

## Design of interface shape for protective capillary barriers

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**Abstract.** Sloping interfaces of fine over coarse porous material have been considered for use as barriers to infiltration for many years. Previous literature has developed analytical solutions for flow over such interfaces, numerical simulation of such flow, and the effects of anisotropy on the diversion capacity of such a system. In all of these analyses, however, the interface was assumed to consist of a constant slope, facilitating certain calculations. In this note it is shown that both from a performance perspective (as measured by the ability to divert fluid) and a design perspective (ease of computing diversion capacity) the shape of the interface should be curved, typically in a parabolic shape. Use of this shape is predicted to double barrier diversion capacity. Interfaces made up of easy to construct segmented approximations to the parabola are predicted to provide significantly enhanced performance as well.

In the design of a capillary barrier protective cover there are several parameters which must be established: (1) What is the maximum allowable flux through the system (leakage)? (2) What is the horizontal distance over which water is to be diverted? (3) What is the maximum sustained flux that the system will experience during the design life? and (4) What materials are available for use? In this paper we will discuss the implications of the first three considerations.

When a capillary barrier experiences uniform and constant application of water, the diverted flux increases linearly with horizontal position down-dip. In the case of a barrier where the slope in the down-dip direction is constant, first failure will always be near the down-dip end of the interface, while the upper portion of the interface could handle much more flow without local penetration. Indeed, the upper portion of the interface could divert the incident flux even if the interface had a lower local slope, while if the slope were greater toward the down-dip portion, the diversion could be increased prior to breakthrough. It would appear logical to increase the slope of the barrier in proportion to the expected local flux, which increases linearly with down-slope distance.

From a design perspective it seems reasonable to suppose that the optimal use of the total fall in elevation has been achieved when the breakthrough flux through the interface is constant along the entire length of the interface. It will be shown that this assumption immediately dictates the geometry of the interface and greatly simplifies the hydraulic analysis. Consider the limiting condition where the maximum allowable flux is penetrating the barrier. Let us now posit to have selected a barrier geometry such that this flux penetrates the barrier at all points. This uniform leakage flux condition implies that the pressure is also uniform along the textural interface. Since this condition exists at all points along the barrier, the vertical moisture profile in the upper media is uniform across the barrier, and there will be no gradients in pressure in directions tangent to the interface. Thus all barrier parallel flux will be driven by gradients in the gravitational potential, which, given the uniform vertical pressure profile, equal to the local slope of

the interface. For a given location the lateral rate of flow  $Q$  is given by

$$Q = \int_0^T K(z')S dz' = S \int_0^T K(z') dz' \quad (1)$$

where  $z'$  is the vertical distance from the interface,  $T$  is the thickness of the fine material,  $K(z')$  is the hydraulic conductivity a distance  $z'$  above the interface, and  $S$  is the local slope of the interface. In our case the vertical moisture profile is identical at all locations, so the value of the integral is fixed for the entire interface. Thus (1) may be written

$$Q = CS \quad (2)$$

where

$$C = \int_0^T K(z') dz' \quad (3)$$

This integral may be computed for isotropic or anisotropic media using the results of Ross [1990], Steenhuis *et al.* [1991], or Stormont [1995] for the conductivity profile at the down-dip limit with the incident flux set equal to the maximum penetration flux.

In the case of the linear-roof design (Figure 1),  $Q = qx$ , where  $x$  is the horizontal distance from the up-slope crest and  $q$  is the incident flux. Putting this into (3), we immediately find that

$$S = \frac{xq}{C} \quad (4)$$

If the interface is parameterized by coordinates  $(x^*, z^*)$ , then

$$S = \frac{dz^*}{dx^*} \quad (5)$$

which may be substituted into (4)

$$\frac{dz^*}{dx^*} = \frac{x^*q}{C} \quad (6)$$

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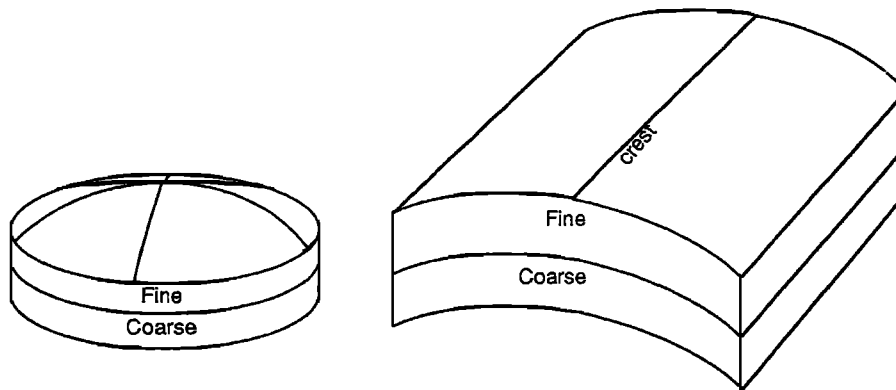


Figure 1. Definition sketches for (left) circular and (right) linear-roof capillary barrier designs.

which may be solved for the interface elevation as a function of horizontal distance from the crest

$$z^* = \frac{x^{*2}q}{2C} \quad (7)$$

Equation (7) suggests that the use of a parabolic interface for linear-roof capillary barriers will provide optimal diversion.

To estimate the improvement in barrier performance obtained using the curved interface, consider a barrier which has a total fall of  $H$  over a horizontal distance  $L$  (Figure 2). For the straight barrier, (2) suggests a diversion capacity of  $CL/H$ , while for a parabolic system, (7) suggests a diversion of  $2CL/H$ ; the parabolic interface provides a factor of 2 increase in diversion capacity for a given total elevation loss.

Within the uniform leakage framework it is also straightforward to compute the optimum shape of a circular-roof diversion. In such a setup, for any radial distance  $r$  from the peak, the total flux intercepted by the barrier will be  $\pi r^2 q$  which will transect a circumference of media with total length  $2\pi r$ . The local lateral flow will be  $qr/2$ , which, by comparison to the linear-roof analysis, implies an optimal interface parameterized by  $z = r^2 q/C$ . The slope should be half that of a linear-roof for any given distance from the crest. Clearly, this analysis

could be extended to arbitrary roof geometry by setting the local interface slope to be a constant times the area of capture that contributes to the flow at each point.

In practical terms, the construction of a smoothly changing slope presents logistical difficulties. Notice that the curved system could be approximated by a series of  $n$  straight segments of equal horizontal extent and linearly increasing slope. Using such an approximation to a barrier with total horizontal extent  $H$  and total fall  $L$ , the slope of the most down-dip segment would be

$$S = \frac{Hn^2}{L \sum_{i=1}^n i} \quad (8)$$

In such a segmented system, first leakage would occur at the lowest end of each segment simultaneously, since the diverted flux/slope ratio will be minimal and equal for all segments at these points. Using three and five straight segments, the diversion capacity is predicted to be at least 75% and 83%, respectively, of the diversion of a smoothly curved interface, demonstrating a rapid convergence to the optimal performance.

## References

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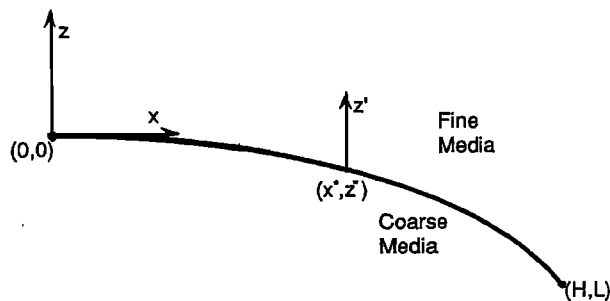


Figure 2. Parameter identification.