1.5σ or beyond. Geometric moving average control chart gives best results at intermediate shift levels of about 1.0σ.

Of the three sample sizes (3, 4 and 5) used in this study, sample size of five yields the highest profit per hour. However, too large a sample size may result in a decrease of profit per hour if the testing causes the destruction of items and the cost of sampling per item is very high.

Small sampling interval of one hour yields the highest profit per hour among three sampling intervals (1, 2 and 4 hours) used in this study. Too small sampling

interval could yield lower profit per hour if the increased cost of more frequent sampling, more investigations caused by false alarms, and more frequent shut down of the production process exceeds the savings from early detection of the shift, particularly, when the cost of sampling, the cost of searching for an assignable cause, and the income per hour of production are very high.

An Economic Comparison of $\overline{X}$, Cumulative Sum
and Geometric Moving Average Control Charts
for Controlling Process Mean

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

AN ECONOMIC COMPARISON OF $\bar{X}$, CUMULATIVE SUM
AND GEOMETRIC MOVING AVERAGE CONTROL CHARTS
FOR CONTROLLING PROCESS MEAN

CHAPTER I

INTRODUCTION

Background

In production processes, it is necessary that products should satisfy customers who expect to purchase good quality products. Failure to satisfy the customers may reflect in the customer's attitude toward the company, loss of business reputation, loss of future business, and finally loss of market share. Furthermore, improper operation with slow or no detection of malfunctioning, results in rework to correct the nonconforming production items; this increases the scrap cost which results in a net loss of labor, material, and defective products that cannot be economically repaired or used.

Quality is defined as level of performance, fitness for use, meeting an expectation, degree of excellence, and conformance to a standard. There are two principal aspects of quality : functional performance and appearance criterion. Functional performance characteristics include satisfactory operation, reliability, durability, and

2

maintainablity.   Appearance characteristics refer to the
cosmetic features such as color, cleanliness, smoothness,
shape, size, and texture.

There are two types of variations that affect
production quality : variations produced by "chance
causes" (or random variations) and variations produced by
"assignable causes" (Figure 1.1). Variations due to chance
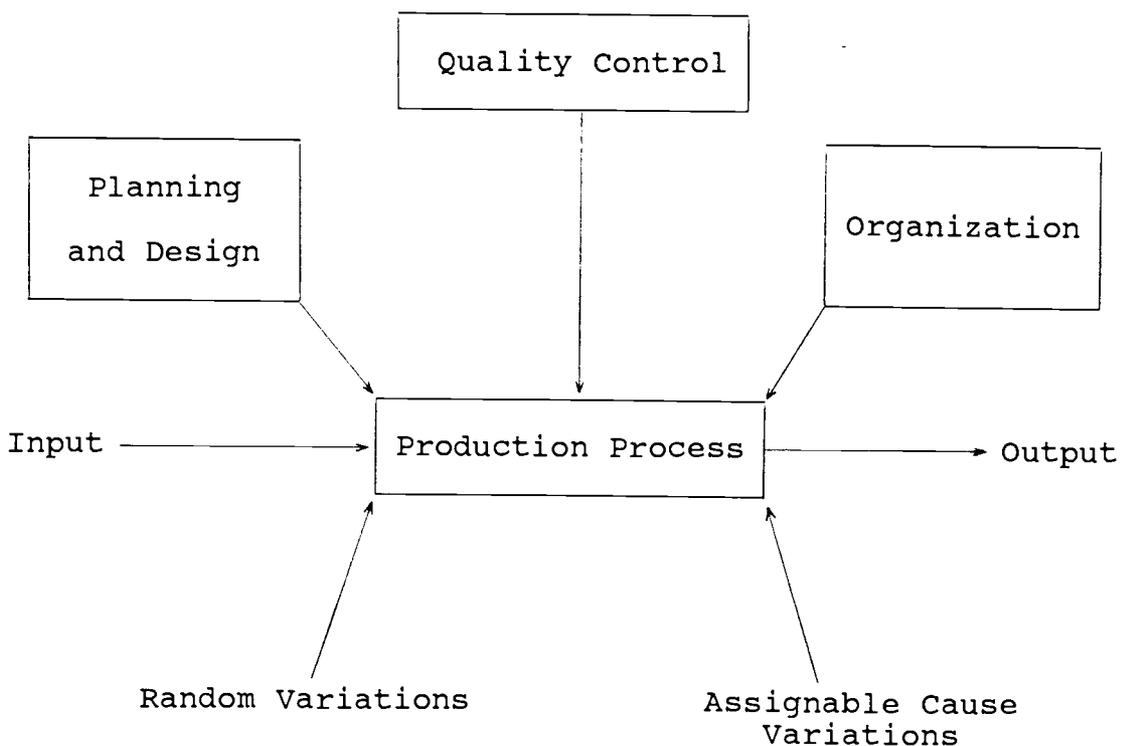
Figure 1.1 Production Process and Quality Control.

causes consist of uncontrollable environmental influences
that cannot be eliminated. Variations due to assignable
causes that may occur as a result from defective raw

materials, improperly adjusted machines, operation error, cumulative effects of heat, tool wear, shock, vibration, contamination, and/or other phenomena can usually be identified and corrected by using statistical quality control methods.

In this study, "control" means the checking of some product outcomes against a standard. Quality Control refers to a spectrum of managerial methods for attempting to maintain the quality of manufactured products at a desired level. Statistical Quality Control refers to all methods that use statistical principles and techniques for controlling product quality.

Modern manufacturing and service industries are faced with a considerable challenge due to increasing customer quality requirements. To meet this challenge, modern statistical quality control methods are used. The basic objective of modern statistical quality control is to control the production process at a desired quality level at the most economical costs. One of the main instruments in quality control used to control quality by distinguishing between variation produced by chance causes and the real change is a control chart.

Control Chart

A control chart is a graphical display of a quality

characteristic that has been measured from a production sample. It is used to make inferences about the control status of a production process. Figure 1.2 shows a basic control chart. The center line represents the target value of the production process. The two horizontal lines above and below the center line shown in Figure 1.2 are called the upper control limit (UCL) and the lower control limit (LCL), respectively.
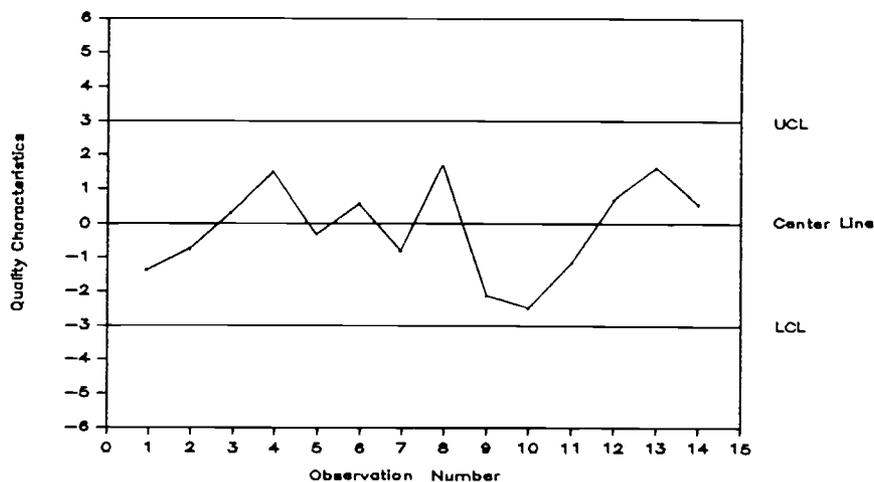
Figure 1.2 A Basic Control Chart.

While the process operates in statistical control, almost all sample average plotted points fluctuate between the upper and lower control limits. Whenever a plotted point falls outside these control limits, it is assumed that there is an occurrence of a shift in process mean

that needs to be investigated to determine if the shift is
due to an assignable cause.

Control Limits Versus Specification limits   Basically,
control limits on a control chart refer to the desired
quality performance.   On the other hand, specification
limits are details of what the product characteristics
should be.   The specification limits are determined by the
management, the product designers or the manufacturing
engineers based on the requirements of the customers.
There is no mathematical or statistical relationship
between the control limits and the specification limits.
Thus, product specification limits might be less than,
equal to, or greater than the quality control limits.

Development of Some Control Chart Procedures   The first
control chart was introduced in 1924 by Shewhart of the
Bell Telephone Laboratories.   Shewhart used $\pm$ 3 standard
deviation as control limits.   Because of its simplicity,
Shewhart's chart (commonly referred to as $\bar{X}$-control chart)
has been widely used to maintain statistical control in
industry.

In 1954, Page introduced the Cumulative Sum control
chart which is an inspection scheme based on cumulative
sums of the differences between each sample average and a
reference value.   He showed that this procedure is much

more sensitive than Shewhart control chart procedure under the situation of small and moderate shifts in the process mean.

In 1959, Roberts developed a graphical procedure called Geometric Moving Average control chart in which the most recent observation is assigned a certain weight, and the weights for all previous observations decrease in a geometric progression.

Objective

It is not clear that in a process with a shift of moderate magnitude in process mean which type of control chart is appropriate based on economic criteria , since each type of control chart has advantages and disadvantages in a specified situation. For example, some control charts fail to detect small shifts, while others are ineffective to detect the large shifts in process mean.

The objective of this research is to compare three types of control charts, namely, $\bar{X}$, Cumulative Sum, and Geometric Moving Average control charts, for controlling the process mean on an economic basis. All control charts are compared on the basis of profit per hour. The comparison is made under the following assumptions :

1. A single assignable cause produces the shifts

of small and moderate magnitude in process mean.

    2. The assignable cause of variation may occur at any time during the operation of the production process.

    3. All three control charts are capable of detecting variation on either side of the target value.

    4. When the process goes out-of-control, it remains out-of-control until the assignable cause is found and removed.

Approach

    A simulation model is developed to simulate the control chart functions in a production process. The simulation is executed in BASIC on an IBM PC/XT. The profit per hour obtained by using each combination of the type of control chart, sample size, sampling interval, and the magnitude of shift in process mean is collected and analyzed.

    Each control chart selected for comparison has been matched to make sure that all of them have the same characteristics when the process operates in-control. Generally, all control charts are matched by the false alarm rate, which is the average number of observations taken before the control chart gives an indication that the process is out-of-control when the process is actually operating in-control. An indication by a control chart of

an out-of-control condition when the process actually is operating in-control is usually called a "false alarm".

In this study, all control charts are matched so that all of them have equal number of false alarms, equal false alarm rate, and give equal profit per hour when the process operates in-control for a specified period of time. The effect of the type of control chart, sample size, sampling interval, and the magnitude of shift in process mean on profit per hour during the production process are observed by using simulation. The results obtained from simulation are analyzed by using Analysis of Variance. The analysis investigate the effect of the type of control chart, sample size, sampling interval, and the magnitude of shift in process mean on the economic design of a control chart.

CHAPTER II

LITERATURE REVIEW

Population and Sample Parameters

In plotting a control chart, samples of size n are usually drawn from the production items that represent the product population. The sample average is then computed, and plotted against observation number on a control chart. If the production process is normally distributed with mean μ and variance σ^2, then the sample average computed from samples of size n, taken independently from the product population, are distributed with mean μ and sample variance $\sigma_{\overline{X}}^2$ where $\sigma_{\overline{X}}^2$ is equal to σ^2/n .

$\overline{X}$-Control Chart

$\overline{X}$-control chart usually uses $\pm$ 3 standard deviations (referred to as 3σ) from the mean as control limits. This allows about 99.7 percent of the plotted points to fluctuate in this region while the process is in-control. When a point reaches 3σ limits, it is assumed that there is an occurrance of an assignable cause that results in a shift in process mean.

Freund (1962) suggested that in plotting a control

chart, increasing the sample size will reduce the range of random fluctuation because the sample standard deviation ($\sigma_{\bar{X}}$) becomes smaller. By using $3\sigma_{\bar{X}}$ as the control limits, the control chart will be more powerful to detect a real change without changing the probability of Type I error.

To increase the power in detecting shifts of moderate magnitude in the process mean, several modifications of the $\bar{X}$-control chart are used. According to some of the decision rules suggested by Duncan (1974), an action is taken if one point falls on or outside $3\sigma_{\bar{X}}$ region, two consecutive points fall on or outside $2\sigma_{\bar{X}}$ region, or four consecutive points fall on or outside $1\sigma_{\bar{X}}$ region.

Figure 2.1 shows a plot of $\bar{X}$-control chart with warning limits. The values plotted in Figure 2.1 are shown
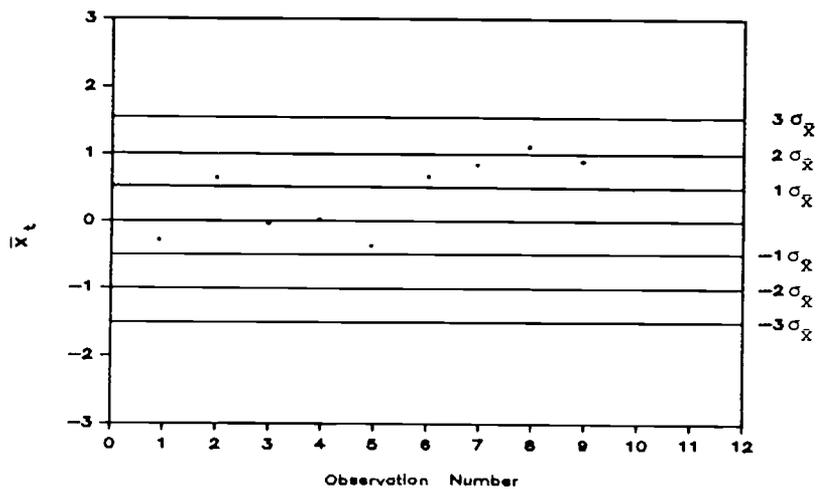
Figure 2.1 $\bar{X}$-Control Chart with Warning Limits.

Table 2.1   Example Data for Control Charts.

Observation Number	Sample Average $\bar{X}_t$	Cumulative Sum $\sum(\bar{X}_t - \mu)$	Geometric Moving Ave. $r = 0.25$ $Z_t = r*\bar{X}_t + (1 - r)*Z_{t-1}$
1	-0.28	-0.28	-0.07
2	0.62	0.34	0.10
3	-0.03	0.31	0.07
4	0.01	0.32	0.05
5	-0.42	-0.10	-0.06
6	0.65	0.55	0.11
7	0.79	1.34	0.28
8	1.15	2.49	0.50
9	0.87 *	3.36 *	0.59 *
10	0.24	0.24	0.06

in Table 2.1. The sample average represents an average of four samples.   The first five observations are random samples from a normal distribution with a mean of zero and a standard deviation of one.   The last five   observations are random samples from a normal distribution with a mean of one and a standard deviation of one. Points 1 through 5 fluctuate around the mean of zero.   The process shifts at point 6;   however action is taken at point 9 when there have been four consecutive points outside $1\sigma_x$ warning limits.   The symbol   *   shown in Table 2.1 indicates that,

at this point, the control chart gives an out-of-control indication.

Economic Design of $\bar{X}$-Control Chart

Several studies have been done to determine the optimal values of $\bar{X}$-control chart parameters. Duncan (1956) is the pioneer investigator of the economic design of $\bar{X}$-control chart. He formulated an economical model to determine the optimal sample size, sampling interval and control limit of $\bar{X}$-control chart to maximize the average net income of a production process under the assumption that a single assignable cause which takes form of a shift of constant magnitude is present. He assumed that the standard deviation of the process remains stable and the time of occurance of the assignable cause follows the negative exponential distribution. Furthermore, he assumed that the process is not shut down while the search for the assignable cause is in progress. An approximation to the optimal design was found. Sensitivity analysis for anticipated changes in the parameters of the model were also made.

Goel (1968) developed a computer algorithm to find the exact optimal parameters of Duncan's model. Comparison of Goel's results with Duncan's algorithm yields smaller cost but in many cases the difference between the two is

insignificant.

Knappenberger and Grandage (1969) developed a method for determining the optimal parameters of a $\bar{X}$ -control chart that minimize the expected cost per unit produced. The authors assumed that the time the process remains in control follows an exponential distribution, and the expected cost per unit produced consists of the sum of three components -- the expected cost of sampling and testing, the expected cost of searching for an assignable cause and repairing the process, and the expected cost associated with the production of defective items. However, the assumption that the delay time required to take the samples, inspect the samples, perform computations, plot the sample average, and test whether or not the control chart indicates that the process is in-control is negligible, seems to be unrealistic especially with large sample sizes and complicated inspection procedures. A two-staged procedure was developed to obtain the optimal parameters of $\bar{X}$-control chart. Two computer programs were developed to perform the two-stage procedure.

Duncan (1971) extended his single assignable cause model to multiple assignable causes. Each asssignable cause produces a shift of known magnitude in the process mean and the time of the occurance of the assignable causes are assumed to be independently, exponentially

distributed.   In this case, the optimal parameters of an $\bar{X}$-control chart are obtained.

Gibra (1971) developed a model to determine the optimal parameters, sample size, sampling interval, and the factor determining the spread of the control limits of $\bar{X}$-control chart.   He suggested in the model discussed, the control chart was used not only as a device for detecting an out-of-control condition but also for maintaining a prescribed quality level of product.   Gibra assumed that the standard deviation remains stable and the process is subject to the occurance of a single assignable cause.   The time that the process remains in control follows the exponential distribution, and the process is allowed to continue in operation during the search process.

Chiu and Wetherill (1974) proposed a simple semi-economic scheme for the design of $\bar{X}$-control chart that could be used by the practitioner. The authors found that 25 semi-economic plans are generally very close to the exact optimal plans worked out by Goel (1968).   Their semi-economic plans were also applied to the multiple cause model of Duncan (1971) and the modified model of Taylor (1968), and were shown to yield satisfactory results.

Hu (1986) investigated the economic design of the $\bar{X}$-control chart based on the modified model of Duncan

(1956). He assumed that the process failure rate is not constant but may increase over time, and the time between process shifts has a Weibull distribution. Under certain assumptions, the optimal parameters (sample size, sampling interval, and a factor determining the control limits) have been found. He found that this economic design is relatively insensitive to a failure mechanism which has an increasing failure rate. He suggested that when the appropriate assumption are met, the economic design of the $\bar{X}$ -control chart may be obtained from a relative simple model based on the exponential distribution even when the true failure mechanisms of the process do not have constant failure rates.

Cumulative Sum Control Chart

Cumulative sum (Cusum) control chart accumulates the sum of the difference between the sample average and a reference value, k. This modification is more sensitive to detect a shift of moderate magnitude in process mean than the ordinary Shewhart control chart with only 3σ action limits, where each observation is viewed independently.

One-Sided and Two One-Sided Cumulative Sum Control Charts

Cumulative sum control charts could be designed to

detect shifts in either direction or in one direction only from the reference value. Page (1954) introduced the concept of cumulative sum control chart. For the situation where the only interest is an increase in the process mean, Page (1961) suggested the method of plotting S_{t1}, the cumulative sum of the differences between each sample average and a reference value k_1, on an one-sided cusum chart. The cumulative sum quantity

$$S_{t1} = \sum_{t=1}^{t} (\overline{X}_t - k_1)$$

is plotted against the observation number, t, where $\overline{X}_t$ is the sample average of tth observation. The graph shows a downward trend when the process mean is less than k_1 and an upward trend when the process mean is greater than k_1. A decision that there is a positive shift in process mean is taken after the t-th observation if

$$S_{t1} \geq h ,$$

where h is the control limit. Whenever the cusum value, S_{t1} becomes negative or an action is taken, the cumulation is reset to zero.

For two-sided control, the second one-sided cumulative sum control chart with reference value k_2, where $k_1 > k_2$, and control limit -h is necessary for checking whether there is a negative shift in process mean. The point

$$S_{t2} = \sum_{t-1}^{t} (\overline{X}_t - k_2)$$

is plotted against t on a seperate control chart simultaneously, and an action is taken if

$$S_{t2} \leq -h .$$

As soon as the action is taken or the cumulative sum value, S_{t2}, for detecting a negative shift becomes positive, the cumulation is reset to zero. Usually, the reference values k_1 and k_2 are chosen about halfway between the target value, μ and the rejectable quality values, μ_1 and μ_2 respectively, where $\mu_1 > \mu > \mu_2$.

Cumulative Sum Control Chart Using V-Mask

The other method for dectecting two-sided variation is given by Barnard (1959). The difference between the sample average and the target value (μ), is accumulated as the cumulative sum value S_t. The cumulative sum control chart is formed by plotting the quantity

$$S_t = \sum_{t-1}^{t} (\overline{X}_t - \mu)$$

against the observation number t, and each point is then tested by a symmetric V-mask with an angle 2θ at its vertex originating a distance d ahead from the current point. The degree of half angle of V-mask, θ , is

specified so as to distinguish between the variations due to chance and real variations at the value of W*tan θ. The distance d that V-mask vertex is placed ahead of the current point is specified to set up the upper and lower arms of the V-mask. These are the decision limits. When a previous point is above the upper arm of the mask, then a conclusion can be drawn that there is a shift in the process mean below the target value.

Some experiments by Barnard (1959) showed that the changes of moderate magnitude in process mean are most easily noticed when one step on the horizontal axis and one step on the vertical axis are both equal to 2σ. Traux (1961) made a similar conclusion. According to this study, a scale factor (W) of one step on horizontal axis and one step on vertical axis both equal to 2- to 3-σ gives very satisfactory graphical interpretation.

Figure 2.2 shows a cumulative sum control chart. The values of S_t are plotted against the observation number. The plotted values of S_t are shown in the second column of Table 2.1. They are based on the sample averages of the first column, and since the mean (μ) is zero, S_t is a sum of sample averages. The plot in Figure 2.2 uses θ of 14 degree and d of 8.5*W, where W is equal to $2\sigma_{\bar{x}}$. The ninth point shows an evidence of a positive shift in the process mean since the fifth point falls below the lower limb of V-mask. This necessitates corrective action.

19
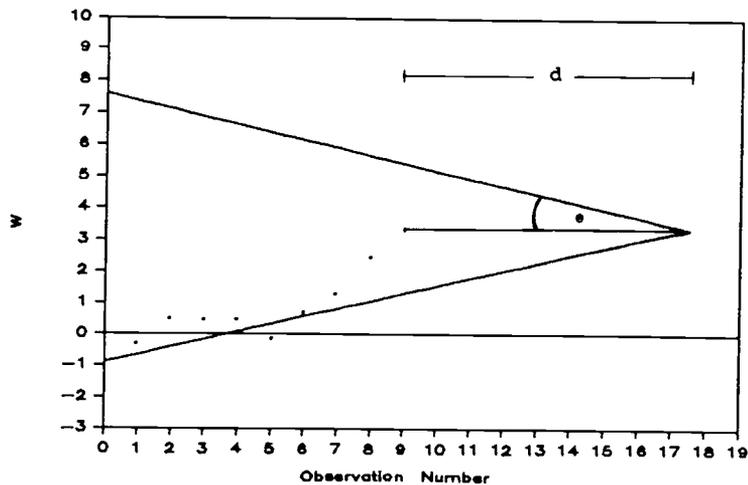
Figure   2.2   Cumulative Sum Control Chart   Using
                V-Mask.

Equivalence of a V-Mask and Two One-Sided Cumulative Sum
Control Charts

The reference values of two one-sided cumulative sum
control charts [Page, 1961] in terms of θ and   d   of
cumulative sum control chart using V-mask are :

$$k_1 = \mu + W * \tan \theta, \quad \text{and}$$

$$k_2 = \mu - W * \tan \theta.$$

The decision limits for two one-sided cumulative sum
control charts using k_1 and k_2 as reference values that
run concurrently on separate charts in terms of θ and   d
of cumulative sum control chart using V-mask are :

$$h = d * \tan \theta , \quad \text{for } k_1 , \text{ and}$$

$$- h \ \ = - d * \tan \theta \ , \ \ \ \text{for} \ \ k_2 \ .$$

Two one-sided cumulative sum control charts are equivalent to a cumulative sum control chart using V-mask with the additional rule that after the first trial after any control chart gives the alarm signal, the cumulative sum values S_t's of both two one-sided cusum charts are reset to zero.

This study uses cumulative sum control chart with V-mask. However, a program was developed using two one-sided cusum charts that performed function equivalent to that of V-mask. The listing of this program is shown in Appendix B. This program may substituted for the cumulative sum control chart using V-mask subroutine (lines 1980 through 2240 of the main program given in Appendix A).

Economic Design of Cumulative Sum Control Chart

Taylor (1968) studied the economic design of cumulative sum control chart for controlling the mean of a normally distributed quality characteristic with known variance. The cumulative sum control chart is maintained to detect a single assignable cause of variation which takes the form of a shift of known magnitude. The process is shut down while a search for the trouble is made. If

the signal is not a false alarm, additional time is required to repair the process. The optimal value of V-mask design parameter θ is obtained by simulation, and the optimal value of the distance that V-mask is placed ahead of the most recent point d is obtained by trial and error under the assumption that the sample size and the sampling interval were known.

Goel and Wu (1973) presented a procedure for the economic design of cumulative sum control chart to control the process mean with a normally distributed characteristic. The authors followed the same assumption and approach used by Duncan (1956) in formulating the expected cost function. The expected elapsed time between the first sample after the occurance of the shift and the last sample prior the detection was determined using the results derived by Taylor (1968). The optimal values of the sample size, sampling interval, and V-mask decision limits were obtained using a computer search technique.

Geometric Moving Average Control Chart

The geometric moving average control chart combines the new sample average with the past data before making any decision about a possible shift in process mean. A weighting factor r, which takes a value between 0 and 1, is assigned to the most recent observation. All previous

observations' weights decrease in a geometric progression from the most recent back to the first.

A point, Z_t, on a geometric moving average control chart at time t is given by :

$$Z_t = r * \bar{X}_t + (1 - r) * Z_{t-1}$$

where
$$t = 1, 2, 3, \ldots$$
$$0 < r \leq 1,$$
$$Z_o = \mu,$$

$\bar{X}_t$ is the current process average , and

Z_{t-1} is the value of the immediate successor point.

Roberts (1966) showed that the standard deviation of the Z_t's in order to compute their control limits under the assumption that all $\bar{X}_t$'s have equal variance $(\sigma_{\bar{X}}^2)$ is given by :

$$\text{Variance } (Z_t) = \frac{(r*[1 - (1 - r)^{2t}])*\sigma_{\bar{X}}^2}{2 - r}$$

where t is the observation number and t > 0. σ_{Zt} will increase to its limiting value after the first few observations :

$$\sigma_{Zt} = [\frac{r}{2 - r}]^{1/2} * \sigma_{\bar{X}}$$

where the sample standard deviation, $\sigma_{\bar{X}} = \sigma/\sqrt{n}$.

Figure 2.3 shows a geometric moving average control

chart, where values of Z_t taken from Table 2.1 are plotted against the observation number. The plot is based on r = 0.25, n = 4, σ = 1, and $σ_{\bar{x}}$ = 1/2 . Control limits of ± 2.71$σ_{Zt}$ are used. These are obtained from the matching procedure described in Chapter III. The ninth point outside the upper control limit (UCL) indicates that there has been a shift in process mean above the target value.
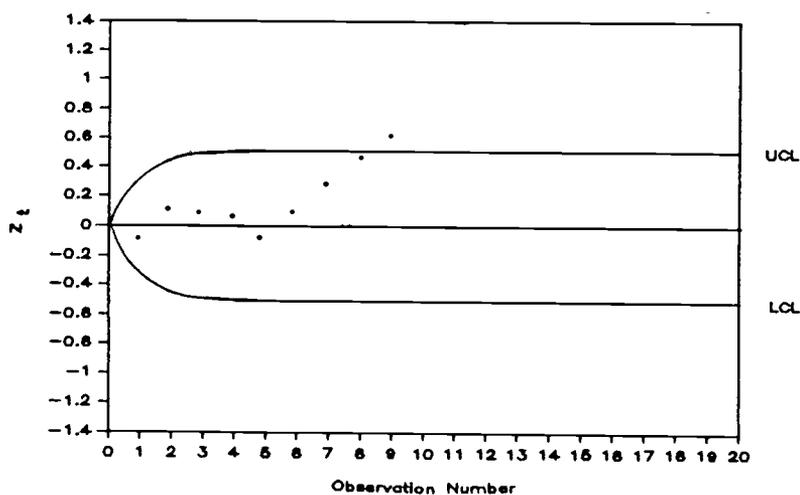
Figure 2.3 Geometric Moving Average Control Chart.

Comparison of Control Charts

Several studies have been done that compare control charts on different criteria. Freund (1962) used average run length (ARL) to compare Shewhart, cumulative sum, geometric moving average, and the acceptance control

charts. The acceptance control chart is used in quality assurance for acceptance or rejection of a product based on a given standard . The study concluded that the cumulative sum or geometric moving average control chart should be used where greater sensitivity in detecting small process shifts is required. The Shewhart control chart should be used in process development work to determine whether or not a "state of statistical control" has been achieved.

Johnson and Leone (1962) made a comparison of average run length for $\bar{X}$ and cumulative sum control chart. They found that the cumulative sum control chart gives more rapid indication of a shift in the process mean for very small magnitude of shifts. They also indicated that this advantage decreases sharply as the magnitude of shift increases. The results of this comparison are in general agreement with the comparison made by Traux (1961).

Ewan (1963) made the comparisons of the standard Shewhart control charts with cumulative sum and weighted mean charts by using average run length. He found that the cumulative sum control charts are more efficient than the Shewhart control charts for the shifts between 0.5σ to 2σ in the process mean. In this region, the shifts can be detected approximately twice as quickly by the cusum chart, or in the same period of time the cusum charts can detect the shifts with much smaller sample sizes.

However, the differences in terms of average run length between cumulative sum and weighted mean charts were small.

Roberts (1966) compared five control chart procedures, namely, Moving Average, Geometric Moving Average, Cumulative Sum, Girshick-Rubin, and Run Sum Charts on the basis of average run length. The comparison was made under the assumptions that the standard deviation, sample size, and sampling interval are fixed. Rough comparisons show the comparable effectiveness of the various types of tests, though the Girshick-Rubin test appears to have a slight advantage over the others under the idealized conditions assumed. Roberts suggested that, in selecting a control chart procedure for a particular application, simplicity and understandability of the computing, plotting, and testing operations are of primary importance.

Goel (1968) compared economically optimal $\bar{X}$- and cumulative sum control charts for the same cost data and technical factors. He found that there is little difference of optimum costs between these two types of control charts.

Chiu and Cheung (1977) compared the $\bar{X}$-control charts with warning limits and optimal cumulative sum control charts on an economic basis. The authors found that the modified $\bar{X}$-charts and cumulative sum control charts are

almost equivalent in most economic respects, and both are slightly better than the ordinary $\bar{X}$-control charts.

Although many studies have been done on the comparison of control charts, most of these comparisons use average run length as a basis, rather than an economic measure. The objective of this research is to fill this void by providing a procedure for economically comparing the types of control charts discussed in this chapter -- $\bar{X}$, cumulative sum, and geometric moving average control charts.

CHAPTER III

MODELING APPROACH

Production Process

To develop an economic model for a production
process, it was necessary to make certain assumptions
about the behavior of the process. The assumptions
summarized below should not greatly effect the generality
of the results.

The process is assumed to start in-control. After a
certain period of time, a significant shift in the system
parameters may occur indicating possible malfunctioning of
the system. Alternatively, the control chart may indicate
an out-of-control condition without an assignable cause, a
situation referred to as a false alarm. In either case,
the process is stopped and search for an assignable cause
is initiated. If the out-of-control indication was false
alarm, the system is restarted once the search for an
assignable cause fails. Otherwise, the problem is
rectified and the system restarted. The production
process models with, and without the presence of an
assignable cause are shown in Figures 3.1 and 3.2,
respectively.
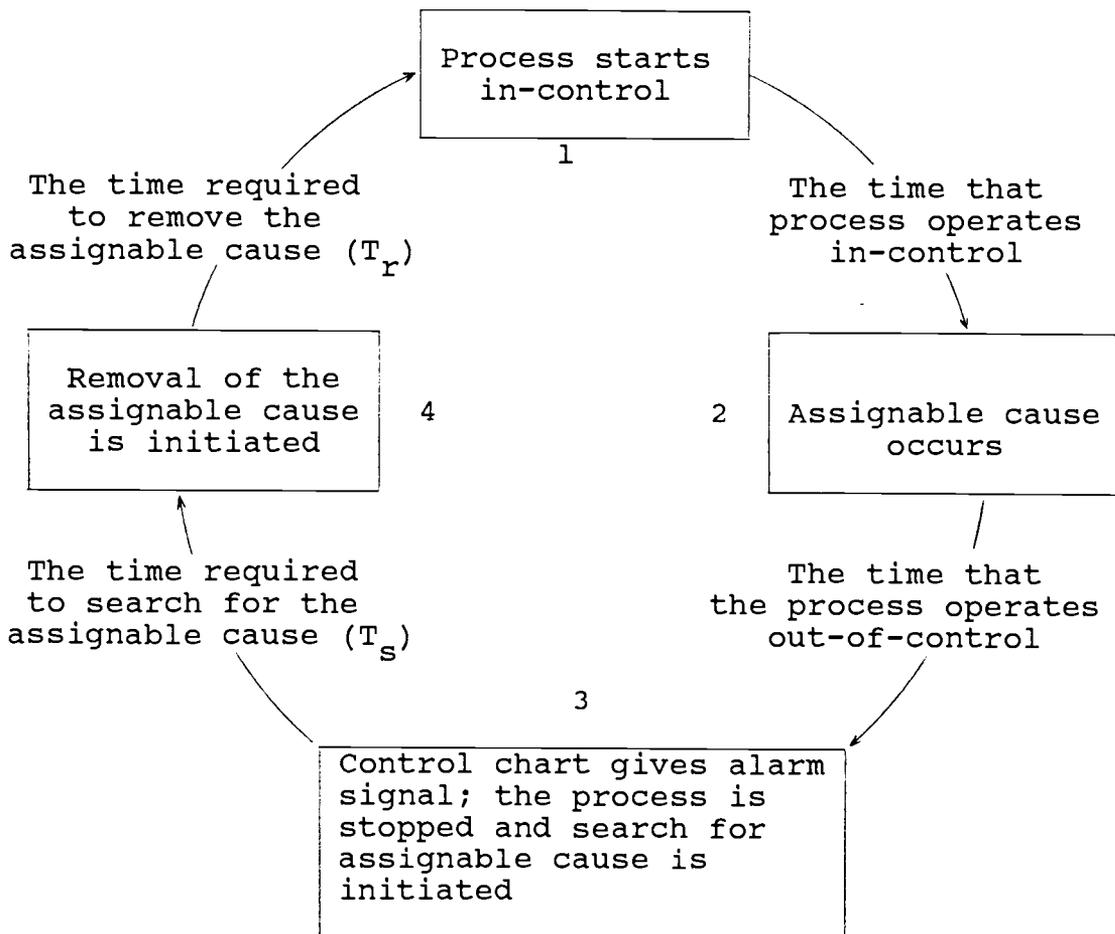
Figure 3.1 The Simulation Cycle When the Assignable
            Cause Exists.
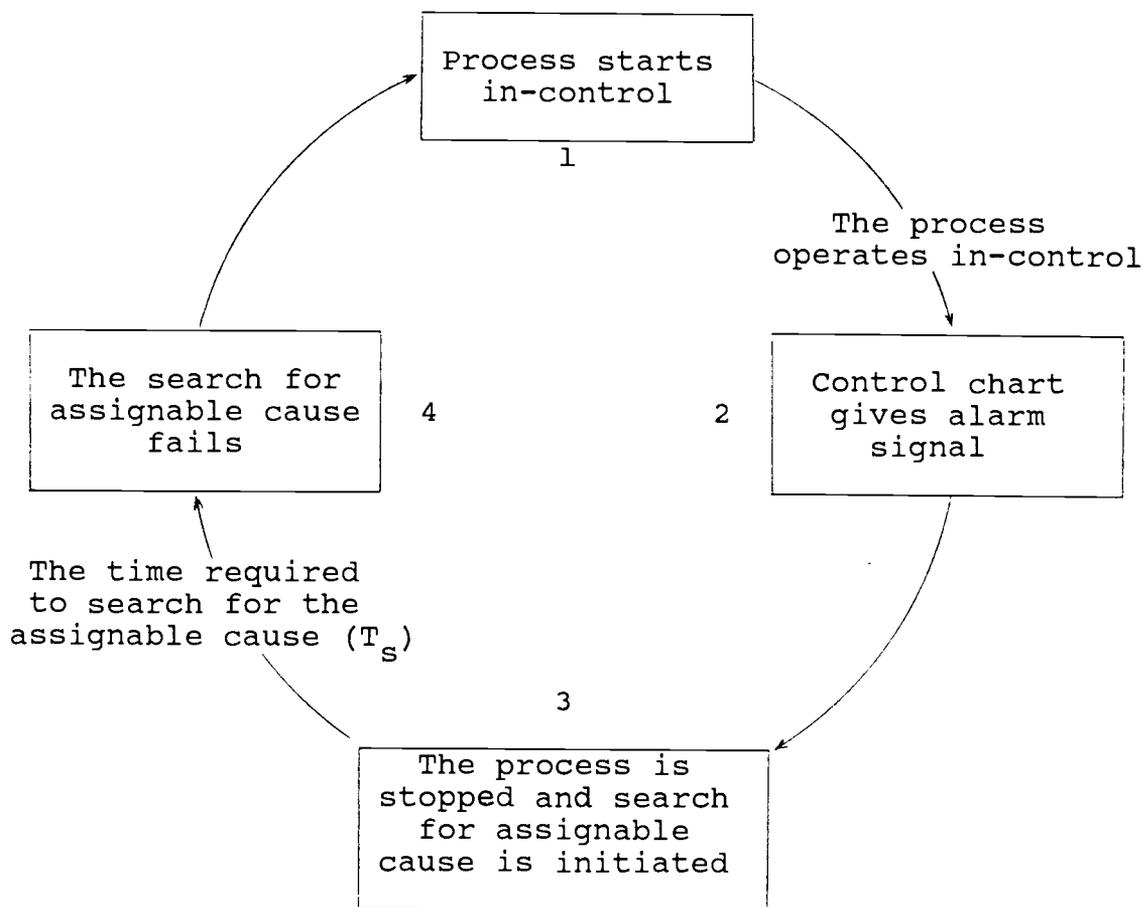
Figure 3.2 The Simulation Cycle When the Assignable
Cause does not Exist.

Statistical Basis of the Control Charts   The production
process is assumed to start in-control with mean μ and
standard deviation σ. The process remains in-control for
a random length of time before a single assignable cause
which takes the form of a shift of known magnitude, δ , in
the process mean occurs.   In terms of the standard
deviation, this shift is given by $\sigma*\delta$. The time that the
process operates in-control is assumed to follow an

exponential distribution with a mean $1/\lambda$ hours. The parameter λ, is assumed to be 0.05. Samples of size n are taken at intervals of s hours. These samples are used to calculate sample average, which is then plotted against the observation number on control charts. The delay time (T_d) in hours required to take samples, inspect the samples, perform computations, plot the sample average, and check whether or not the control chart indicates that the process is in-control, is assumed to consist of both fixed set up time and and variable delay time. The variable delay time is proportional to the sample size. The total delay time in hours per observation in this study is assumed to be

$$0.02 + 0.02*n \quad .$$

Whenever the control chart indicates an out-of-control condition, the process is stopped for a certain period of time (T_s). It is assumed to take 30 minutes to search for an assignable cause. If the signal is not a false alarm, then an additional period of time (T_r), assumed to be one hour, is required to repair the process before it can be restarted.

Selection of Control Chart Parameters

In this study, $\overline{X}$-control chart with warning limits

is used in detecting the shifts of moderate magnitude in process mean. The action is taken if one point falls on or outside $3\sigma_{\bar{x}}$ region, two consecutive points fall on or outside $2\sigma_{\bar{x}}$ region, or four consecutive points fall on or outside $1\sigma_{\bar{x}}$ region.

For the cumulative sum control chart, selecting a too large value of θ results in the insentivity of detecting the assignable cause. On the other hand, if too small value of θ is used, the cumulative sum control chart will be oversensitive. In this study, an half angle of V-mask vertex of 14 degrees is used to distinguish between the variations due to chance and variations due to the real assignable cause at the reference value about half way between the target value and a shift of about 0.5σ when the sample size of four is used. This value (θ) is fixed in the study. The distance d that V-mask vertex is placed ahead of the current point is selected so that the desired characteristics of the cusum chart matches the other two control charts when the process is in-control. The matching procedure is described at the end of this chapter.

For the geometric moving average control chart, the weighting factor r may take a value between 0 and 1. Selecting inappropriate value of r will result in insensitivity of giving the alarm signal when the assignable cause is present. Roberts (1959) suggested

that for early detection of smaller changes, the parameter r decreases from unity to smaller values. Ewan (1963) suggested that to detect large shifts, a high value of r could be used; for small shifts, a smaller value of r is desirable. However, no research has been done to find the optimum value of r under the situation when several small and moderate magnitude of shifts in the process mean are encountered. In this study, a weighting factor value of 0.25 is used. This value was used by Roberts (1959) in the comparison of control charts using average run length as the criterion.

Cost Parameters   Generally, the costs considered in the economic design of control charts are the costs of sampling, testing and maintaining the control chart, the costs of searching for an assignable cause if the control chart indicates an out-of-control condition, the costs of repair or removing the assignable cause, and the costs of operating out-of-control.

Montgomery (1985) suggested that the costs of sampling, testing and maintaining the control chart includes the expenses of inspectors' and technicians' salaries and wages, the cost of testing equipment, and the cost of possibly destroying items sampled. This cost is assumed to consist of both fixed and variable components. Let

a   =   the overhead   cost of sampling, testing and
          maintaining   the   control   chart   per
          observation, and

b   =   the   variable cost of   sampling, testing and
          maintaining the control chart, proportional to
          the sample size, n .

The   cost   of sampling,   testing and maintaining   a   control
chart is then given by

$$a + b * n .$$

Furthermore, let

C   =   cost per   hour   during the   time the   process
          operates out-of-control

C_r =   cost of removing an assignable cause

C_s =   cost of searching for an assignable cause

N_o =   number of observations   during the simulation
          cycle

N_t =   number of true alarms   during   the simulation
          cycle

N_f =   number of false alarms   during the simulation
          cycle

P   =   income per   hour   while   the process operates
          in-control

V   =   average time (hours) between process   shifts
          and detection

The loss cost during the simulation cycle is the summation

of the cost of sampling, testing and maintaining the
control chart, the cost of operating out-of-control, the
cost to search for the assignable cause and the cost of
removing an assignable cause when the process is found
out-of-control. The loss cost function during a
production process completed at the end of any cycle as
shown in Figures 3.1 and 3.2 is given by :

$$\text{Loss cost} = N_o(a + b*n) + N_t(C*V) + C_s(N_t + N_f) + N_t(C_r)$$
$$= N_o(a + b*n) + N_t(C*V + C_r) + C_s(N_t + N_f) \quad.$$

The total income from a production process completed at
the end of any cycle, as defined in Figures 3.1 and 3.2
may be obtained from

$$\text{Income} = P(N_t/\lambda + N_t*V)$$
$$= P*N_t(1/\lambda + V)$$

The profit from a production process at the end of
any cycle is equal to the difference between income and
the loss cost. The profit per hour is equal to the profit
divided by the number of hours of production time.

In this research, the overhead cost of sampling,
testing and maintaining a control chart (a) is assumed to
be $ 0.5 per observation and the variable cost of
sampling, testing and maintaining a control chart (b)
takes a value of $ 0.1 per sample. An observation may
consist of more than one samples, depending upon the

sample size. The cost in dollars per time unit during the time the process operates out-of-control (C), is proportional to the magnitude of shift in process mean, and is assumed to be given by

$$C = 50 * |\delta|,$$

where δ is the magnitude of shift in process mean in term of process standard deviation.

The cost of searching for an assignable cause (C_s) is assumed to be $ 20. The cost of repairing or removing an assignable cause is not included in the loss cost because the experiments performed in this study are based on an equal number of shift occurances during the simulation for all three control charts.

Experimental Methodology

The profit per hour for each experiment under the same cost data and technical factors was obtained by using a simulation model. The simulation model was developed to simulate the control chart functions. The simulation was executed in BASIC on an IBM PC/XT.

Simulation Model   The overall structure of the simulation model is shown in Figure 3.3 . The control chart functions associated with the three control charts are

given in Figures 3.4 through 3.6 . A listing of the program is provided in Appendix A. The major functions performed in Figure 3.3 are summarized below :

1. Parameter initialization and input : the first part of the program consists of initializing cost parameters and production process parameters, such as the process mean and standard deviation, and obtaining input parameters for the simulation such as type of control chart, control chart parameters, sample size, sampling interval, and the magnitude of shift in process mean.

2. Time advance mechanism : the simulation is a discrete event simulation where time advances from one event to another. The events taking place in the system are (a) start of the process, (b) sampling, (c) occurance of assignable cause, (d) search for an assignable cause, and (e) removal of the assignable cause.

3. Production process modeling : as state earlier, the process is assumed to start in-control. After a certain period of time, a shift in the process mean may occurs indicating a malfunction. When the control chart indicates an out-of-control situation, the production process is stopped and search for an assignable cause is initiated. If an assignable cause is found, the process is repaired and then restarted. However, if the control chart gives an out-of-control indication without an assignable cause, the process is immediately restarted

after the search for an assignable cause fails.

    4. Output Statistics : The following statistics are collected during simulation :

        1. Number of observations during simulation

        2. Number of shifts, where shift is deviation from the target value, defined as $\sigma*\delta$

        3. Number of false alarms

        4. Average time the process operates in-control

        5. Average time between process shift and the action being taken

        6. Average run length (ARL)

        7. Income obtained from the production process

        8. Loss cost during the production process

        9. Profit per hour obtained from production.

Figure 3.4 shows the specific functioning of $\overline{X}$-control chart. For each observation, a check is made if the point is on or outside $3\sigma_{\overline{X}}$ limits, two consecutive points are on or outside the $2\sigma_{\overline{X}}$ limits, or four consecutive points are on or outside $1\sigma_{\overline{X}}$ limits. The process continues if none of the conditions are met. However, if any one of the three conditions is satisfied, the operation is suspended and search for an assignable cause is initiated.

Figure 3.5 shows the specific functioning of cumulative sum control chart. For each observation, a check is made if there is any previous point on or outside

the V-mask arms which represent the decision limits. The process continues if the condition is not met. However, if any previous point is on or outside the decision limits, the operation is suspended, and search for an assignable cause is initiated.

Figure 3.6 shows the specific functioning of two one-sided cumulative sum control charts. For each observation, a check is made if the cumulative sum value of any of the two one-sided cusum charts reaches the decision limit. The process continues if the condition is not met. Furthermore, after the first trial after any control chart gives the alarm signal, both cumulative sum values are reset to zero. A listing for two one-sided cusum charts is provided as an alternative to the V-mask procedure in Appendix B.

Figure 3.7 shows the specific functioning of geometric moving average control chart. For each observation, a check is made if the plotted point falls outside the control limits. The process continues if the condition is not met. However, if any plotted point falls on or outside control limits, the operation is suspended, and search for an assignable cause is initiated.
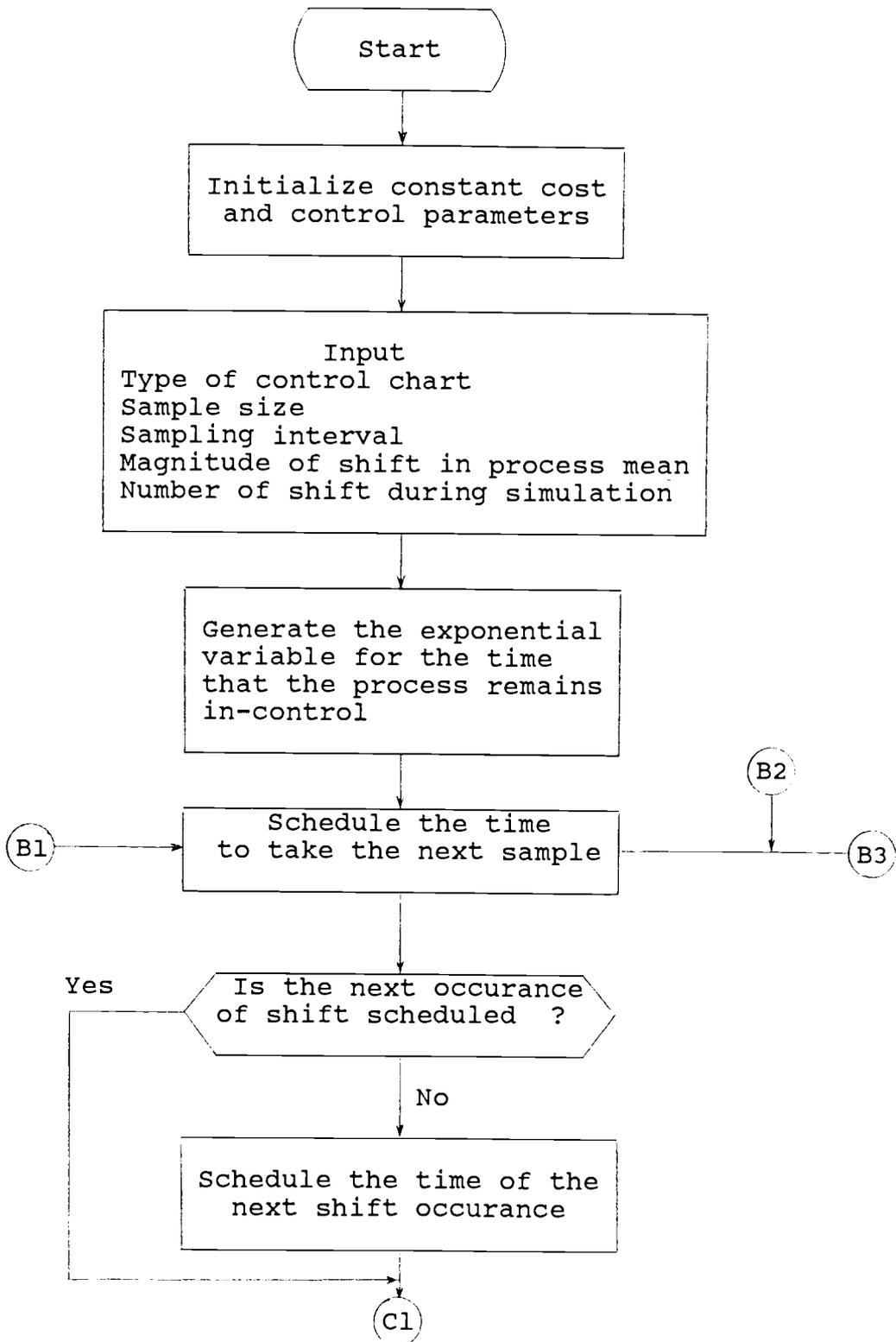
Figure 3.3 Simulation Model.
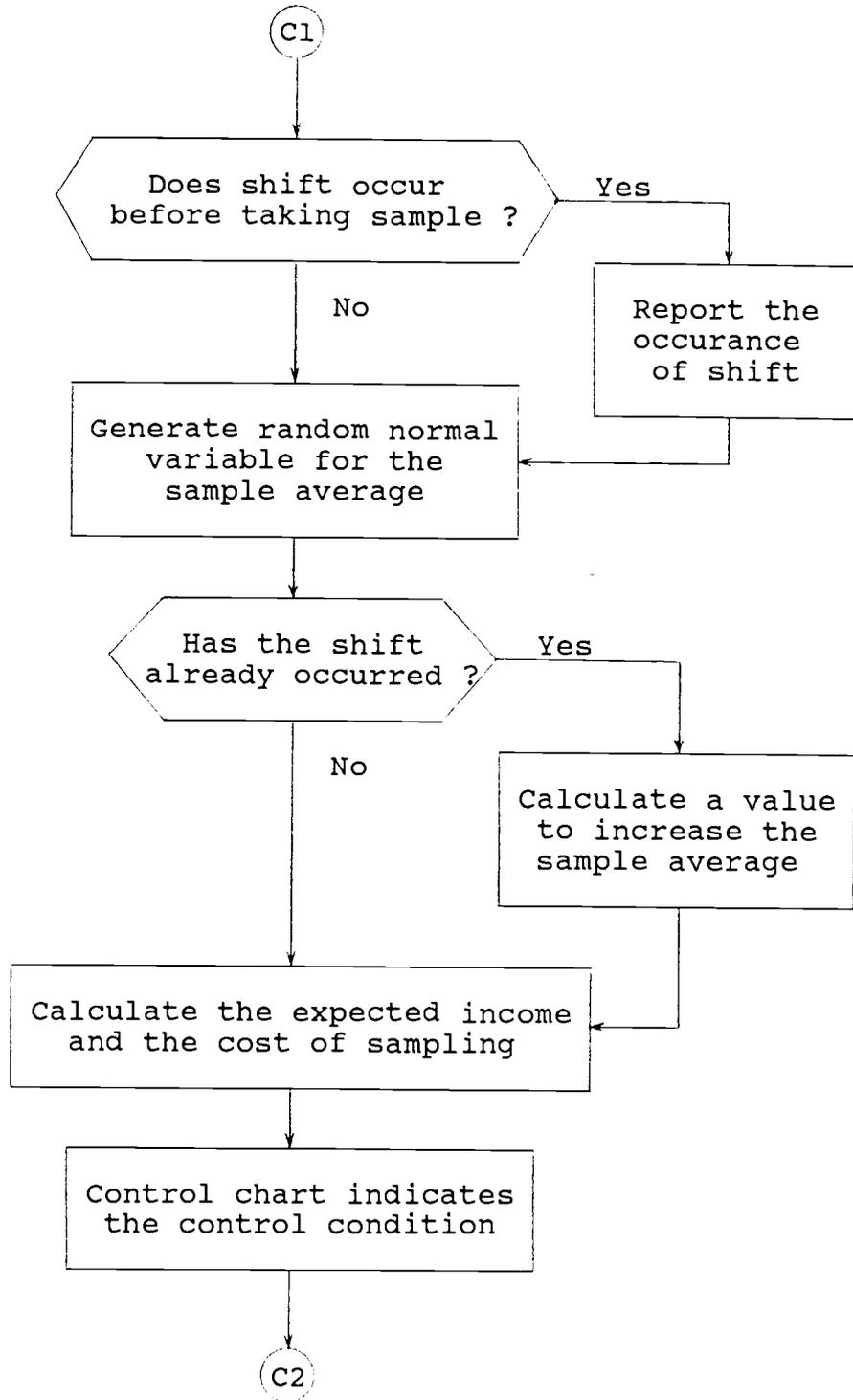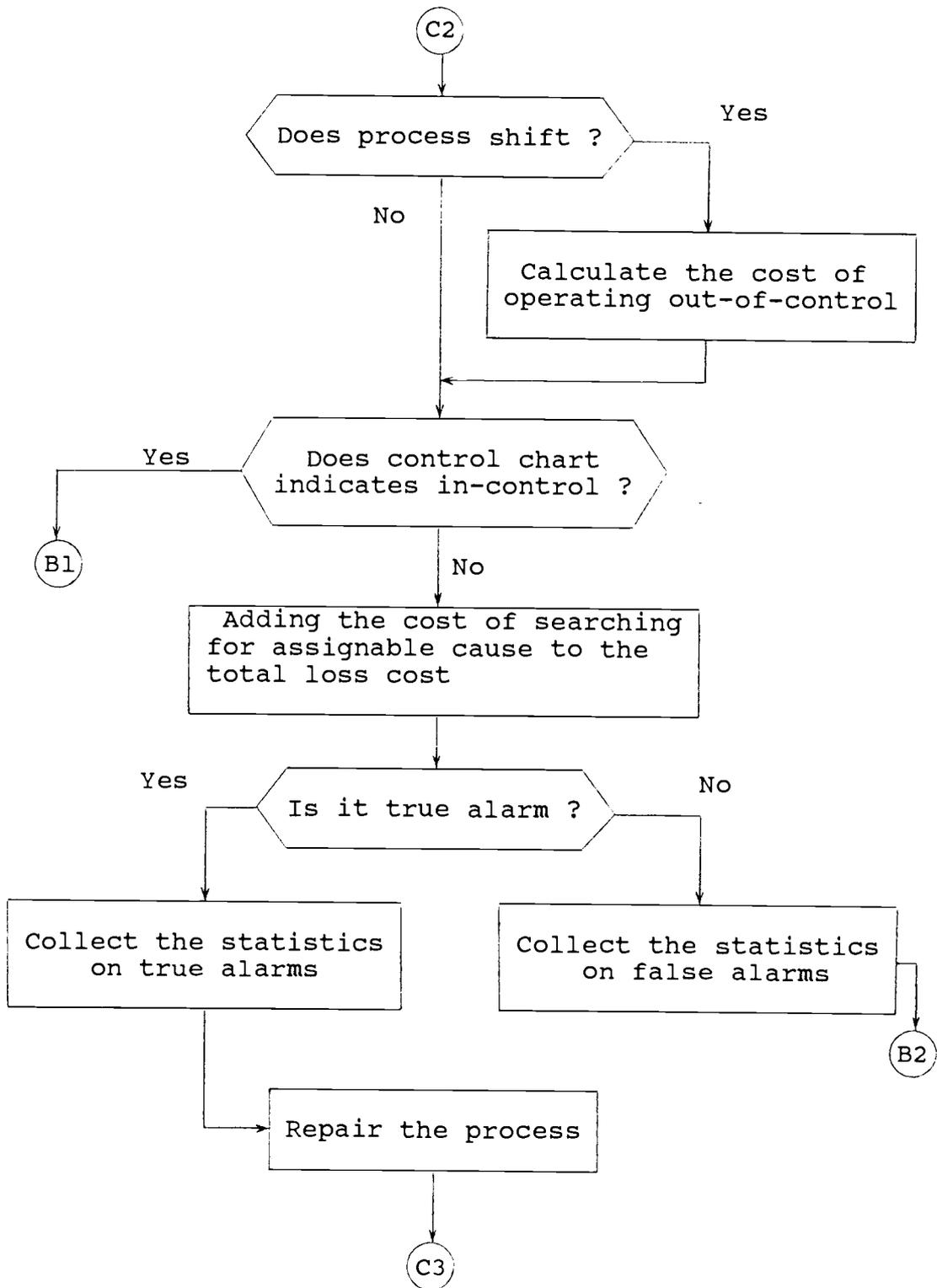
Figure 3.3   Simulation Model (continued).

Figure 3.3   Simulation Model (continued).

(C3)

Does the number of true alarms reaches the desired number   ?   No

(B3)

Yes

Calculate :
average time the process is in-control
average time between shift & detection
average run length (ARL)
profit per hour

Print results :
Number of observations
Number of true alarms
Number of false alarms
Average time between shift & detection
Average run length
Profit per hour

Stop
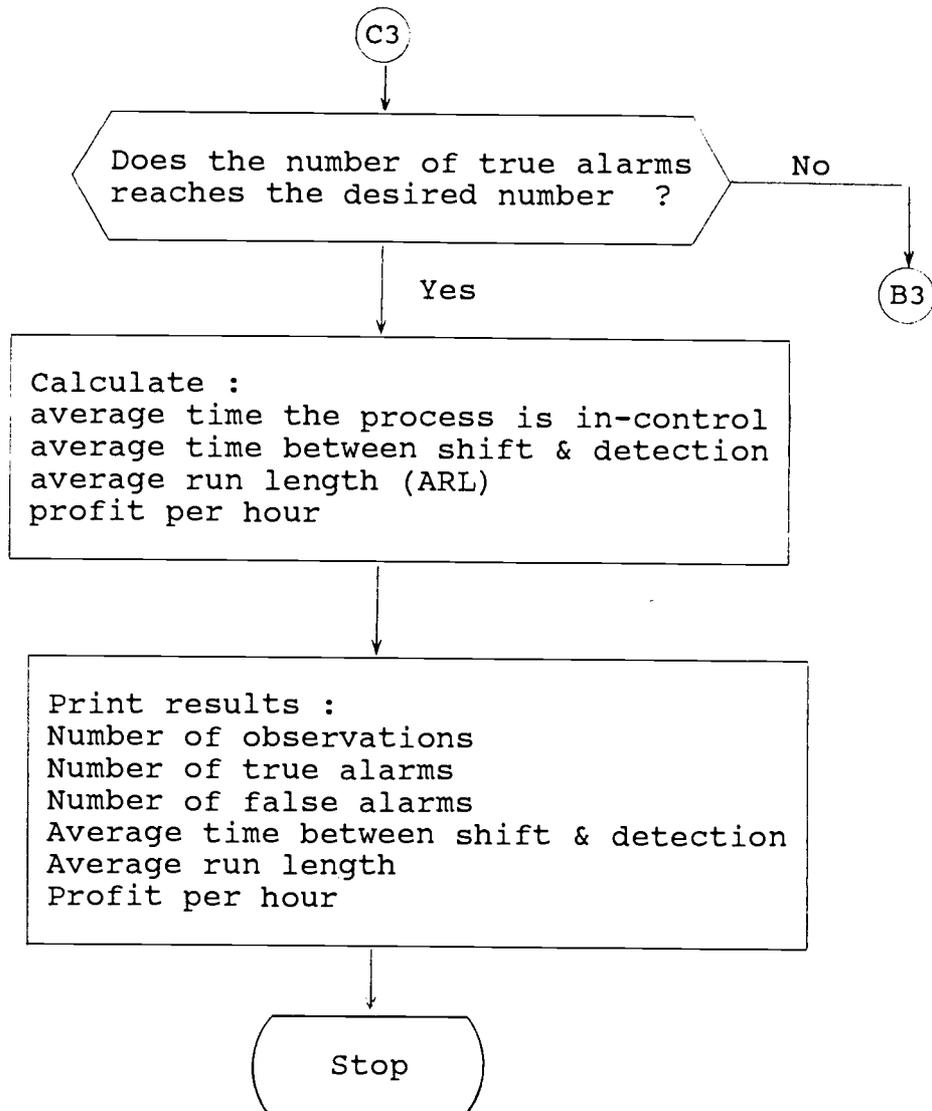
Figure 3.3   Simulation Model (continued).

Figure 3.4   Flow Chart for $\bar{X}$-Control Chart.
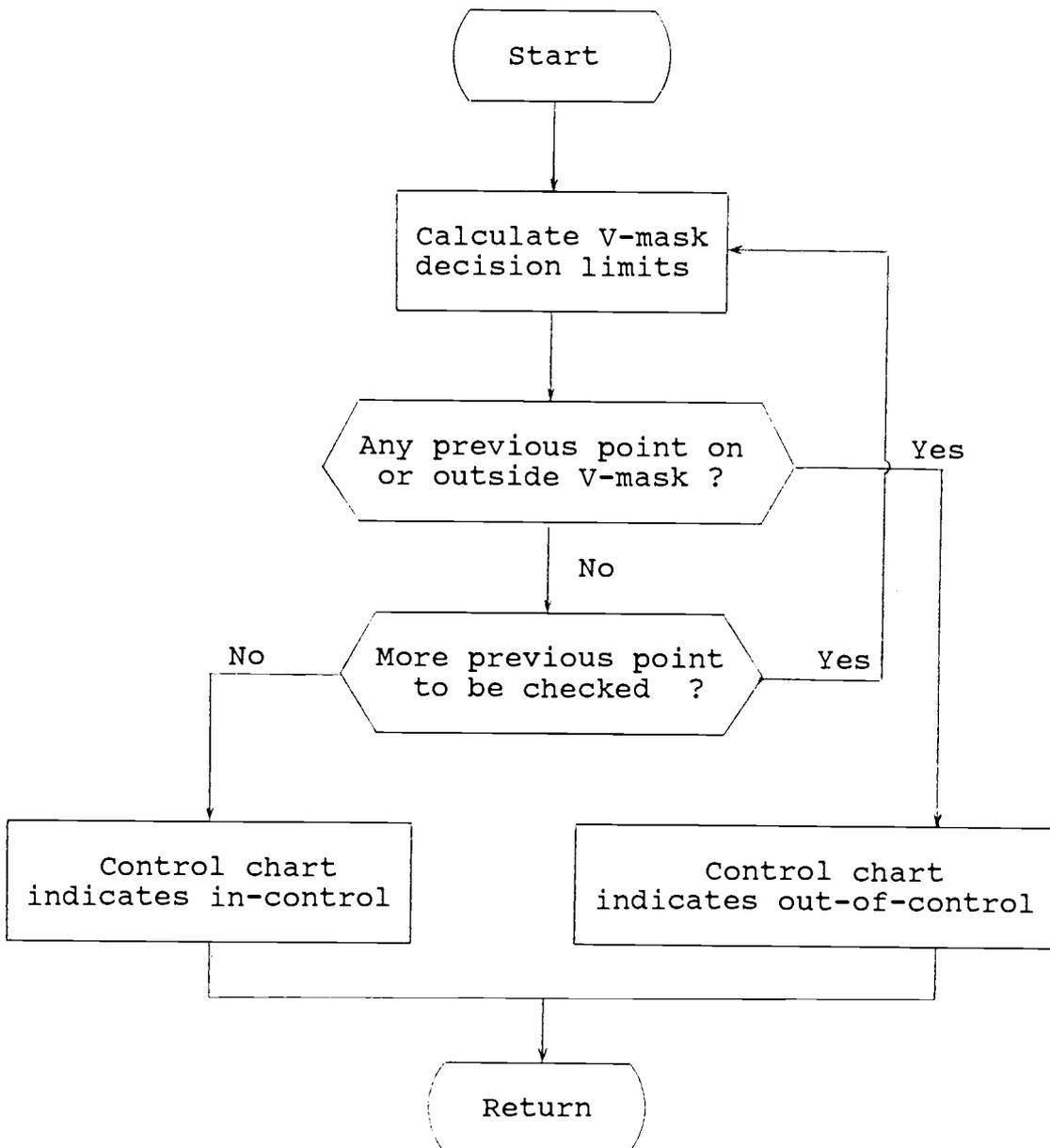
Figure 3.5   Flow Chart for Cumulative Sum   Control
              Chart Using V-mask.
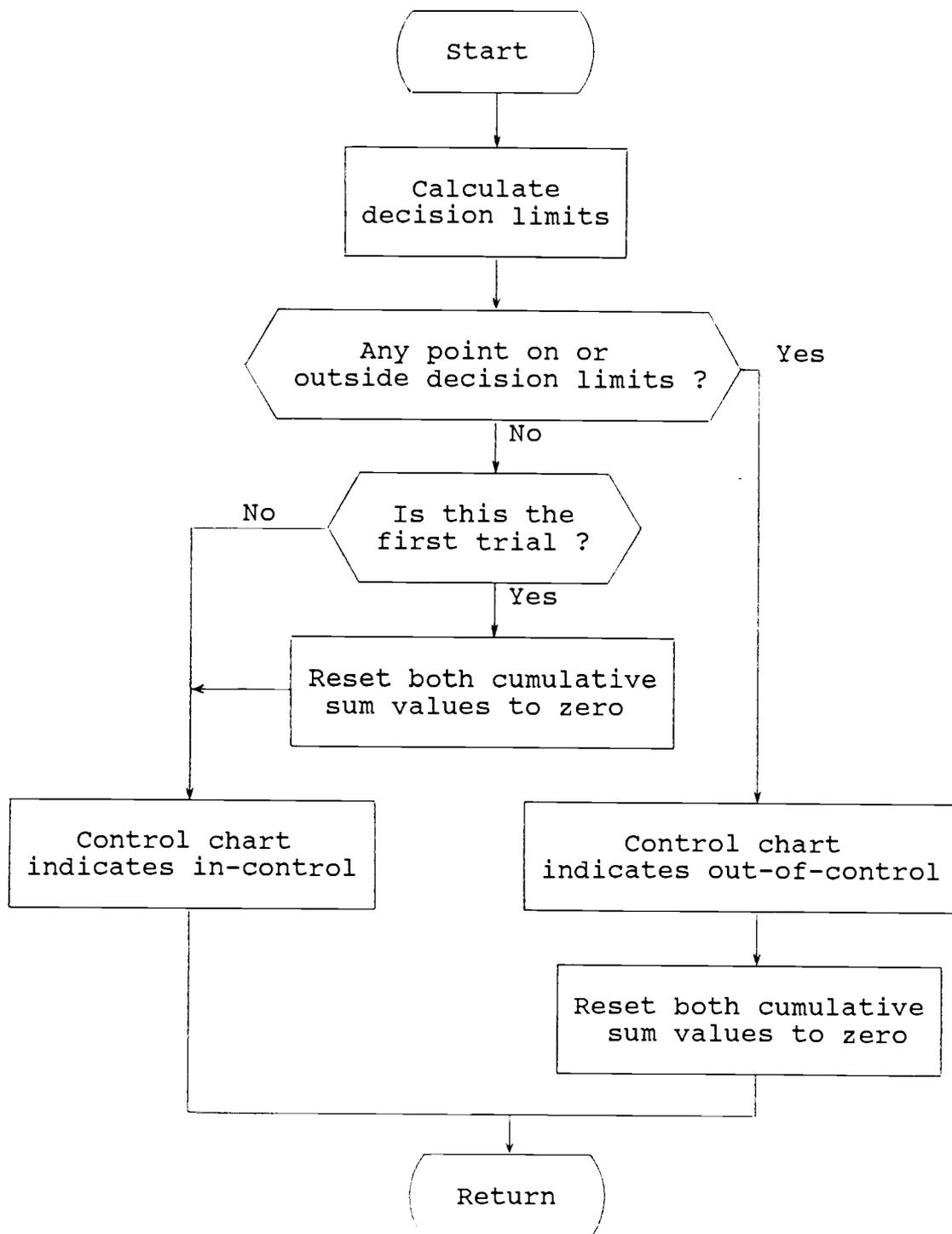
Figure 3.6   Flow Chart for Two One-sided Cumulative
               Sum Control Charts.

46
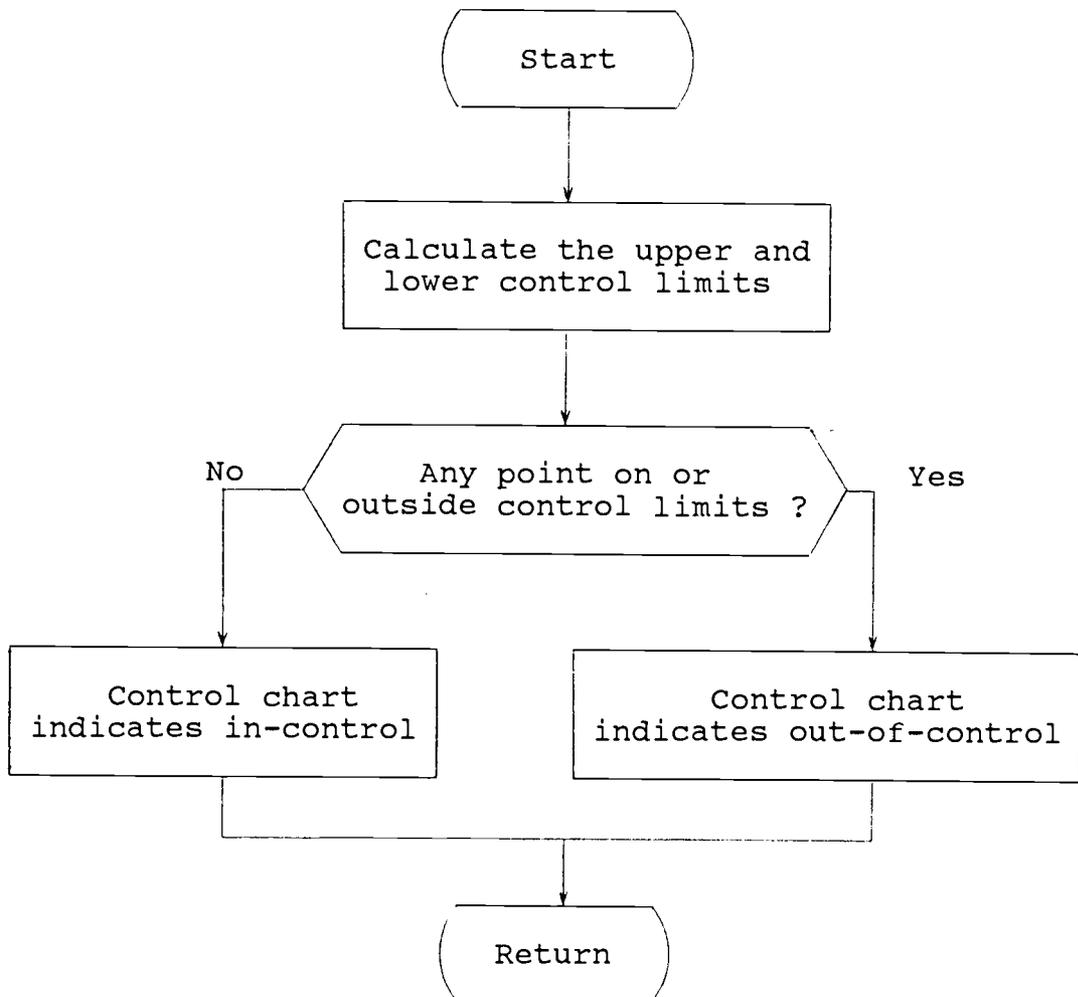
Figure 3.7   Flow Chart for Geometric Moving   Average
              Control Chart.

Random Normal Variables Generation

The method introduced by Box and Muller (1958) is used in the program to generate random normal numbers with mean zero and sample variance of 1/n for the sample average of size n , taken from the production items every specified sampling interval.

The random normal number ($\overline{X}$) representing the sample average of size n from the population N(μ, σ^2) is obtained from the expressions

$$\overline{X} = \mu + Z_1\sigma_{\overline{X}} , \quad \text{and}$$
$$\overline{X} = \mu + Z_2\sigma_{\overline{X}} ,$$

this is because $\overline{X}$ are distributed N(μ, $\sigma_{\overline{X}}$), where $\sigma_{\overline{X}} = \sigma/\sqrt{n}$ [Mendenhal, Scheaffer and Wackerly, 1981]. The selection of $\overline{X}$ alternates between the above two expressions. The values of Z_1 and Z_2 are shown by Box and Muller to be:

$$Z_1 = (-2 \ln U_1) * \cos(2\pi U_2) , \quad \text{and}$$
$$Z_2 = (-2 \ln U_1) * \sin(2\pi U_2) ,$$

where U_1 and U_2 are random numbers between 0 and 1.

Random Exponential Variables Generation

As stated earlier, the random length of time that the

process operates in-control before an assignable cause occurs follows an Exponential distribution with mean $1/\lambda$. The method used to generate the random exponential variables (y) in this simulation is taken from the exponential distribution function

$$f(y) = \lambda e^{-\lambda y}$$

where λ is the Exponential distribution parameter,

$\lambda > 0$ and $y > 0$

the cumulative distribution $F(y)$ is obtained by

$$F(y) = \int_0^y f(y) \, dy$$

$$= -[e^{-\lambda y}]_0^y$$

$$= 1 - e^{-\lambda y}.$$

If U_3 = Random number from Uniform distribution, $U(0,1)$, then

$U_3 = F(y)$, or

$$U_3 = 1 - e^{-\lambda y}$$

$$e^{-\lambda y} = 1 - U_3$$

$$-\lambda y \ln e = \ln(1-U_3)$$

$$y = -[\ln(1 - U_3)]/\lambda .$$

Matching the Control Charts

Before comparison, the three control charts were
matched so that all of them have equal number of false
alarms, equal average run length, and equal profit per
hour when the process operates in-control. Table 3.1
shows the profit per hour, number of false alarms and

Table 3.1 Matching of Control Charts.

Type of Control Chart	Profit/hr $	Number of false alarms	False Alarm rate
$\overline{X}$	98.91569	100	247.5
Cumulative Sum	98.91569	100	247.2
Geometric Moving Average	98.91569	100	247.2

false alarm rate obtained from matching three types of
control charts when the process operates in-control for
24,810 simulation hours. The matching is performed using
the following operational parameters :

- Sample size (n) = 3

- Sampling interval (s) = 1 hour

- Delay time required to search for an
  assignable cause (T_s) = 30 minutes

- Time required to remove the assignable cause

(T_r) = 1 hour

- Cost of searching for an assignable cause (C_s) = 20 dollars

- Income per hour during the process operates in-control (P) = 100 dollars per hour

The control chart characteristics obtained from matching are shown in Table 3.2 . For the cumulative sum

Table 3.2   Control Chart Parameters Obtained from Matching.

Type of Control Chart	Control Chart Parameters
$\overline{X}$	$\pm\ 3\sigma_{\overline{X}}$ action limits
	$\pm\ 2\sigma_{\overline{X}}$ warning limits
	$\pm\ 1\sigma_{\overline{X}}$ warning limits
Cumulative Sum	θ = 14 degree, d = 8.5*W
Geometric Moving Average	r = 0.25, UCL = $+2.71\sigma_{Zt}$
	LCL = $-2.71\sigma_{Zt}$

control chart, a value of d equivalent to 8.5*W (where W is the scale factor equal to $2\sigma_{\overline{X}}$ as described in Chapter II) match the cusum chart with the $\overline{X}$-control chart. For the geometric moving average control chart at a weighting factor value of 0.25, the upper and lower

control limits $\pm 2.71\sigma_{Zt}$ give the desired characteristics. This characteristics are then used in the simulation study.

Selection of Factors and Factor Levels

In this research, the effect of four factors on profit per hour during the production process were observed and analyzed (Table 3.3). These four factors

Table 3.3 Factors and Factor Levels Used in Study.

Factor Name	Levels			
	I	II	III	IV
Type of Control Chart (T)	$\overline{X}$	Cusum	Geometric	
Sample Size (N)	3	4	5	
Sampling Interval (S)	1 hour	2 hours	4 hours	
Magnitude of shift in process mean (M)	0.5σ	1.0σ	1.5σ	2.0σ

were the type of control chart, sample size, sampling interval and the magnitude of shift in process mean. For the first factor, there were three types of control

charts being compared -- $\bar{X}$, cumulative sum, and geometric moving average control charts. Three levels of sample sizes (3,4 and 5) were used, and the values of sampling interval used were 1,2 and 4 hours. The values for the magnitude of shift in process mean in terms of the process standard deviation were 0.5σ, 1.0σ, 1.5σ and 2.0σ . The simulation model was run to detect 600 shifts in the process mean for every combination of the four factors, a total of 108 combinations. Additionally, two runs were performed for each combination. The results were analyzed using Analysis of Variance. The results and their analysis is presented in the next chapter.

CHAPTER IV

ANALYSIS OF RESULTS

The simulation model was executed to obtain a total of 216 observations, two for each combination of factor levels. Each observation yields profit per hour when the simulation was run to detect 600 shifts in the process mean. These values were then analyzed using Analysis of Variance (ANOVA).

The results obtained from the simulation are shown in Appendix C. The ANOVA table is shown in Table 4.1. Graphical interpretation and analysis of results are presented in this chapter.

ANOVA

The ANOVA technique measures the total variability in the data by the sum of squares deviations from the mean value. The total variability consists of variability due to each factor, possible interaction among the factors, and a random error component.

The objective of ANOVA is to identify the factors that effect the system performance, and to investigate the interaction between the components. The factors analyzed in this study were the type of control chart, sample size,

sampling interval, and the magnitude of shift in process mean. Table 4.1 shows the Analysis of Variance of all four factors studied and their interactions. The results from the simulation model were analyzed using Statistical Interactive Programming System (SIPS) developed by Rowe and Brenne (1981). Also shown in Table 4.1 is the Statistical F-test, which is the ratio between mean of squared deviations of a factor or factor combination and mean squared error. This test is performed to test whether or not a factor or an interaction has a significant effect at 0.05 level of significance under the assumptions that, the response variables (profit per hour) are independent, the regression model is linear, and the error terms are independent and normally distributed with mean of zero and variance σ^2. These assumptions are approximately met in this study.

Analysis of Results

Table 4.1 shows that the only insignificant results are the three-factor interaction between type of control chart, sample size and sampling interval, and the four-factor interaction. The symbol * indicates the significant F-test values. Snedecor and Cochran (1980) suggested that when three-factor interaction is present, it indicates that the corresponding three-way table of

Table 4.1   Analysis of Variance (ANOVA) Table.

Source	df.	SS.	MS.	F	
Total	215	4,977.18			
Type of Control Chart (T)	2	32.9874	16.4937	865	*
Sample Size (N)	2	175.2406	87.6203	4,598	*
Sampling Interval (S)	2	3,554.16	1,777.08	93,249	*
Magnitude of the Shift (M)	3	589.821	196.607	10,316	*
T * N	4	3.29441	0.823602	43.22	*
T * S	4	19.4000	4.85001	254.50	*
T * M	6	164.540	27.4233	1,439	*
N * S	4	18.3157	4.57892	240.27	*
N * M	6	4.07575	.679292	35.64	*
S * M	6	389.173	64.8621	3,403	*
T * N * S	8	0.04095	.005119	0.2686	
T * N * M	12	2.81060	.234217	12.29	*
T * S * M	12	14.2954	1.19128	62.51	*
N * S * M	12	6.10819	.509016	26.71	*
T * N * S * M	24	.861312	.035888	1.88	
Error	108	2.05819	.0190573		

factor means must be examined for the interpretation of the results, particularly in experiments that have very large main effects. The results are summerized in Figure 4.1 through 4.5, and are analyzed below; the numerical values used in plotting these figures are given in Appendix E.

Main Effects The main effects are all significant and contain high percentage of the sum of squares deviation. Table 4.2 shows the percentage of sum of squares deviation contributed by each main effect. The table shows that almost three-fourths of the main effect variation is due to sampling interval.

Table 4.2 Main Effects Variations.

Main Effect		Sum of Squares	% Variation
Type of Control Chart	(T)	32.9874	0.66
Sample Size	(N)	175.2406	3.52
Sampling Interval	(S)	3,554.16	71.41
Magnitude of Shift in Process Mean	(M)	589.821	11.85
Total		4,977.18	

Figure 4.1 (a), shows that for the overall experiment, the profit per hour obtained from the geometric moving average chart is slightly higher than the $\bar{X}$-chart which in turn has a slightly higher profit per hour than the cusum chart. However, interpretation of the main effects gives only the general view of each type of control chart performed under various factor-level combinations in this study. Explanation of the two-factor and three-factor interaction effects will provide more specific information.

The results from Figure 4.1 (b) show that for the overall experiment, the sample size of five yields highest profit per hour among three sample sizes used in this study. The larger the sample size, the smaller the sample standard deviation. A small sample standard deviation increases effectiveness in rapidly detecting a shift in process mean. However, it should be noted that a large sample size could decrease the profit, especially when testing causes the destruction of items and the cost of sampling per item is very high.

Large sampling interval lets the process stay in an out-of-control condition for a longer period of time before the observation is taken and the shift is detected. Figure 4.1 (c) shows that for the experiments conducted in this study, using the smallest sampling interval of one hour results in the fastest shift detection which in turn
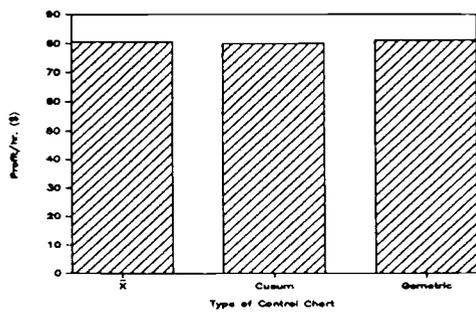
a
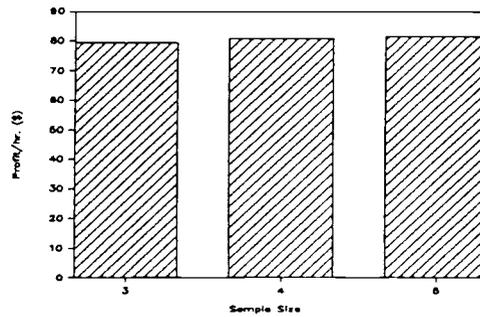
b.
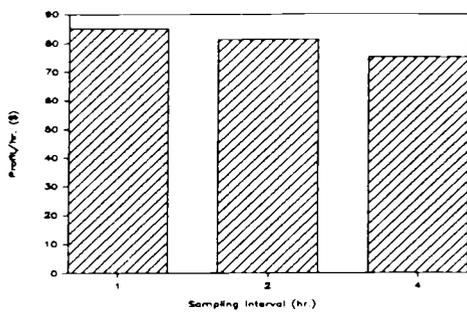
c
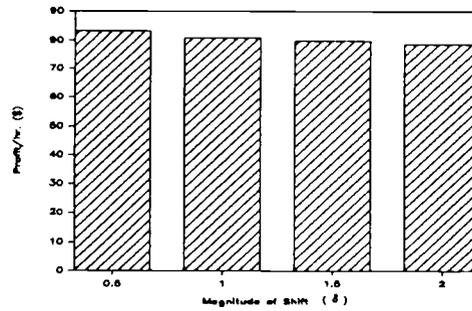
d

Figure 4.1   Main Effects.

reduces the cost of operating out-of-control and yields the highest profit per hour. The results obtained from Table 4.1 shows very strong significant effect among the three levels of sampling interval used in this study. However, it should be noted that too small a sampling interval could yield lower profit if the increasing cost of more frequent sampling, more investigations caused by false alarms, and more frequent shut down of the production process exceed the savings from early detection of the shift, particularly when the cost of sampling, the cost of searching for an assignable cause, and the income per hour of production are very high.

Since the penalty cost per hour during the process operates out-of-control depends upon the magnitude of shift as described earlier in chapter III, the larger the magnitude of shift, the higher the penalty cost of operating out-of-control. High penalty cost results in a high loss cost and a low profit according to the loss cost and profit function expressed in chapter III. Figure 4.1 (d) shows that for the overall experiments, the highest profit per hour is obtained when the magnitude of shift is 0.5σ. As the magnitude of shift increases, the profit per hour decreases gradually.

Two-Factor Interaction Effects   Since all two-factor interaction effects are significant, the corresponding

two-way table of factor means must be examined for the interpretation of the results. Figure 4.2 (a) shows that for a specified type of control chart, increasing the sample size results in an increasing of profit per hour. $\bar{X}$-chart shows the greatest increase in profit as the sample size increases.

Figure 4.2 (b) shows that sampling interval also has a strong effect on profit per hour. Using a sampling interval of one hour yields higher profit per hour as compared to higher sampling intervals. $\bar{X}$-chart has a slight disadvantage to cusum chart when sampling interval of one hour is used but for larger sampling intervals, $\bar{X}$-chart shows advantage over the cusum chart.

Figure 4.2 (c) shows the interaction effects between the type of control chart and the magnitude of shift in process mean. To produce the highest profit per hour, the cusum chart is appropriate for a small shift (0.5σ). The geometric moving average control chart produces the highest profit per hour when the magnitude of shift is about 1.0σ. For the shifts of moderate magnitude about 1.5σ to 2.0σ, the $\bar{X}$-chart is the most appropriate, while the cusum chart gives the lowest profit per hour. This is because the cusum chart is sensitive to the small shift in process mean, the geometric chart is appropriate for the intermediate shift levels of about 1.0σ, and the $\bar{X}$-chart is effective when the shift of larger magnitude is
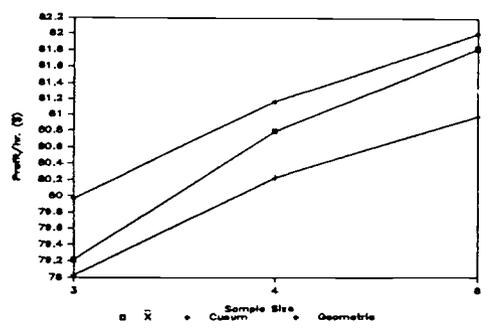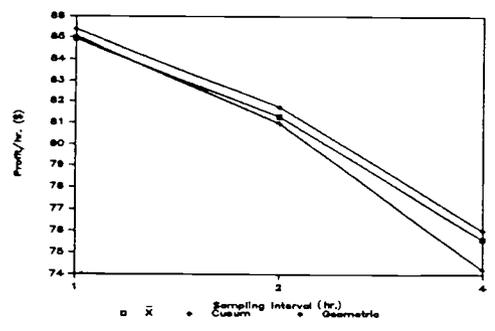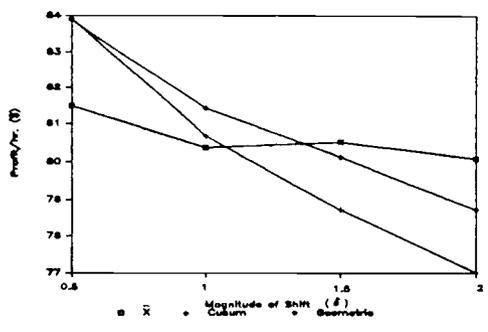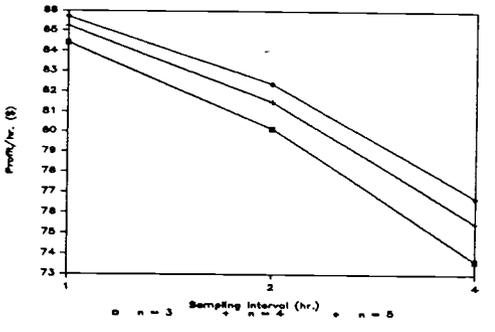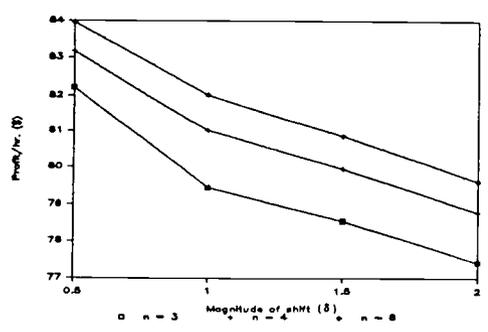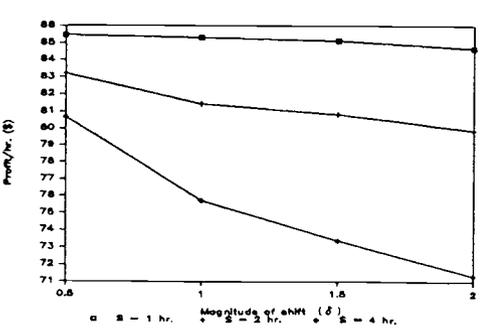
Figure 4.2 Two-Way Interaction Effects.

present. Generally, $\bar{X}$-chart is very powerful when the production process has not been stable and the magnitude of shift is large.

The lines connected between different magnitudes of shift for cusum and geometric chart in Figure 4.2 (c) decrease gradually as the magnitude of shift increases because the penalty cost per hour during out-of-control operation is proportional to the magnitude of shift, and both control charts decrease their power as the magnitude of shift increases. However, the line for $\bar{X}$-chart in Figure 4.2 (c) does not gradually decrease, but fluctuates around the shift of 1.5σ. As stated earlier, as the magnitude of shift increases, $\bar{X}$-chart is more effective and detects the shift faster. Thus, for a particular sample size and sampling interval, as the magnitude of shift increases from 1.0σ to 1.5σ, inspite of the higher penalty cost per hour during out-of-control operation, the total loss cost per hour during the overall production process with the shift of 1.5σ is lower due to the very rapid shift detection of $\bar{X}$-chart at that particular condition.

As shown in Figure 4.2 (d), the sampling interval also has a strong effect on profit per hour. Under the assumptions made in this study, using large sample size and small sampling interval is strongly preferred. Figure 4.2 (e) shows that, for a certain magnitude of shift,

using a larger sample size yields higher profit per hour. For a given sample size, the smaller the shift is, the higher the profit per hour.

Figure 4.2 (f) shows that for a given magnitude of shift, using a sampling interval of one hour is most favorable. It should be noticed that, using sampling interval of four hours yields far less profit per hour than other sampling intervals, especially in the presence of the shift in process mean above 0.5σ. This is because the time during which the process operates out-of-control is relatively large with large sampling interval. This in turn results in higher out-of-control penalty costs while the time of in-control operation and the expected income per hour remain constant.

Three-Factor Interaction Effects   Generally, three-factor interaction effects are complicated and negligible except in experiments that have very large main effects. Since the main effects in this study are exceptionally large, and the three-factor interactions are significant, the corresponding three-way table of factor means must be examined to obtain a clear interpretation of the results.

Among all combinations of the type of control chart, sample size and the magnitude of shift in process mean, using sample size of five is the most appropriate, as shown in Figures 4.3 (a), (b) and (c). In the presence of

Sample Size = 3

a

Sample Size = 4

b
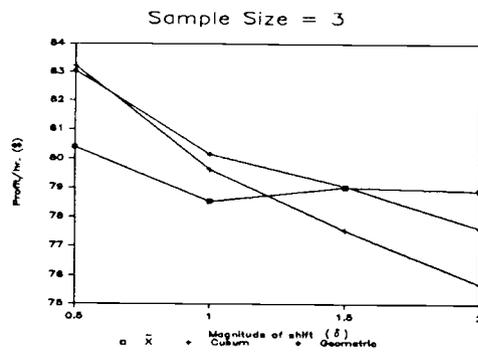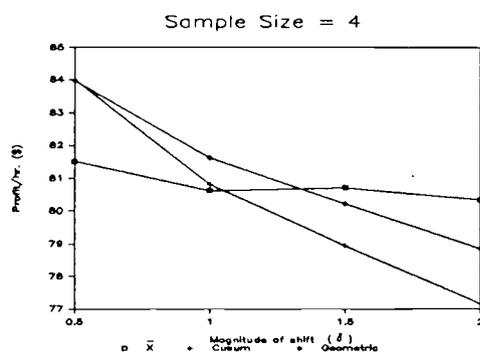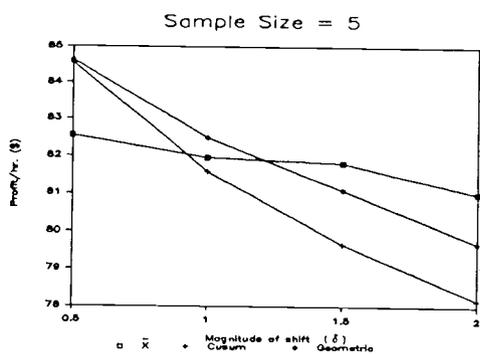
Sample Size = 5

c

Figure 4.3   Three-Way Interaction Effects.
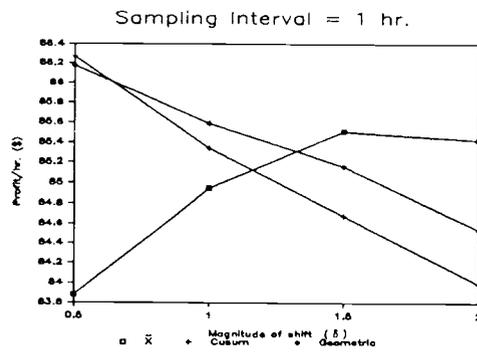
d
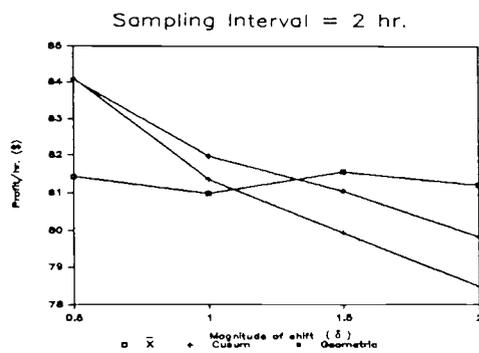
e
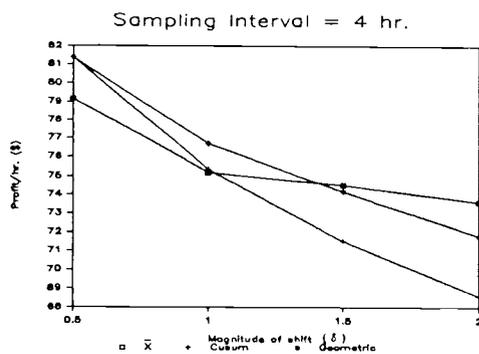
f

Figure 4.3   Three-Way Interaction Effects (continued).
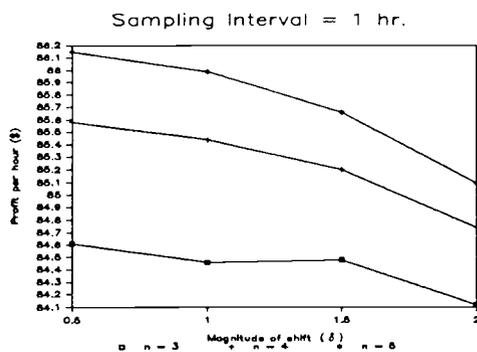
g
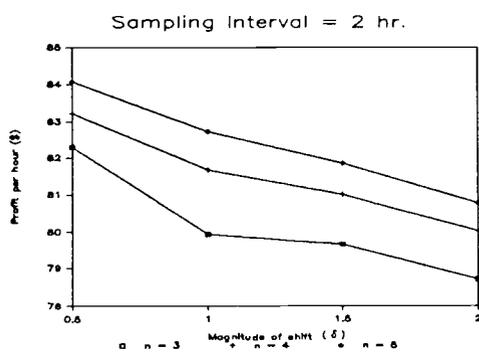
h
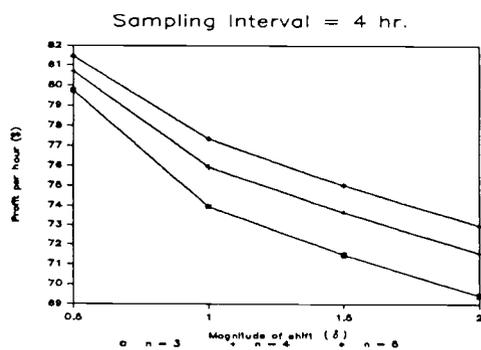
i

Figure 4.3   Three-Way Interaction Effects (continued).

small magnitude of shift about 0.5σ to 1.0σ, and a sample size of five, the geometric moving average control chart is the most favorable. For moderate shift about 1.5σ to 2.0σ, the X̄-chart is most appropriate. The profit per hour obtained from a cumulative sum control chart for this sample size is slightly lower than a geometric chart with a shift of 0.5σ, but it seems to decrease sharply as the magnitude of shift increases.

With the sample size of three or four as shown in Figures 4.3 (a) and (b), the cusum chart is the best at the shift of 0.5σ. For the shift of 1.0σ, the geometric chart is the best, and the X̄-chart is most appropriate at the shift of 1.5σ or larger.

Among all combinations of the type of control chart, sampling interval and the magnitude of shift, shown in Figures 4.3 (d), (e) and (f), using sampling interval of one hour is preferable. With a shift of 0.5σ, the cumulative sum control chart shows a slight advantage over geometric chart. Under this condition, the X̄-chart has a disadvantage due to its weakness in detecting small shift. For the shift of 1.0σ, geometric chart is more favorable than cusum chart and X̄-chart, respectively. However, X̄-chart becomes preferable when the shift becomes larger, and gives the highest profit per hour among the three types of control chart when the shift in process mean is about 1.5σ to 2.0σ. Figures 4.3 (g), (h) and (i) shows

the interaction effect between sample size, sampling interval and magnitude of shift in process mean. The results show that, for a specific combination of sample size and magnitude of shift, using sampling interval of one hour yields the highest profit per hour. For a specific combination of sampling interval and magnitude of shift, using larger sample size is more favorable. Figure 4.3 (d) shows that, when the shift changes from 0.5σ to 1.0σ, $\bar{X}$-chart obtained a higher profit per hour. Although the higher penalty cost per hour was applied, the profit per hour still increases because $\bar{X}$-chart detects the shift faster as the magnitude of shift increases. When the sampling interval of two hours was used as shown in Figure 4.3 (e), the process stayed out-of-control twice longer while the mean time that the process operates in-control remains fixed. In this case, the profit per hour at the shift of 1.0σ decreases.

Figures 4.3 (g), (h) and (i) show the interaction effect between sample size, sampling interval and the magnitude of shift in process mean. The results show that, for a specific combination of sample size and magnitude of shift, using sampling interval of one hour yields the highest profit per hour. For a specific combination of sampling interval and magnitude of shift, using sample size of five is preferable.

Analysis of Average Run Length, and Time Between Shift and Detection

The average run length (ARL) is the average value indicating the number of observations that have been taken since the shift occured before the control chart gives the alarm signal. The time between process shift and detection depends upon the average run length and the sampling interval used. Since the average run length and the time between process shift and detection indicate how fast the control chart detects the shift, additional analysis of average run length and time between process shift and detection are made to observe their behavior under the influence of the type of control chart, sample size, sampling interval, and the magnitude of shift in process mean.

Figures 4.4 (a) and (b) show the average run length analysis. When the sample size is equal three and four, the cusum chart is more effective than the geometric or the $\bar{X}$-chart in detecting the shift of 0.5σ. For the larger shift of 1.0σ, geometric chart increases its effectiveness and obtains the smallest ARL. At the shift of 1.5σ to 2.0σ, $\bar{X}$-chart is more powerful and is the fastest in detecting the shift.

With the sample size of five as shown in Figure 4.4 (c), geometric chart is the best at the shift of 0.5σ to 1.0σ, $\bar{X}$-chart is the best at the shift of 1.5σ to 2.0σ.
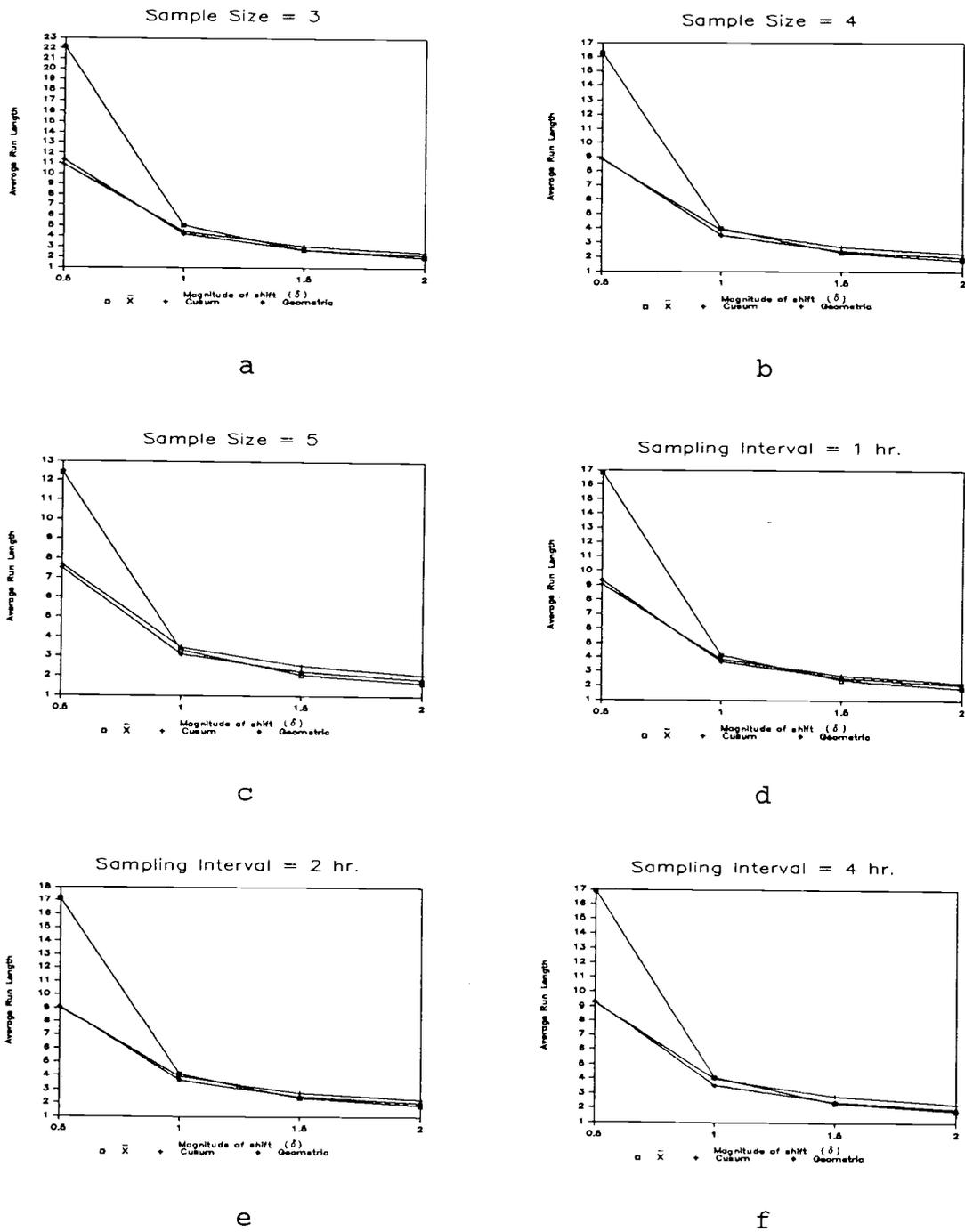
Figure 4.4   Average Run Length Analysis.

It should be noticed that geometric chart increases its
effectiveness very fast as the sample size increases.

As shown in Figures 4.4 (d), (e) and (f), with
sampling interval of one, two, and four hours, cusum chart
always obtains the smallest ARL when the small shift of
0.5σ is present. Geometric chart is the fastest in
detecting the shift about 1.0σ. When the shift becomes
larger, 1.5σ to 2.0σ, $\bar{X}$-chart dominates the other charts.
However, the sampling interval does not show any effect on
the average run length. With a particular type of control
chart, sample size, and magnitude of the shift, using
different sampling intervals always gives similar average
run lengths.

The results for time between shift and detection
shown graphically in Figures 4.5 (a) through 4.5 (f) are
similar to the average run length results presented above.
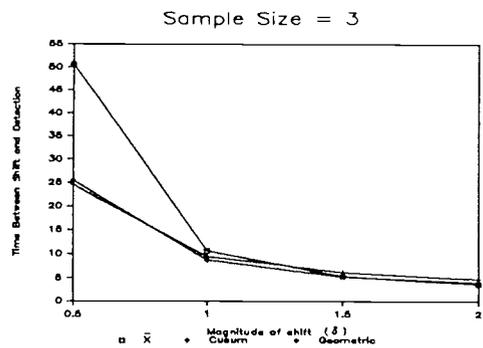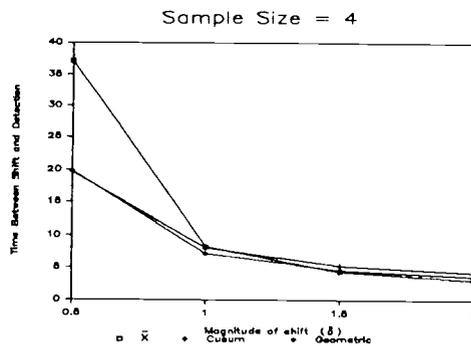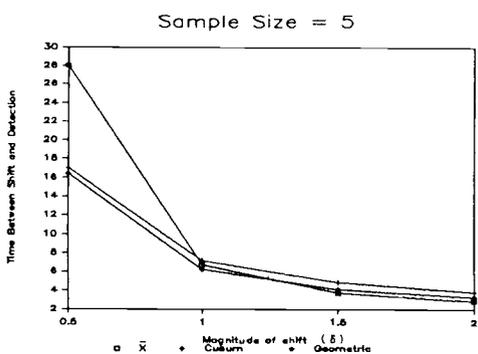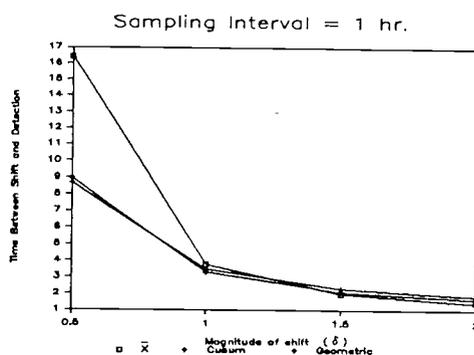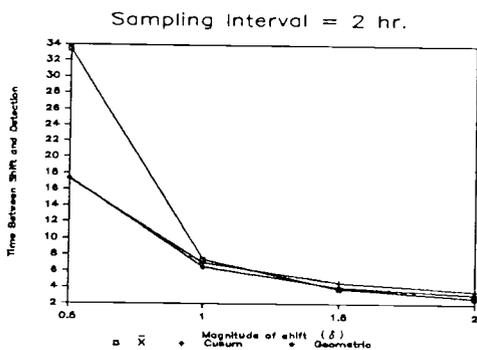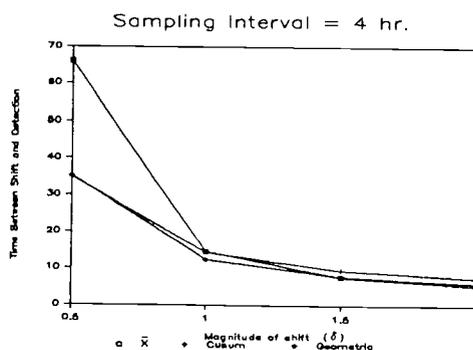
Figure   4.5   Analysis   of   Time   Between   Shift   and
                Detection.

CHAPTER V

CONCLUSIONS AND RECOMENDATIONS

Conclusions

The simulation were performed on 108 factor-level combinations, two replicates for each combination giving a total of 216 observations. The simulation was performed to detect 600 shifts in process mean to obtain profit per hour during the production process operation.

The effects of four factors, the type of control chart, sample size, sampling interval and the magnitude of shift in process mean on profit per hour were observed and analyzed using Analysis of Variance. F-test was performed to test whether or not a factor or an interaction has a significant effect on profit per hour at 0.05 level of significance.

The results show that sampling interval has a very high effect on the economic design of a control chart. The magnitude of shift in process mean, the sample size, and the type of control chart also has an influence on the design of a control chart, but to a lesser degree.

Cumulative sum control chart has advantage over the other two types of control charts when the shift of small magnitude of about 0.5σ is present, but the advantage decreases as the magnitude of shift increases. $\bar{X}$-chart is

ineffective to detect small shifts. Geometric moving average chart is the most favorable control chart at the shift of about 1.0σ. With the shift of about 1.5σ to 2.0σ, $\bar{X}$-chart is the most preferable, and gives the highest profit per hour. For shifts of these magnitudes, cumulative sum chart is very ineffective and gives the lowest profit per hour. In term of average run length, and the time between process shift and detection, the results are the same.

To summerize, cumulative sum control chart should be used when a small shift about 0.5σ is present. With the shift about 1.0σ, geometric moving average chart should be used. $\bar{X}$-chart is the most appropriate with the shift about 1.5σ or larger.

Sample size of five yields the highest profit per hour in this study. However, too large a sample size may result in a decrease of profit per hour if the testing causes the destruction of items and the cost of sampling per item is very high.

Small sampling interval of one hour yields the highest profit per hour in this study; too small sampling interval could yield lower profit per hour if the increasing cost of more frequent sampling, more investigations caused by false alarms, and more frequent shut down of the production process exceed the savings from early detection of the shift, particularly, when the cost of sampling, the

cost of searching for an assignable cause, and the income per hour of production are very high.

Recomendations for Future Research

Extension of this study leads to several areas. Some of the proposed extensions are :

1. The simulation program developed in this study can be modified to handle the multiple assignable causes situation.

2. The comparison process can be repeated using a wider range of values for the four factors used in the study.

3. The procedure can be expanded to include other types of control charts in the comparison process.

BIBLIOGRAPHY

Allan, D. H. W., "Statistical Quality Control", Reinhold
    Publishing Corporation, 1959.

Adam, E.E., and Ebert, R.J., "Production and Operations
    Management", Englewood Cliffs, NJ : Prentice-Hall,
    Inc., 1982.

Barnard, G.A., "Control Charts and Stochastics Processes",
    Journal of the Royal Statistical Society, Vol. 21,
    No. 2, 1959, pp 239-257.

Box, G.E.P., and Muller, M.E., "A Note on the Generation
    of Random Normal Deviates", Annals of Mathematical
    Statistics, Vol. 29, 1958, pp 610-611.

Chiu, W.K., and Cheung, K.C., "An Economic Study of $\overline{X}$-
    Charts with Warning Limits", Journal of Quality
    Technology, Vol. 9, No. 4, 1977, pp 166-171.

Chiu, W.K., and Wetherill, G.B., "A Simplified Scheme for
    the Economic Design of $\overline{X}$-Charts", Journal of Quality
    Technology, Vol. 6, No. 2, 1974, pp 63-69.

Duncan, A.J., "The Economic Design of $\overline{X}$-Charts Used to
    Maintain Current Control of a Process", Journal of
    American Statistical Association, Vol. 51, No. 274,
    1956, pp 228-242.

Duncan, A,J., "The Economic Design of $\overline{X}$-Charts When there
    Is a Multiplicity of Assignable Causes", Journal of
    the American Statistical Association, Vol.66, No.333,

1971, pp 107-121.

Duncan, A.J., "Quality Control and Industrial Statistics", Richard D. Irwin Inc., 1974.

Ewan, W.D., "When and How to Use Cu-Sum Charts", Technometrics, Vol. 5, No. 1, 1963, pp 1-22.

Ewan, W.D. and Kemp, K.W., "Sampling Inspection of Continuous Processes with no Autocorrelation Between Successive Results", Biometrika, Vol. 47, No. 3 and 4, 1960, pp 363-380.

Freund, R.A., "Graphical Process Control", Industrial Quality Control, Vol. 18, No. 7, 1962, pp 15-22.

Goel, A.L., "A Comparative and Economic Investigation of $\bar{X}$ and Cumulative Sum Control Charts", Ph.D. Dissertation, University of Wisconsin, Madison, WI, 1968.

Goel, A.L., Jain, S.C. and Wu, S.M., "An Algorithm for the Determination of the Economic Design of $\bar{X}$-Charts Based on Duncan's Model", Journal of the American Statistical Association , Vol.63, No.321, 1968, pp 304-320.

Goel, A.L., and Wu, S.M., "Determination of A.R.L. and a Contour Nomogram for Cusum Charts to Control Normal Mean", Tecnometrics, Vol. 13, No. 2, 1971, pp 221-230.

Goel, A.L. and Wu, S.M., "Economically Optimal Design of Cusum Charts", Management Science, Vol. 19, No. 11, 1973, pp 1271-1282.

Goldsmith, P.L. and Whitfield, H., "Average Run Lengths in Cumulative Chart Quality Control Schemes", Technometrics, Vol. 3, No. 1, 1961, pp 11-20.

Gibra, I.N., "Economically Optimal Determination of the Parameters of X-Control Chart", Management Science, Vol. 17, No. 9, 1971, pp 635-646.

Gibra, I.N., "Recent Developments in Control Chart Techniques", Journal of Quality Technology, Vol. 7, No. 4, 1975, pp 183-192.

Hayes, G.E. and Romig, H.G., "Modern Quality Control", Glencoe Publishing Co., Inc., 1982.

Hu, P.W., "Economic Design of an X-Bar Chart with Non-Exponential Times Between Process Shifts", IE News, Vol. 21, No. 1, 1986.

Johnson, N.L. and Leone, F.C., "Cumulative Sum Control Charts : Mathematical Principles Applied to Construction and Use", Part I, Industrial Quality Control , Vol. 18, No. 12, 1962, pp 15-21.

Johnson, N.L. and Leone, F.C., "Cumulative Sum Control Charts : Mathematical Principles Applied to Construction and Use", Part II, Industrial Quality Control, Vol. 19, No. 1, 1962, pp 29-36.

Johnson, N.L. and Leone, F.C., "Cumulative Sum Control Charts : Mathematical Principles Applied to Construction and Use, Part III, Industrial Quality Control, Vol. 19, No. 2, 1962, pp 22-28.

Knappenberger, H.A., and Grandage, A.H., "Minimum Cost Quality Control Tests", AIIE Transactions, Vol. 1, No. 1, 1969, pp 24-32.

Lester, H.R., Enrick, L.N., and Mottley, E.H., "Quality Control for Profit", Marcel Dekker, Inc., 1985.

Mendenhal, W., Scheaffer, R.L., and Wackerly, D.D., "Mathematical Statistics with Application", Wadsworth, Inc., 1981.

Montegomery, C.D., "Introduction to Statistical Quality Control", John Wiley & Sons, Inc., 1985.

Page, E.S., "Continuous Inspection Schemes", Biometrika, Vol. 41, 1954, pp 100-114.

Page, E.S., "Control Chart With Warning Lines", Biometrika, Vol. 42, 1955, pp 243-254.

Page, E.S., "Cumulative Sum Charts", Tecnometrics, Vol. 3, No. 1, 1961, pp 1-9.

Page, E.S., "Comparison of Process Inspection Schemes", Industrial Quality Control, Vol. 21, No. 5, 1964, pp 245-249.

Roberts, S.W., "Properties of Control Chart Zone Test", Bell System Technical Journal, Vol. 37, 1958, pp 83-114.

Roberts, S.W., "Control Chart Tests Based on Geometric Moving Averages", Technometrics, Vol. 1, No. 3, 1959, pp 239-250.

Roberts, S.W., "A Comparison of Some Control Chart

Procedures", Technometrics, Vol. 8, No. 3, 1966, pp 411-430.

Rowe, K.E., and Brenne, R.N., "Statistical Interactive Programming System (SIPS) : Command Reference Manual for CYBER 70/73 and HONEYWELL 440", Department of Statistics, Oregon State University, Corvallis, Oregon, 1981.

Snedecor, G.W., and Cochran, W.G., "Statistical Methods", 7th Edition, Iowa State University Press, Ames, Iowa, 1980.

Taylor, H.M., "The Economic Design of Cumulative Sum Control Charts", Technometrics, Vol. 10, No. 3, 1968, pp 479-488.

Truax, H.M., "Cumulative Sum Charts and Their Application to the Chemical Industry", Industrial Quality Control, Vol. 18, No. 6, 1961, pp 18-25.

APPENDICES

APPENDIX A

SIMULATION PROGRAM LISTING

```
10 CLS : PRINT
20 PRINT "      This program is developed for simulation of
   control charts for controlling process mean in
   production process."
30 PRINT : PRINT "      Please enter information needed for
   simulation."
40 DIM EP(1000), HA(5000), CUSUM(5000)
50 REM INITIALIZE CONTROL PARAMETERS
60 MEAN = 0        : REM MEAN OF THE PROCESS
70 SIGMA = 1       : REM STANDARD DEVIATION OF THE PROCESS
80 LAMBDA = .05 : REM EXPONENTIAL DISTRIBUTION PARAMETER
                     OF THE TIME BETWEEN THE PROCESS SHIFTS
90 A = .5          : REM OVERHEAD COST OF SAMPLING ($)
100 B = .1         : REM COST OF SAMPLING PROPORTIONAL TO THE
                     NUMBER OF SAMPLES PER OBSERVATION ($)
110 TS = .5        : REM TIME REQUIRED TO SEARCH FOR
                     ASSIGNABLE CAUSE OF VARIATION (Hr.)
120 TR = 1         : REM TIME REQUIRED TO REMOVE AN
                     ASSIGNABLE CAUSE OF VARIATION (Hr.)
130 CS = 20        : REM COST OF SEARCHING FOR ASSIGNABLE
                     CAUSE (Hr.)
140 P = 100        : REM INCOME PER HOUR DURING THE PROCESS
                     OPERATES IN-CONTROL ($)
150 GOTO 190
160 DELAY = .02*N + .02 : REM DELAY TIME PROPORTIONAL TO
    THE SAMPLE SIZE PLUS CONSTANT ($)
170 C = 50*ABS(MAG)      : REM LOSS COST PROPORTIONAL TO
    MAGNITUDE OF SHIFT IN PROCESS MEAN ($)
180 GOTO 590
190 TAB = 17 : PRINT
200 REM ENTER THE INFORMATION NEEDED FOR SIMULATION
210 INPUT "EXPERIMENT # "; NB
220 PRINT "PLEASE SELECT THE TYPE OF CONTROL CHART FOR
    THIS SIMULATION"
230 PRINT "     1......X-BAR CHART"
240 PRINT "     2......CUMULATIVE SUM CHART (V-mask)"
250 PRINT "     3......GEOMETRIC MOVING AVERAGE CHART"
260 INPUT CODE
270 IF CODE = 1 OR CODE = 2 OR CODE = 3 THEN 290
280 PRINT "CONTROL CHART CODE ERROR." : GOTO 260
290 INPUT "SAMPLE SIZE   "; N
300 INPUT "SAMPLING INTERVAL  (hours)   "; S
310 INPUT "MAGNITUDE OF SHIFT IN PROCESS MEAN IN TERM  OF
    ? ...SIGMA    "; MAG
320 IF CODE = 1 THEN THREE = 3*SIGMA/SQR(N) :
    TWO = 2*SIGMA/SQR(N) : ONE = 1*SIGMA/SQR(N) : GOTO 430
330 IF CODE <> 2 THEN 390
340 INPUT "DEGREE OF HALF V-MASK ANGLE = "; DEGREE
350 PRINT "LET  W  be the scale factor represented one
    unit of vertical and horizontal plotting scale."
360 INPUT "LEAD DISTANCE OF V-MASK AHEAD OF CURRENT POINT
    IN TERM OF   ? ..W     "; D
```

```
370 RADIAN = DEGREE*3.141593/180 : GOSUB 1980 : GOTO 430
380 REM INPUT WEIGHTING FACTOR OF GEOMETRIC MOVING AVERAGE
    CHART
390 INPUT "r = "; R : IF R <= 0 OR R > 1 THEN PRINT
    "R   must be positive value between 0 and 1" : GOTO 380
400 REM INPUT PARAMETER TO DETERMINE UCL & LCL OF
    GEOMETRIC MOVING AVERAGE CHART
410 INPUT "STANDARD CONTROL UNIT ="; UNIT :
    IF UNIT < 0 THEN UNIT = UNIT*(-1)
420 T = 0 : ZB = MEAN
430 IF MAG = 0 THEN INPUT "SIMULATION TIME (hours) "; HR :
    RANDOMIZE(1) : GOTO 460
440 INPUT "NUMBER OF SHIFTS BEFORE STOP SIMULATION = ";
    SVAL
450 SVAL = INT(SVAL) : IF SVAL <= 0 THEN PRINT "**ERROR**"
    : GOTO 440
460 INPUT "Do you want to see trace on screen when control
    chart indicates out-of-control.  (Y or N)  "; S$
470 INPUT "Do you want to make change about the given
    information.  (Y or N)  "; R$
480 IF R$ = "Y" OR R$ = "y" THEN CLS : PRINT : GOTO 200
490 IF MAG <> 0 THEN GOSUB 930
500 CLS
510 PRINT TAB(TAB) "START SIMULATION" : PRINT
520 REM GENERATE TIME OF THE PROCESS
530 TNOW = 0
540 PRINT "TIME "; TNOW; " MIN."; TAB(TAB) "PROCESS START
    IN-CONTROL"
550 PROCESS$ = "OK"
560 E = 1
570 M = HR*60
580 GOTO 160
590 REM SET THE TIME FOR NEXT SAMPLE AND NEXT OCCURANCE OF
    THE SHIFT IN PROCESS MEAN
600 TSAMP = TNOW + S*60 : IF MAG = 0 THEN 690
610 IF TEXPON = 1 THEN 630
620 TF = TSTART + EP(E)*60 : TEXPON = 1
630 IF PROCESS$ = "SHIFT" THEN 790
640 REM PROCESS IS OK NOW.  THE SHIFT WILL OCCUR NEXT.
650 IF FALARM > REC THEN TF = TF + TS*60 : REC = FALARM
660 IF TSAMP < TF THEN 690
670 GOSUB 730
680 GOTO 800
690 REM PROCESS IS OK NOW, CONTINUE SAMPLING
700 IF MAG <> 0 THEN 720
710 IF TSAMP >= M THEN GOSUB 2810 :
    IF AN$ = "N" OR AN$ = "n" THEN 810
720 GOTO 800
730 REM SUBROUTINE TO REPORT WHEN PROCESS SHIFTS
740 PROCESS$ = "SHIFT"
750 PRINT "TIME "; TF; TAB(TAB) "** PROCESS SHIFTS **"
760 REM COLLECT STATISTICS OF THE TIME PROCESS OPERATES
```

```
     IN-CONTROL
770 TINCTRL = TINCTRL + EP(E)*60 :
     PRINT TAB(TAB) "TIME IN CONTROL "; EP(E)*60
780 NSHIFT = NSHIFT + 1 : RETURN
790 REM PROCESS IS ALREADY SHIFT BUT CONTROL CHART HAS NOT
     DETECTED IT YET, CONTINUE SAMPLING
800 GOSUB 1010 : GOTO 590
810 REM CALCULATE THE  EXPECTED INCOME BEFORE SIMULATION
     COMPLETED
820 IF MAG = 0 THEN GOSUB 2440
830 REM PRINTOUT THE RESULTS OF SIMULATION
840 GOSUB 2490
850 GOTO 2950
860 REM SUBROUTINE TO GENERATE NORMAL DISTRIBUTION
870 R1 = RND(1) : R2 = RND(2)
880 IF FLAG$ = "FC" THEN 900
890 Z = SQR(-2*LOG(R1))*SIN(6.283185*R2) : FLAG$ = "FC" :
     GOTO 910
900 Z = SQR(-2*LOG(R1))*COS(6.283185*R2) : FLAG$ = "FS"
910 AVE = MEAN + Z*SIGMA/SQR(N)
920 RETURN
930 REM SUBROUTINE TO GENERATE EXPONENTIAL DISTRIBUTION
     FOR THE TIME PROCESS OPERATES IN-CONTROL
940 PRINT "GENERATING EXPONENTIAL DISTRIBUTION FOR THE
     TIME PROCESS IS IN-CONTROL......"; SVAL; "VALUES...
     PLEASE WAIT..."
950 FOR J = 1 TO SVAL
960     R3 = RND(3)
970     EP(J) = ( -LOG(1-R3))/LAMBDA
980     PRINT "EP("; J; ") ="; EP(J)
990 NEXT J :
     RANDOMIZE(1) : REM THIS RANDOMIZE SEED MUST BE
                        CHANGED FOR REPLICATION
1000 RETURN
1010 REM SUBROUTINE SAMPLING
1020 GOSUB 860
1030 TNOW = TSAMP : TLAST = TSAMP
1040 LCOST = LCOST + A + B*N : SUM = 0
1050 IF PROCES$ = "OK" THEN 1070
1060 GOSUB 2910 : AVE = AVE + DELTA
1070 SAMPLE = SAMPLE + 1
1080 PRINT "TIME "; TNOW; TAB(TAB) "SAMPLE AVE = "; AVE
1090 ON CODE GOSUB 1780, 2050, 2250
1100 IF PLOT$ = "IN" THEN 1130
1110 GOSUB 1290
1120 GOTO 1240
1130 REM CONTROL CHART INDICATES IN-CONTROL, CALCULATE
     EXPECTED INCOME
1140 INCOME = INCOME + S*P
1150 REM CONTROL CHART INDICATES IN-CONTROL AND THE
     PROCESS IS REALLY IN-CONTROL
1160 IF MAG = 0 THEN RL = RL + 1 :
```

```
      PRINT TAB(TAB) "RUN LENGTH = "; RL
1170 IF PROCESS$ = "OK" THEN 1240
1180 REM CONTROL CHART INDICATES IN-CONTROL BUT ACTUALLY
      THE PROCESS SHIFTS, CALCULATE LOSS COST
1190 IF RL = 0 THEN 1210
1200 LCOST = LCOST + S*C : GOTO 1230
1210 IF TF >= TSAMP THEN 1240
1220 LCOST = LCOST + (TNOW - TF)*C/60
1230 RL = RL + 1 : PRINT TAB(TAB) "RUN LENGTH = "; RL
1240 REM RETURN TO THE MAIN PROGRAM
1250 PRINT TAB(TAB) "INCOME     ="; INCOME
1260 PRINT TAB(TAB) "LOSS COST  ="; LCOST
1270 IF PLOT$ = "OUT" THEN GOSUB 1690
1280 RETURN
1290 REM SUBROUTINE OUT-OF-CONTROL
1300 TNOW = TNEXT
1310 PRINT "TIME "; TNOW; TAB(TAB) "CONTROL CHART
      INDICATES OUT-OF-CONTROL" : BEEP
1320 PRINT TAB(TAB) "STOP THE PROCESS & SEARCH FOR
      ASSIGNABLE CAUSE"
1330 REM CALCULATE THE EXPECTED INCOME
1340 INCOME = INCOME + (S + DELAY)*P
1350 REM SEARCHING FOR ASSIGNABLE CAUSE OF VARIATIONS
1360 IF PROCESS$ = "SHIFT" THEN 1420
1370 REM FALSE ALARM, COLLECT STATISTICS
1380 FALARM = FALARM + 1
1390 PRINT TAB(TAB) "FALSE ALARM # "; FALARM :
      ALARM$ = "FA"
1400 IF MAG = 0 THEN 1510
1410 GOTO 1570
1420 REM TRUE ALARM, COLLECT STATISTICS
1430 TALARM = TALARM + 1
1440 PRINT TAB(TAB) "TRUE ALARM #"; TALARM : ALARM$ = "TA"
1450 TSAD = TSAD + (TNOW - TF)
1460 NDETEC = NDETEC + 1
1470 REM CALCULATE LOSS COST CAUSE BY PROCESS SHIFT
1480 IF RL = 0 THEN 1500
1490 LCOST = LCOST + (S + DELAY)*C : GOTO 1510
1500 LCOST = LCOST + (TNOW - TF)*C/60
1510 REM COLLECT STATISTICS OF AVERAGE RUN LENGTH (ARL)
1520 RL = RL + 1 : PRINT TAB(TAB) "RUN LENGTH ="; RL
1530 IF MAG <> 0 THEN TSUMRL = TSUMRL + RL :
      PRINT TAB(TAB) "AVE. RUN LENGTH = "; TSUMRL/TALARM :
      GOTO 1570
1540 FSUMRL = FSUMRL + RL :
      PRINT TAB(TAB) "FALSE ARL.   ="; FSUMRL/FALARM
1550 LPRINT "TIME "; TNOW/60; "Hr. ";
      TAB(25) "FALSE ALARM #"; FALARM
1560 LPRINT TAB(25) "RUN LENGTH ="; RL;
      TAB(45) "FALSE ARL. ="; FSUMRL/FALARM : LPRINT
1570 REM CALCULATE LOSS COST CAUSE BY SEARCHING FOR
      ASSIGNABLE CAUSE
```

```
1580 TNEXT = TNOW + TS*60
1590 IF MAG <> 0 THEN 1640
1600 IF TNEXT < M THEN 1640
1610 GOSUB 2810 : IF AN$ <> "N" AND AN$ <> "n" THEN
     GOTO 1640
1620 PRINT TAB(TAB) "INCOME      ="; INCOME
1630 PRINT TAB(TAB) "LOSS COST  ="; LCOST : TNOW = M :
     GOTO 830
1640 TNOW = TNEXT : LCOST = LCOST + CS
1650 IF PROCES$ = "OK" THEN 1680
1660 REM REPAIR THE PROCESS
1670 PRINT TAB(TAB) "REPAIR THE PROCESS" :
     TNEXT = TNOW + TR*60 : TNOW = TNEXT
1680 RL = 0 : RETURN
1690 REM SUBROUTINE TO RESTART THE PROCESS IN-CONTROL
1700 IF ALARM$ = "FA" THEN 1740
1710 PROCES$ = "OK" : TSTART = TNOW : TEXPON = 0 : E = E+1
1720 PRINT "TIME "; TNOW;
     TAB(TAB) "PROCESS GOES BACK IN-CONTROL"
1730 PRINT TAB(TAB) "PRIFIT/HR =";
     (INCOME - LCOST)*60/TNOW : GOTO 1750
1740 PRINT "TIME "; TNOW; TAB(TAB) "START THE PROCESS"
1750 IF S$ = "Y" OR S$ = "y" THEN PRINT "Enter  CONT  to
     continue." : BEEP : BEEP : STOP
1760 IF MAG <> 0 AND NDETEC = SVAL THEN 830
1770 RETURN
1780 REM SUBROUTINE X-BAR CHART   (with warning limits)
1790 GOSUB 2370
1800 IF AVE - MEAN >= THREE OR AVE - MEAN <= -THREE THEN
     PRINT TAB(TAB) "ONE POINT OUTSIDE 3*sigma x control
     limits" : GOTO 1960
1810 IF AVE - MEAN >= TWO THEN 1850
1820 IF AVE - MEAN < TWO AND AVE - MEAN > -TWO THEN
     UTWO = 0 : LTWO = 0 : GOTO 1880
1830 LTWO = LTWO + 1 : UTWO = 0
1840 PRINT TAB(TAB) "POINTS BELOW 2*sigma x ="; LTWO :
     GOTO 1870
1850 UTWO = UTWO + 1 : LTWO = 0
1860 PRINT TAB(TAB) "POINTS ABOVE 2*sigma x ="; UTWO
1870 IF UTWO = 2 OR LTWO = 2 THEN 1960
1880 IF AVE - MEAN >= ONE THE 1920
1890 IF AVE - MEAN < ONE AND AVE - MEAN > -ONE THEN
     UONE = 0 : LONE = 0 : GOTO 1950
1900 LONE = LONE + 1 : UONE = 0
1910 PRINT TAB(TAB) "POINTS BELOW 1*sigma x ="; LONE :
     GOTO 1940
1920 UONE = UONE + 1 : LONE = 0
1930 PRINT TAB(TAB) "POINTS ABOVE 1*sigma x ="; UONE
1940 IF UONE = 4 OR LONE = 4 THEN 1960
1950 PLOT$ = "IN" : GOTO 1970
1960 PLOT$ = "OUT" : LONE = 0 : UONE = 0 :
     LTWO = 0 : UTWO = 0
```

```
1970 RETURN
1980 REM SUBROUTINE TO CALCULATE VALUES USED FOR
     CUMULATIVE SUM V-MASK
1990 W = 2*SIGMA/SQR(N) : IF SKIP$ = "YES" THEN 2010
2000 STORE = 300
2010 PRINT "CALCULATING VALUES USED FOR V-MASK"; STORE;
     "VALUES,  Please wait."
2020 FOR CU = REF + 1 TO STORE : HA(CU) = CU*W*TAN(RADIAN)
     : NEXT CU
2030 REF = STORE : HB = D*W*TAN(RADIAN)
2040 RETURN
2050 REM SUBROUTINE CUMULATIVE SUM CHART USING V-MASK
2060 REM THIS SUBROUTINE COULD BE SUBSTITUTED BY
     SUBROUTINE TWO ONE-SIDED CUMULATIVE SUM CONTROL
     CHARTS PROVIDED IN APPENDIX B  WHICH RUNS FASTER.
2070 GOSUB 2370
2080 CP = CP + 1
2090 PP = CP
2100 CUSUM = CUSUM + (AVE - MEAN)
2110 CUSUM(CP) = CUSUM
2120 PRINT TAB(TAB) "CUSUM("; CP; ") = "; CUSUM
2130 REM CALCULATE DECISION LIMIT & DEVIATION FROM V-MASK
     HORIZONTAL LINE OF INDIVIDUAL SAMPLE
2140 PP = PP -1
2150 IF (CP - PP) <= REF THEN H = HA(CP - PP) + HB :
     GOTO 2180
2160 STORE = STORE + 500 : SKIP$ = "YES"
2170 GOSUB 1980 : GOTO 2150
2180 IF CUSUM(CP) - CUSUM(PP) >= H OR
     CUSUM(CP) - CUSUM(PP) <= -H THEN 2210
2190 IF PP > 1 THEN 2140
2200 PLOT$ = "IN" : GOTO 2240
2210 PRINT TAB(TAB) "CUSUM("; PP; ") = "; CUSUM(PP);
     "   IS OUTSIDE V-MASK."
2220 PRINT TAB(TAB) "V-MASK UPPER LIMIT ="; CUSUM(CP) + H;
     " & LOWER LIMIT ="; CUSUM(CP) - H :
     PRINT TAB(TAB) "DECISION INTERVAL  h ="; H
2230 PLOT$ = "OUT" : CP = 0 : CUSUM = 0
2240 RETURN
2250 REM SUBROUTINE GEOMETRIC MOVING AVERAGE CHART
2260 GOSUB 2370
2270 T = T + 1 : ZT = R*AVE + (1 - R)*ZB : ZB = ZT
2280 PRINT TAB(TAB) "Z("; T; ") ="; ZT
2290 REM CHECK WHETHER OR NOT THE CURRENT POINT IS INSIDE
     CONTROL LIMITS ?
2300 OMR = 1 - (1-R)^(2*T)
2310 RANGE = SQR(R*OMR/(2 - R))*UNIT*SIGMA/SQR(N)
2320 EXPUCL = MEAN + RANGE : EXPLCL  = MEAN - RANGE :
     PRINT TAB(TAB) "UCL ="; EXPUCL; "& LCL ="; EXPLCL
2330 IF ZT >= EXPUCL OR ZT <= EXPLCL THEN 2350
2340 PLOT$ = "IN" : GOTO 2360
2350 PLOT$ = "OUT" : T = 0 : ZB = MEAN
```

```
2360 RETURN
2370 REM SUBROUTINE FOR CONTROLLING DELAY TIME
2380 TNEXT = TNOW + DELAY*60
2390 IF MAG <> 0 THEN 2420
2400 IF TNEXT >= M THEN GOSUB 2810 :
     IF AN$ = "N" OR AN$ = "n" THEN 820
2410 GOTO 2430
2420 IF PROCES$ = "OK" AND TF <= TNEXT THEN GOSUB 730
2430 RETURN
2440 REM SUBROUTINE TO CALCULATE THE EXPECTED INCOME IN
     PRODUCTION PROCESS BEFORE SIMULATION COMPLETED
2450 REM CALCULATE THE EXPECTED INCOME
2460 INCOME = INCOME + (M - TNOW)*P/60
2470 TNOW = M
2480 RETURN
2490 REM SUBROUTINE PRINTOUT, PRINT OUT PROFIT PER HOUR
     AND STATISTICS
2500 PRINT "TIME "; TNOW; "MIN.";
     TAB(TAB) "STOP SIMULATION"
2510 AINCTRL = TINCTRL/(NSHIFT*60)
2520 ATSAD = TSAD/(NDETEC*60)
2530 IF MAG = 0 THEN 2550
2540 ARL = TSUMRL/TALARM : GOTO 2560
2550 ARL = FSUMRL/FALARM
2560 PROFIT = INCOME - LCOST
2570 HR = TNOW/60 : PROFPH = PROFIT/HR : PRINT : BEEP
2580 IF CODE <> 1 THEN 2600
2590 PRINT "X-BAR CHART    (with warning limits)."
2600 IF CODE = 2 THEN PRINT "CUMULATIVE SUM CHART ";
     "  (d ="; D; "*W "; ", Half Angle ="; DEGREE;
     "degree)."
2610 IF CODE = 3 THEN PRINT "GEOMETRIC MOVING AVERAGE
     CHART. ( r ="; R; ", STANDARD CONTROL UNIT ="; UNIT;
     ")" : PRINT
2620 PRINT "SIMULATION TIME   = "; HR; "HOURS"
2630 PRINT "SAMPLE SIZE       = "; N
2640 PRINT "SAMPLING INTERVAL = "; S; "HR."
2650 PRINT "MAGNITUDE OF SHIFT IN PROCESS MEAN = "; MAG;
     "SIGMA"
2660 PRINT "NUMBER OF OBSERVATIONS  = "; SAMPLE
2670 PRINT "NUMBER OF SHIFTS        = "; NSHIFT
2680 PRINT "NUMBER OF TRUE ALARMS   = "; TALARM
2690 PRINT "NUMBER OF FALSE ALARMS  = "; FALARM
2700 PRINT "AVERAGE TIME THE PROCESS OPERATES IN-CONTROL
     = "; AINCTRL; "HR."
2710 PRINT "AVERAGE TIME BETWEEN SHIFT & DETECTION   = ";
     ATSAD; "HR."
2720 IF MAG = 0 THEN 2740
2730 PRINT "AVE. RUN LENGTH  = "; ARL : GOTO 2750
2740 PRINT "FALSE ALARM RATE = "; ARL
2750 PRINT "INCOME    = $"; INCOME
2760 PRINT "LOSS COST = $"; LCOST
```

```
2770 PRINT "PROFIT     = $"; PROFIT
2780 PRINT "PROFIT PER HOUR  = $"; PROFPH
2790 PRINT "SIMULATION COMPLETED"
2800 RETURN
2810 REM QUESTION ABOUT MORE SIMULATION TIME
2820 BEEP : BEEP
2830 INPUT "Do your want more simulation time  (Y or N) ";
     AN$
2840 IF AN$ <> "N" AND AN$ <> "n" THEN GOSUB 2860
2850 RETURN
2860 REM SUBROUTINE FOR MORE SIMULATION TIME
2870 INPUT "How many more simulation hours do you want ";
     ADD
2880 HR = HR + ADD
2890 M =   HR*60
2900 RETURN
2910 REM SUBROUTINE TO INCREASE THE SAMPLE AVERAGE WHEN
     PROCESS SHIFTS
2920 IF (TSAMP - TF)/(S*60) >= 1 THEN
     DELTA = MAG*SIGMA : GOTO 2940
2930 DELTA = MAG*SIGMA*(TSAMP - TF)/(S*60)
2940 RETURN
2950 PRINT TAB(65) "EXPERIMENT # "; NB
2960 END
```

APPENDIX B

PROGRAM FOR TWO ONE-SIDED CUMULATIVE SUM CONTROL CHARTS

```
1980 REM SUBROUTINE TO CALCULATE VALUES USED FOR TWO
         ONE-SIDED CUMULATIVE SUM CONTROL CHARTS
1990 W = 2*SIGMA/SQR(N)
2000 REM CALCULATE REFERENCE VALUE K1 AND K2
2010 K1 = MEAN + W*TAN(RADIAN) :
         PRINT "REFERENCE VALUE k1 ="; K1
2020 K2 = MEAN - W*TAN(RADIAN) :
         PRINT "REFERENCE VALUE k2 ="; K2
2030 H = D*W*TAN(RADIAN) : PRINT "h = +-"; H
2040 RETURN
2050 REM SUBROUTINE TWO ONE-SIDED CUMULATIVE SUM
         CONTROL CHARTS
2060 GOSUB 2370
2070 CUSUMP = CUSUMP + (AVE - K1)
2080 IF CUSUMP >= H THEN PST$ = "OUT" : GOTO 2160
2090 PST$ = "IN" : IF CUSUMP < 0 THEN CUSUMP = 0
2100 IF PFIR$ = "Y" THEN CUSUMP = 0 : PFIR$ = "N"
2110 CUSUMN = CUSUMN + (AVE - K2)
2120 IF CUSUMN <= -H THEN NEG$ = "OUT" : GOTO 2160
2130 NEG$ = "IN" : IF CUSUMN > 0 THEN CUSUMN = 0
2140 IF NFIR$ = "Y" THEN CUSUMN = 0 : NFIR$ = "N"
2150 PLOT$ = "IN" : PRINT TAB(TAB) "POS.CUSUM ="; CUSUMP;
         ","; TAB(40) "NEG.CUSUM ="; CUSUMN : GOTO 2240
2160 PLOT$ = "OUT"
2170 REM CUSUM THAT DETECTS NEGATIVE SHIFT GIVES ALARM
         SIGNAL
2180 IF NEG$ = "OUT" THEN 2210
2190 REM CUSUM THAT DETECTS POSITIVE SHIFT GIVES ALARM
         SIGNAL
2200 PRINT TAB(TAB) "POSITIVE CUSUM GIVES ALARM SIGNAL.
         (see details below)" : PRINT TAB(TAB)
         "POSITIVE CUSUM ="; CUSUMP; ", H ="; H : GOTO 2220
2210 PRINT TAB(TAB) "NEGATIVE CUSUM GIVES ALARM SIGNAL
         (see details below)" : PRINT TAB(TAB)
         "NEGATIVE CUSUM ="; CUSUMN; ", H ="; -H
2220 CUSUMP = 0 : CUSUMN = 0
2230 PFIR$ = "Y" : NFIR$ = "Y"
2240 RETURN
```

APPENDIX C

EXAMPLE OUTPUT FROM THE SIMULATION

```
X-BAR CHART    (with warning limits).

SIMULATION TIME    =   26070.57   HOURS
SAMPLE SIZE        =   3
SAMPLING INTERVAL =   1   HR.
MAGNITUDE OF SHIFT IN PROCESS MEAN = .5 SIGMA
NUMBER OF OBSERVATIONS   =   25096
NUMBER OF SHIFTS         =   600
NUMBER OF TRUE ALARMS    =   600
NUMBER OF FALSE ALARMS   =   46
AVE. TIME THE PROCESS OPERATES IN-CONTROL  = 20.19206 HR.
AVE. TIME BETWEEN SHIFT & DETECTION        = 21.72058 HR.
AVE. RUN LENGTH    =   22.125
INCOME      =   $ 2514768
LOSS COST =   $ 358856.4
PROFIT      =   $ 2155912
PROFIT PER HOUR    =   $ 82.69522
                                        EXPERIMENT # 1
SIMULATION COMPLETED
```

CUMULATIVE SUM CHART (d = 8.5*W , Half Angle = 14 degree).

```
SIMULATION TIME    =  19276.92    HOURS
SAMPLE SIZE        =  3
SAMPLING INTERVAL =  1  HR.
MAGNITUDE OF SHIFT IN PROCESS MEAN  =  .5 SIGMA
NUMBER OF OBSERVATIONS  =  18308
NUMBER OF SHIFTS        =  600
NUMBER OF TRUE ALARMS   =  600
NUMBER OF FALSE ALARMS  =  36
AVE. TIME THE PROCESS OPERATES IN-CONTROL  = 20.19206 HR.
AVE. TIME BETWEEN SHIFT & DETECTION        = 10.40616 HR.
AVE. RUN LENGTH    =  10.805
INCOME     =  $ 1835888
LOSS COST =  $ 183422.9
PROFIT     =  $ 1652465
PROFIT PER HOUR   =  $ 85.72248
```

EXPERIMENT # 37

SIMULATION COMPLETED

```
GEOMETRIC MOVING AVERAGE CHART.
        ( STANDARD CONTROL UNIT = 2.71*Sigma Zt )
SIMULATION TIME   =  19711.23   HOURS
SAMPLE SIZE       =  3
SAMPLING INTERVAL =  1  HR.
MAGNITUDE OF SHIFT IN PROCESS MEAN =  .5 SIGMA
NUMBER OF OBSERVATIONS  =  18740
NUMBER OF SHIFTS        =  600
NUMBER OF TRUE ALARMS   =  600
NUMBER OF FALSE ALARMS  =  40
AVE. TIME THE PROCESS OPERATES IN-CONTROL  = 20.19206 HR.
AVE. TIME BETWEEN SHIFT & DETECTION        = 11.12668 HR.
AVE. RUN LENGTH    =  11.53
INCOME    =  $ 1879120
LOSS COST =  $ 194654.9
PROFIT    =  $ 1684465
PROFIT PER HOUR   =  $ 85.45713
                                    EXPERIMENT # 73
SIMULATION COMPLETED
```

APPENDIX D

SUMMARY OF RESULTS

No.	Factor Levels				Profit Per Hour ($)	Average Run Length	Time Between Shift and Detection (Hr.)
	T	N	S	M			
1	1	3	1	1	82.6952	22.1250	21.7206
1	1	3	1	1	82.7068	22.0866	21.6853
2	1	3	1	2	83.7454	5.0867	4.6778
2	1	3	1	2	83.8552	5.0117	4.6120
3	1	3	1	3	84.7507	2.7100	2.3078
3	1	3	1	3	84.6348	2.7650	2.3591
4	1	3	1	4	84.8372	2.0167	1.6110
4	1	3	1	4	84.9633	1.9833	1.5791
5	1	3	2	1	80.3415	21.9117	42.9662
5	1	3	2	1	80.2037	22.6133	44.3775
6	1	3	2	2	78.8786	5.1883	9.5199
6	1	3	2	2	79.1965	5.0033	9.1512
7	1	3	2	3	80.1567	2.6783	4.5074
7	1	3	2	3	80.1573	2.6750	4.4981
8	1	3	2	4	80.0282	1.9983	3.1445
8	1	3	2	4	80.0803	1.9900	3.1309
9	1	3	4	1	78.2779	21.7567	85.2219
9	1	3	4	1	78.1959	22.3967	87.7887
10	1	3	4	2	72.8346	4.9433	17.9761
10	1	3	4	2	72.7776	4.9700	18.0825
11	1	3	4	3	72.7119	2.6233	8.6965
11	1	3	4	3	71.7398	2.7833	9.3358
12	1	3	4	4	71.7426	1.9717	6.0893
12	1	3	4	4	71.7772	1.9683	6.0760
13	1	4	1	1	84.0652	15.6917	15.3158
13	1	4	1	1	83.9662	16.0683	15.6875
14	1	4	1	2	85.1712	3.9617	3.5770
14	1	4	1	2	85.1118	3.9983	3.6186
15	1	4	1	3	85.5875	2.3300	1.9428
15	1	4	1	3	85.6051	2.3350	1.9513
16	1	4	1	4	85.5815	1.7650	1.3828
16	1	4	1	4	85.5157	1.7950	1.4113
17	1	4	2	1	81.3295	16.9533	33.0740
17	1	4	2	1	81.4139	16.6183	32.4006
18	1	4	2	2	81.3602	3.9200	7.0116
18	1	4	2	2	81.1002	4.0433	7.2514
19	1	4	2	3	81.8000	2.2750	3.7223
19	1	4	2	3	81.8238	2.2733	3.7153
20	1	4	2	4	81.2956	1.7900	2.7421
20	1	4	2	4	81.6277	1.7417	2.6481
21	1	4	4	1	79.1689	16.185	62.9628
21	1	4	4	1	79.1592	16.2317	63.1494

22	1	4	4	2	75.4585	3.9183	13.8898
22	1	4	4	2	75.5560	3.8850	13.7628
23	1	4	4	3	74.7018	2.3133	7.4763
23	1	4	4	3	74.7961	2.3033	7.4289
24	1	4	4	4	74.0619	1.7450	5.2040
24	1	4	4	4	73.9428	1.7583	5.2561
25	1	5	1	1	85.0000	12.2017	11.8377
25	1	5	1	1	84.8715	12.6767	12.3111
26	1	5	1	2	85.8305	3.4083	3.0428
26	1	5	1	2	85.9002	3.3283	2.9641
27	1	5	1	3	86.2511	2.0283	1.6618
27	1	5	1	3	86.2095	2.0350	1.6724
28	1	5	1	4	85.8434	1.6533	1.2920
28	1	5	1	4	85.7545	1.6800	1.3168
29	1	5	2	1	82.6155	12.3900	23.9665
29	1	5	2	1	82.6766	12.2700	23.7249
30	1	5	2	2	82.8019	3.2750	5.7320
30	1	5	2	2	82.6291	3.3450	5.8628
31	1	5	2	3	82.8274	2.0333	3.2562
31	1	5	2	3	82.5942	2.0883	3.3652
32	1	5	2	4	82.2663	1.6267	2.4456
32	1	5	2	4	81.9915	1.6717	2.5285
33	1	5	4	1	80.0884	12.4300	47.9501
33	1	5	4	1	80.0488	12.5517	48.4504
34	1	5	4	2	77.3040	3.3117	11.4768
34	1	5	4	2	77.2298	3.3333	11.5699
35	1	5	4	3	76.8378	2.0050	6.2635
35	1	5	4	3	76.2235	2.0900	6.6029
36	1	5	4	4	75.2838	1.6333	4.7696
36	1	5	4	4	74.8612	1.6700	4.9224
37	2	3	1	1	85.7225	10.8050	10.4062
37	2	3	1	1	85.6107	10.9700	10.5651
38	2	3	1	2	84.7749	4.3400	3.9368
38	2	3	1	2	84.7329	4.3550	3.9524
39	2	3	1	3	84.1267	2.9567	2.5548
39	2	3	1	3	84.1131	2.9717	2.5706
40	2	3	1	4	83.3855	2.4267	2.0173
40	2	3	1	4	83.3693	2.4283	2.0268
41	2	3	2	1	83.3899	10.6767	20.5005
41	2	3	2	1	83.4234	10.6200	20.3866
42	2	3	2	2	80.1921	4.5167	8.1703
42	2	3	2	2	80.2163	4.5050	8.1534
43	2	3	2	3	78.8835	2.9917	5.1333
43	2	3	2	3	78.8087	3.0033	5.1576
44	2	3	2	4	77.1528	2.4633	4.0737
44	2	3	2	4	77.2532	2.4467	4.0404
45	2	3	4	1	80.6721	10.8700	41.6815
45	2	3	4	1	80.5045	11.3267	43.5084
46	2	3	4	2	73.9143	4.5033	16.2156
46	2	3	4	2	73.9406	4.4950	16.1820
47	2	3	4	3	69.6335	3.1483	10.7956
47	2	3	4	3	69.7440	3.1317	10.7219

48	2	3	4	4	66.5235	2.5150	8.2626
48	2	3	4	4	66.6291	2.5067	8.2219
49	2	4	1	1	86.3608	8.7367	8.3531
49	2	4	1	1	86.3917	8.7150	8.3288
50	2	4	1	2	85.4404	3.8017	3.4155
50	2	4	1	2	85.4093	3.7850	3.4025
51	2	4	1	3	84.7861	2.6583	2.2743
51	2	4	1	3	84.8261	2.6330	2.2516
52	2	4	1	4	84.1447	2.1550	1.7758
52	2	4	1	4	83.9540	2.2117	1.8261
53	2	4	2	1	84.2026	8.7933	16.7543
53	2	4	2	1	84.0982	8.9633	17.0988
54	2	4	2	2	81.6283	3.8000	6.7685
54	2	4	2	2	81.5309	3.8500	6.8646
55	2	4	2	3	79.9224	2.7150	4.6008
55	2	4	2	3	80.2755	2.6333	4.4341
56	2	4	2	4	78.6708	2.2033	3.5670
56	2	4	2	4	78.5978	2.2117	3.5903
57	2	4	4	1	81.5701	8.7150	33.0819
57	2	4	4	1	81.4313	8.9817	34.1495
58	2	4	4	2	75.4988	3.9083	13.8491
58	2	4	4	2	75.3911	3.9450	14.0023
59	2	4	4	3	71.9517	2.7450	9.1948
59	2	4	4	3	71.9488	2.7417	9.1888
60	2	4	4	4	68.8533	2.2567	7.2486
60	2	4	4	4	68.7192	2.2700	7.3023
61	2	5	1	1	86.7739	7.6067	7.2427
61	2	5	1	1	86.7627	7.6017	7.2351
62	2	5	1	2	85.9023	3.3783	3.0116
62	2	5	1	2	85.8107	3.4150	3.0561
63	2	5	1	3	85.1060	2.4683	2.1001
63	2	5	1	3	85.1078	2.4583	2.0926
64	2	5	1	4	84.4964	2.0250	1.6595
64	2	5	1	4	84.5640	1.9917	1.6326
65	2	5	2	1	84.8777	7.4717	14.1306
65	2	5	2	1	84.6810	7.7867	14.7618
66	2	5	2	2	82.3625	3.4667	6.1175
66	2	5	2	2	82.2606	3.5033	6.1947
67	2	5	2	3	80.9126	2.4633	4.1179
67	2	5	2	3	80.8152	2.4900	4.1644
68	2	5	2	4	79.5506	2.0433	3.2712
68	2	5	2	4	79.6194	2.0300	3.2510
69	2	5	4	1	82.0684	7.7283	29.1494
69	2	5	4	1	82.0045	7.8350	29.5829
70	2	5	4	2	76.7764	3.4767	12.1425
70	2	5	4	2	76.4867	3.5667	12.5091
71	2	5	4	3	73.2168	2.5300	8.3626
71	2	5	4	3	72.7621	2.6033	8.6551
72	2	5	4	4	70.2014	2.1117	6.6882
72	2	5	4	4	70.5523	2.0750	6.5423
73	3	3	1	1	85.4571	11.5300	11.1267
73	3	3	1	1	85.4452	11.5617	11.1617

74	3	3	1	2	84.8045	4.3400	3.9358
74	3	3	1	2	84.8293	4.2767	3.8713
75	3	3	1	3	84.6562	2.7583	2.3523
75	3	3	1	3	84.6116	2.7650	2.3598
76	3	3	1	4	84.0573	2.2400	1.8305
76	3	3	1	4	84.1046	2.2100	1.8052
77	3	3	2	1	83.2470	11.0250	21.1971
77	3	3	2	1	83.2257	11.0583	21.2675
78	3	3	2	2	80.7658	4.2317	7.6140
78	3	3	2	2	80.3322	4.4250	8.0017
79	3	3	2	3	80.1403	2.6817	4.5080
79	3	3	2	3	79.7965	2.7617	4.6713
80	3	3	2	4	78.9010	2.1750	3.4978
80	3	3	2	4	78.8652	2.1800	3.5080
81	3	3	4	1	80.5275	11.2433	43.1756
81	3	3	4	1	80.3450	11.7650	45.2560
82	3	3	4	2	75.1983	4.0183	14.2765
82	3	3	4	2	75.0139	4.0900	14.5623
83	3	3	4	3	72.5861	2.6433	8.7765
83	3	3	4	3	72.5339	2.6533	8.8098
84	3	3	4	4	70.1546	2.1283	6.7100
84	3	3	4	4	69.7142	2.1750	6.9020
85	3	4	1	1	86.3517	8.8567	8.4696
85	3	4	1	1	86.3342	8.9767	8.4100
86	3	4	1	2	85.7368	3.5600	3.1806
86	3	4	1	2	85.7721	3.5350	3.1510
87	3	4	1	3	85.2774	2.4467	2.0628
87	3	4	1	3	85.1486	2.4950	2.1113
88	3	4	1	4	84.6155	2.0300	1.6488
88	3	4	1	4	84.6043	2.0350	1.6506
89	3	4	2	1	84.1862	8.7950	16.7553
89	3	4	2	1	84.1875	8.8017	16.7716
90	3	4	2	2	82.2445	3.5367	6.2381
90	3	4	2	2	82.1978	3.5550	6.2781
91	3	4	2	3	81.0691	2.4450	4.0583
91	3	4	2	3	81.2427	2.4050	3.9723
92	3	4	2	4	80.1020	1.9733	3.0995
92	3	4	2	4	79.8505	2.0067	3.1821
93	3	4	4	1	81.4544	8.9217	33.9100
93	3	4	4	1	81.3874	9.0667	34.4898
94	3	4	4	2	77.0685	3.3900	11.7835
94	3	4	4	2	76.7756	3.4883	12.1755
95	3	4	4	3	74.3317	2.3683	7.6961
95	3	4	4	3	74.2790	2.3767	7.7293
96	3	4	4	4	72.1238	1.9267	5.9300
96	3	4	4	4	71.8126	1.9583	6.0558
97	3	5	1	1	86.7507	7.6917	7.3285
97	3	5	1	1	86.7657	7.5967	7.2366
98	3	5	1	2	86.1854	3.1817	2.8233
98	3	5	1	2	86.2070	3.1633	2.8005
99	3	5	1	3	85.5995	2.2817	1.9160
99	3	5	1	3	85.6859	2.2483	1.8812

100	3	5	1	4	84.9828	1.8917	1.5321
100	3	5	1	4	84.8810	1.9150	1.5560
101	3	5	2	1	84.8774	7.4467	14.0816
101	3	5	2	1	84.8094	7.5150	14.2169
102	3	5	2	2	83.2157	3.1017	5.3891
102	3	5	2	2	83.1538	3.1233	5.4360
103	3	5	2	3	81.9939	2.2150	3.6164
103	3	5	2	3	82.0527	2.2033	3.5958
104	3	5	2	4	80.5689	1.8817	2.9464
104	3	5	2	4	80.6991	1.8583	2.9093
105	3	5	4	1	82.1842	7.5233	28.3298
105	3	5	4	1	82.3234	7.2900	27.3967
106	3	5	4	2	78.0560	3.0917	10.6093
106	3	5	4	2	78.1198	3.0717	10.5297
107	3	5	4	3	75.5733	2.1800	6.9631
107	3	5	4	3	75.6311	2.1750	6.9420
108	3	5	4	4	73.7107	1.7717	5.3300
108	3	5	4	4	73.2535	1.8150	5.4969

APPENDIX E

OUTPUT FROM STATISTICAL ANALYSIS

Table E-1 Main Effects on Profit Per Hour.

a)

Type of Control Chart	Profit Per Hour ($)
$\overline{X}$	80.62
Cumulative Sum	80.08
Geometric Moving Ave.	81.04

b)

Sample Size	Profit Per Hour ($)
3	79.41
4	80.73
5	81.60

c)

Sampling Interval (hr.)	Profit Per Hour ($)
1	85.13
2	81.34
4	75.28

d)

Magnitude of Shift	Profit Per Hour ($)
0.5σ	83.10
1.0σ	80.83
1.5σ	79.79
2.0σ	78.60

Table E-2 Two-Way Tables of Profit Per Hour ($).

a)

Type of Control Chart	Sample Size		
	3	4	5
$\overline{X}$	79.22	80.80	81.83
Cumulative Sum	79.03	80.23	80.99
Geometric Moving Ave.	79.97	81.17	81.97

b)

Type of Control Chart	Sampling Interval (hr.)		
	1	2	4
$\overline{X}$	84.94	81.30	75.62
Cumulative Sum	85.07	80.97	74.21
Geometric Moving Ave.	85.37	81.74	76.01

c)

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	81.49	80.38	80.53	80.08
Cumulative Sum	83.92	80.68	78.72	77.01
Geometric Moving Ave.	83.88	81.43	80.12	78.72

Table E-2   Two-Way Tables of Profit Per Hour ($) (continued).

d)

Sample Size	Sampling Interval (hr.)		
	1	2	4
3	84.42	80.15	73.65
4	85.24	81.49	75.48
5	85.72	82.37	76.70

e)

Sample Size	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
3	82.22	79.44	78.54	77.42
4	83.17	81.02	79.96	78.78
5	83.90	82.02	80.86	79.62

f)

Sampling Interval (hr.)	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
1	85.45	85.29	85.12	84.65
2	83.21	81.45	80.85	79.84
4	80.63	75.74	73.40	71.33

Table E-3 Three-Way Tables of Profit Per Hour ($).

a)   Sample Size = 3

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	80.40	78.55	79.02	78.90
Cumulative Sum	83.22	79.63	77.55	75.72
Geometric Moving Ave.	83.04	80.16	79.05	77.63

b)   Sample Size = 4

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	81.52	80.63	80.72	80.34
Cumulative Sum	84.01	80.82	78.95	77.16
Geometric Moving Ave.	83.98	81.63	80.22	78.85

c)   Sample Size = 5

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	82.55	81.96	81.82	81.00
Cumulative Sum	84.53	81.60	79.65	78.16
Geometric Moving Ave.	84.62	82.49	81.09	79.68

Table E-3 Three-Way Tables of Profit Per Hour ($)
(continued).

d)   Sampling Interval = 1 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	83.88	84.95	85.51	85.42
Cumulative Sum	86.27	85.34	84.68	83.99
Geometric Moving Ave.	86.18	85.59	85.16	84.54

e)   Sampling Interval = 2 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	81.43	80.99	81.56	81.21
Cumulative Sum	84.11	81.36	79.94	78.47
Geometric Moving Ave.	84.09	81.98	81.05	79.83

f)   Sampling Interval = 4 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	79.16	75.19	74.50	73.61
Cumulative Sum	81.37	75.33	71.54	68.58
Geometric Moving Ave.	81.37	76.70	74.16	71.79

Table E-3   Three-Way Tables of Profit Per Hour   ($)
(continued).

g)   Sampling Interval = 1 hr.

Sample Size	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
3	84.61	84.46	84.48	84.12
4	85.58	85.44	85.20	84.74
5	86.15	85.99	85.66	85.09

h)   Sampling Interval = 2 hr.

Sample Size	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
3	82.30	79.93	79.66	78.71
4	83.24	81.68	81.02	80.02
5	84.09	82.74	81.87	80.78

i)   Sampling Interval = 4 hr.

Sample Size	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
3	79.75	73.95	71.49	69.42
4	80.69	75.96	73.67	71.59
5	81.45	77.33	75.04	72.98

Table E-4   Three-Way Tables of Average Run Length.

a)   Sample Size = 3

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	22.15	5.03	2.71	1.99
Cumulative Sum	10.88	4.45	3.03	2.46
Geometric Moving Ave.	11.36	4.23	2.71	2.18

b)   Sample Size = 4

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	16.29	3.95	2.30	1.77
Cumulative Sum	8.82	3.85	2.69	2.22
Geometric Moving Ave.	8.87	3.51	2.42	1.99

c)   Sample Size = 5

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	12.42	3.33	2.05	1.66
Cumulative Sum	7.67	3.47	2.50	2.05
Geometric Moving Ave.	7.51	3.12	2.22	1.86

Table E-4   Three-Way Tables of Average Run Length (continued).

d)   Sampling Interval = 1 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	16.81	4.13	2.37	1.81
Cumulative Sum	9.07	3.85	2.69	2.21
Geometric Moving Ave.	9.34	3.68	2.50	2.05

e)   Sampling Interval = 2 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	17.13	4.13	2.34	1.80
Cumulative Sum	9.05	3.94	2.72	2.23
Geometric Moving Ave.	9.11	3.66	2.45	2.01

f)   Sampling Interval = 4 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	16.92	4.06	2.35	1.79
Cumulative Sum	9.24	3.98	2.82	2.29
Geometric Moving Ave.	9.30	3.52	2.40	1.96

Table E-5   Three-Way Tables of the Time Between
Shift and Detection (Hr.) .

a)   Sample Size = 3

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	50.63	10.67	5.28	3.60
Cumulative Sum	24.51	9.43	6.16	4.77
Geometric Moving Ave.	25.53	8.71	5.25	4.04

b)   Sample Size = 4

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	37.10	8.18	4.37	3.11
Cumulative Sum	19.63	8.05	5.32	4.22
Geometric Moving Ave.	19.80	7.13	4.60	3.59

c)   Sample Size = 5

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	28.04	6.77	3.80	2.88
Cumulative Sum	17.02	7.17	4.91	3.84
Geometric Moving Ave.	16.43	6.26	4.15	3.29

Table E-5 Three-Way Tables of the Time Between Shift
and Detection (Hr.) (continued).

d) Sampling Interval = 1 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	16.43	3.75	1.98	1.43
Cumulative Sum	8.69	3.46	2.31	1.82
Geometric Moving Ave.	8.96	3.29	2.11	1.67

e) Sampling Interval = 2 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	33.42	7.42	3.84	2.77
Cumulative Sum	17.27	7.04	4.60	3.63
Geometric Moving Ave.	17.38	6.49	4.07	3.19

f) Sampling Interval = 4 hr.

Type of Control Chart	Magnitude of Shift (δ)			
	0.5	1.0	1.5	2.0
$\overline{X}$	65.92	14.46	7.63	5.39
Cumulative Sum	35.19	14.15	9.49	7.38
Geometric Moving Ave.	35.43	12.32	7.82	6.07