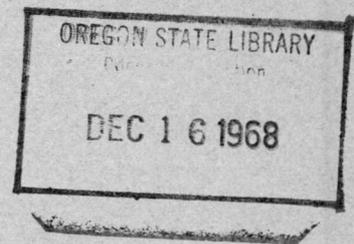


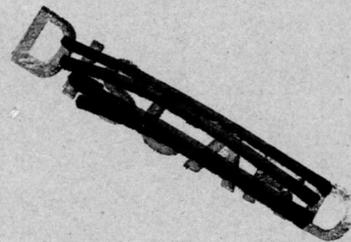
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# Engineering Calculations of Momentum, Heat and Mass Transfer Through Laminar Boundaries



E. ELZY  
and  
G. A. MYERS

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ENGINEERING CALCULATIONS OF  
MOMENTUM, HEAT AND MASS TRANSFER THROUGH LAMINAR BOUNDARY LAYERS

by

Eugene Elzy, Assistant Professor  
Department of Chemical Engineering

and

G. A. Myers, Research Assistant  
Department of Chemical Engineering

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Oregon State University  
Corvallis, Oregon

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## 1. INTRODUCTION

The calculation of heat, mass and momentum transfer is of major importance in many problems of engineering interest. A large number of methods for this calculation have been proposed for the steady two-dimensional or axisymmetrical laminar boundary layer with arbitrary external pressure distribution and constant physical properties. The formally exact power series expansion methods [1,2,3,4,5] and the finite difference technique [6] produce reasonably accurate solutions, but require considerable computation. The approximate methods of Spalding [7,8], Pohlhausen [9], Merk [10,11] and Eckert [10] all require significantly less numerical labor. This reduction in labor is unfortunately accompanied by a great loss of accuracy in many flows, such as decelerating flow.

A new method has recently been proposed by Sisson [12] which appears to be a reasonable compromise between ease of solution and accuracy of results. The present report presents the application of this method to a wide range of problems. The figures necessary for this general type of calculation are also included.

An extensive graphical presentation of the similar solutions to the boundary layer equations is to be found in Appendix A. A wide range of mass transfer rate  $K$ , pressure parameter  $\beta_0$  and dimensionless property ratio  $\Lambda$  has been covered.

## 2. FORMULATION OF THE PROBLEM

### 2.1 The laminar boundary layer equations

The boundary layer equations may be derived from the general equations of change expressing the conservation of momentum, mass and energy [13]. For two-dimensional or axisymmetrical laminar flow of a pure or binary fluid, the steady constant-property boundary layer equations are given by:

Equation of continuity of mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{for two-dimensional flow} \quad (1)$$

$$\frac{\partial ru}{\partial x} + \frac{\partial rv}{\partial y} = 0 \quad \text{for axisymmetrical flow} \quad (2)$$

Equation of motion

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad (3)$$

Equation of energy

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Equation of continuity of a species in a binary mixture

$$u \frac{\partial x_A}{\partial x} + v \frac{\partial x_A}{\partial y} = D_{AB} \frac{\partial^2 x_A}{\partial y^2} \quad (5)$$

subject to the boundary conditions,

at  $y = 0$ :

$$u = 0 \quad (6)$$

$$v = v_w(x) \quad (7)$$

$$T = T_w \quad (8)$$

$$x_A = x_{A_w} \quad (9)$$

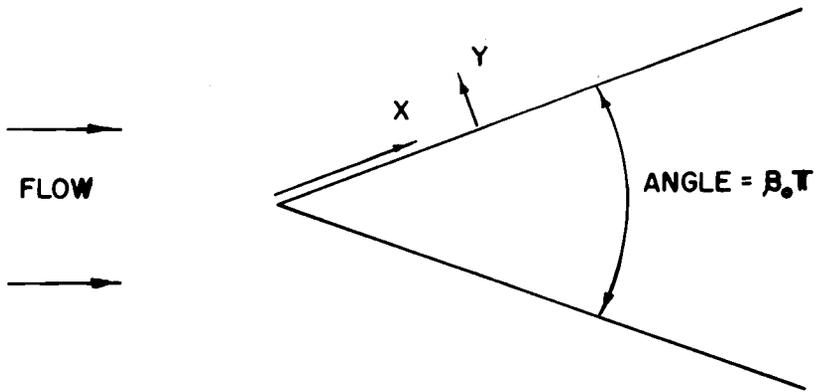
and as  $y \rightarrow \infty$ :

$$u = U(x) \quad (10)$$

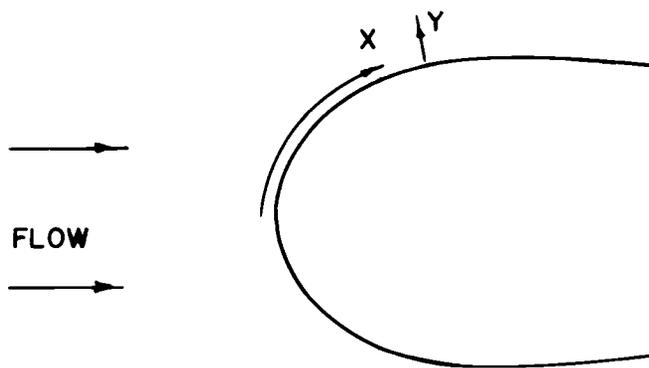
$$T = T_\infty \quad (11)$$

$$x_A = x_{A_\infty} \quad (12)$$

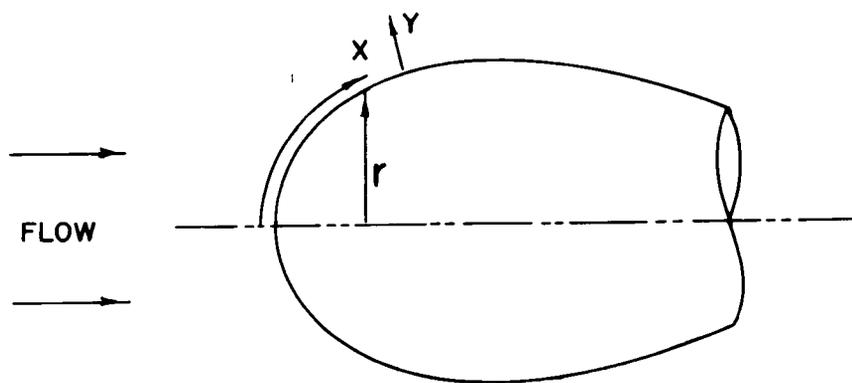
Viscous dissipation, radiation and chemical reaction within the fluid, and the Soret and Dufour effects are neglected. The coordinate system used is shown in Fig. 1.



TWO-DIMENSIONAL WEDGE



ARBITRARY TWO-DIMENSIONAL BODY



ARBITRARY AXISYMMETRICAL BODY

FIGURE 1. COORDINATE SYSTEMS

## 2.2 Transformation of variables

A more convenient representation of the boundary layer equations can be obtained by employing Meksyn's [11] transformation of variables with Mangler's [14] transformation included to reduce the equations for axisymmetrical flow to the same form as for two-dimensional flow. A stream function,  $\psi$ , is defined such that:

$$u = \frac{L}{r} \frac{\partial \psi}{\partial y} \quad (13)$$

$$v = - \frac{L}{r} \frac{\partial \psi}{\partial x} \quad (14)$$

This definition of  $\psi$  immediately satisfies the equation of continuity (1) and (2). Here,  $L$  is an arbitrary reference length on the body in the flow system. The factor  $r/L$  implies Mangler's transformation from axisymmetrical coordinates. For two-dimensional flow:

$$r/L = 1 \quad (15)$$

The  $x$  and  $y$  coordinates are transformed by:

$$\xi = \int_0^x \frac{U(x)}{U_\infty} \frac{r}{L^2} \frac{dx}{L} \quad (16)$$

$$\eta = \left(\frac{Re}{2\xi}\right)^{\frac{1}{2}} \frac{U(x)}{U_\infty} \frac{r}{L} \frac{y}{L} \quad (17)$$

where  $U(x)$  is the velocity of the fluid at the edge of the boundary layer,  $U_\infty$  is a reference velocity and

$$Re = \frac{U_\infty L}{\nu} \quad (18)$$

The stream function is now written:

$$\psi(x, y) = U_\infty L \left(\frac{2\xi}{Re}\right)^{\frac{1}{2}} f(\eta, \xi) \quad (19)$$

Substitutions of the definitions (16), (17) and (19) into equation (3) yields a third order, non-linear partial differential equation

$$f'''' + f''f + \beta(1 - f'f') = 2\xi \left( f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \quad (20)$$

where the prime indicates differentiation with respect to  $\eta$ . The parameter  $\beta$  is defined by:

$$\beta = \frac{2\xi}{U(x)} \frac{dU(x)}{d\xi} \quad (21)$$

The velocity components in terms of the transformation variables are:

$$u(x,y) = U(x) f'(\eta, \xi) \quad (22)$$

$$v(x,y) = -\frac{r}{L} \frac{U(x)}{(2\xi Re)^{\frac{1}{2}}} \left( f + 2\xi \frac{\partial f}{\partial \xi} + (\beta - 1)\eta f' \right) \quad (23)$$

The boundary conditions on equation (20) may now be written:

at  $\eta = 0$ ,

$$f' = 0 \quad (24)$$

$$f + 2\xi \frac{\partial f}{\partial \xi} = -\frac{v_w L}{r} \frac{(2\xi Re)^{\frac{1}{2}}}{U(x)} \quad (25)$$

and as  $\eta \rightarrow \infty$ ,

$$f' \rightarrow 1 \quad (26)$$

Equations (4) and (5) will be transformed by introducing the following dimensionless profile functions:

$$\Pi_T(\eta, \xi) = \frac{T(x,y) - T_w}{T_\infty - T_w} \quad (27)$$

$$\Pi_{AB}(\eta, \xi) = \frac{x_A(x, y) - x_{Aw}}{x_{A\infty} - x_{Aw}} \quad (28)$$

and defining:

$$\Lambda_T = \frac{\nu}{\alpha} \quad (29)$$

$$\Lambda_{AB} = \frac{\nu}{D_{AB}} \quad (30)$$

The equations for thermal energy and mass diffusion are then transformed to an identical form:

$$\Pi'' + \Lambda f \Pi' = 2\xi\Lambda \left( f' \frac{\partial \Pi}{\partial \xi} - \Pi' \frac{\partial f}{\partial \xi} \right) \quad (31)$$

With the assumption that  $T_w$ ,  $T_\infty$ ,  $x_{Aw}$  and  $x_{A\infty}$  are independent of  $x$ , the boundary conditions on equation (31) become:

at  $\eta = 0$ ,

$$\Pi = 0 \quad (32)$$

as  $\eta \rightarrow \infty$ ,

$$\Pi \rightarrow 1 \quad (33)$$

These non-linear partial differential equations (20) and (31), then, with their boundary conditions (24), (25), (26) and (32), (33), respectively, describe momentum, heat and mass transfer within a steady two-dimensional or axisymmetrical constant-property laminar boundary layer. The problem in any practical application is that of finding an adequate solution to these equations for a particular external velocity distribution.

### 3. DEVELOPMENT OF THE SOLUTION

#### 3.1 A series solution of the momentum equation [12]

The calculation of the momentum transfer in laminar boundary layer flows involves the substitution of two series into the momentum equation (20).

The following series are defined:

$$\beta(\xi) = \beta_0(\xi) + \sum_{n=1}^{\infty} a_n(\xi) \beta_n(\beta_0) \quad (34)$$

and

$$f(n, \xi) = f_0(n, \beta_0) + \sum_{n=1}^{\infty} a_n(\xi) f_n(n, \beta_0) \quad (35)$$

For convenience,  $a_1$  is defined as

$$a_1 = 2\xi \frac{d\beta_0}{d\xi} \quad (36)$$

At the present time, the  $a_n$  for  $n \geq 2$  are not of interest. However, they will have similar arbitrary definitions. Substitution of series (34) and (35) into equation (20) produces the ordinary differential equations:

$$f_0'''' + f_0'' f_0 + \beta_0(1 - f_0' f_0') = 0 \quad (37)$$

and

$$\begin{aligned} & f_1'''' + f_0 f_1'' - 2(\beta_0 + 1) f_0' f_1' + 3f_0'' f_1' \\ & = f_0' \frac{\partial f_0'}{\partial \beta_0} - f_0'' \frac{\partial f_0}{\partial \beta_0} - \beta_1(1 - f_0' f_0') \end{aligned} \quad (38)$$

The function  $\beta_1$  may now be defined by setting the boundary conditions on equation (38) such that

at  $\eta = 0$ ,

$$f_1 = 0 \quad (39)$$

$$f_1' = 0 \quad (40)$$

$$f_1'' = 0 \quad (41)$$

and as  $\eta \rightarrow \infty$ ,

$$f_1' \rightarrow 0 \quad (42)$$

Similarly, if the  $a_n$  are chosen so that the  $\beta_n$  appear as free constants in the equations defining the  $f_n$ , it will be possible to define the  $\beta_n$  by setting the boundary conditions for the  $f_n$  equations such that, for  $n \geq 1$ ,

at  $\eta = 0$ ,

$$f_n = 0 \quad (43)$$

$$f_n' = 0 \quad (44)$$

$$f_n'' = 0 \quad (45)$$

and as  $\eta \rightarrow \infty$ ,

$$f_n' \rightarrow 0 \quad (46)$$

Thus it may be seen that the  $f_n$  for  $n \geq 1$  make a zero contribution to the boundary conditions (24), (25) and (26) on the momentum equation (20). They also make no contribution to  $f''$  at  $\eta = 0$ , so momentum transfer calculations may be made using only  $f_0''$  at  $\eta = 0$ .

The boundary conditions on equation (37) may now be written:

at  $\eta = 0$ ,

$$f_0 = - \frac{v_w L}{r} \frac{(2\xi Re)^{\frac{1}{2}}}{U(x)} = -K \quad (47)$$

$$f_0' = 0 \quad (48)$$

and as  $\eta \rightarrow \infty$ ,

$$f_0' \rightarrow 1 \quad (49)$$

It may also be shown that

$$f_0'''(0, \beta_0) = f_0'''(0, \xi) \quad (50)$$

### 3.2 Extension to heat and mass transfer

Equation (31) describing heat and mass transfer may now be considered. The following series is defined:

$$\Pi(\eta, \xi) = \Pi_0(\eta, \beta_0) + \sum_{n=1}^{\infty} a_n \Pi_n(\eta, \beta_0) \quad (51)$$

Substituting equations (35), (36) and (51) into equation (31), it is found that

$$\Pi_0''' + \Lambda f_0' \Pi_0' = 0 \quad (52)$$

and

$$\Pi_1''' + \Lambda f_0' \Pi_1' - 2\Lambda f_0' \Pi_1 = \Lambda \left( f_0' \frac{\partial \Pi_0}{\partial \beta_0} - \Pi_0' \frac{\partial f_0}{\partial \beta_0} \right) - 3\Lambda \Pi_0' f_1 \quad (53)$$

Boundary conditions on these and the higher order equations are

at  $\eta = 0$ ,

$$\Pi_0 = 0 \quad (54)$$

and for  $n \geq 1$ ,

$$\Pi_n = 0 \quad (55)$$

and as  $\eta \rightarrow \infty$ ,

$$\Pi_0 \rightarrow 1 \quad (56)$$

and for  $n \geq 1$ ,

$$\Pi_n \rightarrow 0 \quad (57)$$

The function  $\beta_0(\xi)$  is defined implicitly by equation (34), and, with knowledge of the  $a_n$ , the  $\beta_n$  and the function  $\beta(\xi)$ , it would be possible to evaluate the function  $\beta_0(\xi)$ .

Equation (35) could then be used, with knowledge of the  $f_n$ , to calculate the exact values of  $f(\eta, \xi)$ .

### 3.3 Various simplifications

#### Similar solution

When the external velocity distribution satisfies the relationship [7]:

$$\frac{dU(x)}{dx} = C_1 U^n(x) \quad (58)$$

the substitution of the transformed variables into equation (3) yields equation (37) and its corresponding boundary conditions. Upon transformation both the energy equation (4) and the continuity of species equation (5) reduce to equation (52) with its transformed boundary conditions.

The solutions to equations (37) and (52), with  $K$  and  $\beta_0$  constant, are known as "similar solutions" because the velocity profiles at all points within the boundary layer differ only by a constant scale factor [15]. Tabulations of the numerical solutions are available for large ranges of the parameters  $\beta_0$ ,  $K$  and  $\Lambda$  [16,17,18,19]. Appendix A contains a graphical presentation of these solutions. This is discussed more fully in part 4 of this paper.

Flow over a wedge [20], where

$$U(x) = U_1 x^m \quad (59)$$

$$\beta_0 = \frac{2m}{m+1} \quad (60)$$

and

$$v_w(x) = C_2 x^{\left(\frac{m-1}{2}\right)} \quad (61)$$

comprises an important class of similar solutions. Using potential flow theory [21], it may be shown that the parameter  $\beta_0$  corresponds to a wedge with an included angle of  $(\beta_0 \Pi)$ , as illustrated in Fig. 1.

### Merk similar method

The zero-order approximation is the truncation of series (34) after the first term, so that

$$\beta_0(\xi) = \beta(\xi) \quad (62)$$

This method was proposed by Merk [11]. The Merk similar method unfortunately gives poor results for decelerating flow.

### First order approximation

A first-order approximation, which will also be called the present method, is made by truncating series (34) after the second term to obtain

$$\beta(\xi) = \beta_0(\xi) + 2\xi \frac{d\beta_0}{d\xi} \beta_1(\beta_0) \quad (63)$$

Rearrangement of equation (63) produces an ordinary differential equation

$$\frac{d\beta_0}{d\xi} = \frac{\beta - \beta_0}{2\xi\beta_1} \quad (64)$$

with a boundary condition

$$\text{at } \xi = 0$$

$$\beta_0 = \beta \quad (65)$$

At  $\xi = 0$  the right side of equation (64) is indeterminate. By invoking L'Hospital's rule the boundary condition can be found in the form of the derivative with respect to  $\xi$ . Consequently,

$$\text{at } \xi = 0$$

$$\frac{d\beta_0}{d\xi} = \frac{1}{1 + 2\beta_1} \frac{d\beta}{d\xi} \quad (66)$$

Since  $\beta(\xi)$  can be determined by equation (21) and since  $\beta_1$  is a function of  $\beta_0$  and  $K$ , equation (64) may be solved to find an approximation of  $\beta_0(\xi)$ .

The extension to the calculation of heat and mass transfer rates is simplified by the introduction of another approximation. The series (51) defining  $\Pi$  is truncated after the first term to obtain:

$$\Pi(\eta, \xi, K, \Lambda) = \Pi_0(\eta, \beta_0, K, \Lambda) \quad (67)$$

Thus  $\Pi'$  may be evaluated from the similar solutions at the proper value of  $\beta_0$ , just as  $f''$  has been.

Since  $\beta_0$  will in most cases be evaluated using the approximate equation (64), there will be two approximations used to evaluate  $\Pi'$ . However, if values of  $\Pi'_n$  for  $n \geq 1$  were available,  $\Pi'$  could be evaluated more accurately by including additional terms in the series (51). At the present time no values of  $\Pi'_n$  for  $n \geq 1$  are available. It is anticipated that the calculation of heat and mass transfer coefficients using the approximate equations (64) and (67) will be of sufficient accuracy for most practical applications.

### 3.4 The $\beta_1$ ( $\beta_0, K$ ) function

The  $\beta_1$  function depends only on  $\beta_0$  and  $K$  and is determined by solution of equations (37) and (38) with boundary conditions (39) to (42) and (47) to (49). These equations have been solved by Sisson [12] and the  $\beta_1$  function tabulated for a wide range of  $\beta_0$  and  $K$ . A graphical and tabular presentation of the  $\beta_1$  function is provided in Appendix B.

### 3.5 Definitions

#### Transfer coefficients

The local friction coefficient,  $c_f^\bullet$ , defined by

$$c_f^\bullet = \frac{\nu \left( \frac{\partial u}{\partial y} \right)_{y=0}}{\frac{1}{2} U_\infty^2} \quad (68)$$

may now be written in terms of the transformed variables as

$$c_f^\bullet = \frac{2r}{L} \frac{U^2(x)}{U_\infty^2 (2\xi Re)^{\frac{1}{2}}} f''(0, \xi, K) \quad (69)$$

The local Nusselt number,  $Nu^\bullet$ , defined by

$$Nu_T^\bullet = \frac{h^\bullet L}{k} = - \frac{(\frac{\partial T}{\partial y})_{y=0} L}{(T_w - T_\infty)} \text{ for heat transfer} \quad (70)$$

and

$$Nu_{AB}^\bullet = \frac{k_x^\bullet L}{cD_{AB}} = - \frac{(\frac{\partial x_A}{\partial y})_{y=0} L}{(x_{A_w} - x_{A_\infty})} \text{ for mass transfer} \quad (71)$$

may be written as

$$Nu^\bullet = \frac{r}{L} \frac{U(x)}{U_\infty (\frac{2\xi}{Re})^{\frac{1}{2}}} \Pi'(0, \xi, K, \Lambda) \quad (72)$$

These expressions (69) and (72), then, relate the dimensionless boundary layer functions  $f(\eta, \xi, K)$  and  $\Pi(\eta, \xi, K, \Lambda)$  to the dimensionless functions  $c_f^\bullet$  and  $Nu^\bullet$ , which are commonly used in momentum, heat and mass transfer calculations.

### Flux ratios

Other frequently used quantities are the dimensionless flux ratios,  $R$ ,

$$R_V = \frac{N_{A_w} M_A + N_{B_w} M_B}{\frac{1}{2} \rho U_\infty c_f^\bullet} = \frac{(N_{A_w} M_A + N_{B_w} M_B) U_\infty}{\tau_w} \quad (73)$$

$$R_T = \frac{N_{A_w} \tilde{C}_{pA} + N_{B_w} \tilde{C}_{pB}}{h^\bullet} = \frac{(N_{A_w} \tilde{C}_{pA} + N_{B_w} \tilde{C}_{pB}) (T_w - T_\infty)}{q_w} \quad (74)$$

$$R_{AB} = \frac{N_{A_w} + N_{B_w}}{k_x^\bullet} = \frac{x_{A_w} - x_{A_\infty}}{\frac{N_{A_w}}{N_{A_w} + N_{B_w}} - x_{A_w}} \quad (75)$$

where

$$\tau_w = -\rho v \left( \frac{\partial u}{\partial y} \right)_{y=0} \quad (76)$$

$$q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad (77)$$

When all physical properties are constant and equal for each species, the flux ratios may be written as:

$$R_V = \frac{\rho v_w(x)}{\rho U_\infty c_f / 2} = \frac{K}{f''(0, \xi, K)} \quad (78)$$

$$R_T = \frac{\rho \hat{C}_p v_w(x)}{h} = \frac{K \Lambda_T}{\Pi'(0, \xi, K, \Lambda_T)} \quad (79)$$

$$R_{AB} = \frac{c v_w(x)}{k_x} = \frac{K \Lambda_{AB}}{\Pi'(0, \xi, K, \Lambda_{AB})} \quad (80)$$

The last two equations can be summarized in the single expression:

$$R = \frac{K \Lambda}{\Pi'(0, \xi, K, \Lambda)} \quad (81)$$

### Rate factors

The rate factors,  $\phi$ , are the values of the corresponding  $R$  with the transfer coefficients evaluated for the case of no mass transfer,  $K = 0$ .

$$\phi_V = \frac{\rho v_w(x)}{\rho U_\infty c_f / 2} = \frac{K}{f''(0, \xi, 0)} \quad (82)$$

$$\phi_T = \frac{\rho \hat{C}_p v_w(x)}{h} = \frac{K \Lambda_T}{\Pi'(0, \xi, 0, \Lambda_T)} \quad (83)$$

$$\phi_{AB} = \frac{c v_w(x)}{k_x} = \frac{K \Lambda_{AB}}{\Pi'(0, \xi, 0, \Lambda_{AB})} \quad (84)$$

The general expression is

$$\phi = \frac{K\Lambda}{\Pi'(0, \xi, 0, \Lambda)} \quad (85)$$

### Correction factor

The correction factor,  $\theta$ , is the ratio of the transfer coefficient for mass transfer to the transfer coefficient for low or no mass transfer:

$$\theta_V = \frac{c_f^\bullet}{c_f}, \quad \theta_T = \frac{h^\bullet}{h}, \quad \theta_{AB} = \frac{k_x^\bullet}{k_x}$$

and by inserting the definition of the transfer coefficients,

$$\theta = \frac{\Pi'(0, \xi, K, \Lambda)}{\Pi'(0, \xi, 0, \Lambda)} = \frac{\phi}{R} \quad (86)$$

## 4. GRAPHICAL PRESENTATION OF SIMILAR SOLUTIONS

To facilitate the calculation of heat, mass and momentum transfer through laminar boundary layers, the similar solutions have been presented in various graphical forms in Appendix A. Most of the results were obtained by Elzy and Sisson [16]. The solutions, near or at the separation of the boundary layer, were taken from the papers of Stewart and Prober [17] and Evans [18]. Some extrapolation was done near the regions of separated flow. The extrapolated regions are indicated by dashed lines.

The following figures are available in Appendix A:

<u>Figure</u>	<u>Axes</u>	<u>Variable Parameter(s)</u>
A-1	$f''(0, \beta_0, K) \text{ vs } K$	$-1 < \beta_0 < 5$
A-2	$\Pi'(0, \beta_0, K, .7) \text{ vs } K$	$-1 < \beta_0 < 5$
A-3 thru A-11	$\Lambda^{-1/3} \Pi'(0, \beta_0, K, \Lambda) \text{ vs } K$	$-1 < \beta_0 < 5$ and $.1 < \Lambda < 20$
A-12	$\theta_V \text{ vs } \phi_V$	$-1 < \beta_0 < 5$
A-13	$\theta_V \text{ vs } 1 + R_V$	$-.05 < \beta_0 < 5$
A-14 thru A-18	$\theta \text{ vs } \phi$	$0. < \beta_0 < 5$ and $.1 < \Lambda < \infty$
A-19 thru A-23	$\theta \text{ vs } 1 + R$	$0. < \beta_0 < 5$ and $.1 < \Lambda < \infty$
A-24 thru A-32	$K \text{ vs } 1 + R$	$0. < \beta_0 < 5$ and $.1 < \Lambda < 20$
A-33 thru A-41	$\Pi' \Lambda^{-1/3} \text{ vs } 1 + R$	$-.15 < \beta_0 < 5$ and $.1 < \Lambda < 20$

## 5. APPLICATION OF THE FIRST-ORDER APPROXIMATION

### 5.1 Taylor series solution

Evaluation of the local transfer coefficients requires a local knowledge of  $\beta_0$  and  $K$ . The solution of the first-order differential equation (64) provides the function  $\beta_0$ . Since  $\beta_1$  is given in tabular or graphical form, numerical solution of equation (64) is necessary. However, it is possible to expand  $\beta_0$  in a Taylor series about  $\xi = 0$ .

$$\beta_0(\xi) = \sum_{n=0}^{\infty} b_n \xi^n \quad (87)$$

The  $b_n$  are found by repeated use of L'Hospital's rule to be:

$$b_0 = \beta \Big|_{\xi=0} \quad (88)$$

$$b_1 = \left( \frac{1}{1 + 2\beta_1} \frac{d\beta}{d\xi} \right) \Big|_{\xi=0} \quad (89)$$

$$b_2 = \left( \frac{1}{1 + 4\beta_1} \left( \frac{1}{2} \frac{d^2\beta}{d\xi^2} - 2b_1^2 \frac{\partial\beta_1}{\partial\beta_0} - 2b_1 \frac{dK}{d\xi} \frac{\partial\beta_1}{\partial K} \right) \right) \Big|_{\xi=0} \quad (90)$$

$$b_3 = \left( \frac{1}{1 + 6\beta_1} \left( \frac{1}{6} \frac{d^3\beta}{d\xi^3} - 6b_1b_2 \frac{\partial\beta_1}{\partial\beta_0} - b_1^3 \frac{\partial^2\beta_1}{\partial\beta_0^2} - 4b_2 \frac{dK}{d\xi} \frac{\partial\beta_1}{\partial K} - b_1 \left( \left( \frac{dK}{d\xi} \right)^2 \frac{\partial^2\beta_1}{\partial K^2} + \frac{d^2K}{d\xi^2} \frac{\partial\beta_1}{\partial K} \right) \right) \right) \Big|_{\xi=0} \quad (91)$$

etc.

The functions  $f''(0, \xi, K)$  and  $\Pi'(0, \xi, K, \Lambda)$  can also be expanded in a Taylor series about  $\xi = 0$ .

$$f''(0, \xi, K) = \sum_{n=0}^{\infty} c_n \xi^n \quad (92)$$

$$\Pi'(0, \xi, K, \Lambda) = \sum_{n=0}^{\infty} d_n \xi^n \quad (93)$$

where

$$c_0 = f''(0, \xi, K) \Big|_{\xi=0} \quad (94)$$

$$c_1 = \left( b_1 \frac{\partial f''(0, \xi, K)}{\partial \beta_0} + \frac{dK}{d\xi} \frac{\partial f''(0, \xi, K)}{\partial K} \right) \Big|_{\xi=0} \quad (95)$$

$$c_2 = \frac{1}{2} \left( b_1^2 \frac{\partial^2 f''(0, \xi, K)}{\partial \beta_0^2} + 2b_2 \frac{\partial f''(0, \xi, K)}{\partial \beta_0} + \left( \frac{dK}{d\xi} \right)^2 \frac{\partial^2 f''(0, \xi, K)}{\partial K^2} + \frac{d^2 K}{d\xi^2} \frac{\partial f''(0, \xi, K)}{\partial K} \right) \Big|_{\xi=0} \quad (96)$$

$$d_0 = \Pi'(0, \xi, K, \Lambda) \Big|_{\xi=0} \quad (97)$$

$$d_1 = \left( b_1 \frac{\partial \Pi'(0, \xi, K, \Lambda)}{\partial \beta_0} + \frac{dK}{d\xi} \frac{\partial \Pi'(0, \xi, K, \Lambda)}{\partial K} \right) \Big|_{\xi=0} \quad (98)$$

$$d_2 = \frac{1}{2} \left( b_1^2 \frac{\partial^2 \Pi'(0, \xi, K, \Lambda)}{\partial \beta_0^2} + 2b_2 \frac{\partial \Pi'(0, \xi, K, \Lambda)}{\partial \beta_0} + \left( \frac{dK}{d\xi} \right)^2 \frac{\partial^2 \Pi'(0, \xi, K, \Lambda)}{\partial K^2} + \frac{d^2 K}{d\xi^2} \frac{\partial \Pi'(0, \xi, K, \Lambda)}{\partial K} \right) \Big|_{\xi=0} \quad (99)$$

The coefficients rapidly increase in difficulty of computation, which, coupled with the slow convergence of the series, limits the range of an exact solution of equation (64) to the region near  $\xi = 0$ . The first few terms are valuable in certain approximations as will be shown in Section 4.4. Table 1 contains some of the partial derivatives necessary for the evaluation of the above coefficients. The partials are tabulated for several of the most commonly used  $\beta_0$ ,  $K$  and  $\Lambda$ . The evaluation of the partials was carried out numerically with a differentiated Stirling's interpolation formula.

Table 1. Derivatives for Taylor series coefficients

$\beta_0$	K	$\frac{\partial \beta_1^*}{\partial \beta_0}$	$\frac{\partial^2 \beta_1}{\partial \beta_0^2}$	$\frac{\partial f''^{***}}{\partial \beta_0}$	$\frac{\partial^2 f''}{\partial \beta_0^2}$	$\frac{\partial \Pi'(.7)^{***}}{\partial \beta_0}$	$\frac{\partial^2 \Pi'(.7)}{\partial \beta_0^2}$
0	-.5	-.149	.429	1.00	-1.29	.102	-.332
0	.0	-.272	1.34	1.30	-3.05	.209	-.858
.5	-.5	-.0515	.0850	.652	-.389	.0468	-.0509
.5	.0	-.0636	.125	.705	-.507	.0674	-.0948
.5	.5	-.0671	.148	.725	-.580	.0871	-.152
1.0	-1.	-.0214	.0224	.482	-.160	.0220	-.0134
1.0	-.5	-.0257	.0304	.514	-.197	.0300	-.0221
1.0	.0	-.0287	.0376	.537	-.228	.0389	-.0341
1.0	.5	-.0285	.0392	.541	-.239	.0455	-.0455

$\beta_0$	K	$\frac{\partial \Pi'(.01)}{\partial \beta_0}$	$\frac{\partial^2 \Pi'(.01)}{\partial \beta_0^2}$	$\frac{\partial \Pi'(.05)}{\partial \beta_0}$	$\frac{\partial^2 \Pi'(.05)}{\partial \beta_0^2}$	$\frac{\partial \Pi'(0.1)}{\partial \beta_0}$	$\frac{\partial^2 \Pi'(.1)}{\partial \beta_0^2}$	$\frac{\partial \Pi'(10.)}{\partial \beta_0}$	$\frac{\partial^2 \Pi'(10.)}{\partial \beta_0^2}$
0	0	.00859	-.0395	.0339	-.150	.0580	-.252	.754	-3.04
.5	0	.00237	-.00386	.00981	-.0154	.0172	-.0264	.258	-.332
1.	0	.00125	-.00128	.00530	-.00524	.00945	-.00912	.158	-.122

- \*  $\beta_1$  is  $\beta_1(\beta_0, K)$
- \*\*  $f''$  is  $f''(0, \beta_0, K)$
- \*\*\*  $\Pi'(.7)$  is  $\Pi'(0, \beta_0, K, .7)$

## 5.2 Numerical solution

### Discussion of differential equation (64)

The most accurate values of the local coefficients are obtained by solving equation (64) numerically to obtain the function  $\beta_0(\xi)$ . Since the right side of equation (64) is indeterminate at  $\xi = 0$  difficulties are sometimes incurred in numerical solution. For the case of ideal potential flow around a cylinder with no mass transfer, not even such simple techniques as Euler or Runge-Kutta, give a stable solution. However, the external velocity profile  $U(x)/U_\infty = 1 - x/L$  offers no problem. It is possible to avoid the instability by using the Taylor series expansion of  $\beta_0(\xi)$  for a short distance and then continue on to separation with numerical integration.

Sisson [12] circumvented the instability by calculating  $\beta_0(\xi)$  at station  $n+1$  from

$$\beta_{0_{n+1}} = \beta_{0_n} + \frac{h(\beta_{n+\frac{1}{2}} - \beta_{0_n})}{\frac{1}{2}h + 2\xi_{n+\frac{1}{2}}\beta_{1_{n+\frac{1}{2}}}} \quad (100)$$

where

$$\xi_{n+1} = \xi_n + h \quad (101)$$

and

$$\xi_{n+\frac{1}{2}} = \xi_n + \frac{h}{2} \quad (102)$$

The accuracy of equation (100) has been compared with Runge-Kutta and Euler's integration methods for the case of no mass transfer and  $U(x)/U_\infty = 1 - x/L$ . The results are given in Table 2. The close agreement of the methods indicates that equation (100) provides sufficient accuracy for most engineering applications.

Table 2. Comparison of Sisson's integration formula (100) with Runge-Kutta and Euler's methods

$$U(x)/U_{\infty} = 1 - x/L, K = 0 \text{ and } \Delta\xi = .002$$

$\xi$	$\beta_0$ <u>Sisson's formula (100)</u>	$\beta_0$ <u>Runge-Kutta</u>	$\beta_0$ <u>Euler</u>
.02	-.03247	-.03247	-.03246
.04	-.06623	-.06623	-.06621
.06	-.10109	-.10109	-.10105
.08	-.13663	-.13663	-.13661
.1	-.17197	-.17197	-.17200
.11	-.18890	-.18889	-.18900
.116	-.19828	-.19827	-.19850

Procedure for solving differential equation (64)

An outline of the integration procedure is given below for a general single integration step. For the first step,  $\xi$  and  $\beta_0$  will be determined by the boundary condition (65).

Given: values of  $\xi_n$  and  $\beta_{0_n}$

Determine: values of  $\xi_{n+1}$  and  $\beta_{0_n}$

Step 1

Choose a step length  $h$  and calculate  $\xi_{n+1}$  using equation (101).

Step 2

Use equation (21) to calculate  $\beta_{n+\frac{1}{2}}$ , which is  $\beta$  evaluated at  $(\xi_n + \frac{h}{2})$ .

Step 3

For the first trial, evaluate  $\beta_{1_{n+\frac{1}{2}}}$  at  $\beta_{0_n}$  and  $K_{n+\frac{1}{2}}$ .

For successive trials, evaluate  $\beta_{1_{n+\frac{1}{2}}}$  using  $\beta_{0_{n+\frac{1}{2}}}$  evaluated from

$$\beta_{0_{n+\frac{1}{2}}} = \frac{1}{2}(\beta_{0_{n+1}} + \beta_{0_n}) \quad (103)$$

Step 4

Calculate  $\beta_{0_{n+1}}$  using equation (100).

Step 5

If the change in the calculated values of  $\beta_{0_{n+1}}$  is within the desired error limit, accept the last value of  $\beta_{0_{n+1}}$ . Otherwise, repeat Steps 3 and 4.

The procedure may now be repeated for the next integration step by starting at Step 1.

### Procedure for calculation of local coefficients

The present method may be applied to the practical calculation of momentum, heat and mass transfer coefficients using the stepwise procedure outlined below.

Given:  $U(x)$ ,  $v_w(x)$  and the fluid properties

Determine: the local transfer coefficients

#### Step 1

Determine  $\xi(x)$  using equation (16) and  $U(x)$ . Then find a convenient method of determining  $x(\xi)$ :

- a. analytically
- b. as a successive approximation
- c. plot  $x$  vs  $\xi$

#### Step 2

Determine  $\beta(\xi)$  using equation (21) and  $U(x)$ .

#### Step 3

Determine  $K(\xi)$  using equation (47),  $v_w(x)$  and the fluid properties.

#### Step 4

With  $\beta(\xi)$ ,  $K(\xi)$  solve equation (64) using integration formula (100) to obtain  $\beta_0(\xi)$ .

#### Step 5

Evaluate the dimensionless gradients  $f_0''(0, \beta_0, K)$  and  $\Pi_0'(0, \beta_0, K, A)$  at the calculated values of  $\beta_0(\xi)$  and  $K(\xi)$  using Fig. A-1 through Fig. A-11 or tables of similar solutions [16,17,18,19].

#### Step 6

Calculate the transfer coefficients  $c_f^\bullet$  and  $Nu^\bullet$  using equations (69) and (72).

### 5.3 Average coefficients for heat and mass transfer

The average heat or mass transfer coefficient is given by the integral expression:

$$\overline{Nu}^\bullet = \frac{\int_A Nu^\bullet dA}{\int_A dA} \quad (104)$$

Expressing the differential area in terms of  $x$  and then substitution of equation (72) for  $Nu^\bullet$  yields:

$$\overline{Nu}^\bullet / \sqrt{Re} = \frac{\int_{\left(\frac{x}{L}\right)_1}^{\left(\frac{x}{L}\right)_2} \left(\frac{r}{L}\right)^2 \frac{U(x)}{U_\infty} \frac{\Pi'(0, \xi, K, \Lambda)}{\sqrt{2\xi}} d\left(\frac{x}{L}\right)}{\int_{\left(\frac{x}{L}\right)_1}^{\left(\frac{x}{L}\right)_2} \frac{r}{L} d\left(\frac{x}{L}\right)} \quad (105)$$

or introducing the transformed variable  $\xi$ :

$$\overline{Nu}^\bullet / \sqrt{Re} = \frac{\int_{\xi_1}^{\xi_2} \frac{\Pi'(0, \xi, K, \Lambda)}{\sqrt{2\xi}} d\xi}{\int_{\left(\frac{x}{L}\right)_1}^{\left(\frac{x}{L}\right)_2} \frac{r}{L} d\left(\frac{x}{L}\right)} \quad (106)$$

Since the step-by-step procedure of Section 5.2 leads to a tabular function for  $\Pi'(0, \xi, K, \Lambda)$  equation (106) must be evaluated numerically.

Replacing  $\Pi'(0, \xi, K, \Lambda)$  in equation (106) with its Taylor series, equation (93), and then integrating in the numerator gives:

$$\overline{\text{Nu}}^\bullet / \sqrt{\text{Re}} = \frac{\sqrt{2}}{\int_{\left(\frac{x}{L}\right)_1}^{\left(\frac{x}{L}\right)_2} \frac{r}{L} d\left(\frac{x}{L}\right)} (d_0 (\xi_2^{1/2} - \xi_1^{1/2}) + \frac{d_1}{3} (\xi_2^{3/2} - \xi_1^{3/2}) + \frac{d_2}{5} (\xi_2^{5/2} - \xi_1^{5/2}) + \dots) \quad (107)$$

Truncation of this series after one term produces good results. This approximation is compared with numerical integration of the local coefficients in the examples in Sections 8.2, 8.3 and 8.4. The two methods usually differ from three to ten per cent. It is also shown how the addition of two more terms reduces the difference to around one per cent.

A modification of the Taylor series approach to average calculations is the truncation of equation (93) after the third term and the addition of an empirically fitted term.

$$\Pi'(0, \xi, K, \Lambda) = d_0 + d_1 \xi + d_2 \xi^2 + d_s \xi^s \quad (108)$$

The coefficient  $d_s$  and exponent  $s$  may be found by forcing equation (108) to meet two specifications:

- 1) Equation (108) must give the correct  $\Pi'(0, \xi, K, \Lambda)$  for  $\xi = \xi_{\text{sep}}$ .
- 2) When inserted into equation (106), equation (108) must yield the correct  $\overline{\text{Nu}}^\bullet / \sqrt{\text{Re}}$  for the range  $\xi = 0$  to  $\xi = \xi_{\text{sep}}$ .

Values of the exponent  $s$  for various cases are listed in Table 3. It is suggested that  $s$  be taken as equal to ten if no better information is available. Then, knowing  $\Pi'(0, \xi, K, \Lambda)_{\text{sep}}$ ,  $d_s$  can be computed from:

$$d_s = (\Pi'(0, \xi, K, \Lambda)_{\text{sep}} - d_0 - d_1 \xi_{\text{sep}} - d_2 \xi_{\text{sep}}^2) / \xi_{\text{sep}}^{10} \quad (109)$$

Combining equations (106) and (108) and integrating gives:

$$\overline{Nu} \bullet / \sqrt{Re} = \frac{\sqrt{2}}{\int_{\left(\frac{x}{L}\right)_1}^{\left(\frac{x}{L}\right)_2} \frac{r}{L} d\left(\frac{x}{L}\right)} \left( d_0 (\xi_2^{1/2} - \xi_1^{1/2}) + \frac{d_1}{3} (\xi_2^{3/2} + \xi_1^{3/2}) \right. \\ \left. + \frac{d_2}{5} (\xi_2^{5/2} - \xi_1^{5/2}) + \frac{d_s}{21} (\xi_2^{21/2} - \xi_1^{21/2}) \right) \quad (110)$$

Equation (110) has been used in the examples in Sections 8.2, 8.3 and 8.4. In all three cases the error was reduced to less than one per cent. However, when  $\frac{d\Pi'(0, \xi, K, \Lambda)}{d\xi}$  is positive at  $\xi = 0$ , the addition of the empirical term does not greatly improve the results. It can even give slightly worse results as is shown in the example of Section 8.4.

#### 5.4 Calculations involving R, $\phi$ and $\theta$

In many engineering problems R(x) is known instead of  $v_w(x)$ . The following step-by-step procedure may be used.

Given: U(x), R(x) and the fluid properties

Determine: the local transfer coefficients

##### Step 1

Determine  $\xi(x)$  using equation (16) and U(x). Then find a convenient method of determining  $x(\xi)$ :

- a. analytically
- b. as a successive approximation
- c. a plot of  $x$  vs  $\xi$

##### Step 2

Determine  $\beta(\xi)$  using equation (21) and U(x).

##### Step 3

Determine R( $\xi$ ) from R(x) and the relationship between x and  $\xi$  found in Step 1.

Table 3. Exponent  $s$  for various cases

$U(x)/U_\infty$	$r(x)/L$	$K$	$\Lambda$	$s$
$1 - x/L$	1	0	.01	10.1
$1 - x/L$	1	0	.05	8.7
$1 - x/L$	1	0	.1	8.3
$1 - x/L$	1	0	.7	7.2
$1 - x/L$	1	0	10.	7.8
$2 \sin x/R$	1	0	.01	11.5
$2 \sin x/R$	1	0	.05	10.7
$2 \sin x/R$	1	0	.1	9.6
$2 \sin x/R$	1	0	.7	8.8
$2 \sin x/R$	1	0	10.	8.9
$2 \sin x/R$	1	.5	.7	8.4
$2 \sin x/R$	1	-.5	.7	9.2
$1.5 \sin x/R$	$\sin x/R$	0	.01	27.4
$1.5 \sin x/R$	$\sin x/R$	0	.7	11.1
$1.5 \sin x/R$	$\sin x/R$	0	10.	10.8
$1.5 \sin x/R$	$\sin x/R$	.5	.7	8.8
$1.5 \sin x/R$	$\sin x/R$	-.5	.7	13.3

#### Step 4

Choose a step length  $h$  and calculate  $\xi_{n+1}$  and  $\xi_{n+\frac{1}{2}}$  from equations (101) and (102) respectively.

#### Step 5

Use equation (21) to calculate  $\beta_{n+\frac{1}{2}}$ , which is  $\beta$  evaluated at  $\xi_{n+\frac{1}{2}}$ .

#### Step 6

Estimate  $K_{n+\frac{1}{2}}$  by extrapolating from the values of  $K$  at the previous stations. Newton's formula for 3-point extrapolation would be:

$$K_{n+\frac{1}{2}} = 3K_n - 3K_{n-\frac{1}{2}} + K_{n-1} \quad (111)$$

In order to begin at station  $n=0$  use:

$$K_{\frac{1}{2}} = K_0 + \frac{h}{2} \left. \frac{dK}{d\xi} \right|_{\xi=0} \quad (112)$$

#### Step 7

For the first trial, evaluate  $\beta_{1n+\frac{1}{2}}$  at  $\beta_{0n}$  and  $K_{n+\frac{1}{2}}$ . For successive trials, evaluate  $\beta_{1n+\frac{1}{2}}$  by using  $\beta_{0n+\frac{1}{2}}$  calculated from equation (103).

#### Step 8

Calculate  $\beta_{0n+1}$  from equation (100).

#### Step 9

If the change in the calculated values of  $\beta_{0n+1}$  is within the desired error limit, accept the last value of  $\beta_{0n+1}$ . Otherwise, repeat Steps 7 and 8.

#### Step 10

Locate  $K_{n+1}$  in Fig. A-24 through A-32 using  $R(\xi_{n+1})$  and  $\beta_{0n+1}$ .

### Step 11

Interpolate  $K_{n+\frac{1}{2}}$  from:

$$K_{n+\frac{1}{2}} = \frac{3}{8} K_{n+1} + \frac{3}{4} K_n - \frac{1}{8} K_{n-1} \quad (113)$$

If this  $K_{n+\frac{1}{2}}$  agrees with the  $K_{n+\frac{1}{2}}$  used in Step 7 within the prescribed error limits, accept the last  $\beta_{0n+1}$  calculated in Step 8. Otherwise, use the  $K_{n+\frac{1}{2}}$  calculated from equation (113) and repeat Steps 7-11 until agreement for  $K_{n+\frac{1}{2}}$  is attained.

### Step 12

Evaluate the dimensionless gradients  $f_0'(0, \beta_0, K)$  and  $\Pi'(0, \beta_0, K, \Lambda)$  at the calculated values of  $\beta_{0n+1}$  and  $K_{n+1}$  using Fig. A-1 through A-11 or tables of similar solutions [16,17,18,19].

### Step 13

Calculate the local transfer coefficients  $c_f^\bullet$  and  $Nu^\bullet$  using equations (69) and (72).

Repeat Steps 4-13 until separation occurs or the region of interest has been covered.

## 6. MOMENTUM TRANSFER RESULTS OF THE FIRST-ORDER APPROXIMATION AND COMPARISONS WITH OTHER WORK

### 6.1 The circular cylinder with $U(x)/U_\infty = 2 \sin x$ and $K = 0$

Fig. 2 shows the results of momentum transfer calculations made using the first order approximation with the external velocity distribution of  $U(x)/U_\infty = 2 \sin x$ , which is predicted by potential flow theory for a circular cylinder in crossflow. Also shown are the results of calculations made by the approximate methods of Merk [11], Eckert [10], Spalding [7] and Pohlhausen [15], by the series methods of Blasius [15] and Gortler [3] and by the continuation method of Gortler and Witting [22]. The separation points predicted by these different calculations are presented in Table 4.

The present method shows good agreement with the Blasius series and the calculations of Gortler and Witting. The

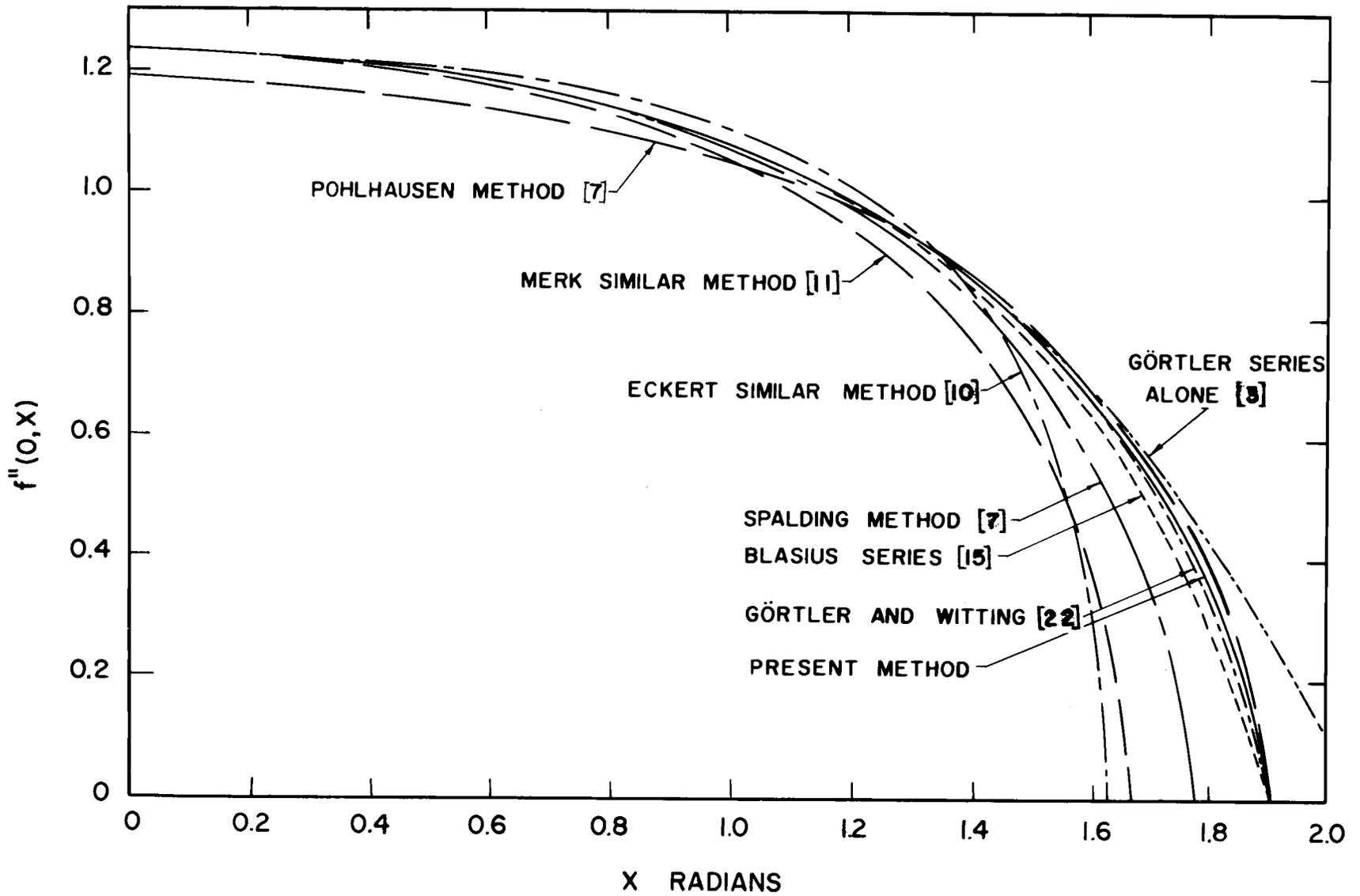


FIGURE 2. COMPARISON OF SEVERAL METHODS FOR MOMENTUM TRANSFER WITH  $U(x)/U_{\infty} = 2 \sin x$ .

Table 4. Separation points with  $U(x)/U_\infty = 2 \sin x$   
and  $K = 0$

Method	Separation Point
Eckert [10]	1.626
Merk [11]	1.661
Spalding [7]	1.768
Blasius [15]	1.898
Gortler and Witting [22]	1.90
Present method	1.902
Pohlhausen [15]	1.912
Gortler series, 6 terms [3]	2.052

Merk and Eckert similar methods are reasonably close out to  $x = 1.4$ , where they fall off rapidly. Spalding's method is in reasonable agreement out to  $x = 1.6$ , where it also falls rapidly. Pohlhausen's method is low in the region near the stagnation point and somewhat high in the region near separation, although it predicts separation closely. With six terms, Gortler's series does not converge adequately for  $x > 1.6$ . Thus, for this case, the first-order approximation appears to give substantially better results for momentum transfer calculations than do other approximate methods.

## 6.2 The velocity distribution $U(x)/U_\infty = 1 - x^n$

The results of momentum transfer calculations by the present method for the external velocity distribution  $U(x)/U_\infty = 1 - x^n$  are shown in Fig. 3. Also shown are the formally exact calculations of Clutter and Smith [6] and Gortler and Witting [22]. Calculations made using the Merk similar method are also shown, for they represent the zero-order approximation and thus are of special interest. Table 5 compares the predicted separation point with that predicted by other investigators. See the example in Section 8.2 for sample calculations.

Calculations made by the present method and by Gortler and Witting [24] for  $U(x)/U_\infty = 1 - x^n$ , with  $n = 2, 3, 4$ , and 5, are shown in Fig. 4. Table 6 gives the separation points predicted by several methods. Except for Spalding's method, the Merk similar method and the present method, all are considered formally exact.

For  $n = 1, 2$ , and 3, there is reasonably good agreement between the present method and the formally exact methods, although it appears that the present method is slightly high in the region just before separation. For  $n = 4$  and 5, the present method is significantly higher than the calculations of Gortler and Witting. However, the latter calculations are less reliable for these values of  $n$  because only two terms could be used in Gortler's series although Witting's numerical continuation was started relatively early [23].

## 6.3 The velocity distribution $U(x)/U_\infty = (1 + x)^{-n}$

Momentum transfer calculations for the external velocity distribution  $U(x)/U_\infty = (1 + x)^{-n}$  have been carried out using the present method. Fig. 5 shows the results for

Table 5. Separation points with  $U(x)/U_\infty = 1 - x$

<u>Investigator</u>	<u>Separation Point</u>
Merk [11]	0.0867
Spalding [7]	0.1073
Leigh [23]	0.1198
Clutter and Smith [6]	0.1200
Present method	0.1241
Gortler and Witting [24]	0.125

Table 6. Separation points with  $U(x)/U_\infty = 1 - x^n$

Exponent n	Method	Separation Point
2	Tani [25]	0.271
	Present method	0.288
	Gortler and Witting [24]	0.290
3	Present method	0.408
	Gortler and Witting [24]	0.409
4	Tani [25]	0.462
	Gortler and Witting [24]	0.485
	Present method	0.495
5	Gortler and Witting [24]	0.552
	Present method	0.560

Table 7. Separation points with  $U(x)/U_\infty = (1 + x)^{-n}$

Exponent n	Method	Separation Point
1	Merk [11]	0.1045
	Clutter and Smith [6]	0.1450
	Present method	0.1534
	Gortler and Witting [22]	0.161
1.5	Present method	0.0984
	Gortler and Witting [22]	0.101
2	Present method	0.0725
	Gortler and Witting [22]	0.075
2.5	Present method	0.0573
	Gortler and Witting [22]	0.058
3	Present method	0.0474
	Gortler and Witting [22]	0.048
3.5	Present method	0.0404
	Gortler and Witting [22]	0.041
4	Present method	0.0353
	Gortler and Witting [22]	0.036

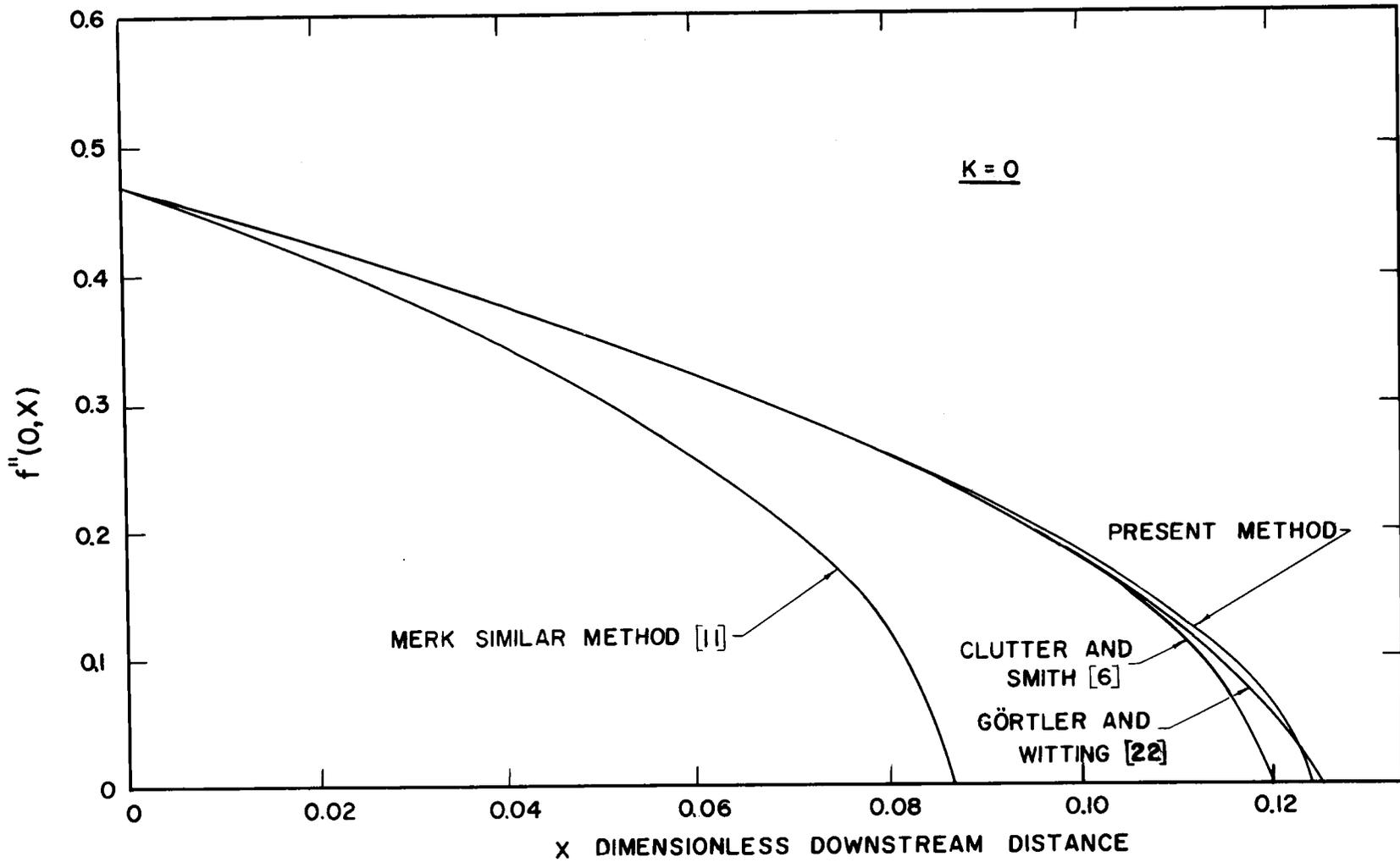


FIGURE 3. COMPARISON OF SEVERAL METHODS FOR MOMENTUM TRANSFER WITH  $U(x)/U_\infty = 1 - x$ .

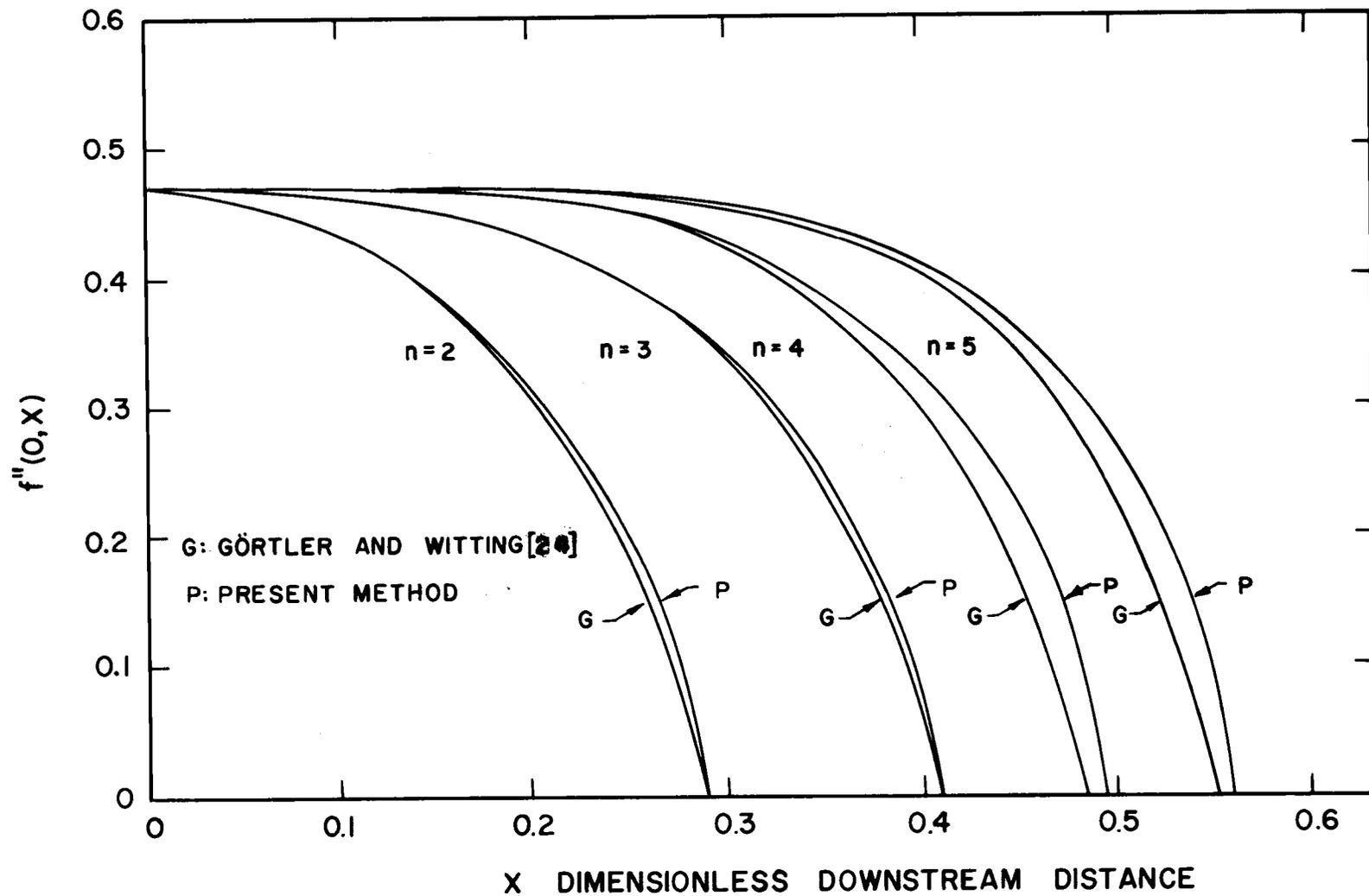


FIGURE 4. COMPARISON OF THE PRESENT METHOD WITH THE CALCULATIONS OF GORTLER AND WITTING WITH  $U(x)/U_\infty = 1 - x^n$ .

$n = 1$ . Also shown are the calculations made by Clutter and Smith [6], by Gortler and Witting [22] and by the Merk similar method [11]. Predicted separation points for  $n = 1, 1.5, 2, 2.5, 3, 3.5$  and 4 are given in Table 7.

For  $n = 1$ , the first-order approximation compares quite well with the calculations of Clutter and Smith, and Gortler and Witting. In fact, the results at every point lie on or between these formally exact calculations. The relatively sharp curvature in the calculations of Clutter and Smith at  $x = 0.13$ , however, indicates that they may have experienced difficulties with their numerical methods and that their results past this point may not be reliable. For the higher values of  $n$ , the present method agrees quite well with the calculations of Gortler and Witting in prediction of the separation points.

#### 6.4 Axisymmetrical flow on a sphere with $U(x)/U_\infty = 1.5 \sin x$

An example of axisymmetrical flow, the sphere with  $U(x)/U_\infty = 1.5 \sin x$ , has also been treated using the present method. The results are shown in Fig. 6. Also shown are the calculations by Clutter and Smith [6] and by the Merk similar method [11]. Table 8 gives the separation points predicted by four different methods.

In the region just before separation,  $1.6 < x < 1.9$ , the present method gives results significantly higher than the formally exact calculations of Clutter and Smith. However, the Blasius-type series employed by Schlichting predicts separation after the first-order approximation does. Since the Blasius series gives good results on the cylinder, the calculations of Clutter and Smith may not be reliable in this region. It may be noted that in this mostly accelerated and, for  $0 < x < 1.2$ , reasonably similar flow the Merk similar method gives reasonably good results.

#### 6.5 A cylinder with injection at the surface

The present method was used to calculate momentum transfer on a cylinder with several different rates of injection at the surface. These results are compared in Fig. 7 with calculations made using the series method of Sparrow, Torrance and Hung [4]. The velocity distribution used was a seventh-degree polynomial correlation of Elzy's [10] data and is given by  $U(x)/U_\infty = 1.79(x/R) - 0.36276(x/R)^3 + 0.02323(x/R)^5 - 0.010153(x/R)^7$ .

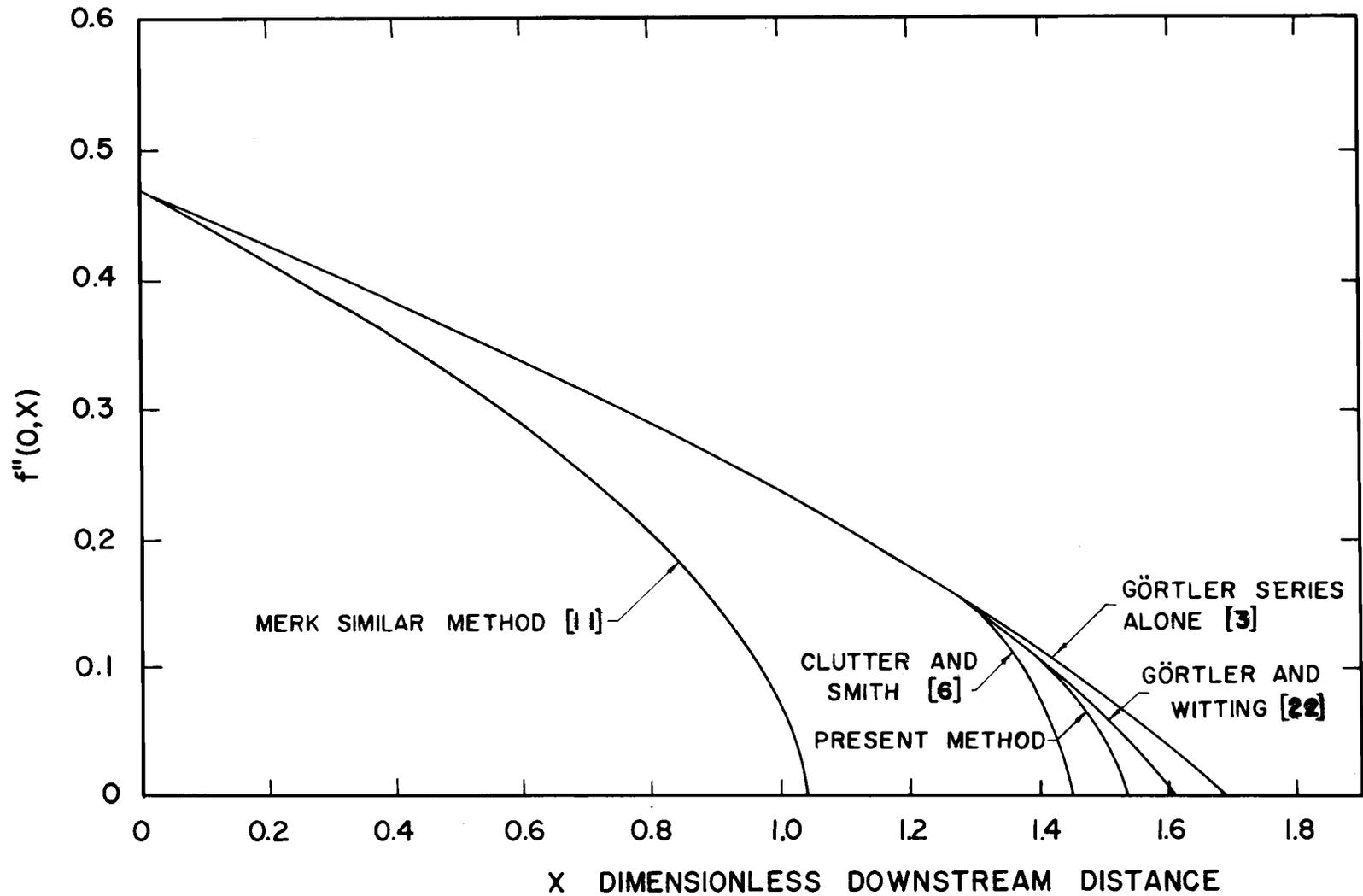


FIGURE 5. COMPARISON OF SEVERAL METHODS FOR  
MOMENTUM TRANSFER WITH  $U(x)/U_{\infty} = (1 + x)^{-1}$ .

Table 8. Separation points on a sphere

Method	Separation Point
Merk [11]	1.693
Clutter and Smith [6]	1.846
Present method	1.894
Blasius-type series [15]	1.913

Table 9. Separation points on a cylinder with injection at the surface

$f_{wd}$	Present Method	Sparrow, Torrance and Hung [4]
0.137	1.482	1.479
0.697	1.434	1.428
1.771	1.350	1.349

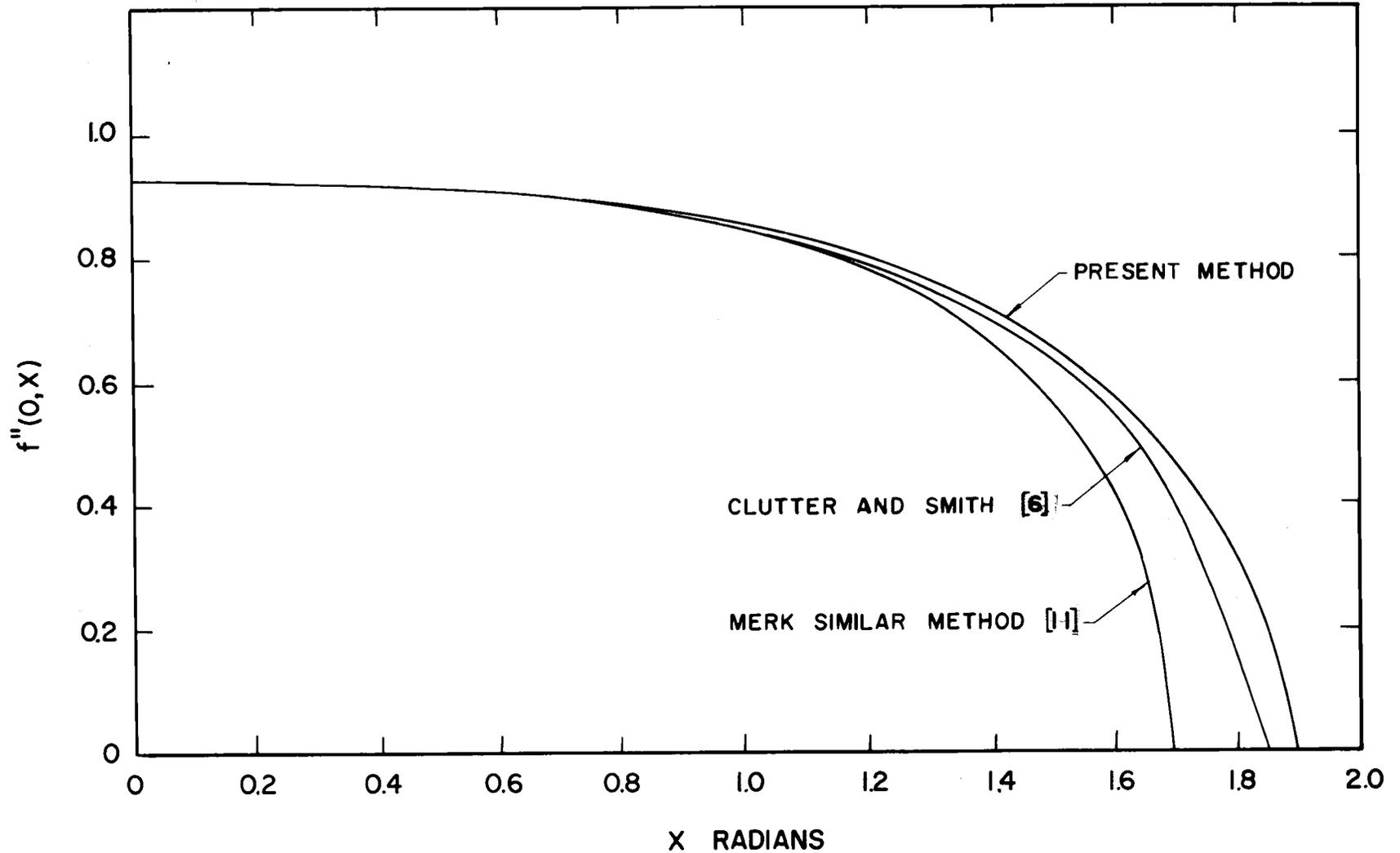


FIGURE 6. COMPARISON OF SEVERAL METHODS FOR MOMENTUM TRANSFER  
WITH AXISYMMETRIC FLOW AND  $U(x)/U_{\infty} = 1.5 \sin x$ .

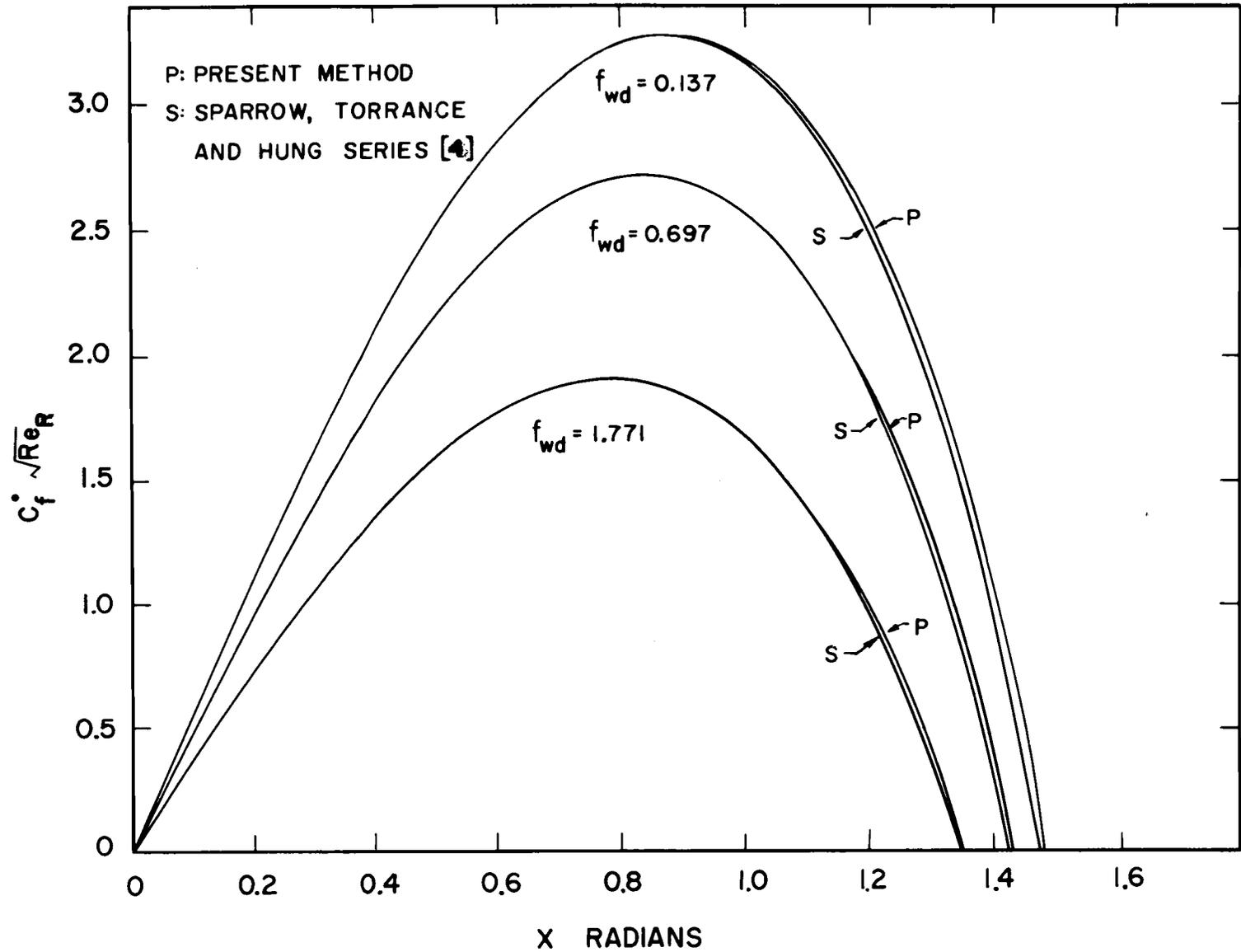


FIGURE 7. COMPARISON OF THE PRESENT METHOD WITH SPARROW, TORRANCE AND HUNG SERIES FOR MOMENTUM TRANSFER ON A CIRCULAR CYLINDER WITH INJECTION AT SURFACE FOR  $U(x)/U_\infty = 1.79x - .36276x^3 + .02323x^5 - .010153x^7$ .

The parameter  $f_{wd}$  is a measure of the normal velocity at the surface and is defined by:

$$f_{wd} = \frac{\rho_w v_w}{\rho_\infty U_\infty} \sqrt{Re_d} \quad (114)$$

In this case  $f_{wd}$  is positive, and consequently the momentum transfer coefficients are reduced.

The separation points predicted by these methods at various rates of injection are given in Table 9. The results obtained with the present method compare quite well with the formally exact calculations made using the series of Sparrow, Torrance and Hung.

## 7. HEAT AND MASS TRANSFER RESULTS OF THE FIRST-ORDER APPROXIMATION AND COMPARISONS WITH OTHER WORK

### 7.1 Introduction

Few accurate analytical solutions are available for heat and mass transfer in non-similar constant-property incompressible laminar boundary layer flow, so the calculations of heat and mass transfer by the present method are also compared to experimental data.

### 7.2 Analytical solutions for the circular cylinder

Fig. 8 shows the comparison between the first-order approximation and Newman's Blasius-series type calculations [26] which are formally exact. The external velocity profile was  $U(x)/U_\infty = 2 \sin x$ ,  $K = 0$  and  $\Lambda = .7$ . It is seen that both methods agree very well. It is instructive to also consider the Schmidt-Wenner [27] velocity profile,  $U(x)/U_\infty = 1.816(x/R) - 0.4094(x/R)^3 - 0.005247(x/R)^5$ , so that the effect of the form of  $U(x)$  can be demonstrated. Fig. 9 shows the results. Blasius series methods require coefficients in the polynomial expansion for  $U(x)$  so that a fifth order polynomial does not allow Newman's [26] method to include higher order terms. As a result Newman's results are not accurate in the range 80-90 degrees, while the present method performs quite well.

The external velocity profile  $U(x)/U_\infty = 2 \sin x$  was used for the calculation of the heat and mass transfer coefficients for the following cases:

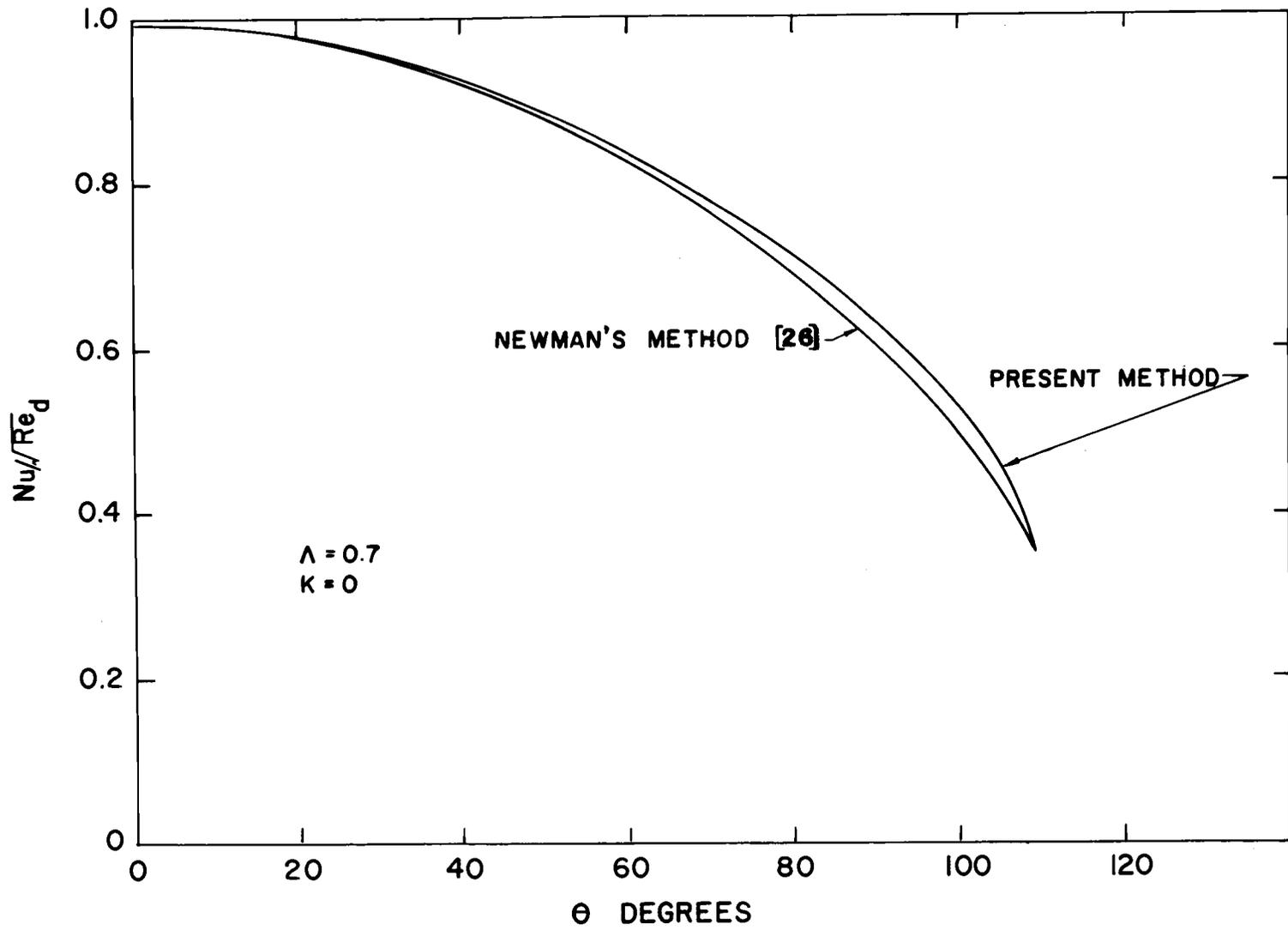


FIGURE 8. COMPARISON OF THE PRESENT METHOD WITH NEWMAN'S SERIES FOR HEAT AND MASS TRANSFER ON A CIRCULAR CYLINDER WITH  $U(x)/U_\infty = 2 \sin x$ .

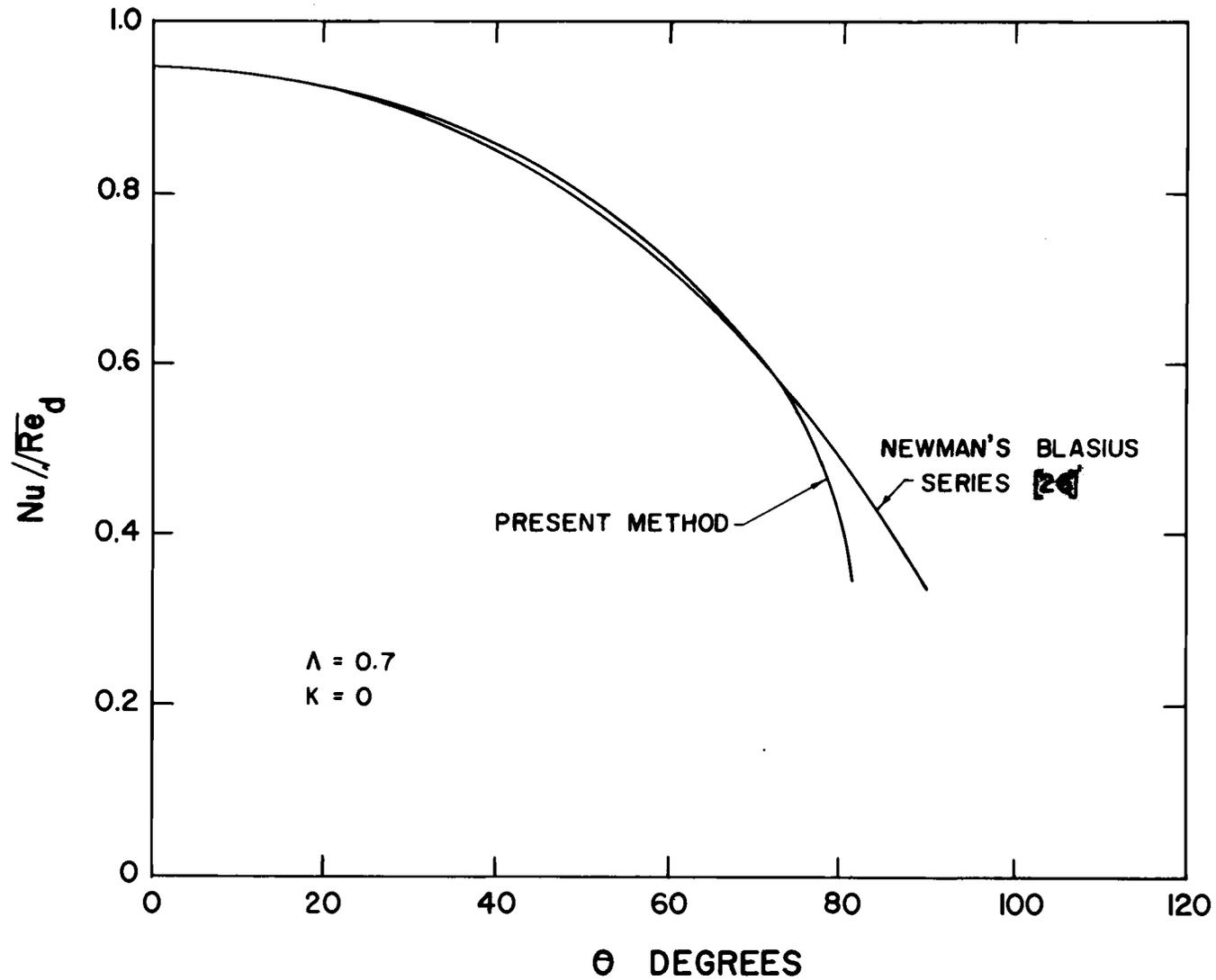


FIGURE 9. COMPARISON OF THE PRESENT METHOD WITH NEWMAN'S SERIES FOR HEAT AND MASS TRANSFER ON A CIRCULAR CYLINDER WITH  $U(x)/U_\infty = 1.816x - .4094x^3 - .005247x^5$ .

- 1)  $K = 0, \Lambda = .01$
- 2)  $K = -.5, \Lambda = .7$
- 3)  $K = 0, \Lambda = .7$
- 4)  $K = .5, \Lambda = .7$
- 5)  $K = 0, \Lambda = 10$

The results are given in Fig. 10.

### 7.3 Experimental heat and mass transfer data for a circular cylinder

Heat and mass transfer data for a circular cylinder in crossflow taken from five investigations [27, 28, 29, 30, 31] is compared in Fig. 11 with calculations made using the present method. The velocity distribution of Schmidt and Wenner [27], correlated by a fifth degree polynomial,  $U(x)/U_\infty = 1.8155(x/R) - 0.4094(x/R)^3 - 0.005247(x/R)^5$ , was used in the calculations. The parameter  $Nu/\Lambda^{0.4}Re^{0.5}$  is recommended by Sogin and Subramanian [29] for the correlation of heat and mass transfer data taken at different values of  $\Lambda$ .

The present method shows reasonably good agreement with the data up to the point of separation, which was calculated to be  $81.0^\circ$  and which appears to vary in the data from about  $77^\circ$  to  $90^\circ$ .

The mass transfer data of Schnautz [28], taken on a cylinder, is shown in Fig. 12. Shown for comparison are calculations made using the Merk [11] and the Eckert [10] similar methods, and the present method, all with  $\Lambda = 0.7$ . Here the Chilton-Colburn analogy [13] has been used to convert Schnautz's data to  $\Lambda = 0.7$ . The velocity distribution of Schmidt and Wenner as given above was used in the calculations. An additional calculation was made with the present method using a fifth degree polynomial velocity distribution,  $U(x)/U_\infty = 1.737(x/R) - 0.2935(x/R)^3 - 0.0593(x/R)^5$ , measured by Elzy [10] in the same wind tunnel used by Schnautz.

Both the Merk and the Eckert similar methods agree well with the data until shortly before they predict separation, at  $x = 1.231$  and  $1.253$  respectively. However, separation does not occur until about  $x = 1.45$ . The present method agrees well with the data. The velocity distribution of Elzy predicts separation slightly earlier, at  $x = 1.355$ , than does the velocity distribution of Schmidt and Wenner, which indicated separation at  $x = 1.416$ . Since the velocity distributions were actually quite similar, it appears that good correlation of velocity data may be quite important.

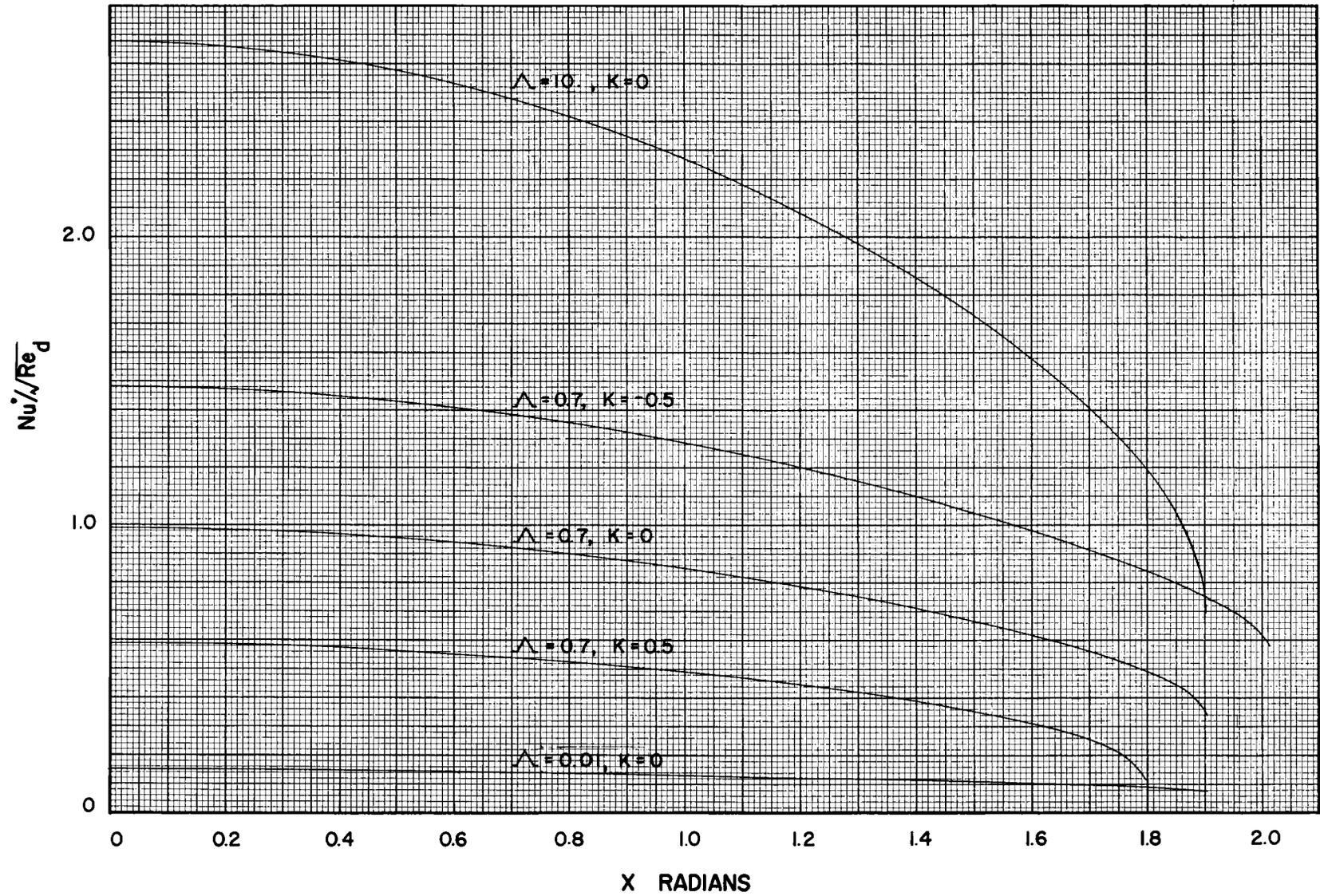


FIGURE 10. HEAT AND MASS TRANSFER ON A CIRCULAR CYLINDER FOR  $U(x)/U_\infty = 2 \sin x$ .

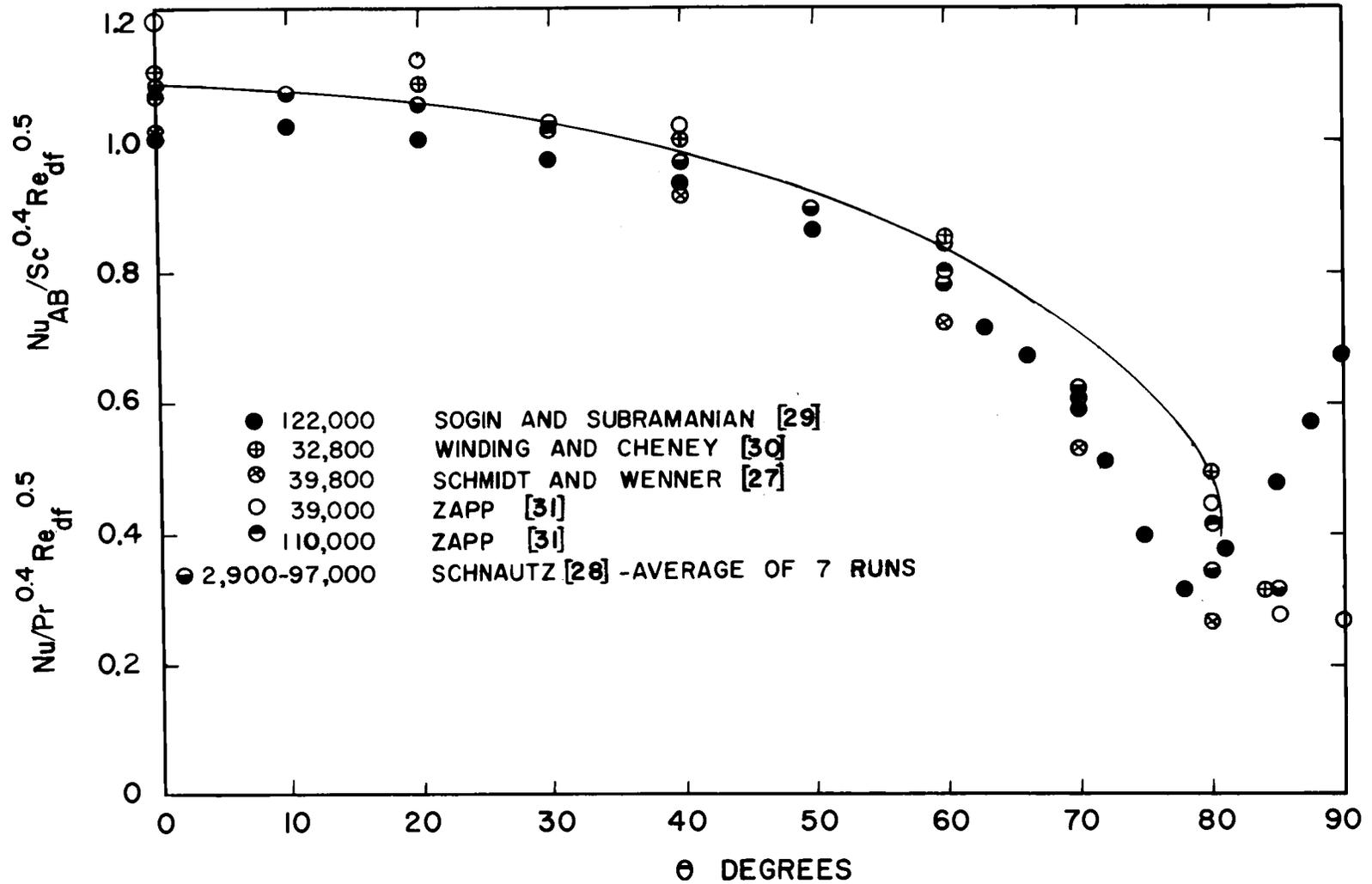


FIGURE 11. COMPARISON OF THE PRESENT METHOD FOR  
 $U(x)/U_\infty = 1.816x - .4094x^3 - .005247x^5$  WITH  
 HEAT AND MASS TRANSFER DATA.

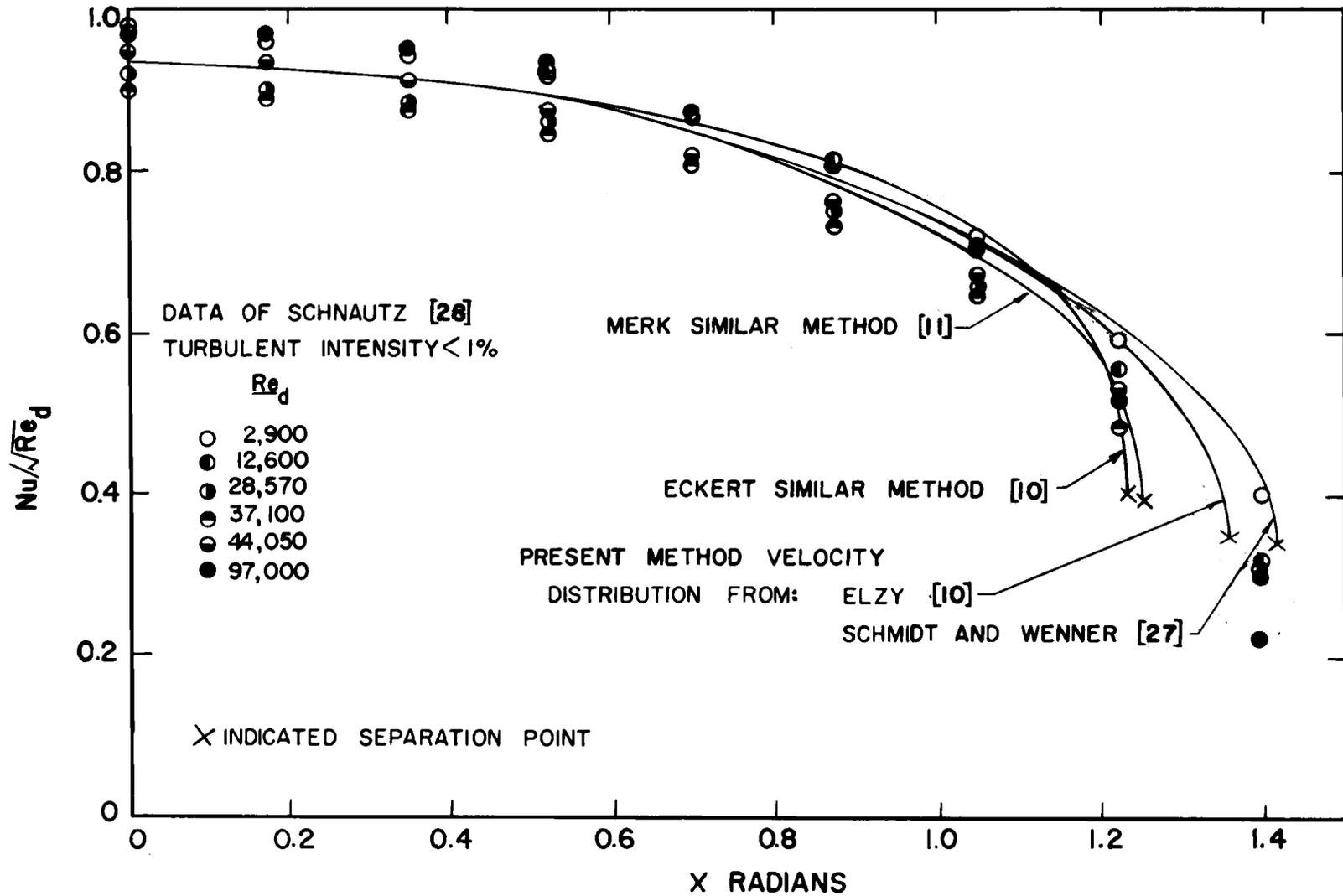


FIGURE 12. COMPARISON OF PRESENT METHOD, AND MERK AND ECKERT SIMILAR METHODS WITH SCHNAUTZ'S MASS TRANSFER DATA.

#### 7.4 Experimental transpiration cooling data on a circular cylinder

The series method of Sparrow, Terrance and Hung [4] and the present method are compared in Fig. 13 with the heat transfer data of Elzy [10] taken on a cylinder with transpiration cooling. The injection parameter,  $f_{wd}$ , is defined by equation (114). Two velocity distributions were used. The velocity profile  $U(x)/U_\infty = 1.665(x/R) + 0.0393(x/R) - 0.1977(x/R)$  was used in the region near the stagnation point. The seventh degree polynomial correlation of Elzy's [10] data, discussed earlier, was used over the rest of the cylinder. See the example in Section 8.3 for sample calculations of a case with a different velocity profile and suction.

#### 7.5 Results for decelerating flow

The external velocity profile  $U(x)/U_\infty = 1 - x$  has been used in the following cases:

- 1)  $K = 0, \Lambda = .01$
- 2)  $K = 0, \Lambda = .7$
- 3)  $K = 0, \Lambda = 10$

The heat and mass transfer results are plotted in Fig. 14. See the example in Section 8.2 for sample calculations of case 2.

#### 7.6 Results for a sphere

The external velocity profile  $U(x)/U_\infty = 1.5 \sin x$  has been used in the following cases:

- 1)  $K = 0, \Lambda = .01$
- 2)  $K = -.5, \Lambda = .7$
- 3)  $K = 0, \Lambda = .7$
- 4)  $K = .5, \Lambda = .7$
- 5)  $K = 0, \Lambda = 10$

The heat and mass transfer results are plotted in Fig. 15. See the example in Section 8.4 for sample calculations of case 4.

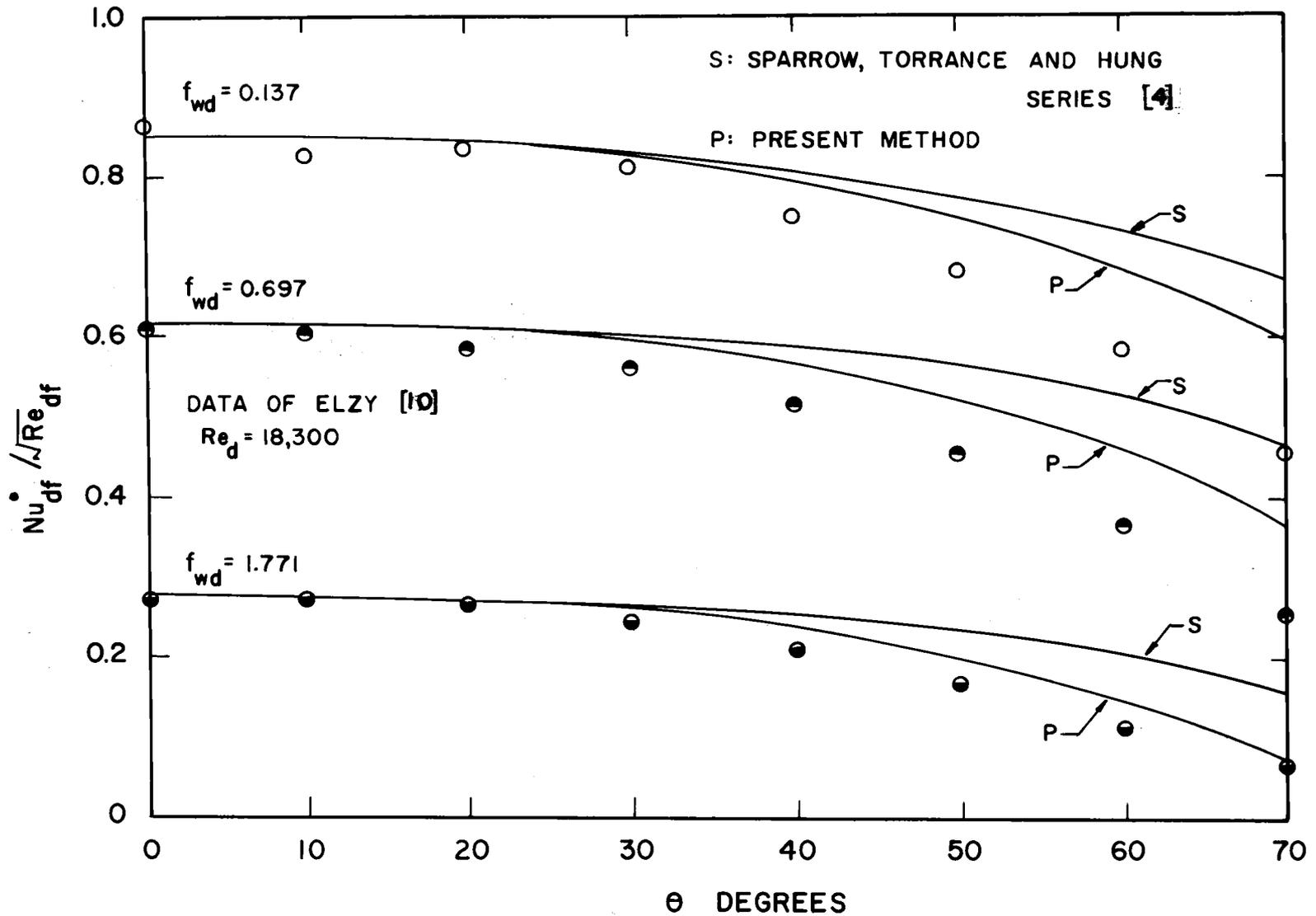


FIGURE 13. COMPARISON OF THE PRESENT METHOD AND SPARROW, TORRANCE AND HUNG SERIES WITH ELZY'S TRANSPIRATION COOLING DATA FOR INJECTION AT THE SURFACE OF A CIRCULAR CYLINDER.

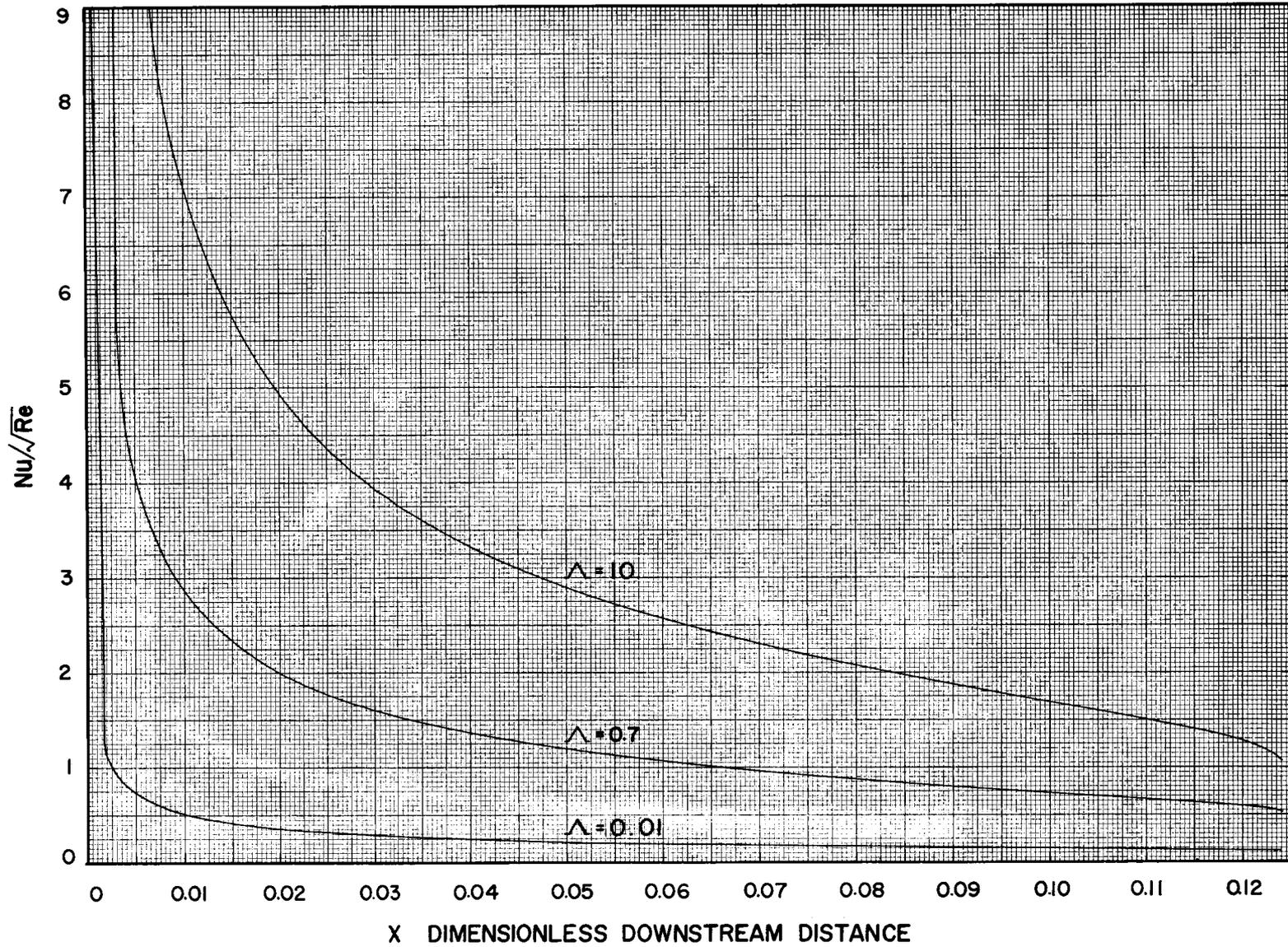


FIGURE 14. HEAT AND MASS TRANSFER IN DECELERATING FLOW WITH  $U(x)/U_\infty = 1 - x$ .

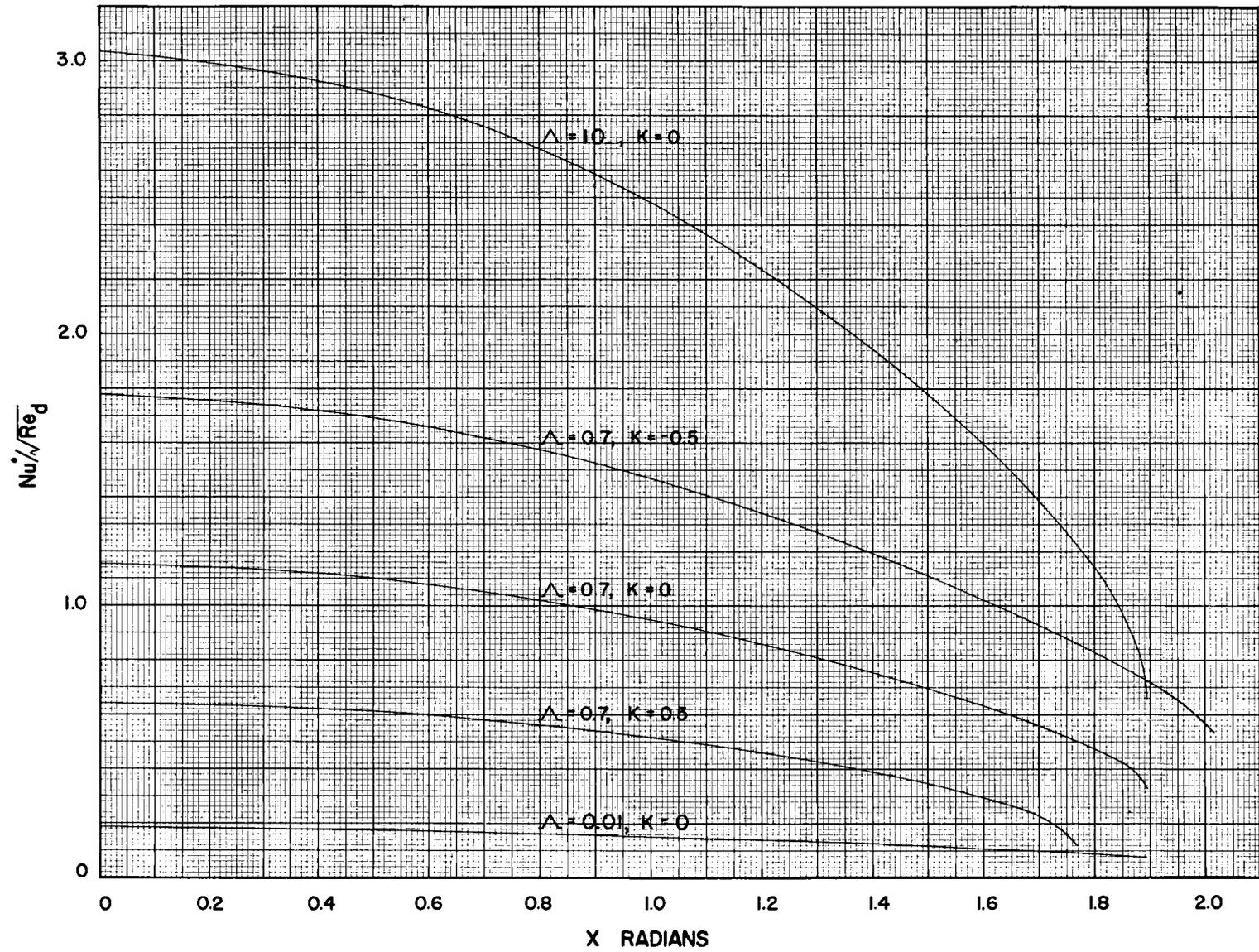


FIGURE 15. HEAT AND MASS TRANSFER ON A SPHERE WITH  $U(x)/U_\infty = 1.5 \sin x$ .

## 8. EXAMPLES

### 8.1 Example 1. Similar solution of flat plate with injection

Air is flowing over a flat plate through which the injection rate is proportional to  $(\frac{x}{L})^{-\frac{1}{2}}$ . Determine the local Nusselt number and local friction factor as a function of the blowing parameter,

$$\frac{v_w}{U_\infty} (2\text{Re}_x)^{\frac{1}{2}},$$

for  $\Lambda = .7$ .

#### Solution

For a flat  $\beta_0 = 0$ ,  $\frac{r}{L} = 1$ , and  $U(x)/U_\infty = 1$ . As  $m = 0$ , equation (61) requires that  $v_w(x) = C_2 x^{-\frac{1}{2}}$  for a similar solution to exist. Since it is given that the injection rate is proportional to  $(\frac{x}{L})^{-\frac{1}{2}}$  or expressing this in terms of  $v_w(x)$ ,

$$v_w(x)/U_\infty = C_2' \left(\frac{x}{L}\right)^{-\frac{1}{2}},$$

the problem has a similar solution. The other expressions needed are:

$$\xi = \int_0^{\frac{x}{L}} \frac{U(x)}{U_\infty} \left(\frac{r}{L}\right)^2 d\left(\frac{x}{L}\right) = \int_0^{\frac{x}{L}} d\left(\frac{x}{L}\right) = \frac{x}{L}$$

$$K = \frac{v_w(x)}{U_\infty} \frac{L}{r} \frac{(2\xi\text{Re})^{\frac{1}{2}}}{U(x)/U_\infty} = \frac{v_w(x)}{U_\infty} \sqrt{2\text{Re}_x}$$

$$c_{f_f}^\bullet = 2 \frac{r}{L} \left(\frac{U(x)}{U_\infty}\right)^2 \frac{f''(0, \xi, K)}{(2\xi\text{Re})^{\frac{1}{2}}} = \left(\frac{2}{\text{Re}_x}\right)^{\frac{1}{2}} f''(0, \xi, K)$$

or

$$c_{f_f}^\bullet \sqrt{\text{Re}_x} = \sqrt{2} f''(0, \xi, K)$$

$$\text{Nu}_x^\bullet = \frac{x}{L} \text{Nu}^\bullet = \frac{r}{L} \frac{x}{L} \frac{U(x)}{U_\infty} \left(\frac{\text{Re}_x}{2\xi}\right)^{\frac{1}{2}} \Pi'(0, \xi, K, \Lambda) = \left(\frac{\text{Re}_x}{2}\right)^{\frac{1}{2}} \Pi'(0, \xi, K, \Lambda)$$

or

$$\text{Nu}_x^\bullet / \sqrt{\text{Re}_x} = \frac{\Pi'(0, \xi, K, \Lambda)}{\sqrt{2}}$$

For the desired  $K$  look up  $f''(0, \xi, K)$  and  $\Pi'(0, \xi, K, \Lambda)$  in Fig. A-1 and Fig. A-2 respectively. The local Nusselt number and local friction factor can now be calculated. The results are provided in Table 10.

## 8.2 Example 2. Decelerating flow

A system with decelerating flow has an external velocity function,  $U(x)/U_\infty = 1 - x/L$ . There is no mass transferred through the solid surface and  $\Lambda = .7$ . Calculate the local friction factor, local Nusselt number, separation point and average Nusselt number for the region from  $\xi = 0$  to separation.

### Solution

The functions  $\xi$  and  $\beta$  are derived for this two-dimensional case, i.e.,  $r/L = 1$ .

$$\xi = \int_0^{\frac{x}{L}} \frac{U(x)}{U_\infty} \left(\frac{r}{L}\right)^2 d\left(\frac{x}{L}\right) = \int_0^{\frac{x}{L}} \left(1 - \frac{x}{L}\right) d\left(\frac{x}{L}\right) = \frac{x}{L} - \frac{1}{2} \left(\frac{x}{L}\right)^2$$

or

$$\frac{x}{L} = 1 - \sqrt{1 - 2\xi}$$

and

$$\beta = \frac{2\xi}{U(x)/U_\infty} \frac{dU(x)/U_\infty}{d\left(\frac{x}{L}\right)} \frac{d\left(\frac{x}{L}\right)}{d\xi} = \frac{2\xi}{1 - \frac{x}{L}} (-1) \frac{1}{1 - \frac{x}{L}} = \frac{2\xi}{2\xi - 1}$$

Table 10. Results of calculations for Example 1

$K = \frac{v_w}{U_\infty} \sqrt{2Re_x}$	$f''(0, \xi, K)$	$c_f \sqrt{Re_x}$	$\Pi'(0, \xi, K, .7)$	$Nu_x / \sqrt{Re_x}$
0	.4696	.6641	.4139	.2927
.1	.3986	.5637	.3618	.2558
.2	.3305	.4674	.3108	.2198
.3	.2658	.3759	.2610	.1846
.4	.2049	.2898	.2126	.1503
.5	.1485	.2100	.1656	.1171
.6	.09747	.1378	.1201	.08492
.8	.01757	.02485	.03399	.02403

Since there is no mass transfer,  $K = 0$ . Table 11 shows the results of Steps 4, 5 and 6 of Section 5.2 using a step increment of  $\Delta\xi = .002$ .

The solution may be approximated with the Taylor series (92) and (93). The following derivations and limits are needed:

$$\beta \Big|_{\xi=0} = \beta_0 \Big|_{\xi=0} = 0$$

$$\frac{d\beta}{d\xi} = - \frac{2}{(1 - 2\xi)^2} \quad \text{so that} \quad \frac{d\beta}{d\xi} \Big|_{\xi=0} = -2$$

$$\frac{d^2\beta}{d\xi^2} = - \frac{8}{(1 - 2\xi)^3} \quad \text{so that} \quad \frac{d^2\beta}{d\xi^2} \Big|_{\xi=0} = -8$$

Tables B-1 and 1 and Fig. A-1 and A-2 supply the following:

$$\beta_1 \Big|_{\xi=0} = .129105$$

$$\frac{\partial \beta_1}{\partial \beta_0} \Big|_{\xi=0} = -.272$$

$$f''(0, \xi, K) \Big|_{\xi=0} = .4696$$

$$\frac{\partial f''(0, \xi, K)}{\partial \beta_0} \Big|_{\xi=0} = 1.30$$

$$\frac{\partial^2 f''(0, \xi, K)}{\partial \beta_0^2} \Big|_{\xi=0} = -3.05$$

$$\Pi'(0, \xi, K, \Lambda) \Big|_{\xi=0} = .4139$$

$$\frac{\partial \Pi'(0, \xi, K, \Lambda)}{\partial \beta_0} \Big|_{\xi=0} = .209$$

$$\frac{\partial^2 \Pi'(0, \xi, K, \Lambda)}{\partial \beta_0^2} \Big|_{\xi=0} = -.858$$

Table 11. Results of calculations for Example 2

$\xi$	$x$	$\beta_0$	$f_0''$	$(c_f \sqrt{Re})_1$	$\Pi_T'(0, \xi, K, \Lambda_T)$	$(Nu/\sqrt{Re})_1$	$(c_f \sqrt{Re})_2$	$(Nu/\sqrt{Re})_2$
0.0	0.0	0.0	0.4696	$\infty$	0.4139	$\infty$	$\infty$	$\infty$
0.01	0.01005	-0.01606	0.4483	6.213	0.4104	2.873	6.21	2.87
0.02	0.02020	-0.03247	0.4257	4.086	0.4066	1.992	4.09	1.99
0.03	0.03046	-0.04920	0.4015	3.082	0.4024	1.593	3.09	1.59
0.04	0.04083	-0.06623	0.3755	2.443	0.3978	1.349	2.45	1.35
0.05	0.05131	-0.08354	0.3476	1.979	0.3925	1.178	2.00	1.18
0.06	0.06192	-0.1011	0.3173	1.612	0.3866	1.047	1.64	1.05
0.07	0.07264	-0.1188	0.2840	1.306	0.3797	.9410	1.36	.951
0.08	0.08348	-0.1366	0.2472	1.038	0.3716	.8514	1.11	.866
0.09	0.09446	-0.1544	0.2053	0.7937	0.3617	.7720	.991	.795
0.10	0.1056	-0.1720	0.1555	0.5563	0.3487	.6973	.723	.733
0.11	0.1168	-0.1889	0.0914	0.3039	0.3297	0.6209	.561	.678
0.116	0.1236	-0.1983	0.0109	0.0348	0.3000	0.5459	.471	.648
0.1164	0.1241	-0.19883	0.0002	0.0005	0.2957	0.5368	.466	.646

1 Results of numerical solution.

2 Results of truncation of Taylor series after third term.

Calculation of the proper coefficients, equations (94-99), gives as approximate results:

$$f''(0, \xi, K) = .4696 - 2.07\xi - 6.10\xi^2$$

$$\Pi'(0, \xi, K, \Lambda) = .4139 - .332\xi - 1.44\xi^2$$

The Nusselt number and friction factor predicted by these expansions are presented in Table 11. The expansion for  $\Pi'(0, \xi, K, \Lambda)$  is also compared with the numerical integration results in Fig. 16.

It is now possible to calculate values of  $\bar{Nu} / \sqrt{Re}$  from  $\xi = 0$  to the separation point calculated earlier ( $\xi_{sep} = .1164$ ). Numerical integration of equation (105) using  $\Pi'(0, \xi, K, \Lambda)$  calculated by numerical means gives  $\bar{Nu} / \sqrt{Re} = 1.53$ . The percent error listed after the following approximate average Nusselt numbers uses this  $\bar{Nu} / \sqrt{Re}$  as the true value.

$$\bar{Nu} / \sqrt{Re} = \frac{d_0 \sqrt{2\xi_{sep}}}{\left(\frac{x}{R}\right)_{sep}} = 1.61 \quad \% \text{ error} = 5.2$$

$$\bar{Nu} / \sqrt{Re} = \frac{\sqrt{2} \left( d_0 \sqrt{\xi_{sep}} + \frac{d_1}{3} (\xi_{sep})^{3/2} + \frac{d_2}{5} (\xi_{sep})^{5/2} \right)}{\left(\frac{x}{R}\right)_{sep}} = 1.54$$

% error = .7

Using  $\xi_{sep} = .1164$  in equation (109) yields,  $d_s = -1.32 \times 10^8$ . Equation (110) gives:

$$\bar{Nu} / \sqrt{Re} = 1.53$$

### 8.3 Example 3. Flow around a cylinder with suction

A cylinder of radius  $R$  rests in a stream of air which is moving perpendicular to the axis of the cylinder. Assume that the external velocity profile is that of Elzy [10],  $U(x)/U_\infty = 1.737\left(\frac{x}{R}\right) - .2935\left(\frac{x}{R}\right)^3 - .0593\left(\frac{x}{R}\right)^5$ . The velocity of the air being pulled through the surface of the cylinder is given by  $\frac{v_w(x)}{U_\infty} \sqrt{Re_d} = -.9319$ .

Calculate the local friction factor, local Nusselt number, separation point and average Nusselt number for the region from  $x = 0$  to separation. Assume that  $\Lambda = .7$ .

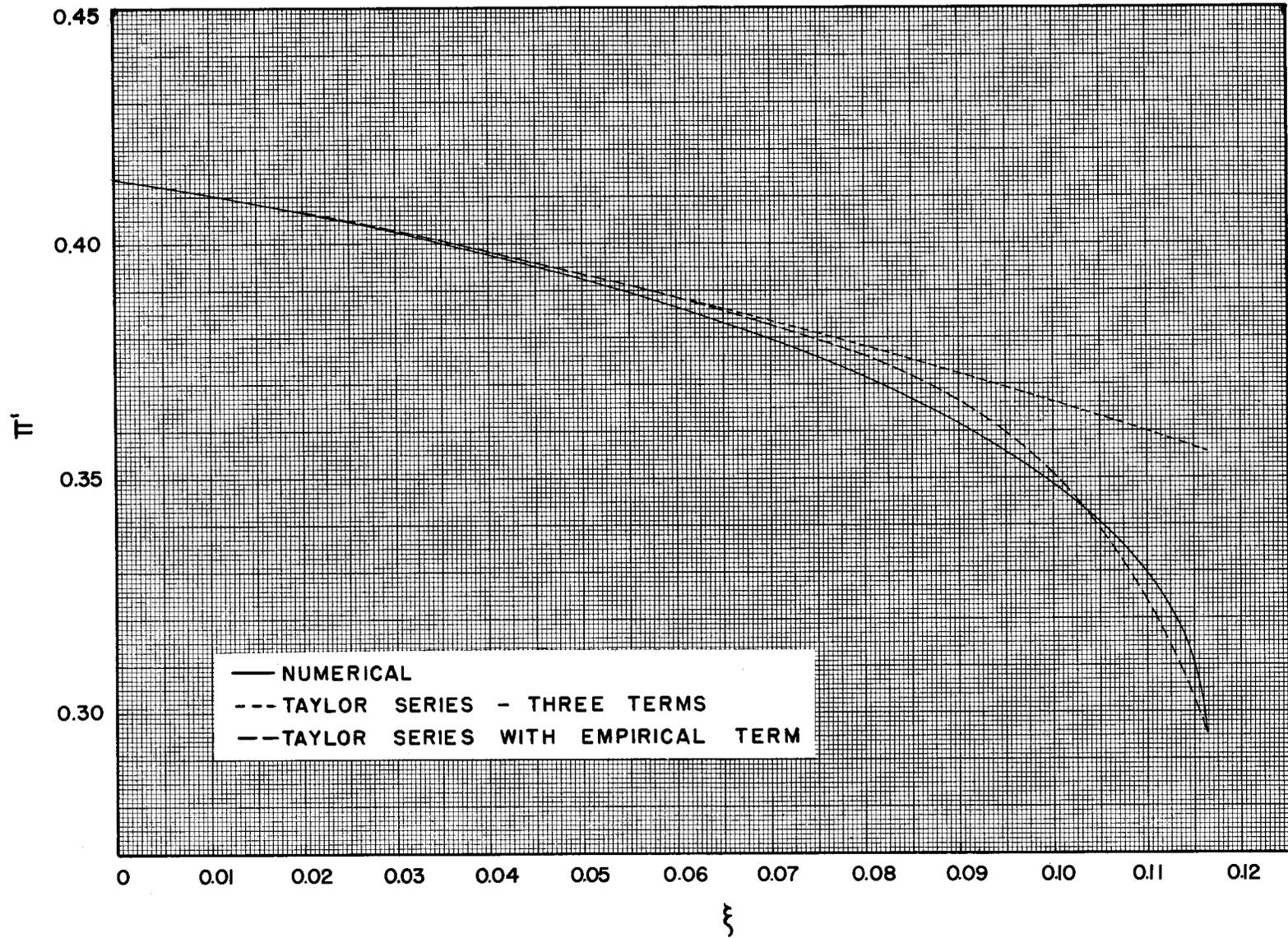


FIGURE 16. DIMENSIONLESS HEAT AND MASS TRANSFER PROFILE FOR EXAMPLE 2 WITH  $U(x)/U_\infty = 1 - x$ .

### Solution

The flow is two-dimensional so  $\frac{r}{R} = 1$  and the functions  $\xi, \beta$  and  $K$  become:

$$\begin{aligned}\xi &= \int_0^{\frac{x}{R}} \frac{U(x)}{U_\infty} \left(\frac{r}{R}\right)^2 d\left(\frac{x}{R}\right) = \int_0^{\frac{x}{R}} \left[ u_1\left(\frac{x}{R}\right) - u_3\left(\frac{x}{R}\right)^3 - u_5\left(\frac{x}{R}\right)^5 \right] d\left(\frac{x}{R}\right) \\ &= .8685\left(\frac{x}{R}\right)^2 - .073375\left(\frac{x}{R}\right)^4 - .0098833\left(\frac{x}{R}\right)^6\end{aligned}$$

$$\begin{aligned}\beta &= \frac{2\xi}{U(x)/U_\infty} \frac{dU(x)/U_\infty}{d\xi} \\ &= \frac{2(.8685\left(\frac{x}{R}\right)^2 - .073375\left(\frac{x}{R}\right)^4 - .0098833\left(\frac{x}{R}\right)^6)}{(1.737\left(\frac{x}{R}\right) - .2935\left(\frac{x}{R}\right)^3 - .0593\left(\frac{x}{R}\right)^5)^2} (1.737 \\ &\quad - .8805\left(\frac{x}{R}\right)^2 - .2965\left(\frac{x}{R}\right)^4)\end{aligned}$$

$$\begin{aligned}K &= \frac{v_w(x)}{U_\infty} \left(\frac{R}{r}\right) \frac{(2\xi Re)^{\frac{1}{2}}}{U(x)/U_\infty} \\ &= -.9319 \frac{(.8685\left(\frac{x}{R}\right)^2 - .073375\left(\frac{x}{R}\right)^4 - .0098833\left(\frac{x}{R}\right)^6)^{\frac{1}{2}}}{1.737\left(\frac{x}{R}\right) - .2935\left(\frac{x}{R}\right)^3 - .0593\left(\frac{x}{R}\right)^5}\end{aligned}$$

The values needed to start the solution are

$$\beta|_{\xi=0} = 1$$

$$K|_{\xi=0} = -.5$$

Proceeding with Steps 4, 5 and 6 of Section 5.2 with  $\Delta\xi = .05$  provides results which are given in Table 12. Substitution of the appropriate results into equation (105) and then integrating numerically gives  $\overline{Nu} \bullet / \sqrt{Re_d} = 1.25$  for the range from stagnation to separation.

Table 12. Results of calculations for Example 3

$\xi$	$x$	$K$	$\beta_0$	$f_0''$	$c_f \sqrt{Re_d}$	$\Pi'(0, \xi, K, \Lambda)$	$(Nu^\bullet / \sqrt{Re_d})_1$	$(Nu^\bullet / \sqrt{Re_d})_2$
0.0	0.0	-.5000	1.0000	1.542	0.0	.7410	1.381	1.38
.1	.3410	-.5077	.9707	1.532	3.264	.7441	1.366	1.37
.2	.4849	-.5163	.9361	1.519	4.427	.7476	1.349	1.35
.3	.5972	-.5260	.8950	1.504	5.171	.7514	1.331	1.33
.4	.6938	-.5370	.8462	1.485	5.658	.7557	1.311	1.31
.5	.7808	-.5494	.7884	1.462	5.946	.7603	1.290	1.28
.6	.8614	-.5637	.7194	1.432	6.064	.7656	1.266	1.26
.7	.9375	-.5803	.6370	1.396	6.023	.7714	1.239	1.23
.8	1.011	-.5997	.5379	1.349	5.827	.7780	1.209	1.19
.9	1.082	-.6227	.4184	1.288	5.473	.7855	1.175	1.15
1.0	1.152	-.6508	.2739	1.208	4.955	.7939	1.137	1.11
1.1	1.222	-.6856	.0998	1.100	4.262	.8034	1.092	1.06
1.2	1.292	-.7305	-.1072	.9479	3.380	.8139	1.038	1.00
1.3	1.365	-.7910	-.3447	.7370	2.333	.8272	.9745	.928
1.4	1.442	-.8788	-.5951	.2401	.6389	.8097	.8587	.840
1.422	1.460	-.9044	-.6516	.0008	.0020	.7875	.8114	.817

1 Results of numerical solution.

2 Results of truncation of Taylor series after third term.

In order to calculate  $\overline{Nu} \bullet / \sqrt{Re_d}$  from the Taylor series the following limits are needed:

$$\left. \frac{d\beta}{d\xi} \right|_{\xi=0} = \frac{3u_3}{u_1^2} = -.2918$$

$$\left. \frac{d^2\beta}{d\xi^2} \right|_{\xi=0} = \frac{2}{3} \frac{1}{u_1^4} (40u_1u_5 - 39u_3^2) = -.5478$$

$$\left. \frac{dK}{d\xi} \right|_{\xi=0} = -6 \frac{u_3}{(2u_1)^{5/2}} \left( \frac{v_w}{U_\infty} \sqrt{Re_d} \right) = .07296$$

$$\left. \frac{d^2K}{d\xi^2} \right|_{\xi=0} = \frac{20}{3} \frac{21u_3^2 - 16u_1u_5}{(2u_1)^{9/2}} \left( \frac{v_w}{U_\infty} \sqrt{Re_d} \right) = -.04517$$

$$\beta_1 \Big|_{\xi=0} = .0421643$$

$$\left. \frac{\partial \beta_1}{\partial \beta_0} \right|_{\xi=0} = -.0257$$

$$\Pi' (0, \xi, K, \Lambda) \Big|_{\xi=0} = .740987$$

$$\left. \frac{\partial \Pi' (0, \xi, K, \Lambda)}{\partial \beta_0} \right|_{\xi=0} = .0300$$

$$\left. \frac{\partial^2 \Pi' (0, \xi, K, \Lambda)}{\partial \beta_0^2} \right|_{\xi=0} = -.0221$$

$$\left. \frac{\partial \beta}{\partial K} \right|_{\xi=0} = .00448$$

$$\left. \frac{\partial \Pi' (0, \xi, K, \Lambda)}{\partial K} \right|_{\xi=0} = -.524$$

$$\left. \frac{\partial^2 \Pi' (0, \xi, K, \Lambda)}{\partial K} \right|_{\xi=0} = -.119$$

The  $\Pi'(0, \xi, K, \Lambda)$  expansion for three terms becomes  $\Pi'(0, \xi, K, \Lambda) = .74099 + .0302\xi + .00384\xi^2$ . This expansion is compared graphically in Fig. 17 with that calculated numerically. The following values of  $\overline{Nu}^\bullet/\sqrt{Re_d}$  can now be calculated. The percent error listed after the results uses the numerically integrated value of  $\overline{Nu}^\bullet/\sqrt{Re_d}$  calculated previously as the true value.

$$\overline{Nu}^\bullet/\sqrt{Re_d} = \frac{2d_0\sqrt{\xi_{sep}}}{\left(\frac{x}{R}\right)_{sep}} = 1.21 \quad \% \text{ error} = 3.$$

$$\overline{Nu}^\bullet/\sqrt{Re_d} = \frac{2(d_0\sqrt{\xi_{sep}} + \frac{d_1}{3}(\xi_{sep})^{3/2} + \frac{d_2}{5}(\xi_{sep})^{5/2})}{\left(\frac{x}{R}\right)_{sep}} = 1.24$$

% error = .8

Now that the separation point is known equation (109) yields,  $d_s = -.000123$ . Equation (110) gives

$$\overline{Nu}^\bullet/\sqrt{Re_d} = 1.24$$

#### 8.4 Example 4. Ideal flow around a sphere with injection

A sphere of radius  $R$  is falling at a constant velocity through air. Potential theory gives  $U(x)/U_\infty = 1.5 \sin(\frac{x}{R})$  for ideal flow around a sphere. Assume that  $K$  remains constant at .5. Calculate the local Nusselt number, friction factor, separation point and average Nusselt number for the region  $x = 0$  to separation. Assume  $\Lambda$  is constant at .7.

##### Solution

For axisymmetrical flow around a sphere:

$$\frac{r}{R} = \sin \frac{x}{R}$$

The functions  $\xi$  and  $\beta$  become:

$$\begin{aligned} \xi &= \int_0^{\frac{x}{R}} \frac{U(x)}{U_\infty} \left(\frac{r}{R}\right)^2 d\left(\frac{x}{R}\right) = \int_0^{\frac{x}{R}} 1.5 \sin^3\left(\frac{x}{R}\right) d\left(\frac{x}{R}\right) \\ &= .5 [2 - \cos\left(\frac{x}{R}\right) (\sin^2\left(\frac{x}{R}\right) + 2)] \end{aligned}$$

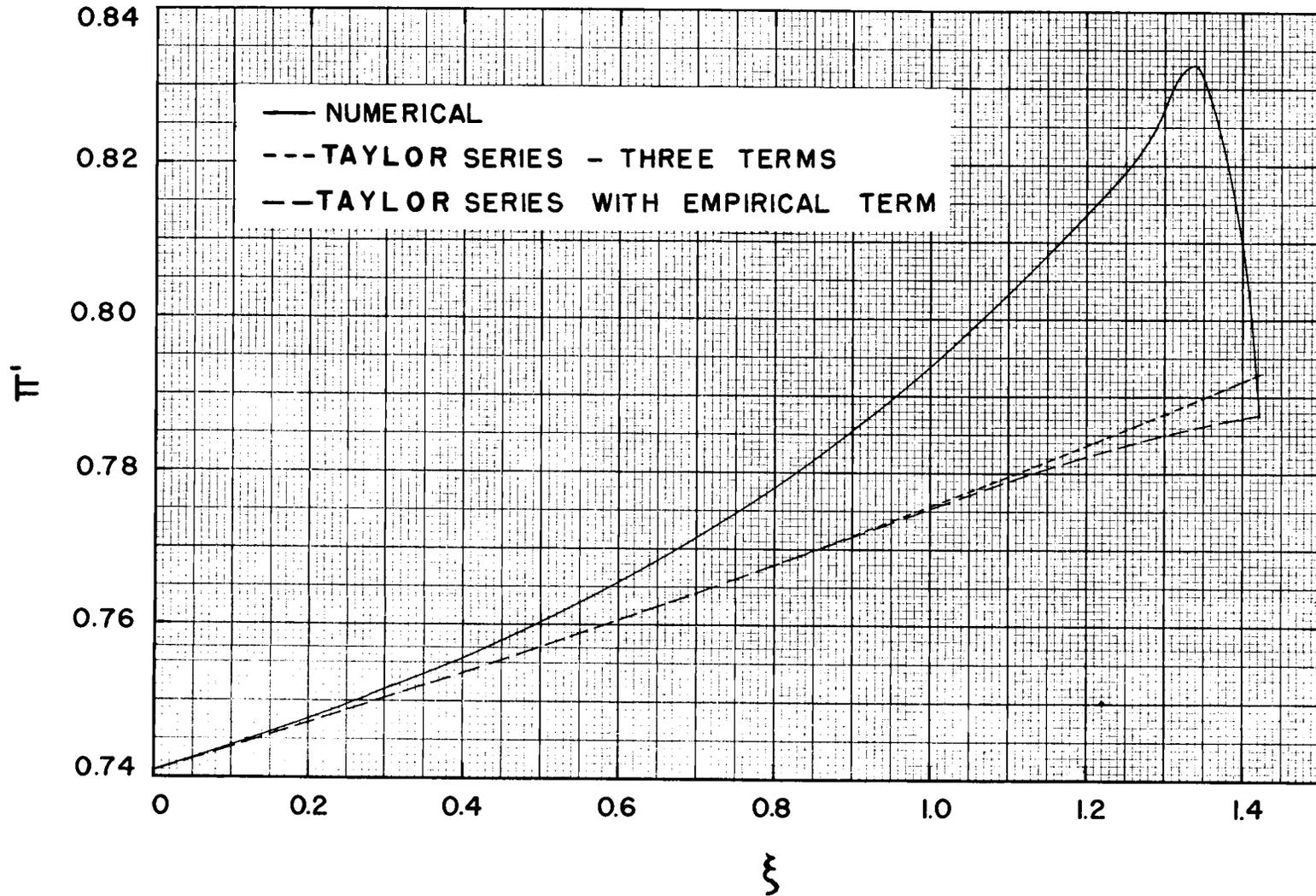


FIGURE 17. DIMENSIONLESS HEAT AND MASS TRANSFER PROFILE FOR EXAMPLE 3 WITH SUCTION AT THE SURFACE OF A CIRCULAR CYLINDER AND  $U(x)/U_\infty = 1.737x - .2935x^3 - .0593x^5$ .

$$\beta = \frac{2\xi}{U(x)/U_\infty} \frac{dU(x)/U_\infty}{d\xi} = \frac{[2 - \cos(\frac{x}{R}) (\sin^2(\frac{x}{R}) + 2)] \cos(\frac{x}{R})}{1.5 \sin^4(\frac{x}{R})}$$

The necessary starting limits are

$$\beta \Big|_{\xi=0} = .5$$

$$K \Big|_{\xi=0} = .5$$

Carrying out Steps 4, 5 and 6 of Section 5.2 with  $\Delta\xi = .01$  gives the results which are tabulated in Table 13. Calculation of the average Nusselt number gives  $\overline{Nu} \bullet / \sqrt{Re_d} = .456$

The following values are needed in order to compute the Taylor series coefficients:

$$\lim_{\xi \rightarrow 0} \frac{d\beta}{d\xi} \rightarrow -\infty$$

$$\lim_{\xi \rightarrow 0} \frac{d^2\beta}{d\xi^2} \rightarrow \infty$$

$$\beta_1 \Big|_{\xi=0} = .0637851$$

$$\frac{\partial \beta_1}{\partial \beta_0} \Big|_{\xi=0} = -.0671$$

$$\Pi' (0, \xi, K, \Lambda) \Big|_{\xi=0} = .262224$$

$$\frac{\partial \Pi' (0, \xi, K, \Lambda)}{\partial \beta_0} \Big|_{\xi=0} = .0871$$

$$\frac{\partial^2 \Pi' (0, \xi, K, \Lambda)}{\partial \beta_0^2} \Big|_{\xi=0} = -.152$$

A plot of  $\beta$  vs  $\xi$  shows that a representative  $\frac{d\beta}{d\xi}$  is attained at  $\xi = .1$ . A good representation of  $\beta$  can be acquired by letting

Table 13. Results of calculations for Example 4

$\xi$	$x$	$\beta_0$	$f_0''$	$c_f \sqrt{Re_d}$	$\Pi(0, \xi, K, \Lambda)$	$(Nu^\bullet / \sqrt{Re_d})_1$	$(Nu^\bullet / \sqrt{Re_d})_2$
0.0	0.0	.5000	.6594	0.0	.2622	.6423	.642
.1	.7535	.4477	.6206	2.828	.2574	.5717	.576
.2	.9168	.4172	.5972	3.004	.2544	.5375	.541
.3	1.034	.3877	.5739	2.993	.2513	.5085	.511
.4	1.131	.3574	.5492	2.895	.2480	.4815	.484
.5	1.216	.3254	.5224	2.741	.2441	.4554	.457
.6	1.294	.2910	.4923	2.545	.2397	.4294	.432
.7	1.367	.2535	.4582	2.314	.2343	.4028	.407
.8	1.436	.2124	.4186	2.050	.2278	.3751	.382
.9	1.504	.1670	.3717	1.752	.2193	.3452	.358
1.0	1.571	.1167	.3153	1.419	.2082	.3123	.333
1.1	1.638	.06138	.2444	1.042	.1921	.2734	.309
1.2	1.705	.00065	.1498	.5987	.1660	.2232	.284
1.290	1.768	-.05017	.0003	.0010	.0948	.1204	.262

1 Results of numerical solution.

2 Results of truncation of Taylor series after third term.

$$\beta = e_0 + e_1 \xi + e_2 \xi^2$$

where  $e_0 = .5$

$$e_1 = \left. \frac{d\beta}{d\xi} \right|_{\xi=.1} = -.3672$$

$$e_2 = -.06803$$

The value of  $e_2$  was selected so that the  $\beta$  approximation gives the true value of  $\beta$  at  $\xi = 0.1$ . In order to calculate  $d_1$  and  $d_2$  let the  $\beta$  derivatives needed be given by

$$\frac{d\beta}{d\xi} = e_1 = -.3672$$

$$\frac{d^2\beta}{d\xi^2} = 2e_2 = -.13606$$

The  $\Pi'(0, \xi, K, \Lambda)$  expansion becomes:

$$\Pi'(0, \xi, K, \Lambda) = .262224 - .0284\xi - .0118\xi^2$$

This is compared with the numerically computed  $\Pi'(0, \xi, K, \Lambda)$  in Fig. 18.

Application of equation (107) to the above expansion with both one and three terms gives the following approximations for  $\overline{Nu}^\bullet / \sqrt{Re_d}$ . The percentage error listed after the results uses the previously found average for the true value.

$$\overline{Nu}^\bullet / \sqrt{Re_d} = \frac{2d_0 \sqrt{\xi_{sep}}}{1 - \cos \left( \frac{x}{R} \right)_{sep}} = .498 \quad \% \text{ error} = 9.2$$

$$\overline{Nu}^\bullet / \sqrt{Re_d} = \frac{2 \left( d_0 \xi_{sep}^{1/2} + \frac{d_1}{3} \xi_{sep}^{3/2} + \frac{d_2}{5} \xi_{sep}^{5/2} \right)}{1 - \cos \left( \frac{x}{R} \right)_{sep}} = .467$$

% error = 2.4

Fig. A-2 provides the value of  $\Pi'(0, \xi, K, \Lambda) = .09459$  at separation when  $K = .5$ . Equation (109) then gives  $d_s = -.008702$ . Now equation (110) gives:

$$\overline{Nu}^\bullet / \sqrt{Re_d} = .457$$

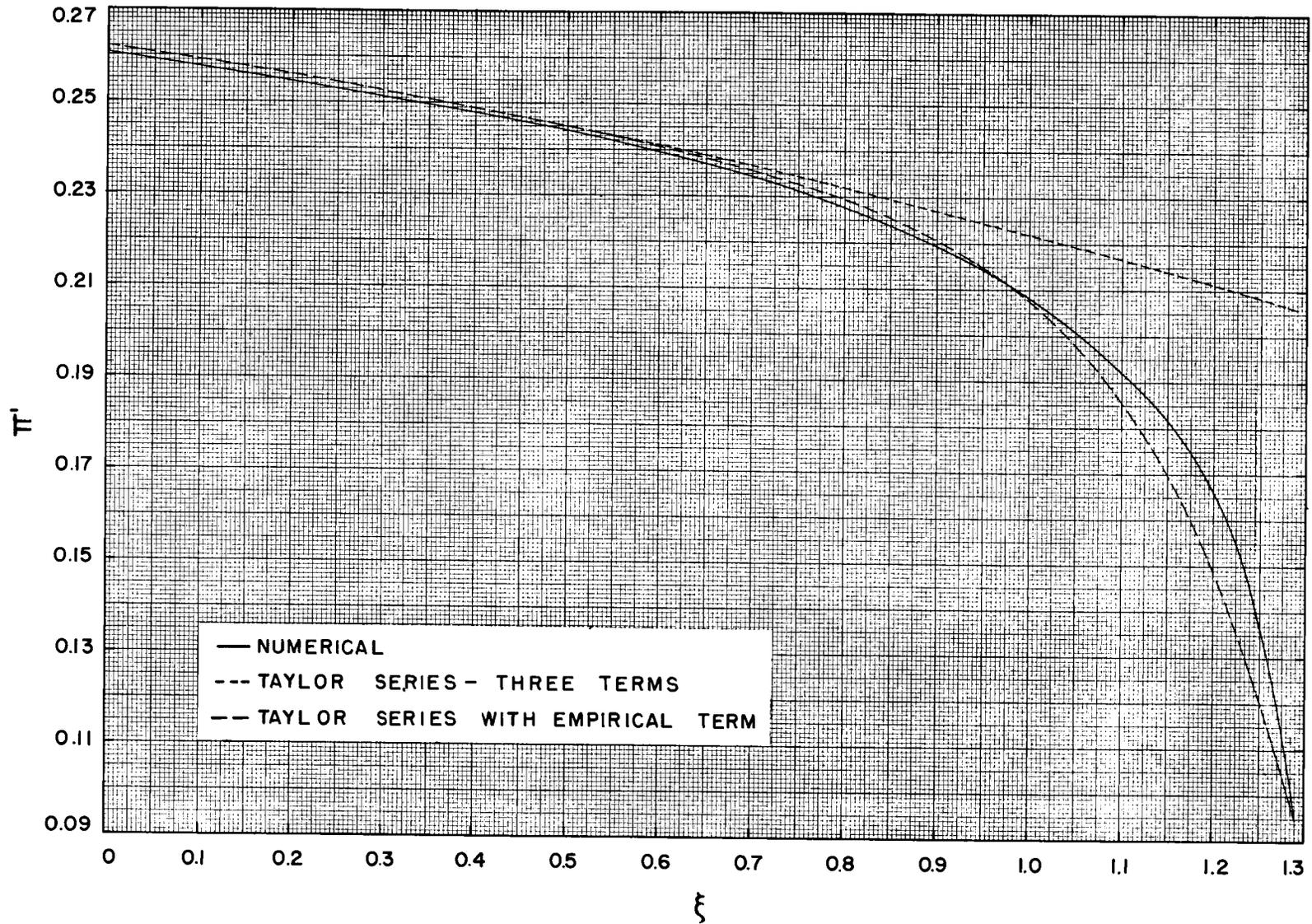


FIGURE 18. DIMENSIONLESS HEAT AND MASS TRANSFER PROFILE FOR EXAMPLE 4 WITH INJECTION AT THE SURFACE OF A SPHERE AND  $U(x)/U_\infty = 1.5 \sin x$ .

8.5 Example 5. Injection from a cylinder with constant  $R_{AB}$

A gaseous species B is moving perpendicular to the axis of a cylinder. Assume the external velocity profile is the same as in Example 3. The conditions are such that the mole fraction of species A remains .544 over the entire surface of the cylinder. The Schmidt number is equal to .7. The solubility of B in A is negligible. Calculate the local friction factor, local Nusselt number, separation point and average Nusselt number for the region from  $x = 0$  to separation.

Solution

The functions  $\xi$  and  $\beta$  are the same as in Example 3. Assuming ideal gas behavior, equilibrium at the surface of the cylinder and complete insolubility of species B in species A, we have:

$$x_{A_w} = .544$$

$$x_{A_\infty} = 0$$

$$N_{B_w} = 0$$

Substitution of these values into equation (75) gives

$$R_{AB} = \frac{x_{A_w} - x_{A_\infty}}{\frac{N_{A_w}}{N_{A_w} + N_{B_w}} - x_A} = \frac{.544 - 0}{1 - .544} = 1.19$$

It can be shown that:

$$\beta_0|_{\xi=0} = \beta|_{\xi=0} = 1.0$$

This value of  $\beta_0$  and the value of  $R_{AB}$  allow  $K|_{\xi=0}$  to be found from Fig. A-27.

$$K|_{\xi=0} = .5$$

In order to start the computation,  $\frac{dK}{d\xi}|_{\xi=0}$  is needed. Since  $R_{AB}$  is constant:

$$\frac{dK}{d\xi} = \frac{R_{AB}}{\Lambda_{AB}} \frac{d\Pi'}{d\xi}$$

and also

$$\frac{d\Pi'}{d\xi} = \frac{\partial \Pi'}{\partial \beta_0} \frac{d\beta_0}{d\xi} + \frac{\partial \Pi'}{\partial K} \frac{dK}{d\xi}$$

Eliminating  $\frac{d\Pi'}{d\xi}$  gives

$$\begin{aligned} \frac{dK}{d\xi} \Big|_{\xi=0} &= \frac{\partial \Pi'}{\partial \beta_0} \frac{d\beta_0}{d\xi} \Big|_{\xi=0} / \left( \frac{\Lambda_{AB}}{R_{AB}} - \frac{\partial \Pi'}{\partial K} \right) \Big|_{\xi=0} \\ &= \frac{1}{1 + 2\beta_1} \frac{d\beta_0}{d\xi} \frac{\partial \Pi'}{\partial \beta_0} \Big|_{\xi=0} / \left( \frac{\Lambda_{AB}}{R_{AB}} - \frac{\partial \Pi'}{\partial K} \right) \Big|_{\xi=0} \end{aligned}$$

The following quantities required to evaluate  $\frac{dK}{d\xi} \Big|_{\xi=0}$  are:

$$\beta_1 \Big|_{\xi=0} = .0419968$$

$$\frac{\partial \Pi'}{\partial \beta_0} \Big|_{\xi=0} = .0455$$

$$\frac{\partial \Pi'}{\partial K} \Big|_{\xi=0} = -.354$$

$$\frac{d\beta_0}{d\xi} \Big|_{\xi=0} = -.2918$$

Therefore,

$$\frac{dK}{d\xi} \Big|_{\xi=0} = -.0130$$

Carrying out Steps 4-13 of Section 5.3 gives results which are tabulated in Table 14. The average Nusselt number is found by numerical integration to be  $\overline{Nu} \bullet / \sqrt{Re_d} = .475$ .

Table 14. Results of calculations for Example 5

$\xi$	$x$	$\beta_0$	$f_0''$	$c_f \sqrt{Re_D}$	$\Pi'(0, \xi, K, \Lambda)$	$Nu/\sqrt{Re_D}$
0.0	0.0	1.000	.9692	0.0	.2933	.5467
0.1	.3410	.9707	.9540	2.033	.2925	.5369
0.2	.4849	.9361	.9357	2.726	.2915	.5261
0.3	.5972	.8950	.9136	3.141	.2902	.5141
0.4	.6938	.8464	.8869	3.379	.2886	.5009
0.5	.7808	.7888	.8545	3.477	.2866	.4862
0.6	.8614	.7205	.8147	3.449	.2841	.4697
0.7	.9375	.6391	.7653	3.303	.2808	.4509
0.8	1.011	.5420	.7031	3.038	.2763	.4294
0.9	1.082	.4265	.6233	2.648	.2700	.4041
1.0	1.152	.2902	.5183	2.126	.2606	.3732
1.1	1.222	.1332	.3751	1.453	.2451	.3331
1.2	1.292	-.0355	.1552	.5535	.2055	.2622
1.243	1.324	-.1032	.0000	.0000	.1703	.2103

## 9. SUMMARY

1. The method of calculation proposed by Sisson [12] works well because the  $\beta_1$  function is defined such that it forces  $f_1'' = 0$  at  $\eta = 0$ . Thus the second derivative of the stream function, which is given by the expansion

$$f''(0, \xi) = f_0''(0, \beta_0) + 2\xi \frac{d\beta_0}{d\xi} f_1''(0, \beta_0) + \dots,$$

can be approximated very well by similar solutions at the value  $\beta_0$ , i.e.,  $f_0''(0, \beta_0)$ . The  $\beta_1$  function has been shown to be a function of the parameters  $\beta_0$  and  $K$ ; if equation (35) is substituted into equation (25), the exact form for equation (39) is given by

$$f_1(0) = -\frac{1}{3} \left. \frac{\partial f_0}{\partial \beta_0} \right|_{\eta=0} \quad \text{or} \quad f_1(0) = \frac{1}{3} \left. \frac{\partial K}{\partial \beta_0} \right|_{\eta=0},$$

so  $\beta_1$  is also a function of the parameter  $(\partial K / \partial \beta_0) |_{\eta=0}$ . Since in most practical problems  $K$  is nearly constant or  $(\partial K / \partial \beta_0) |_{\eta=0}$  is not large, the boundary condition was replaced by  $f_1(0) = 0$  as in equation (39) so that  $\beta_1$  will depend only on  $\beta_0$  and  $K$ . This procedure produces little error in the  $\beta_1$  function as shown by calculations for high rates of injection at constant velocity from a cylinder where  $(\partial K / \partial \beta_0)_{\max} \approx 0.5$ . (See Fig. 7.)

2. The most accurate results are obtained by solving equation (64) by numerical means. Sisson's formula (100) is recommended as it is both accurate and provides a stable solution near  $\xi = 0$ .
3. When the separation point is unknown, the Taylor series expansions (92) and (93) give accurate solutions near  $\xi = 0$ . Equation (107) gives very good approximations of  $\overline{Nu}^\bullet / \sqrt{Re}$ . The results for truncation after one term are in error by no more than ten per cent.
4. If the separation point is known, equation (110) provides an approximation of  $\overline{Nu}^\bullet / \sqrt{Re}$  which is in error by no more than one per cent.

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## NOMENCLATURE

Dimensions are given in terms of mass (M), length (L), time (t), and temperature (T).

- A            area of the surface,  $L^2$ ;
- $a_n$         coefficients in the series (34), (35) and (51), dimensionless;
- $b_n$         coefficients in the series (87), dimensionless;
- $C_1$         constant in equation (58);
- $C_2$         constant in equation (61);
- $c_n$         coefficients in the series (92), dimensionless;
- $C_p$         heat capacity at constant pressure for species  $i$ ,  $L^2t^{-2}T^{-1}$ ;
- $c$            total molar concentration, moles  $L^{-3}$ ;
- $c_f^\bullet$        local friction coefficient defined by equation (68) or (69), dimensionless;
- $d_n$         coefficients in the series (93), dimensionless;
- $D_{AB}$       binary diffusivity,  $L^2t^{-1}$ ;
- $f$            stream function, dimensionless, see equation (20);
- $f_0$         similar stream function, dimensionless, see equation (35);
- $f_n$         auxiliary stream function of order  $n$ , used in equation (35), dimensionless;
- $h^\bullet$         heat transfer coefficient,  $(ML^2t^{-2})L^{-2}t^{-1}T^{-1}$ , see equation (70);
- $k$            thermal conductivity of the fluid,  $(ML^2t^{-2})L^{-1}t^{-1}T^{-1}$ ;
- $K$            dimensionless mass transfer rate defined by equation (47);
- $k_x^\bullet$         mass transfer coefficient, moles  $L^{-2}t^{-1}(\text{mole fraction})^{-1}$ , see equation (71);

L	arbitrary reference length, L;
$m_i$	molecular weight of species i, $M(\text{moles})^{-1}$ ;
m	exponent in equation (59), dimensionless;
$n_i$	molar flux of species i with respect to stationary coordinates, moles $L^{-2}t^{-1}$ ;
$Nu^\bullet$	local Nusselt number defined in equation (72), dimensionless;
$q_w$	energy flux at wall, $Mt^{-3}$ , see equation (77);
$r(x)$	radius of axisymmetrical body, L, see Fig. 1;
R	flux ratio, dimensionless, see equation (76);
Re	Reynolds number, $U_\infty L/\nu$ , dimensionless;
s	exponent in equation (108), dimensionless;
t	time, t;
T	temperature, T;
u	longitudinal velocity, $Lt^{-1}$ ;
$u_n$	coefficients in external velocity distribution, see Section 8.3;
$U_1$	constant in equation (59);
$U(x)$	mainstream velocity, $Lt^{-1}$ ;
$U_\infty$	reference velocity, $Lt^{-1}$ ;
v	transverse velocity, $Lt^{-1}$ ;
x	longitudinal coordinate, L;
$x_A$	mole fraction of species A, dimensionless;
y	transverse coordinate, L;
$\alpha$	thermal diffusivity, $L^2t^{-1}$ ;
$\beta$	flow parameter defined by equation (21), dimensionless;

- $\beta_0$  flow parameter defined by equation (60) or (64), dimensionless;
- $\beta_n$  parameters in the  $f_n$  equations, dimensionless, see equation (34);
- $\eta$  similar coordinate defined by equation (17), dimensionless;
- $\theta$  correction factor, dimensionless, see equation (81);
- $\Lambda$  diffusivity ratio defined in equations (29) and (30), dimensionless;
- $\nu$  kinematic viscosity,  $L^2t^{-1}$ ;
- $\Pi$  profile function defined by equations (27) and (28), dimensionless;
- $\Pi_0$  similar profile function defined by equation (36), dimensionless;
- $\Pi_n$  auxiliary profile function of order  $n$ , used in equation (51), dimensionless;
- $\xi$  coordinate defined by equation (16), dimensionless;
- $\rho$  mass density,  $ML^{-3}$ ;
- $\tau_w$  shear stress at solid surface,  $Mt^{-2}L^{-1}$ , see equation (76);
- $\phi$  rate factor, dimensionless, see equation (80);

#### Subscripts

- $d$  the diameter is used as the reference length  $L$ ;
- $sep$  evaluated at the separation point;
- $V$  quantity relative to momentum transfer;
- $w$  evaluated at wall or surface conditions;
- $\infty$  evaluated at  $\eta = \infty$ ;
- $A$  evaluated for species  $A$ ;
- $T$  or  $AB$  quantity relative to energy or mass transfer, respectively;

### Superscripts

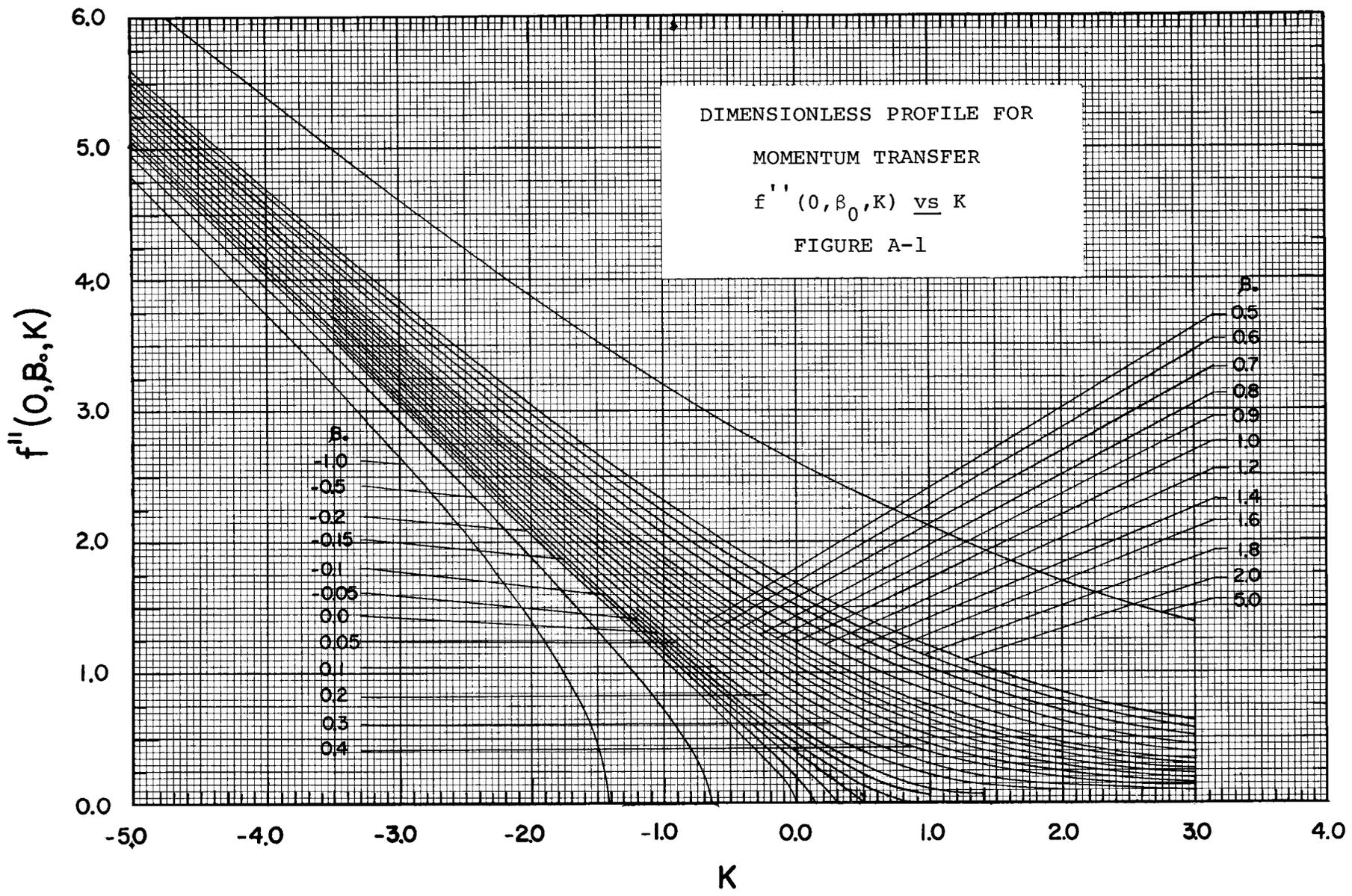
- denotes a quantity evaluated at the prevailing mass transfer conditions;

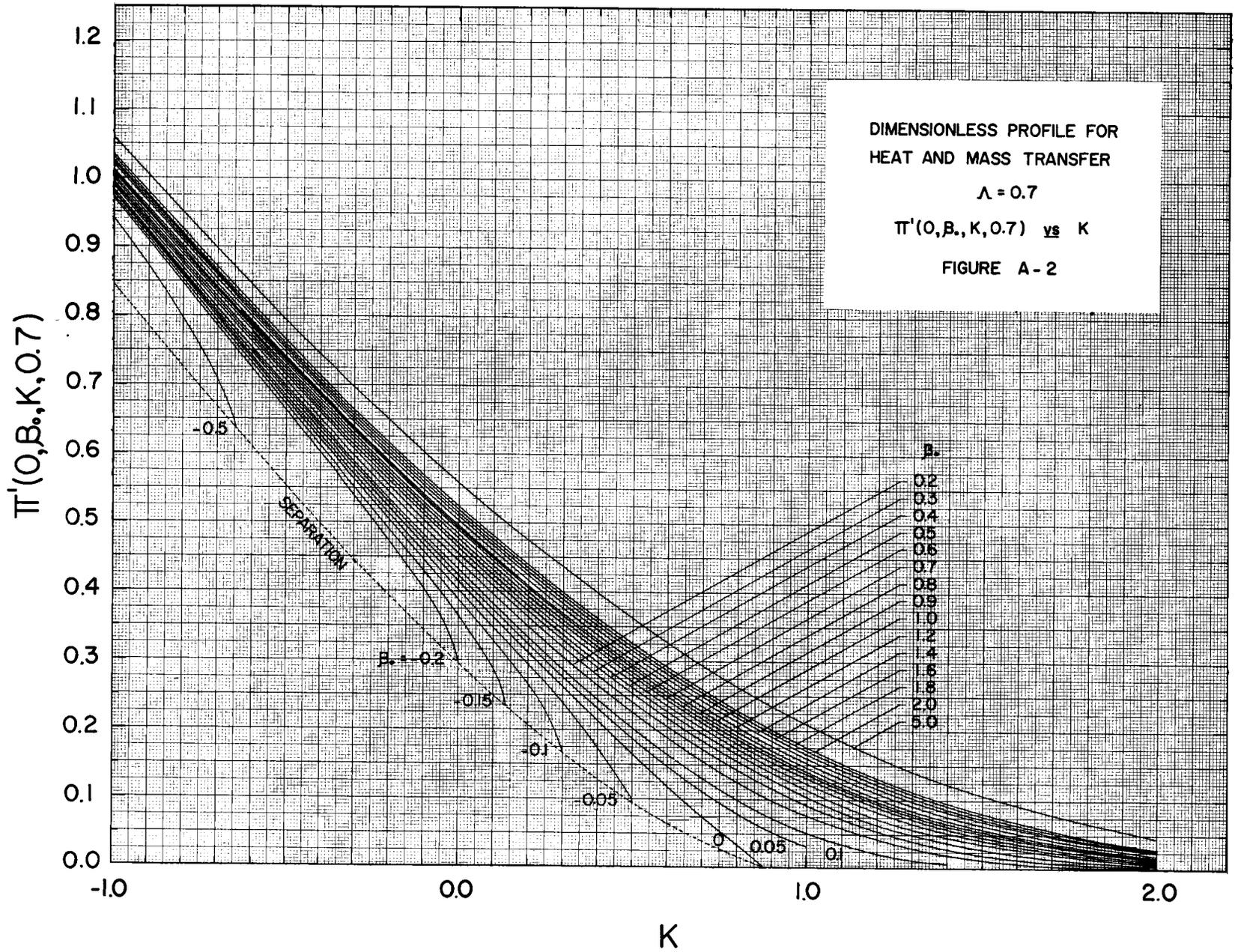
### Overlines

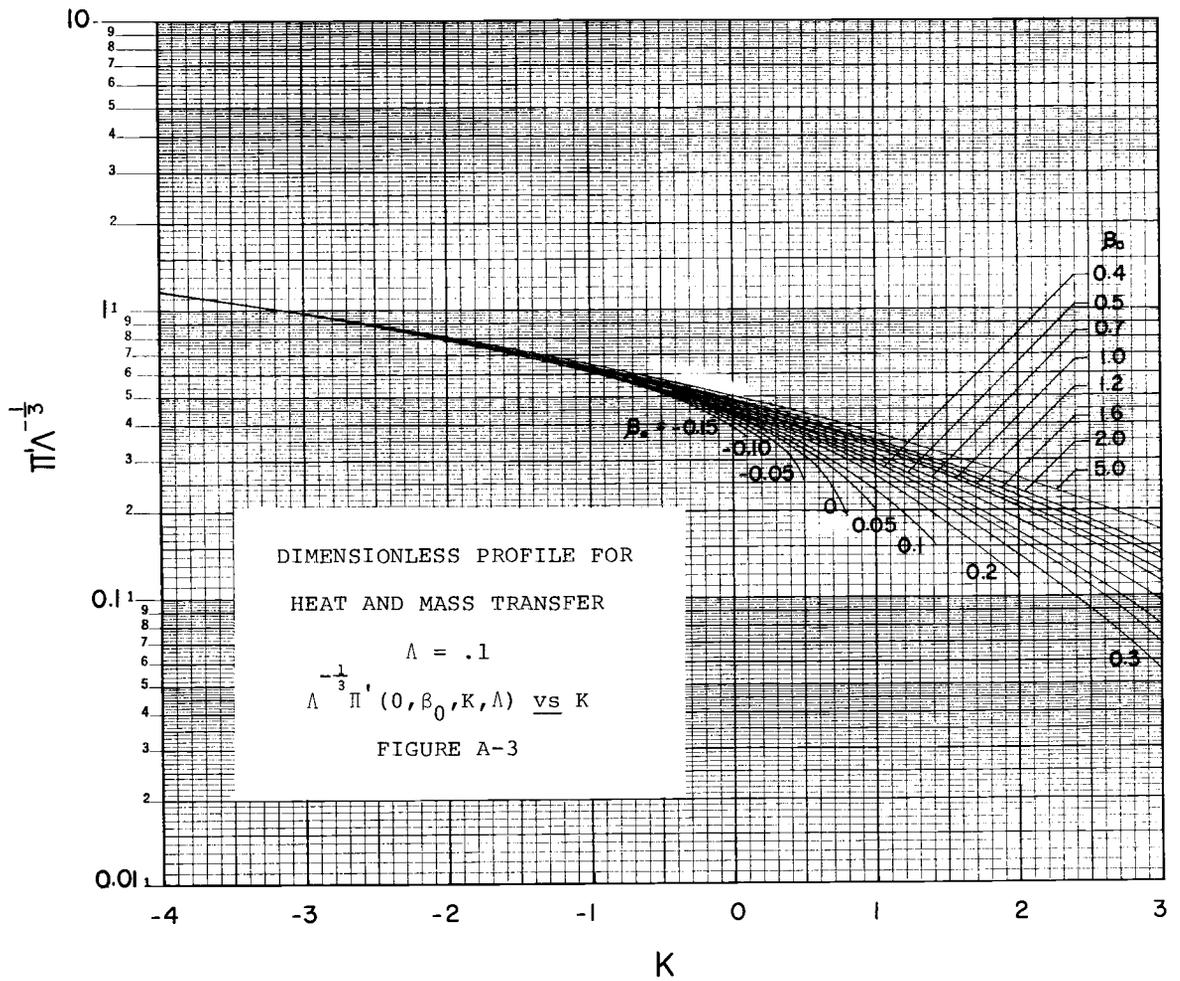
- denotes an average quantity;
- ^ per unit mass;
- per mole.

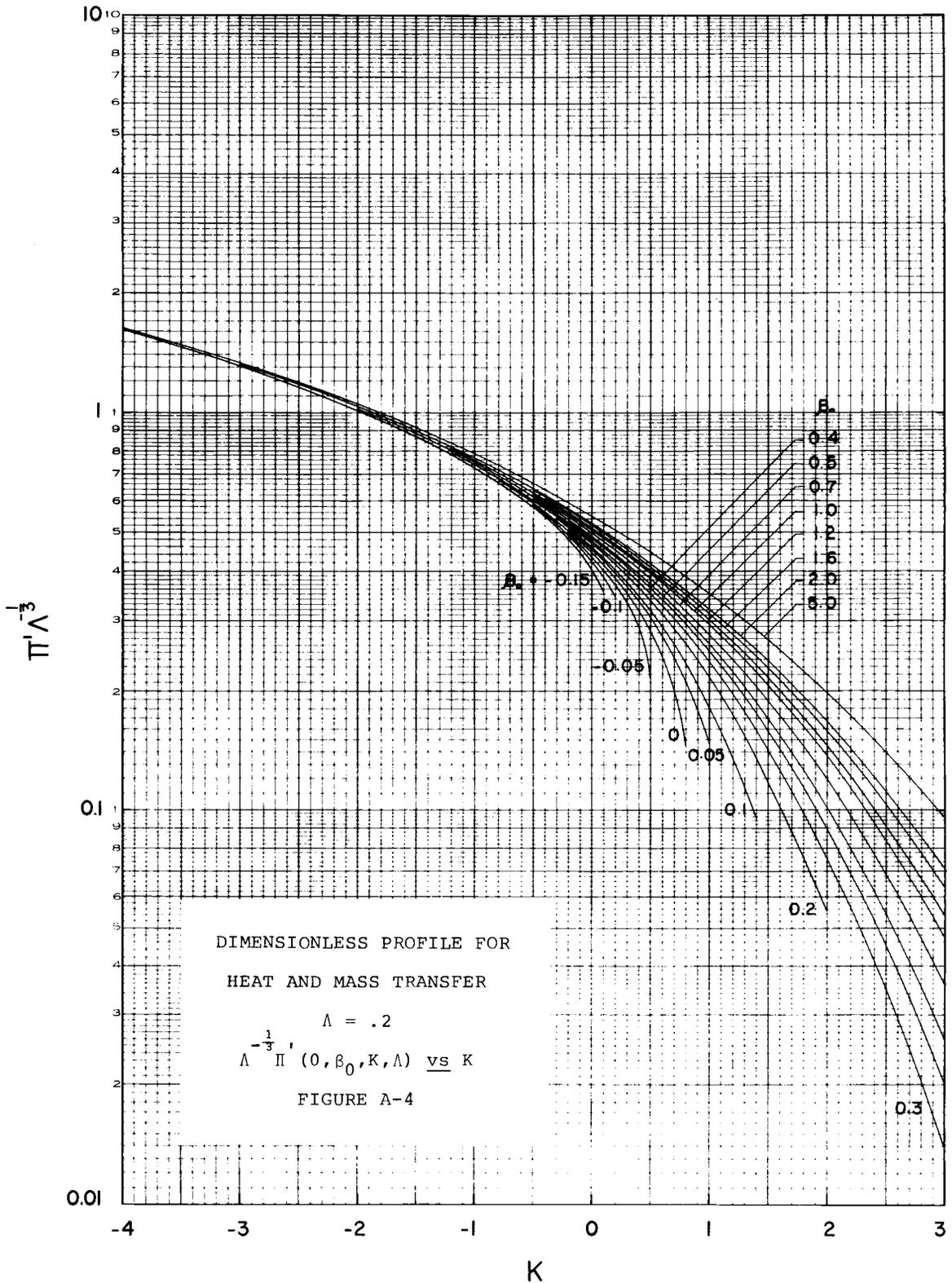
APPENDIX A

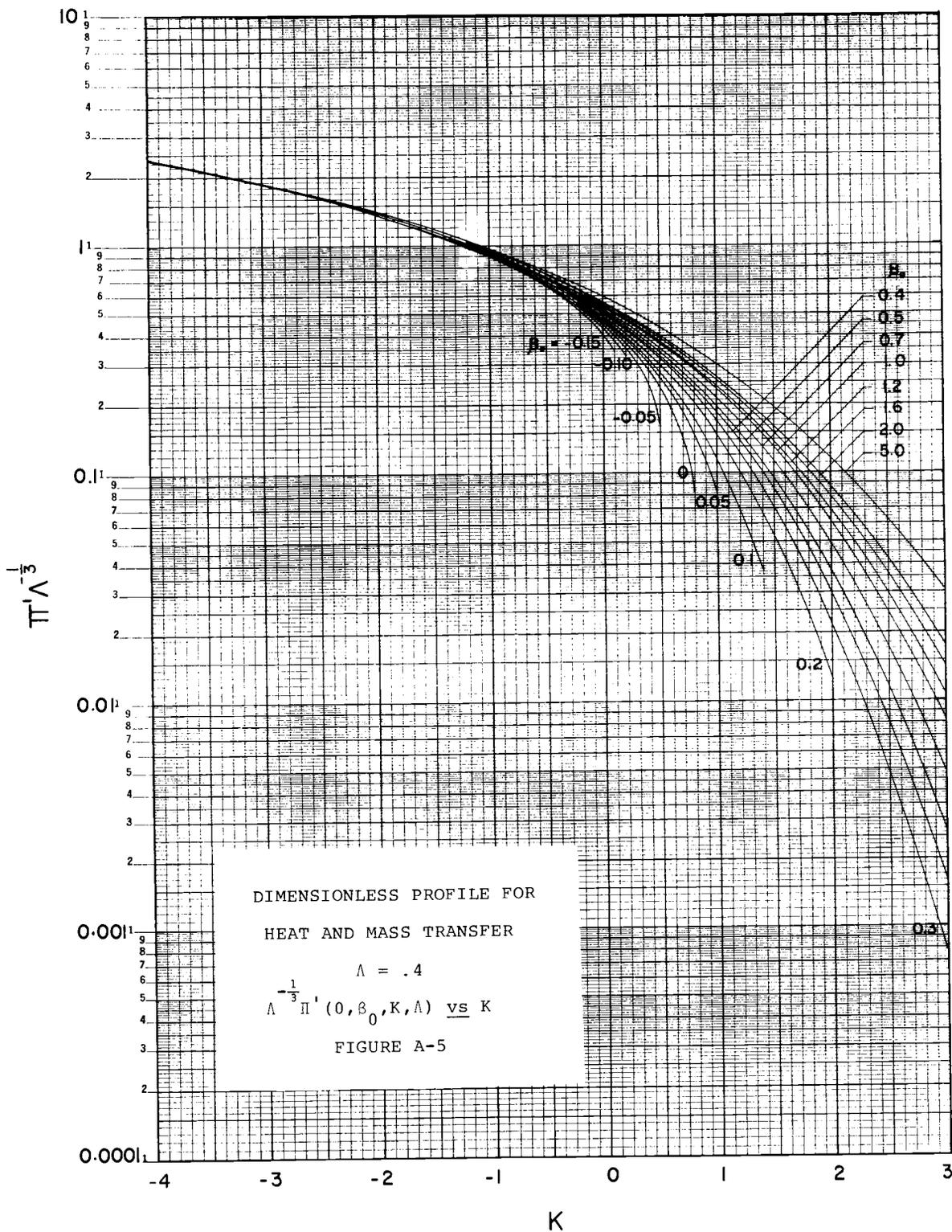
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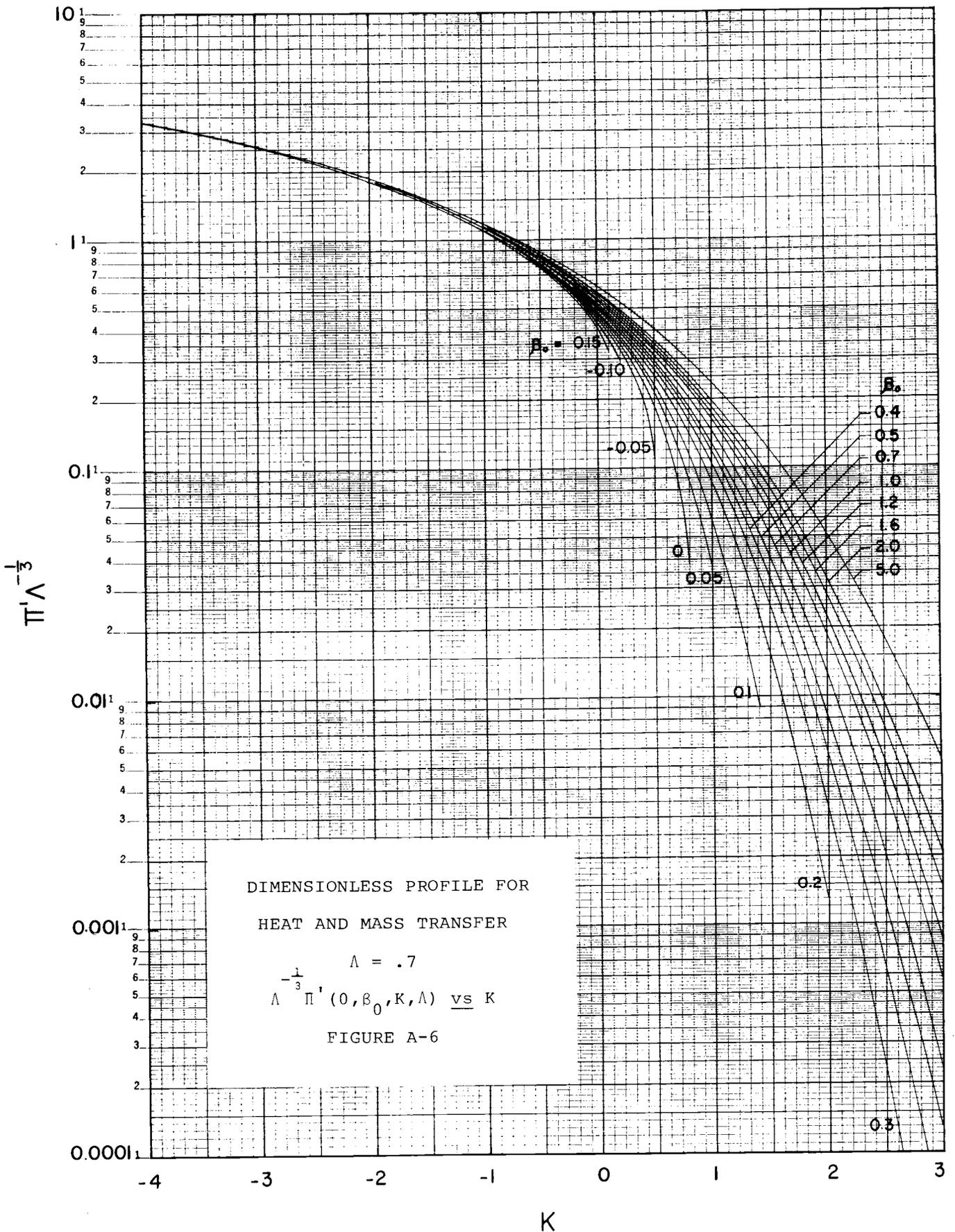


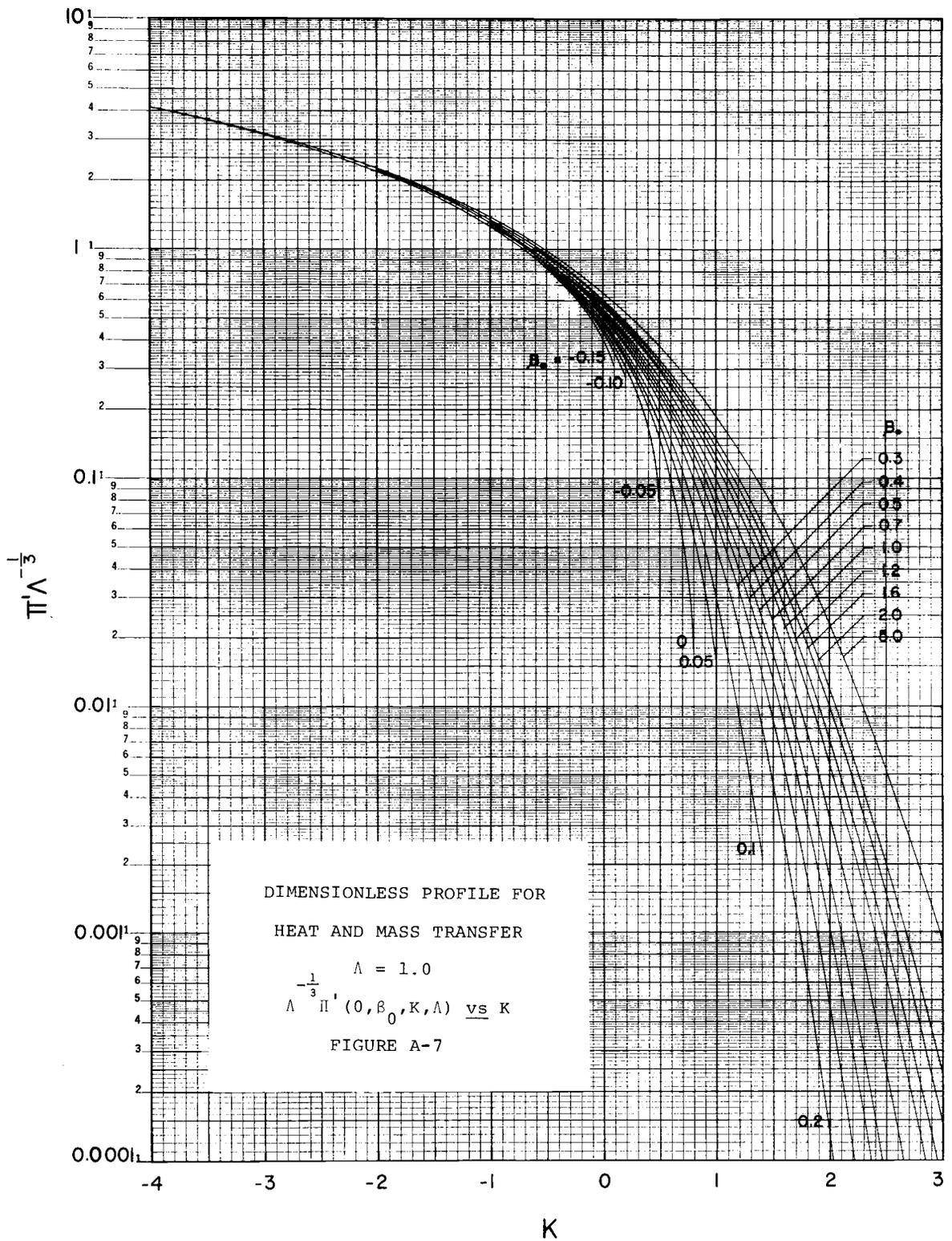


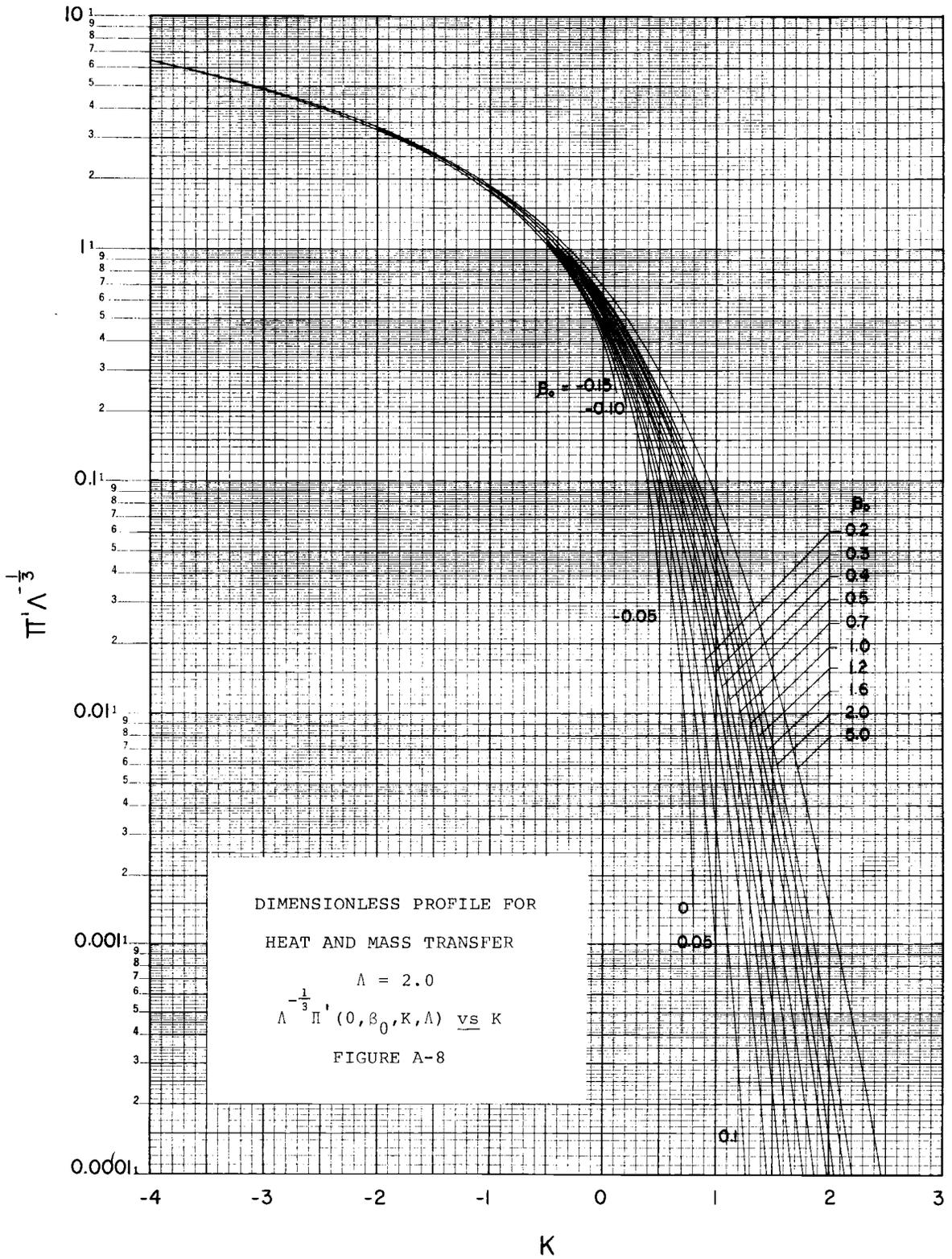


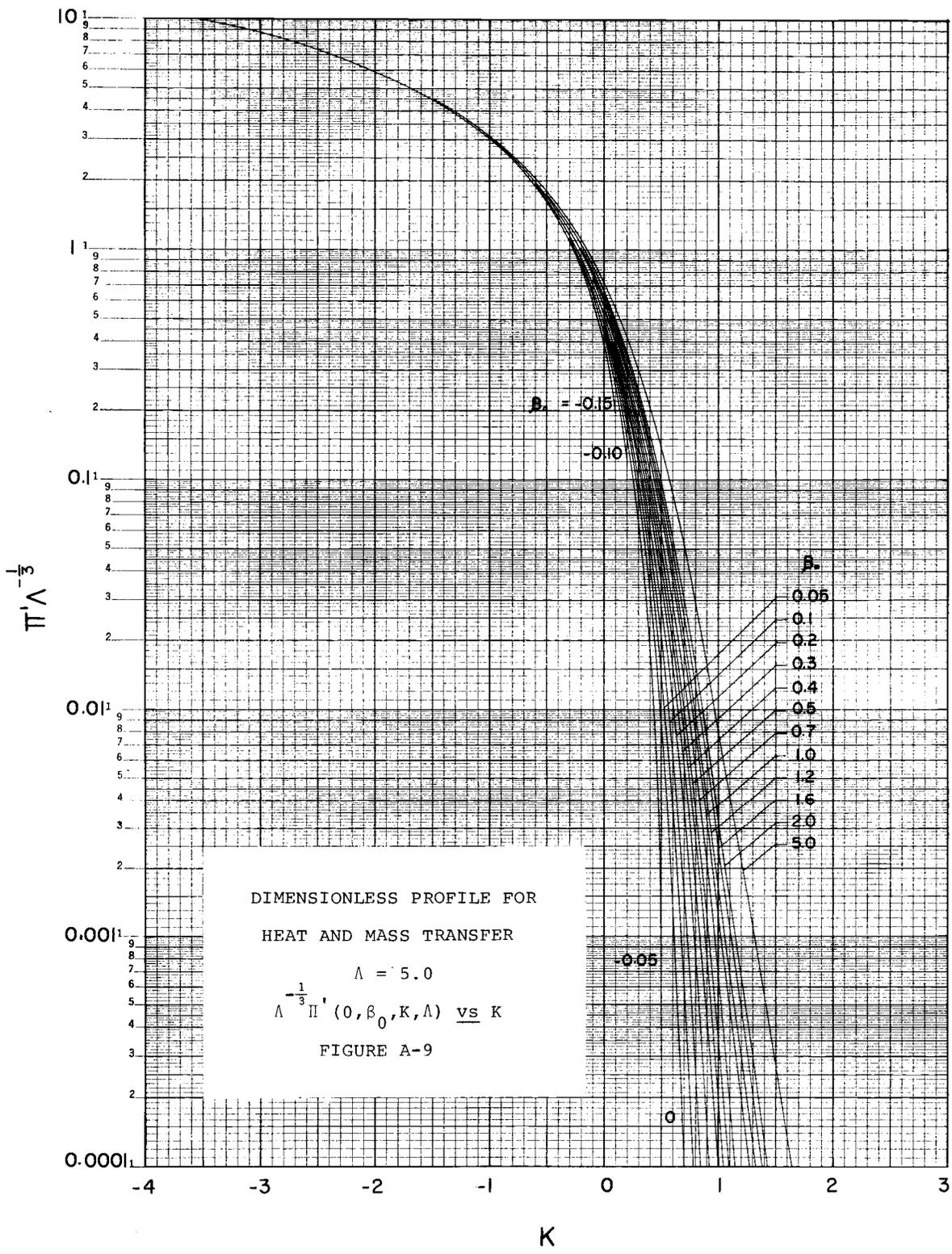


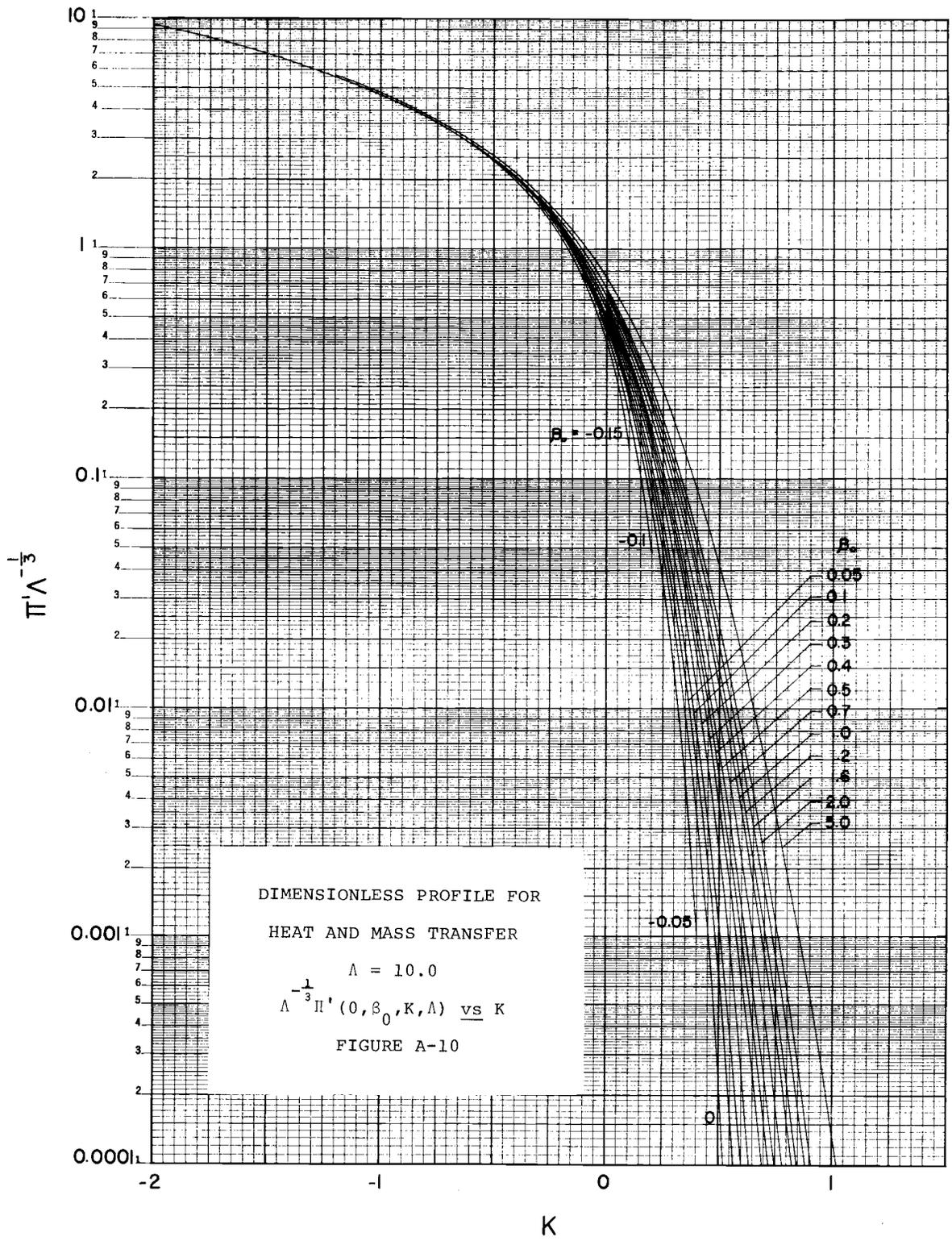


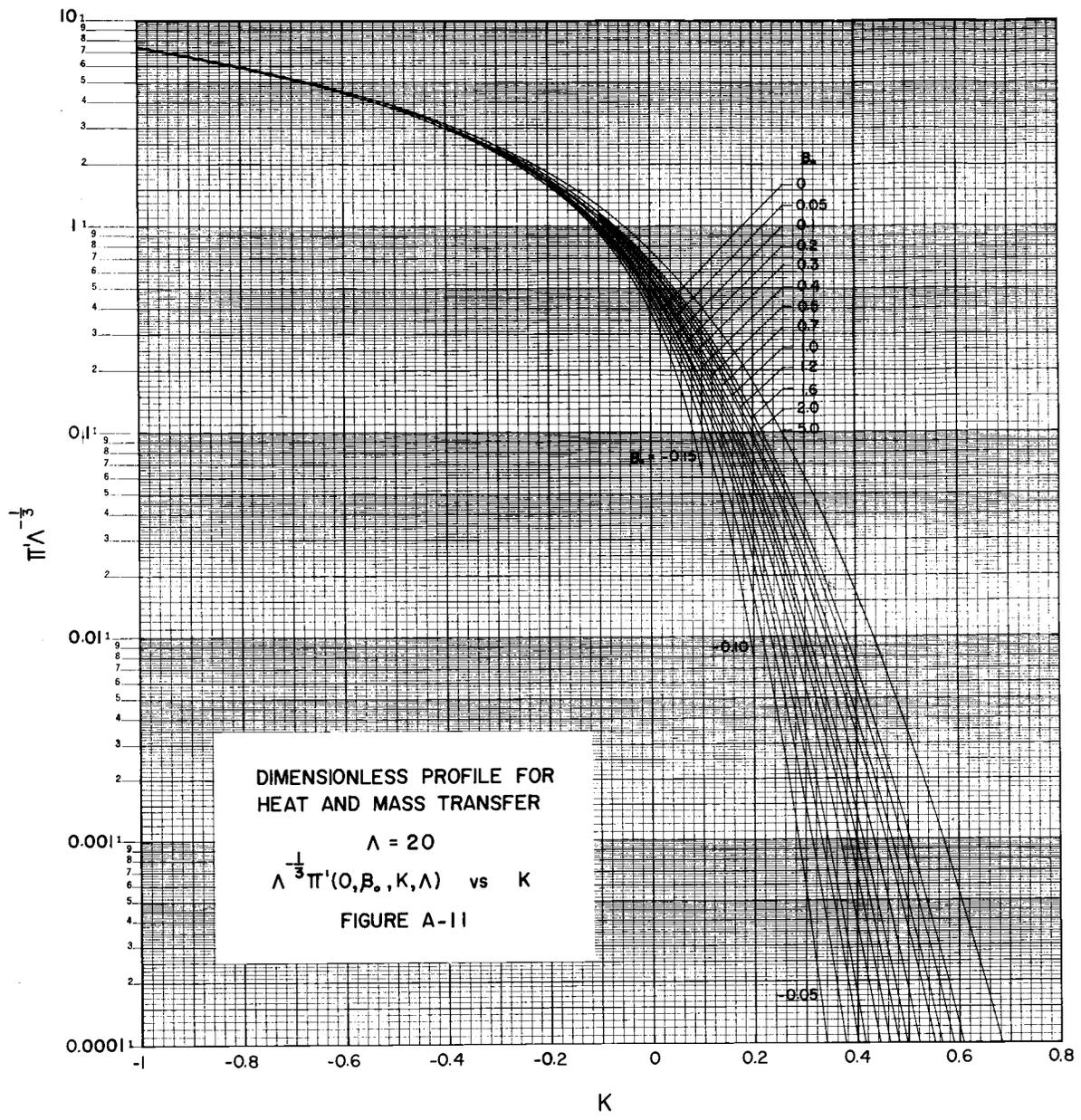


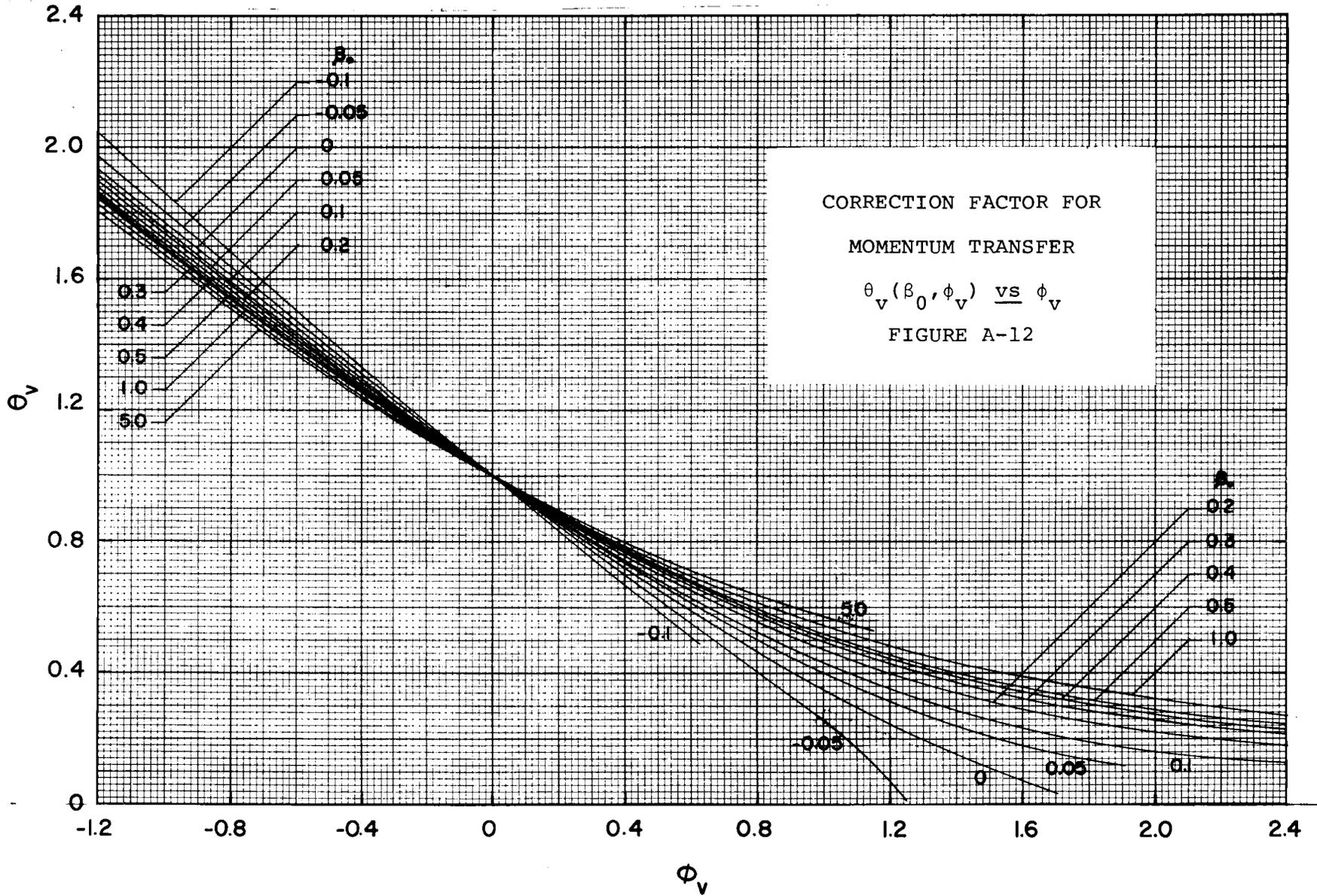


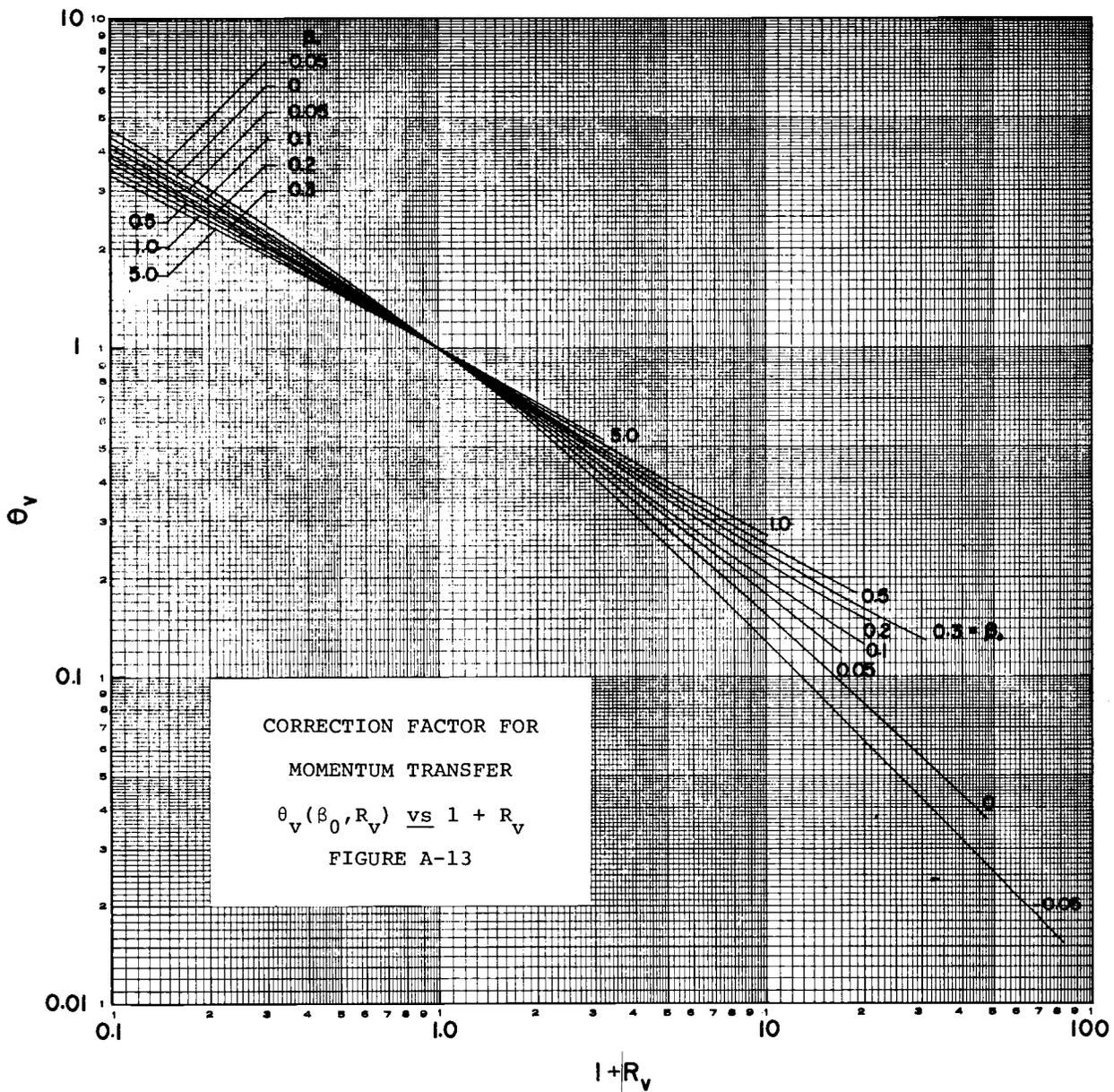


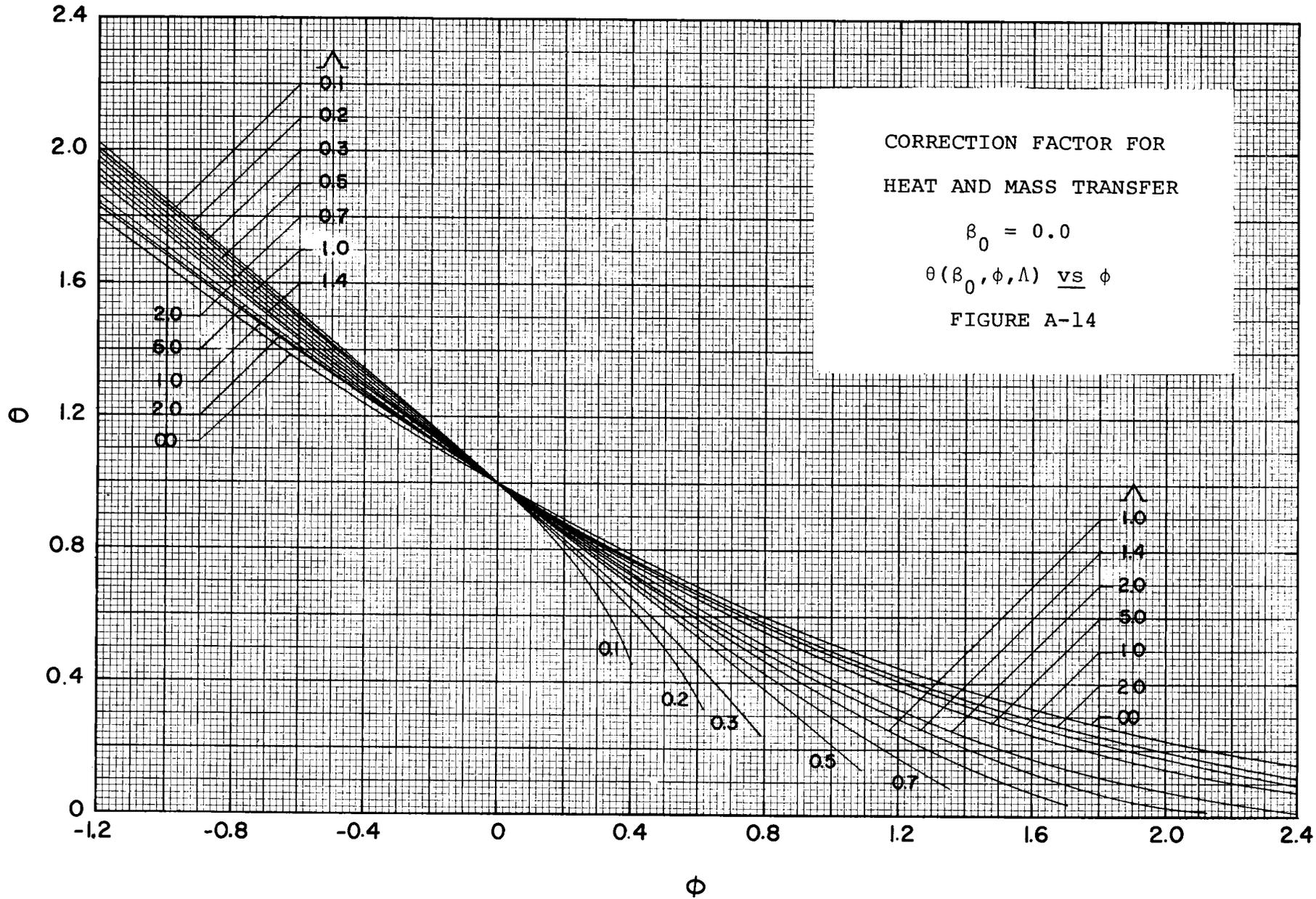


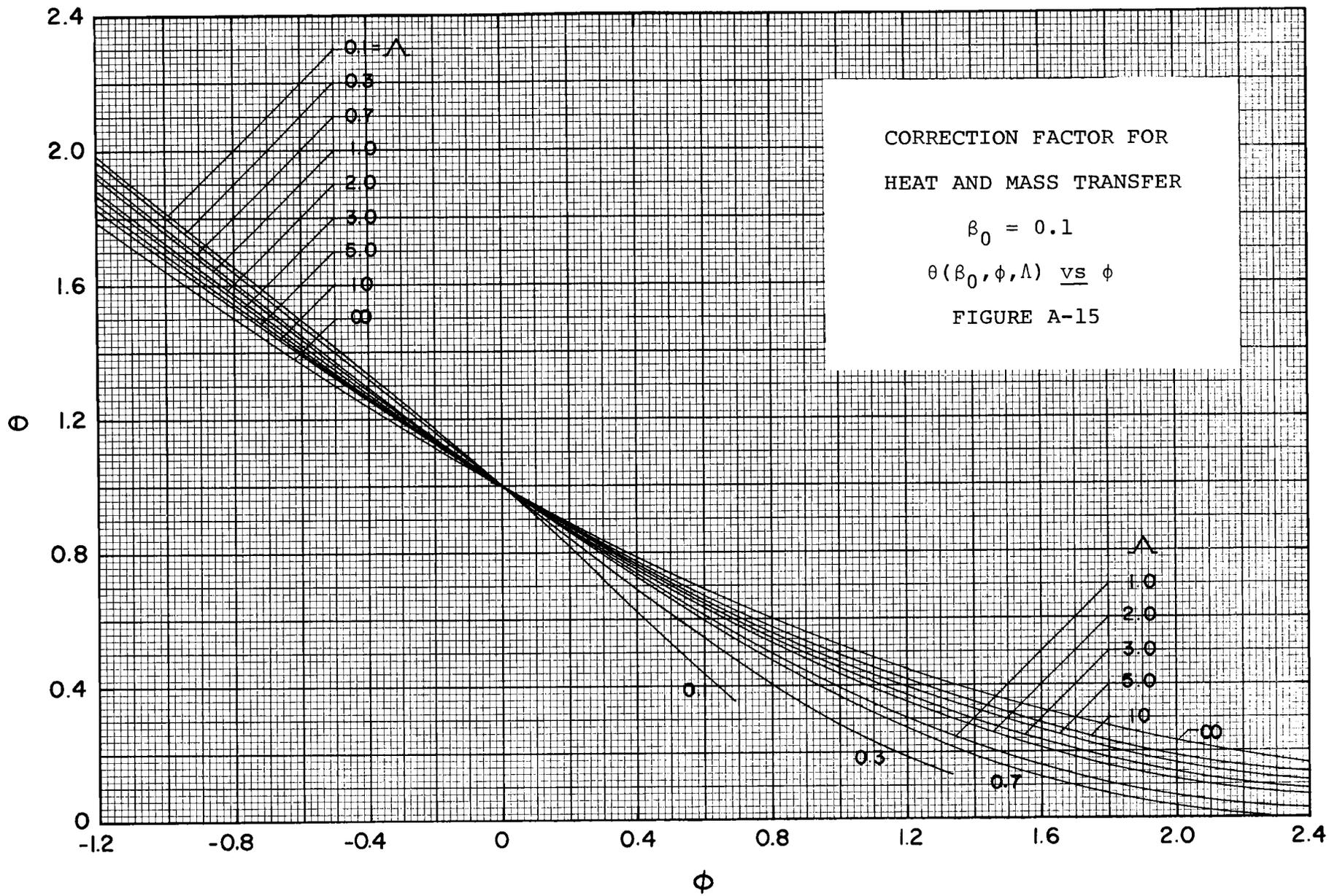


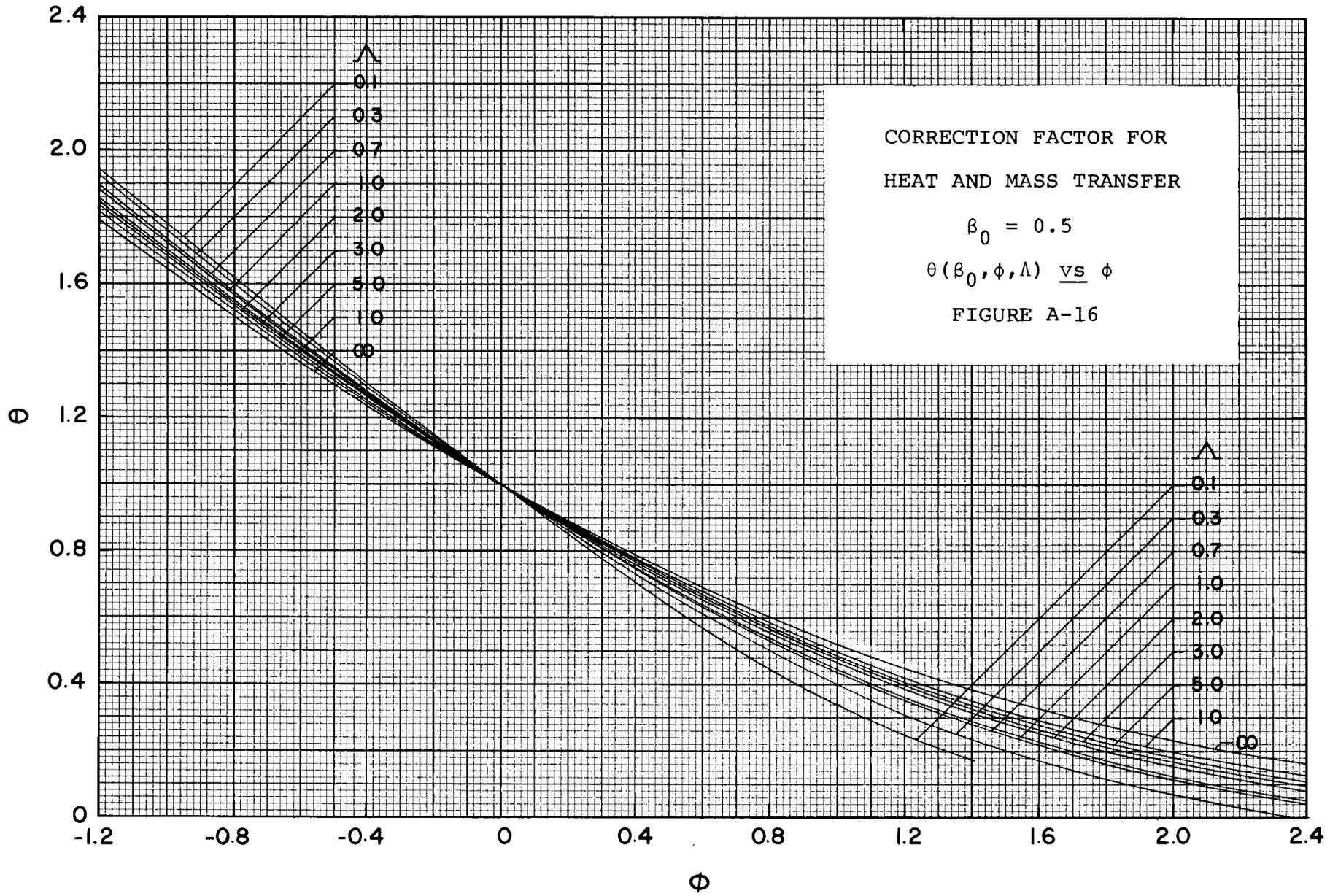


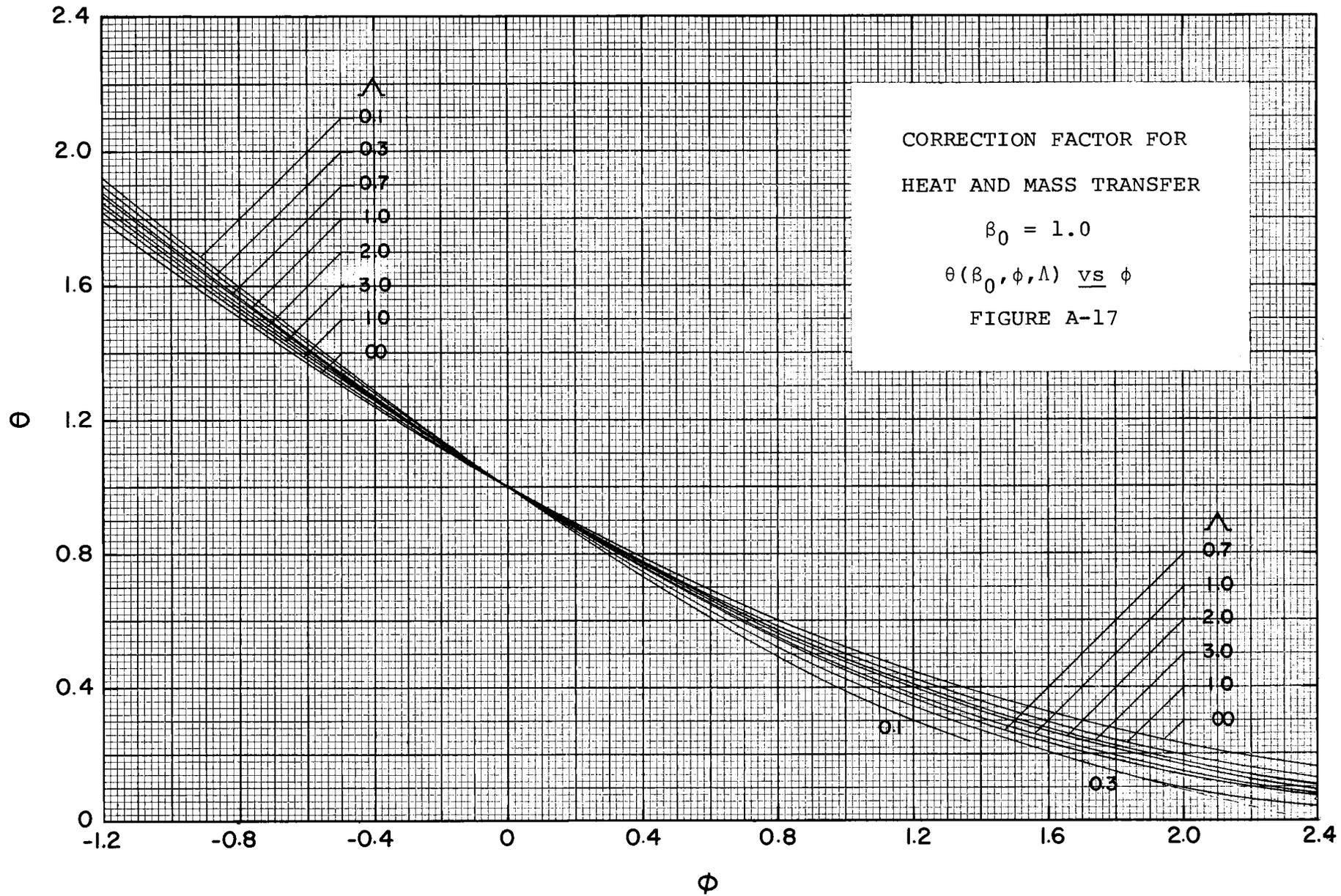


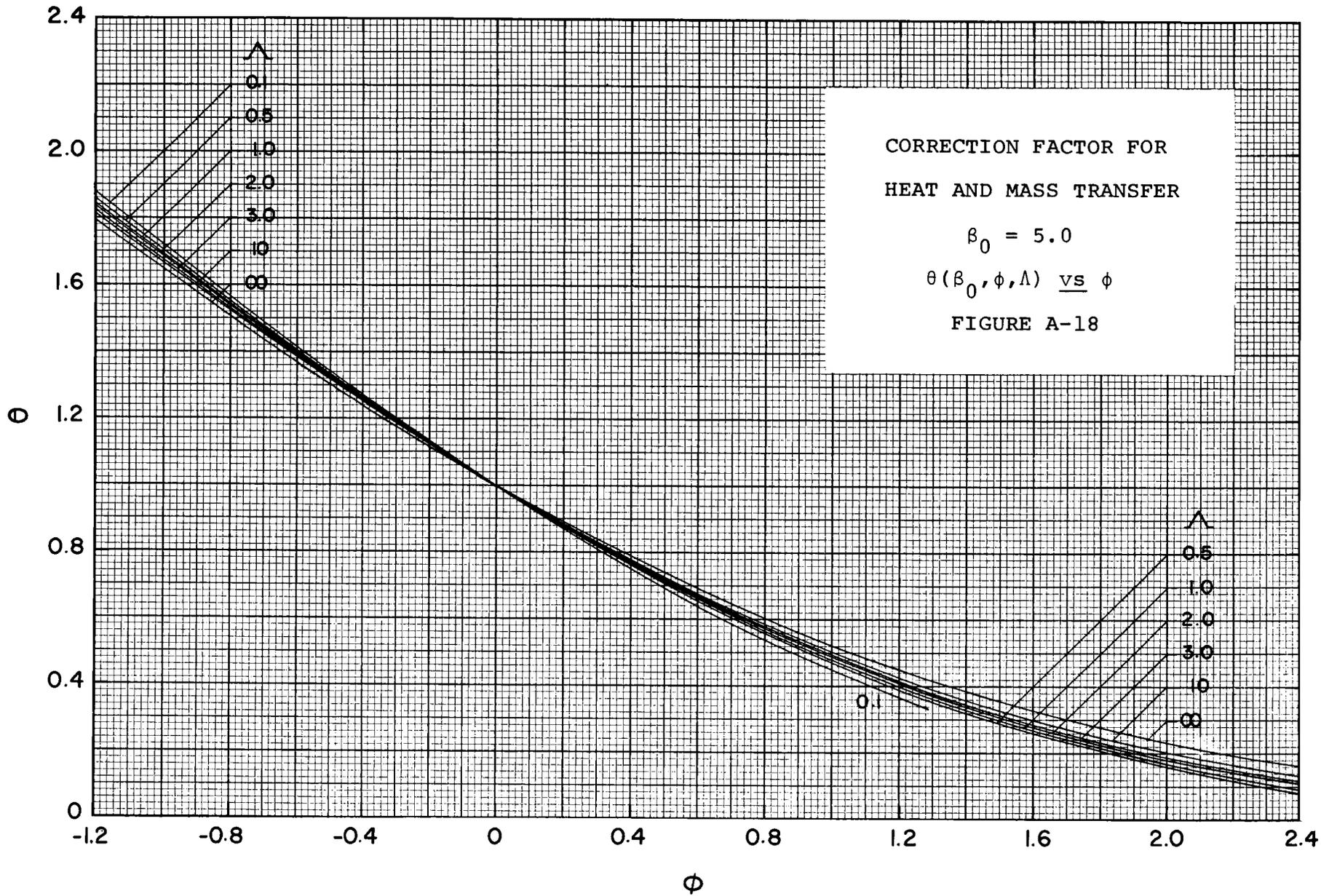


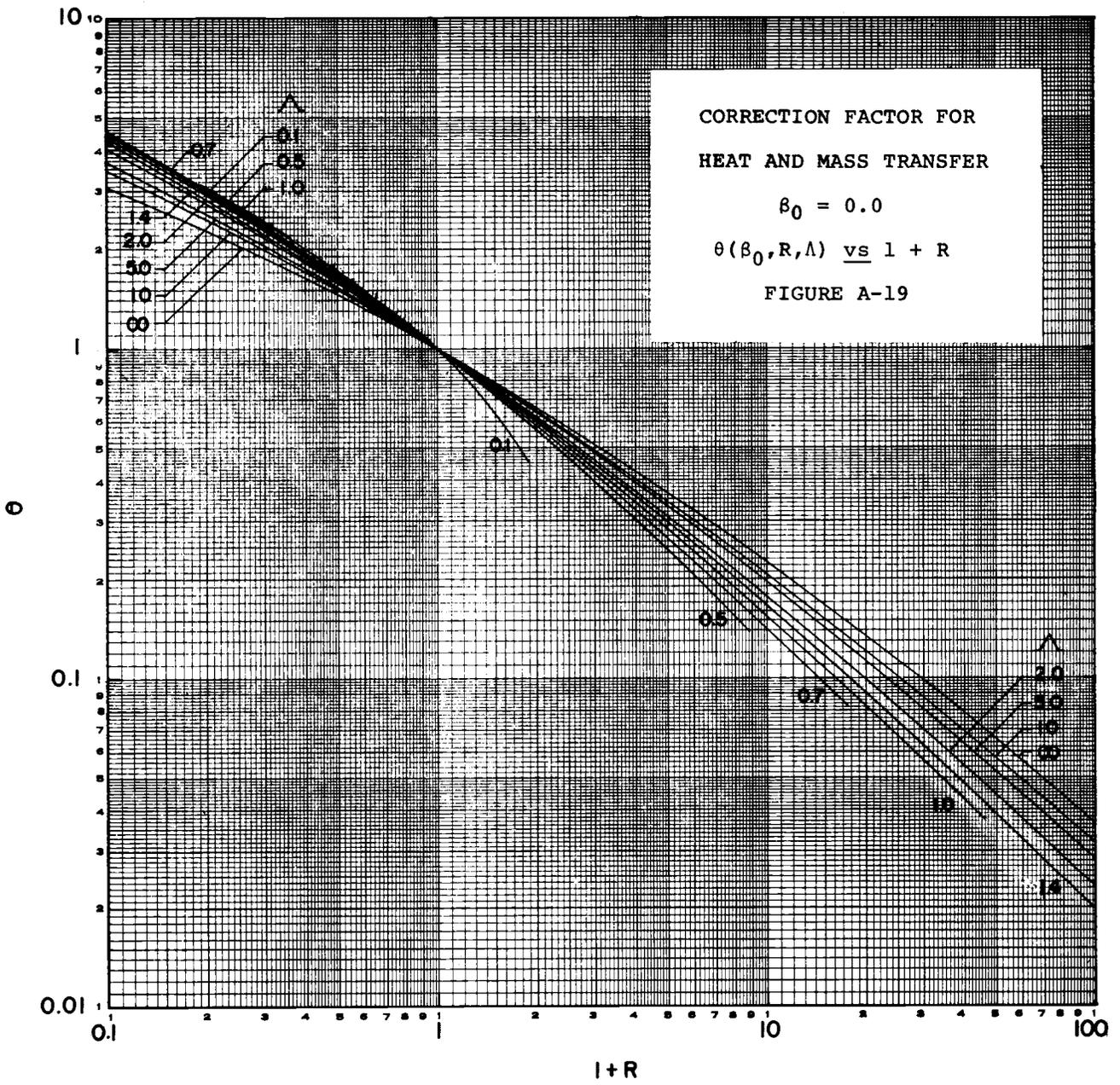


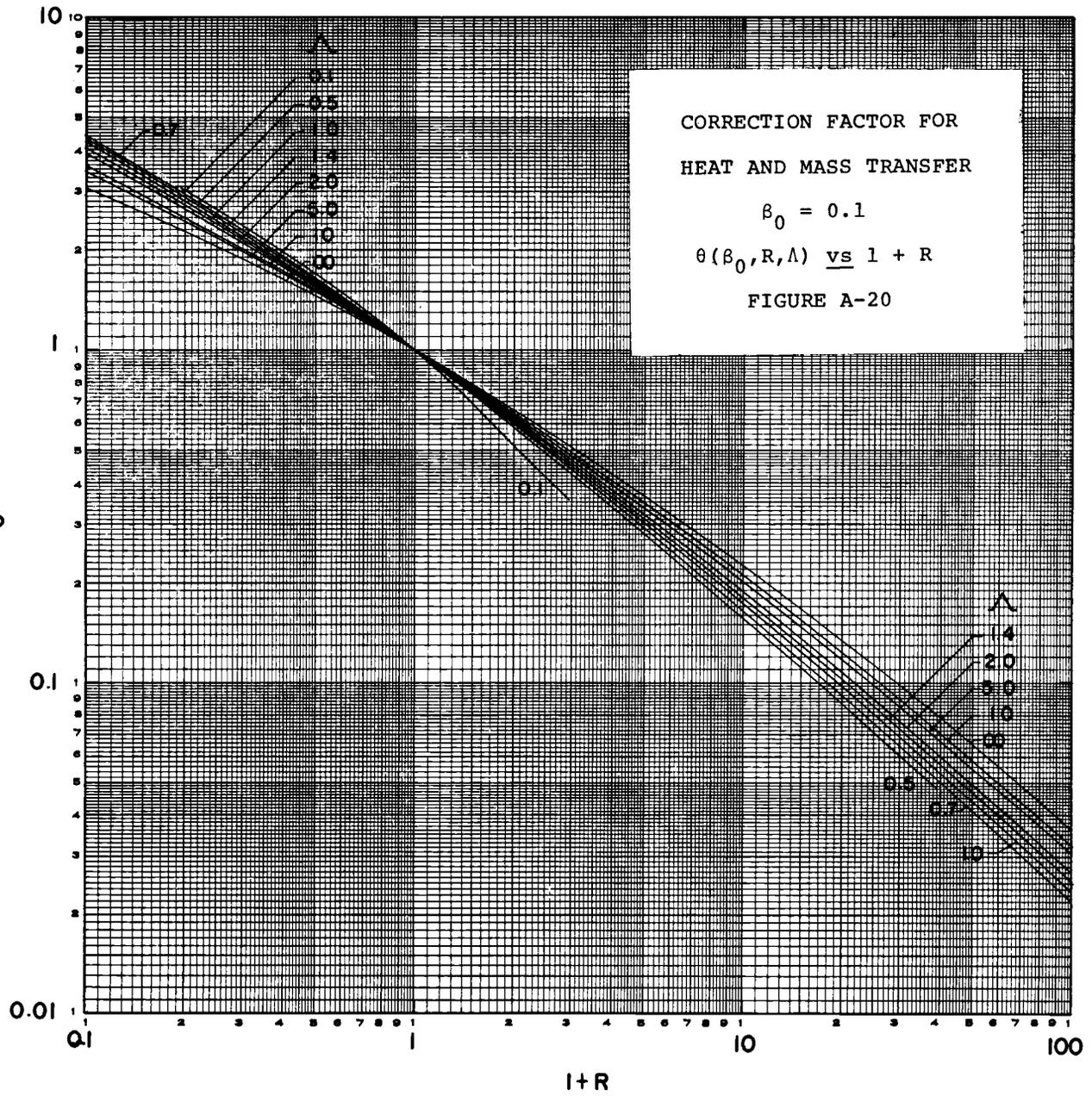


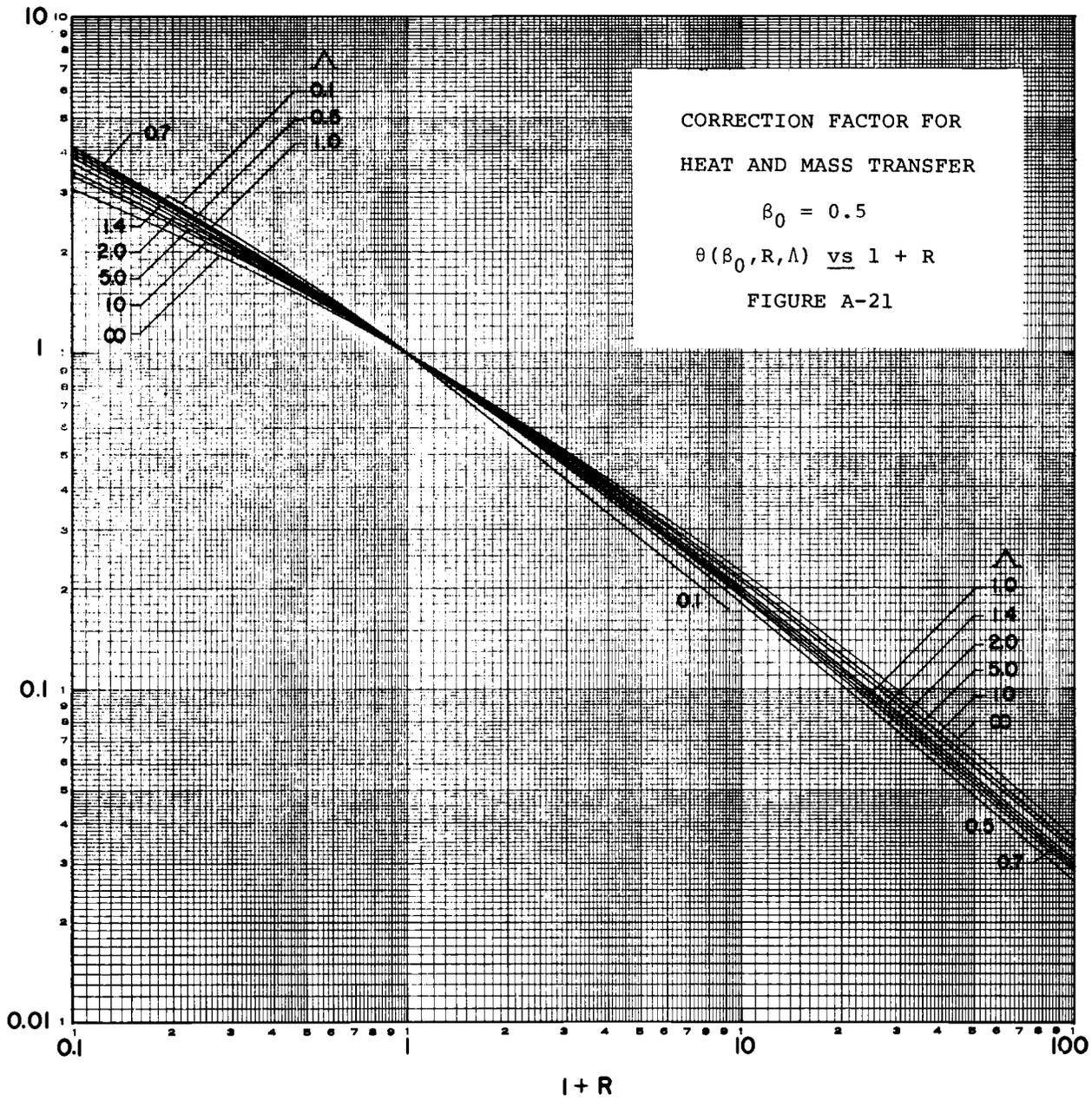


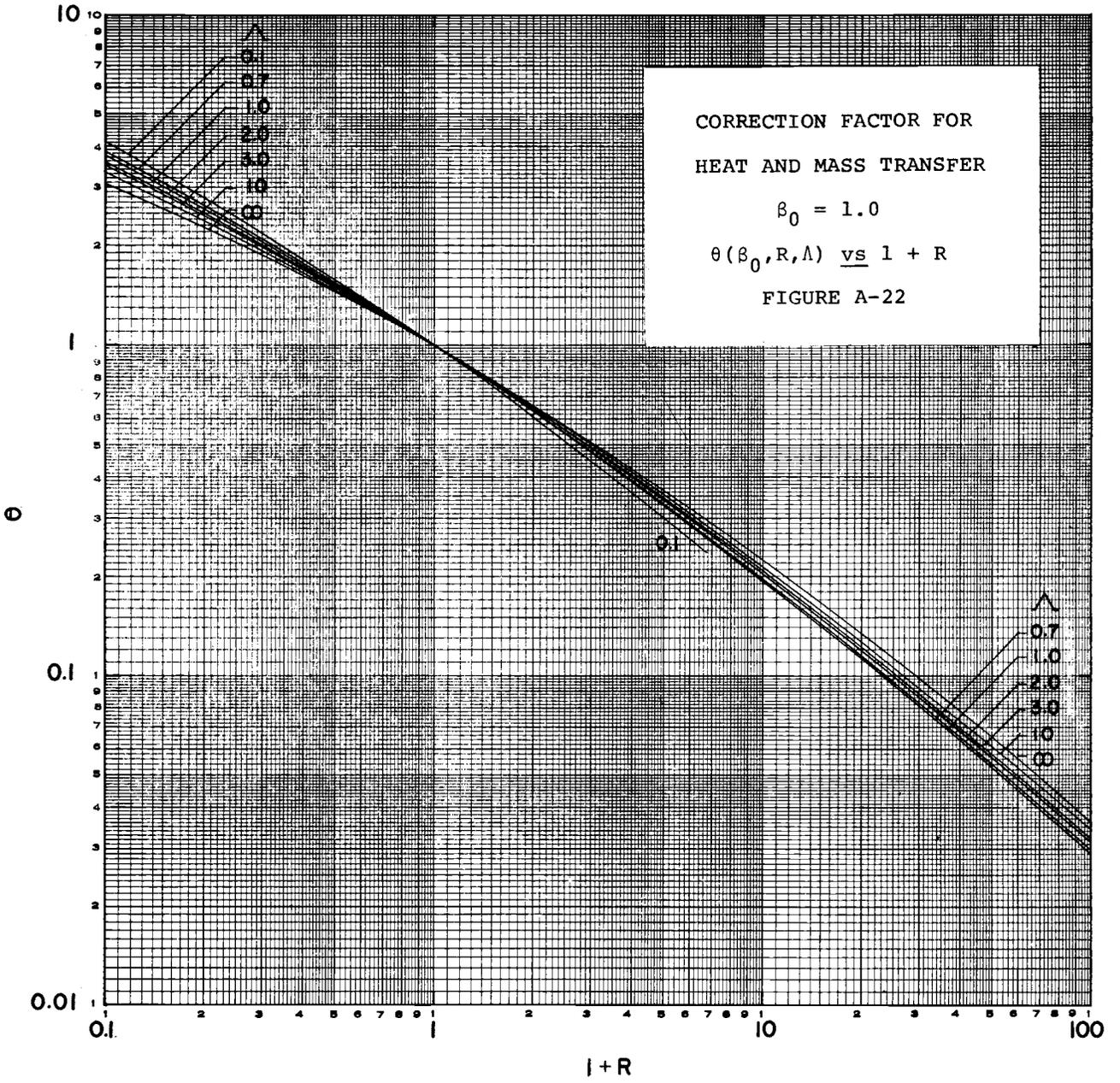


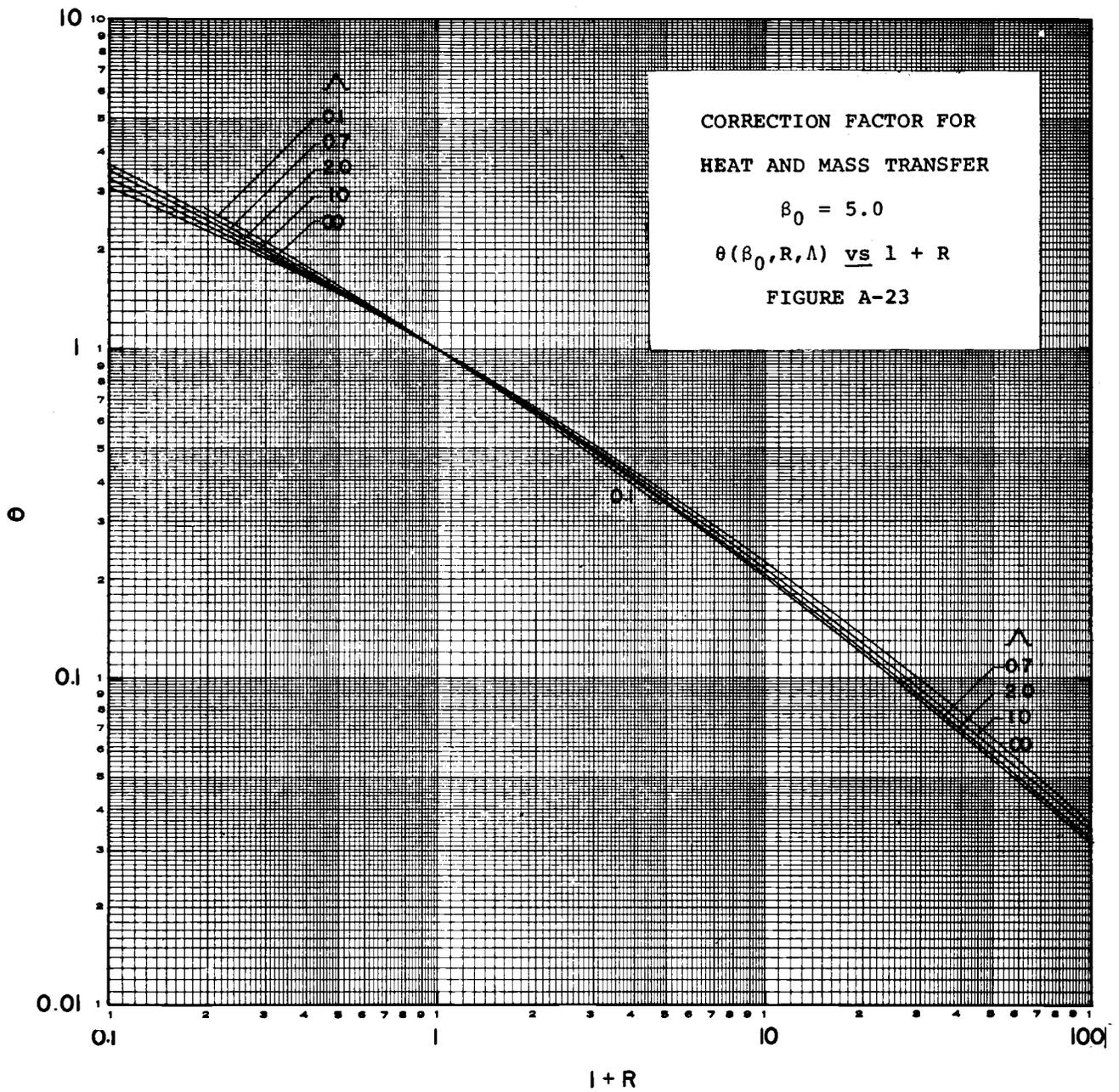


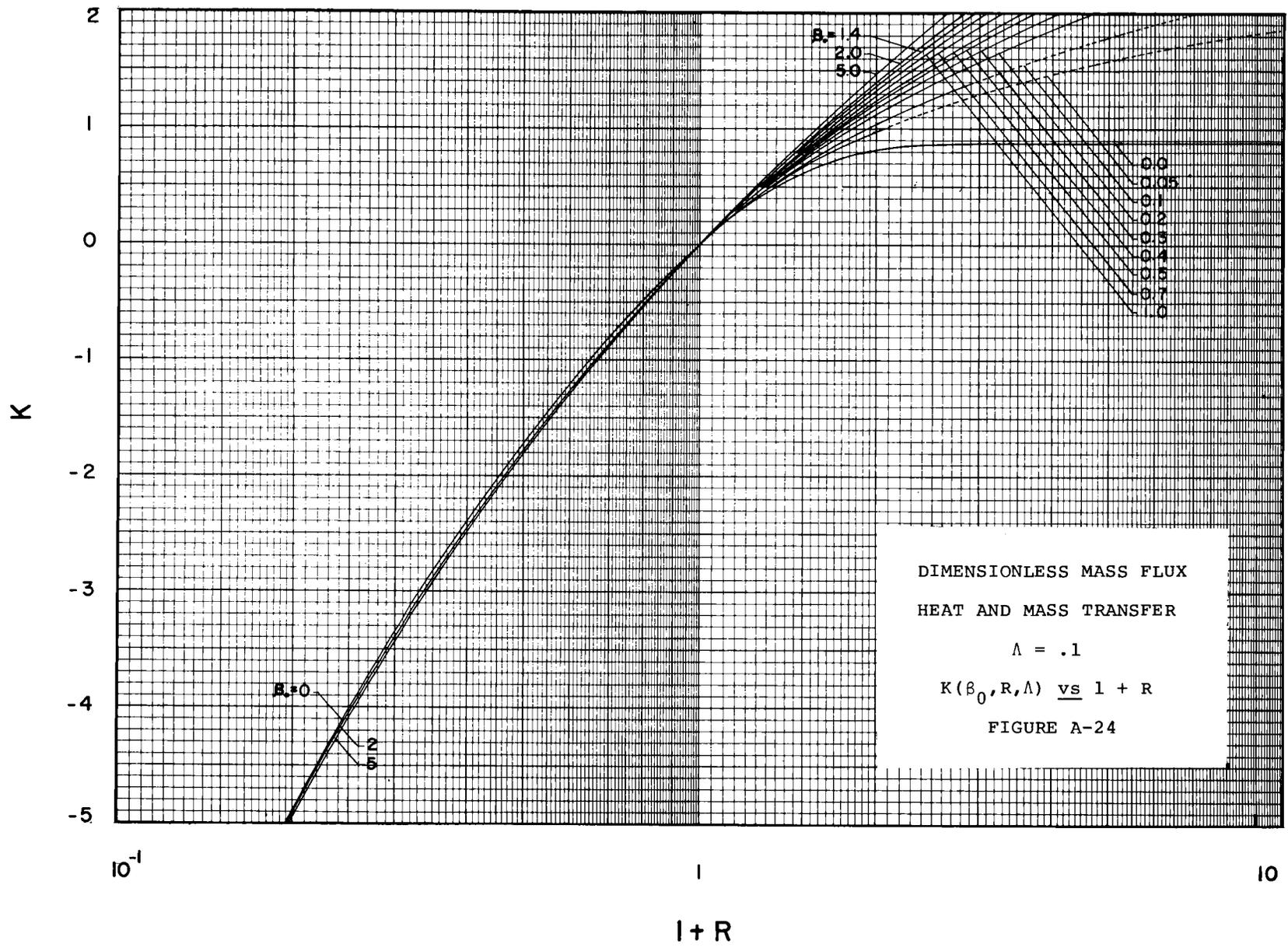




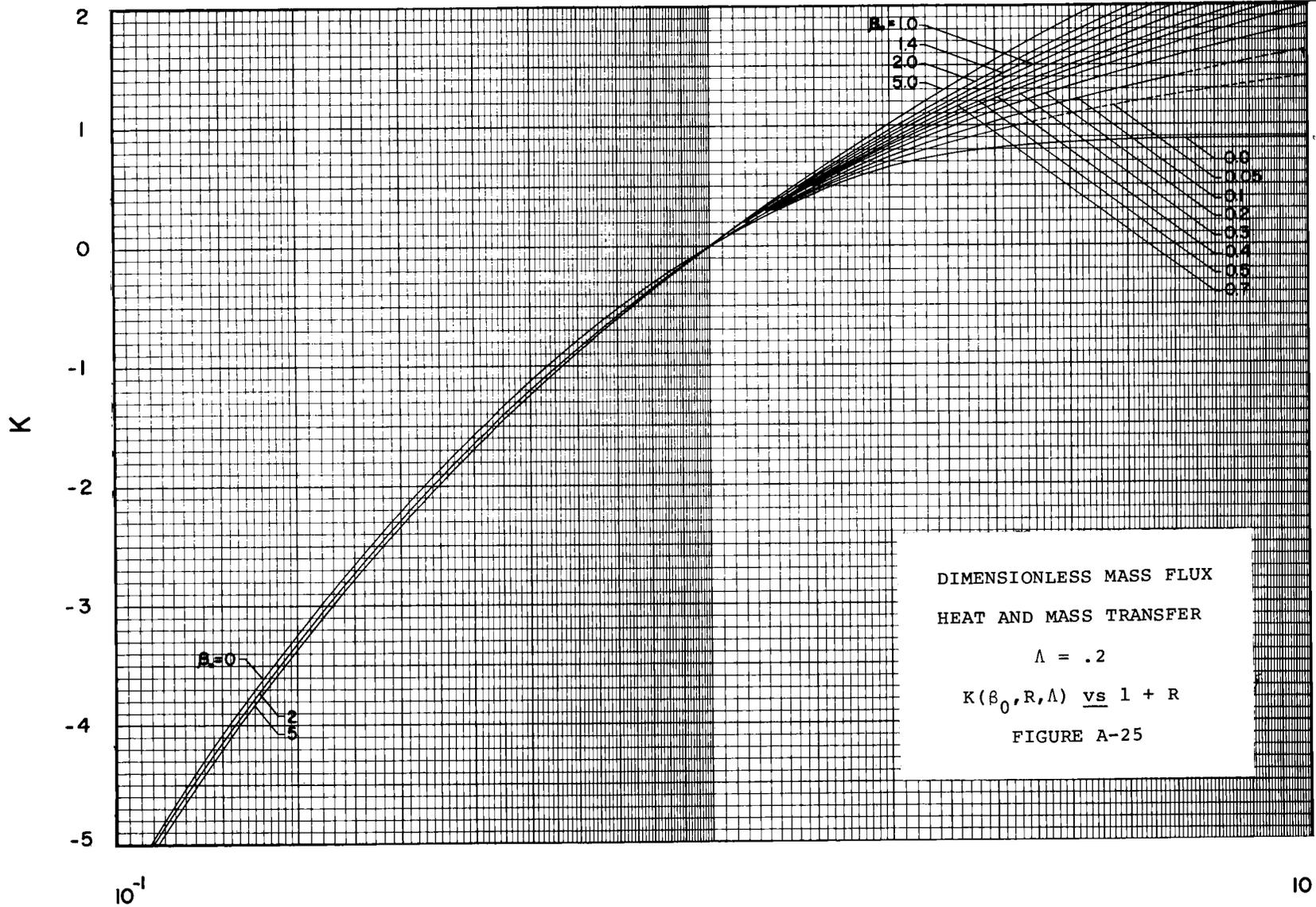


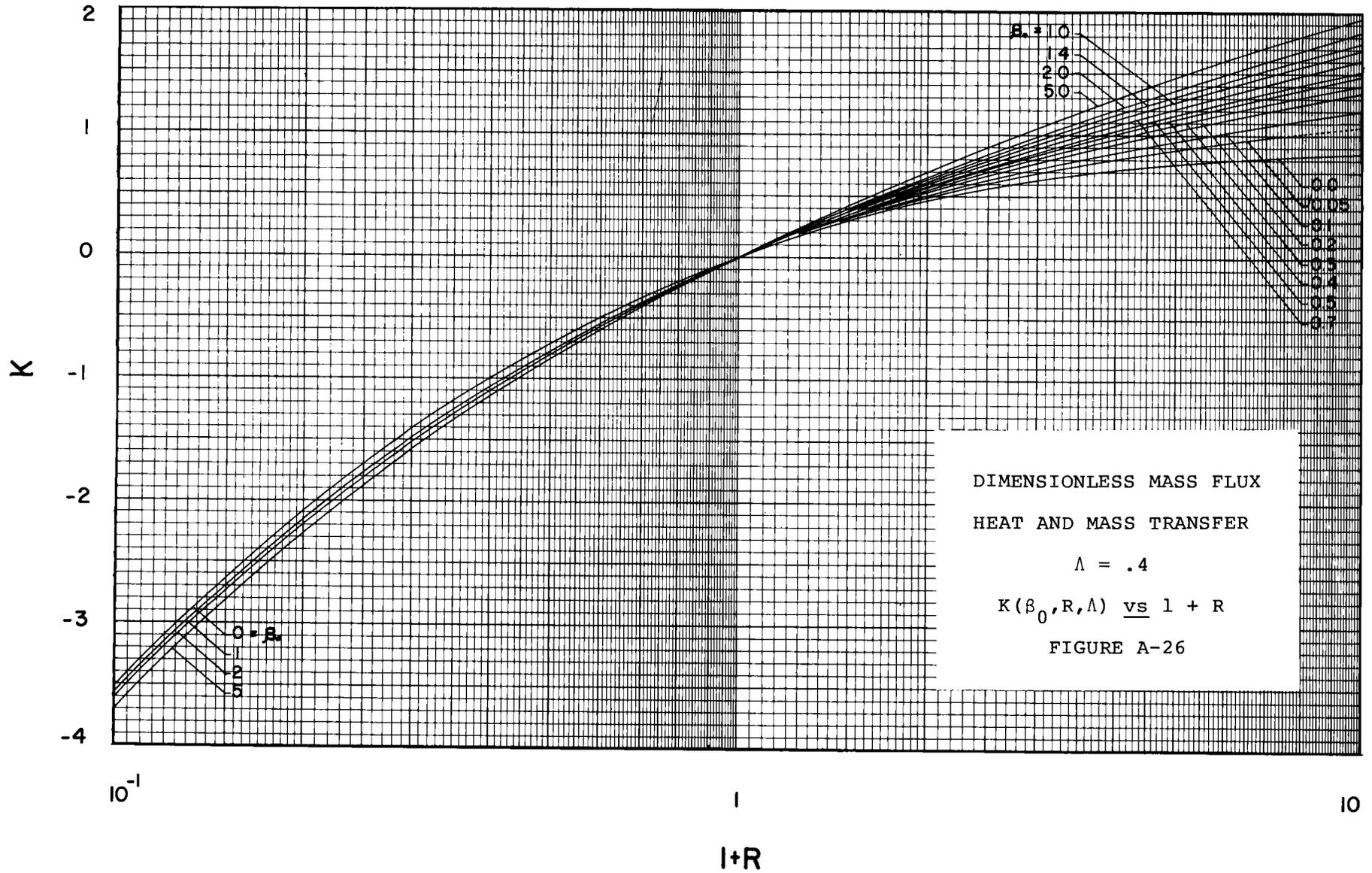


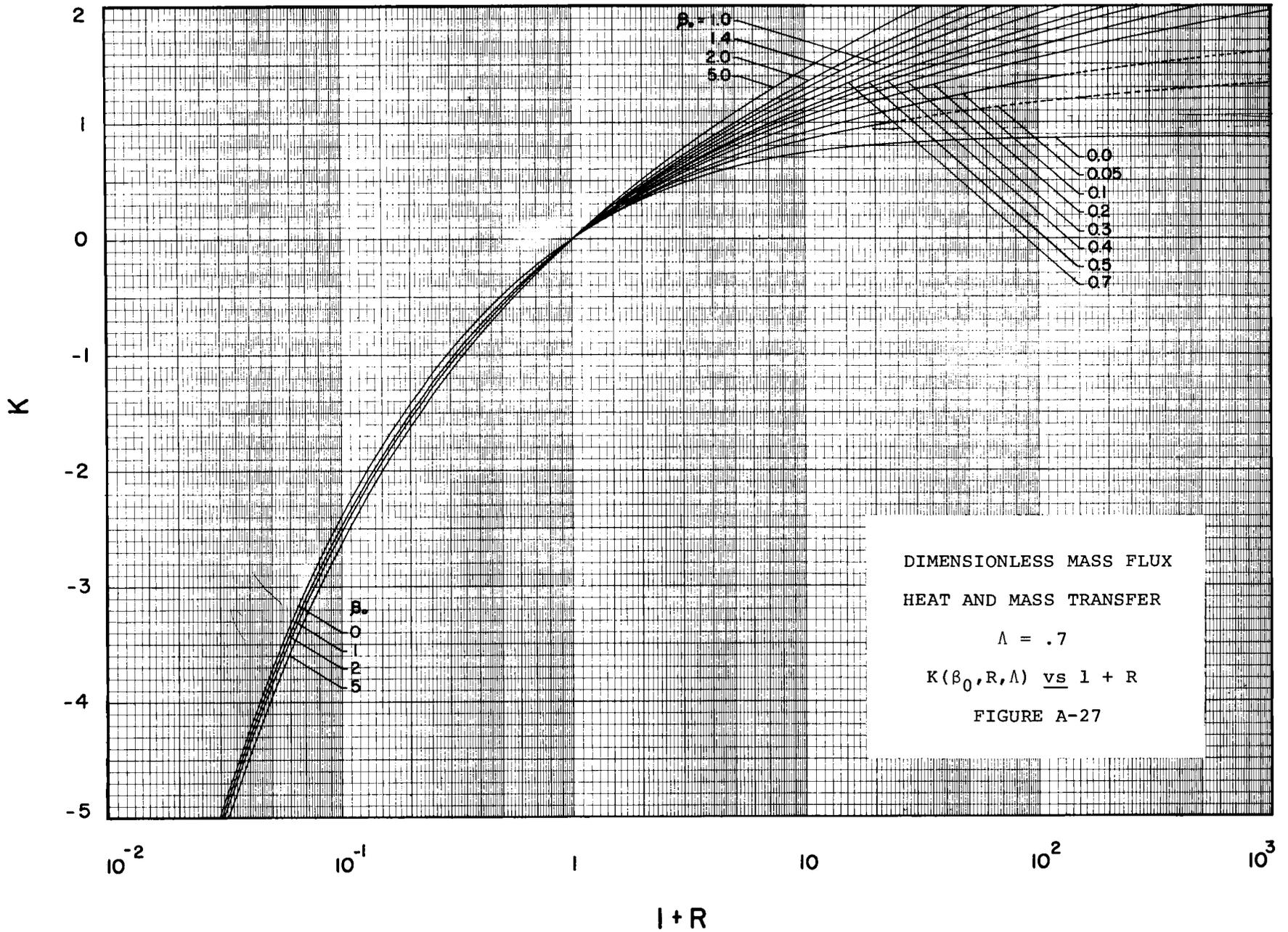


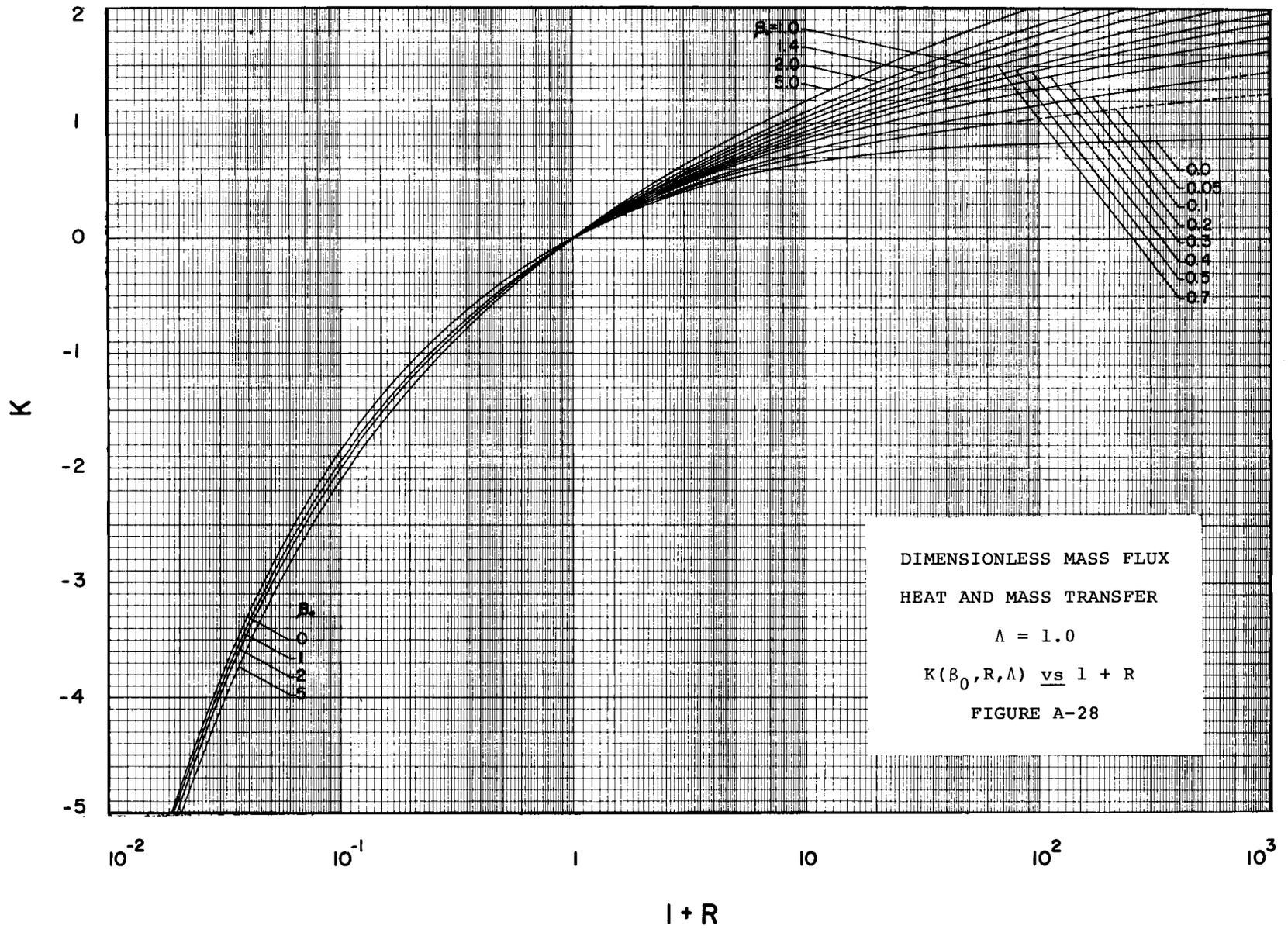


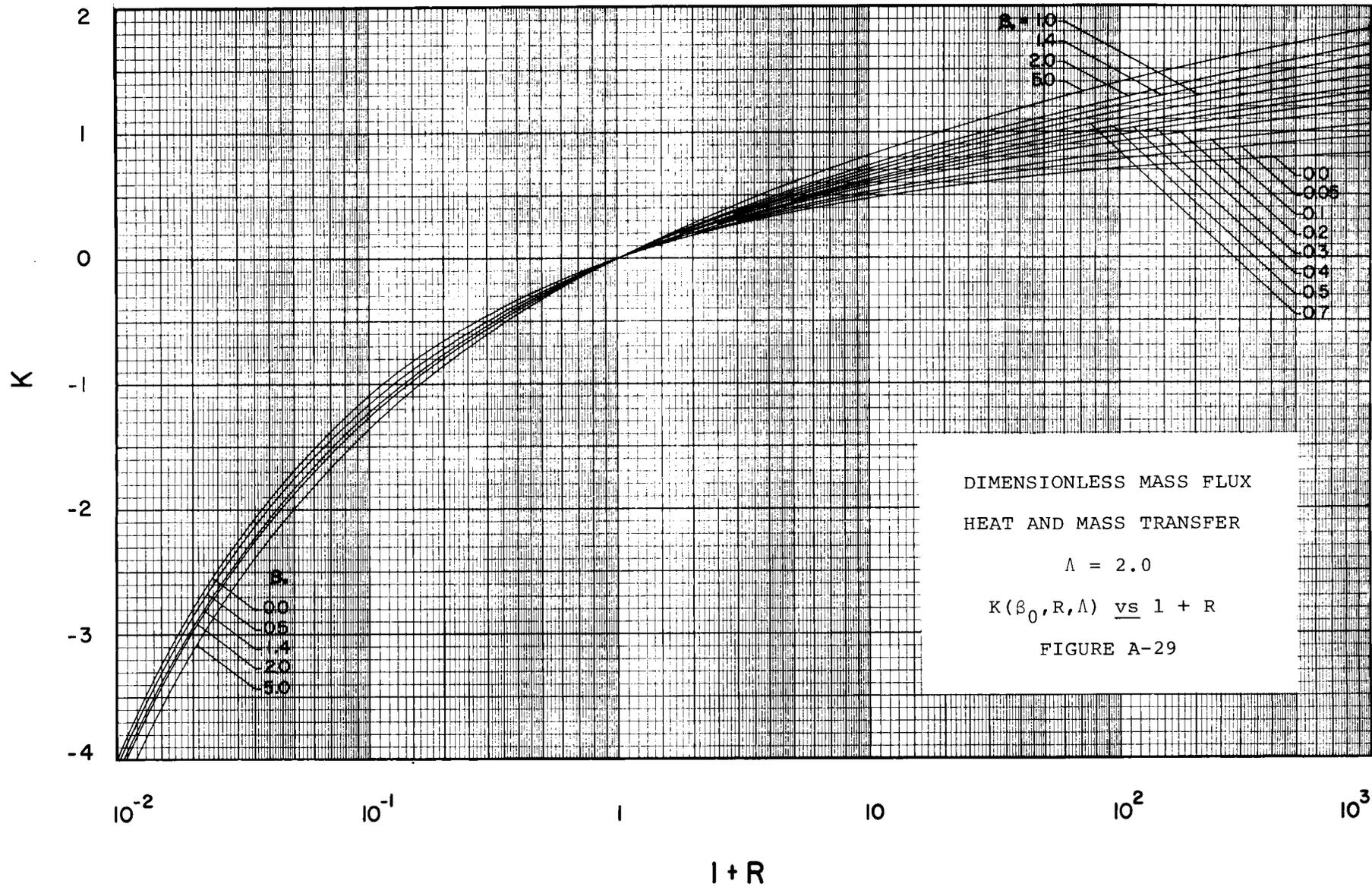
DIMENSIONLESS MASS FLUX  
HEAT AND MASS TRANSFER  
 $\Lambda = .1$   
 $K(\beta_0, R, \Lambda)$  vs  $1 + R$   
FIGURE A-24

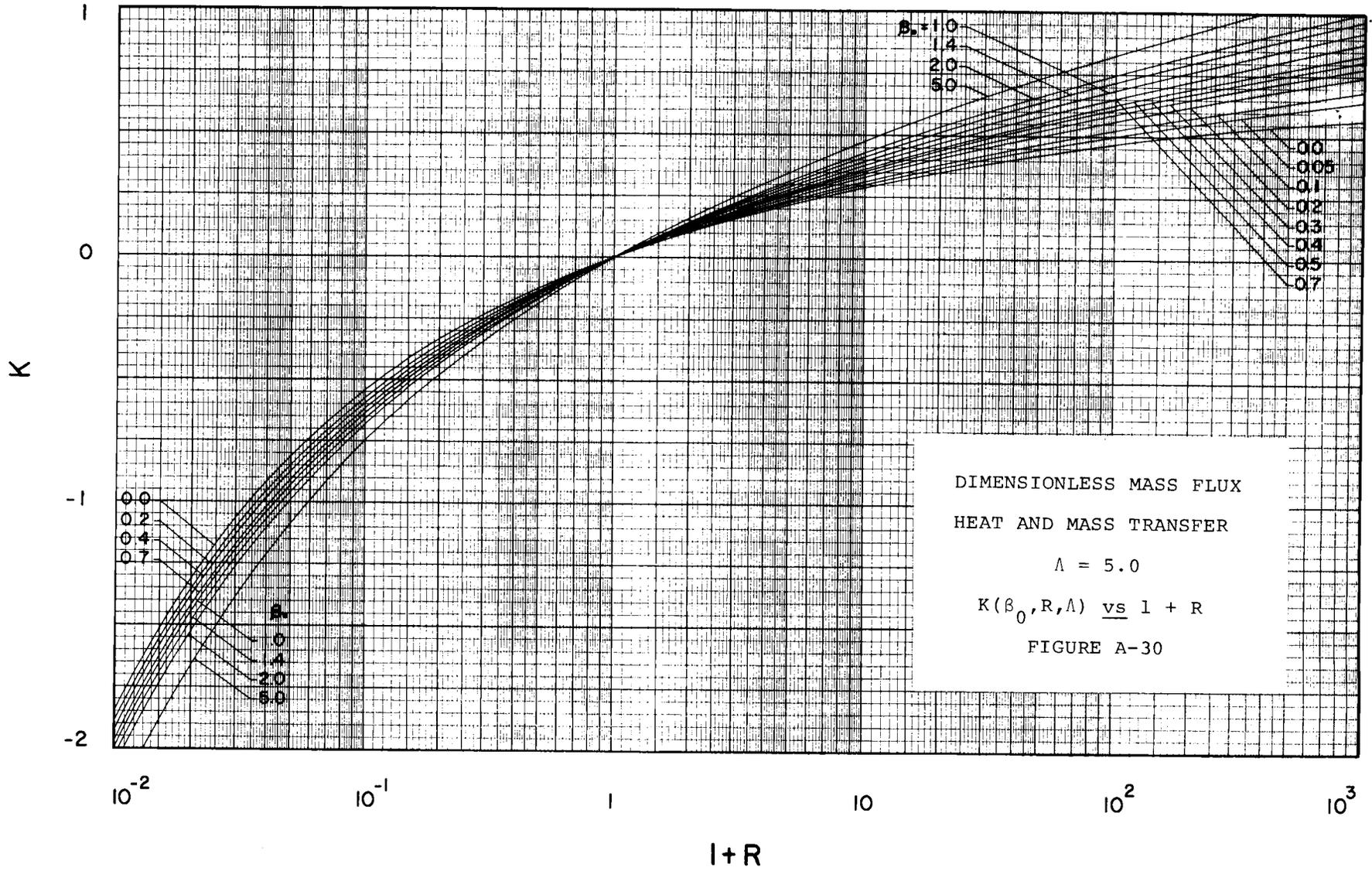


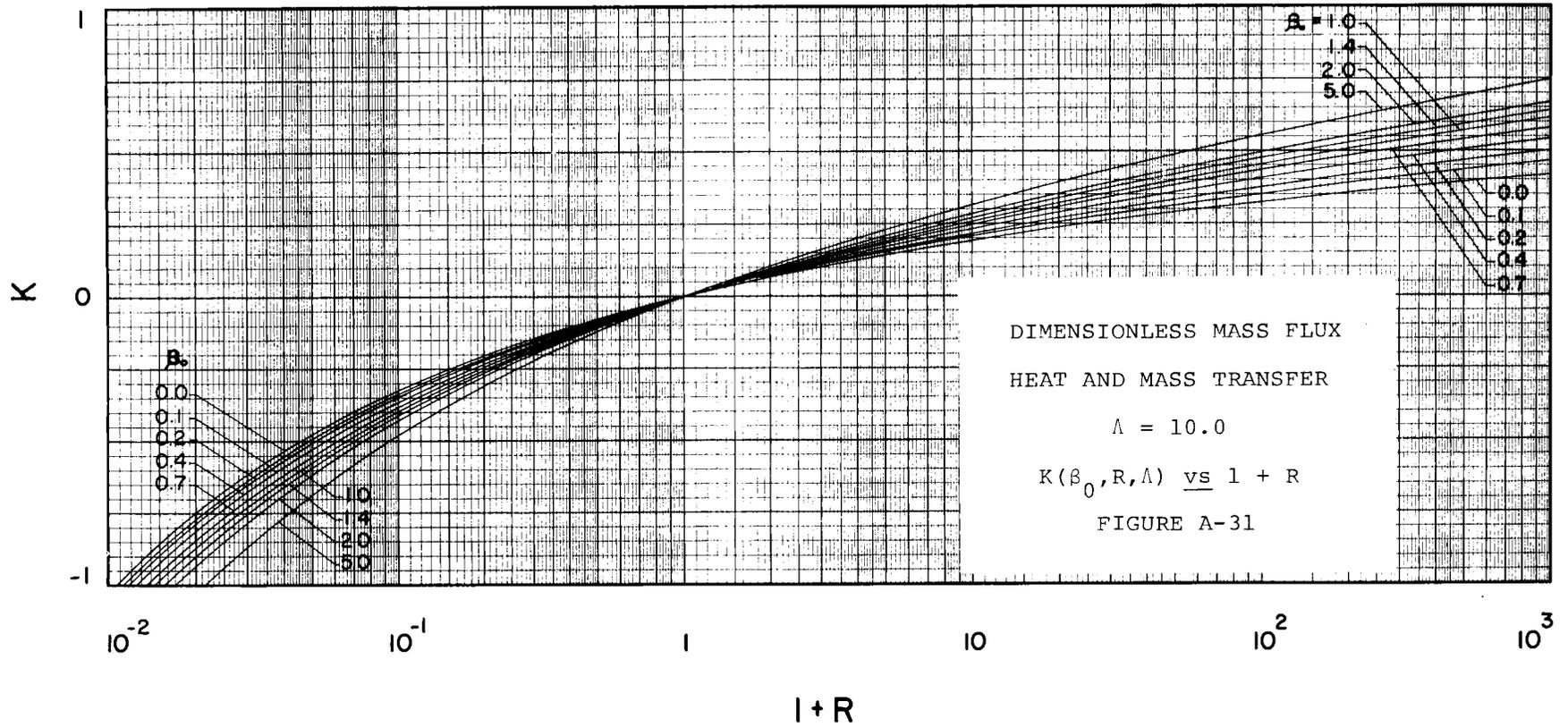


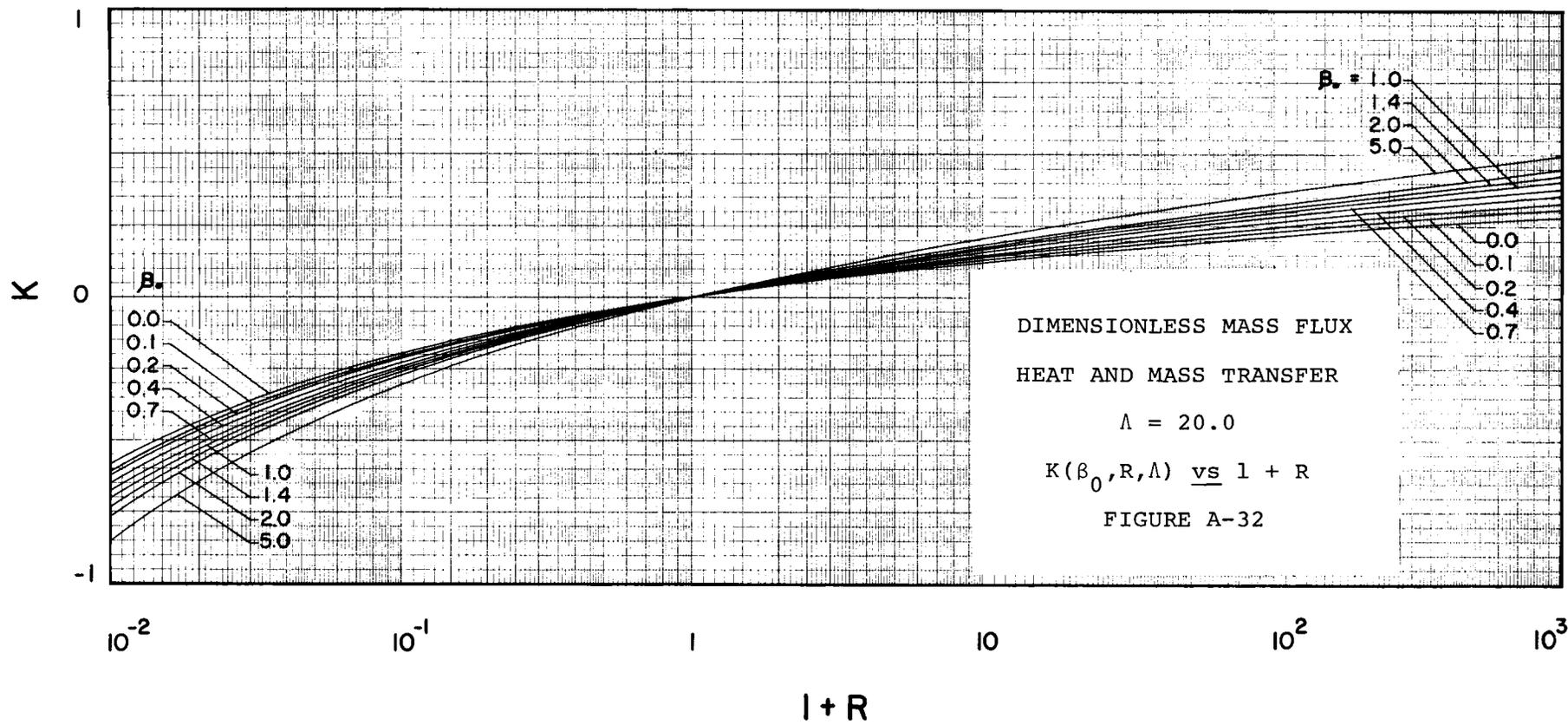


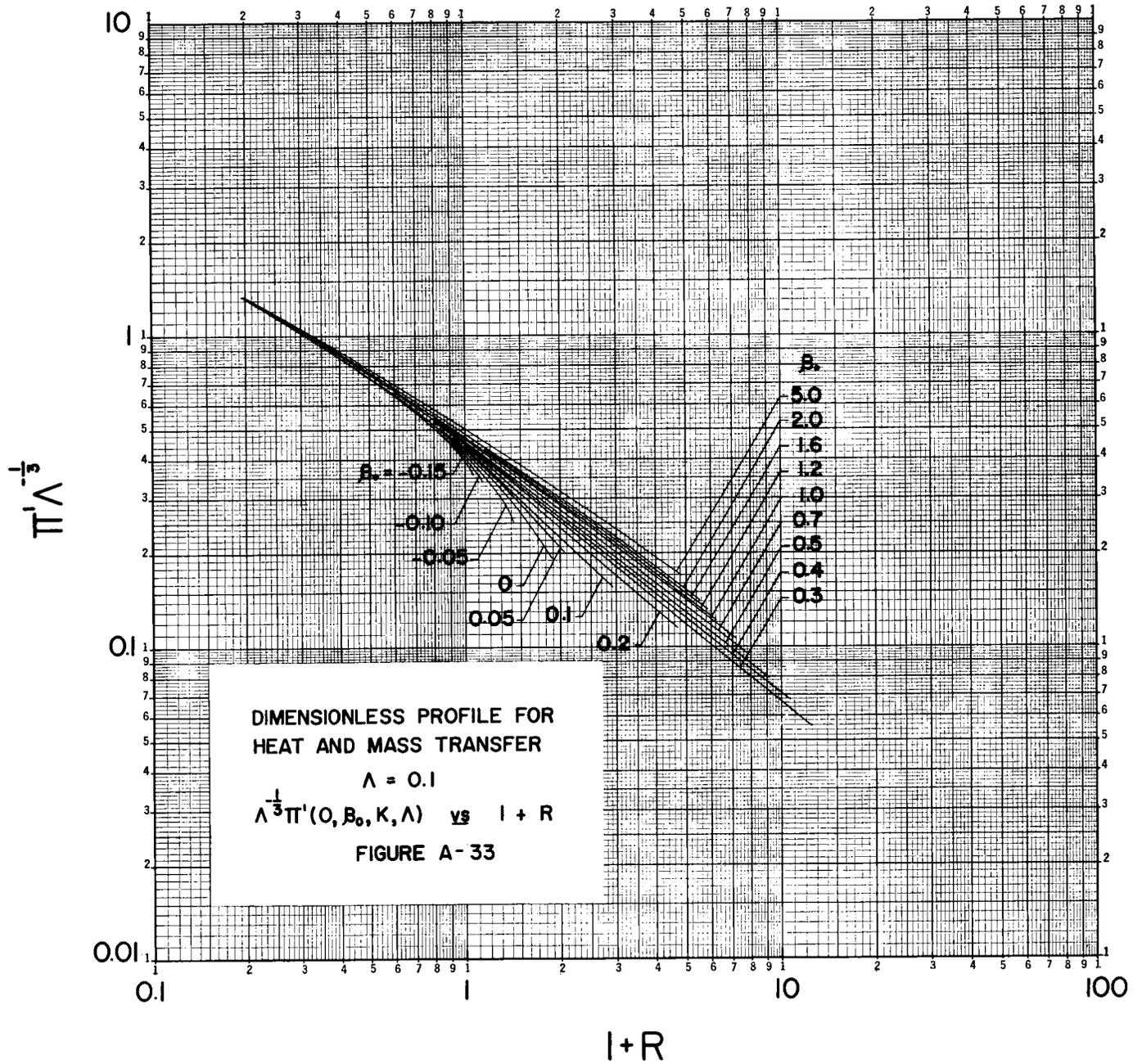




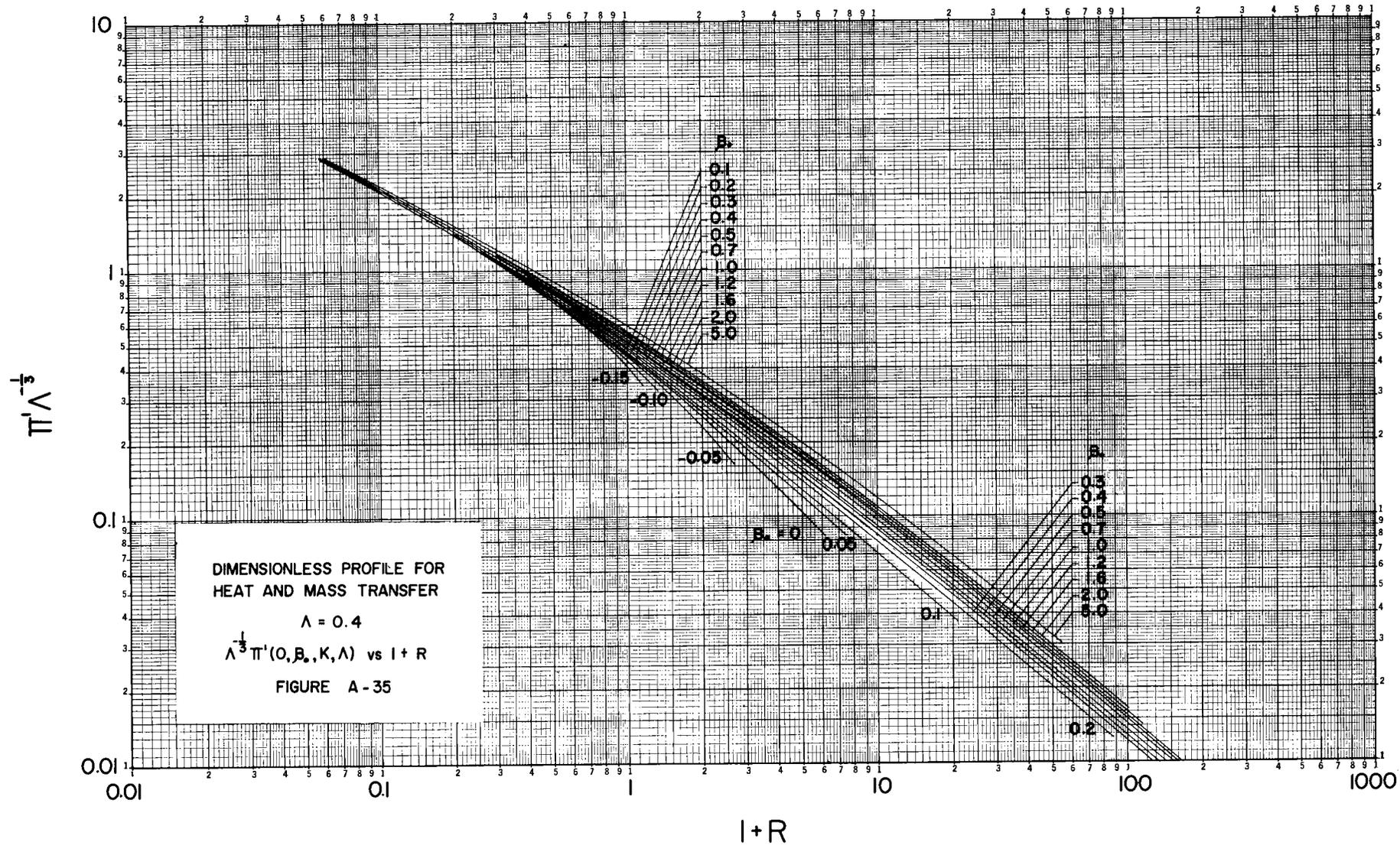


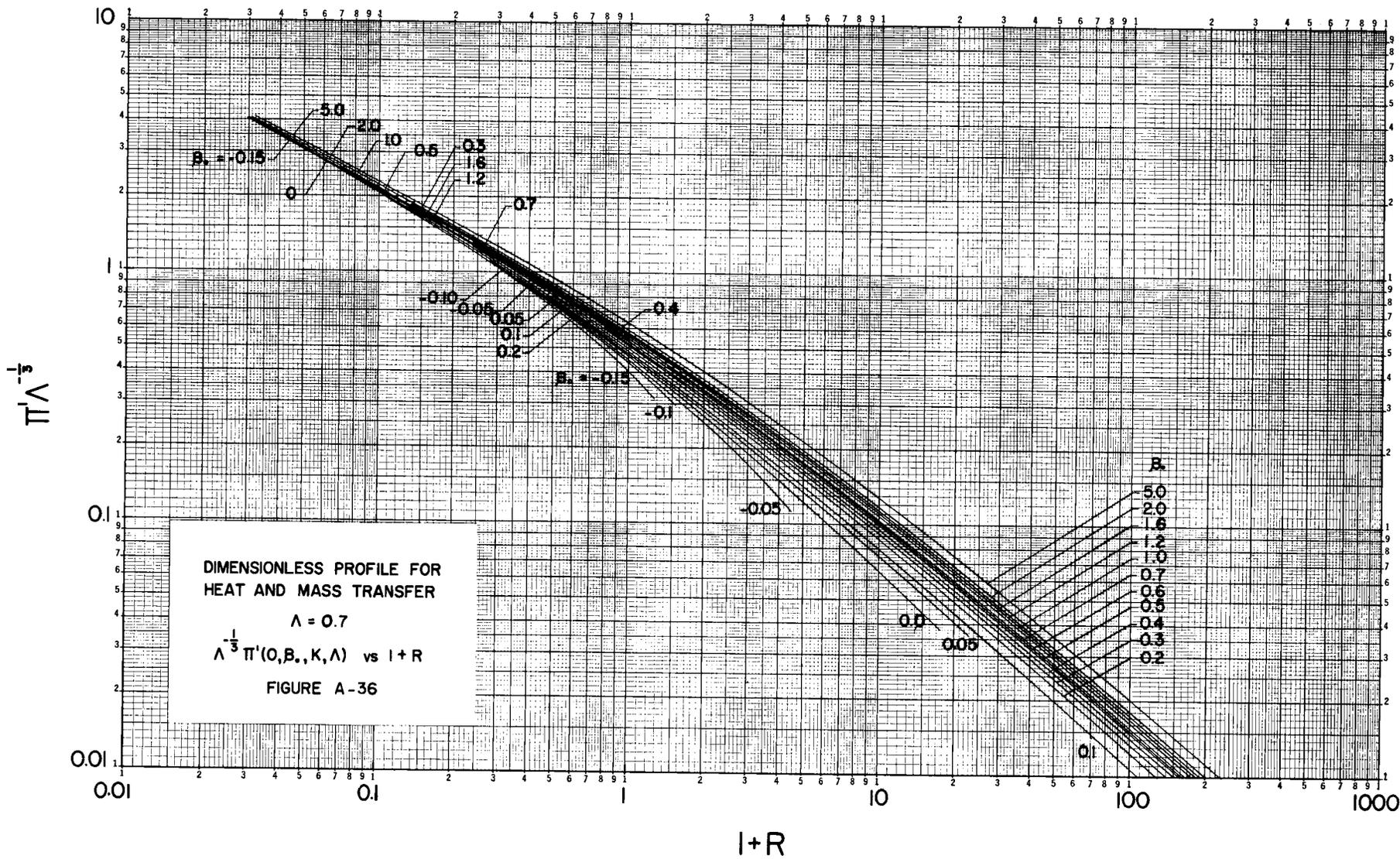


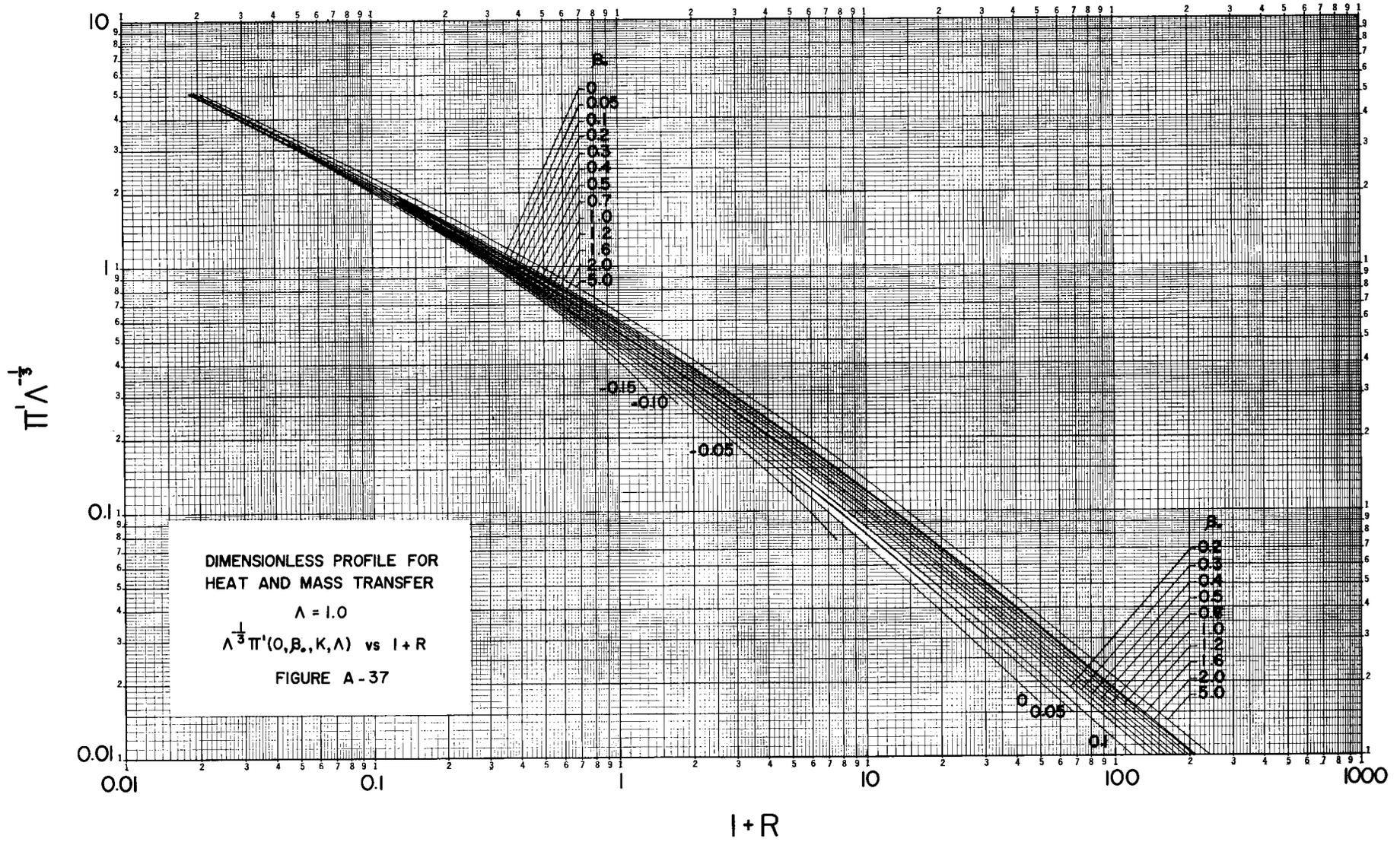




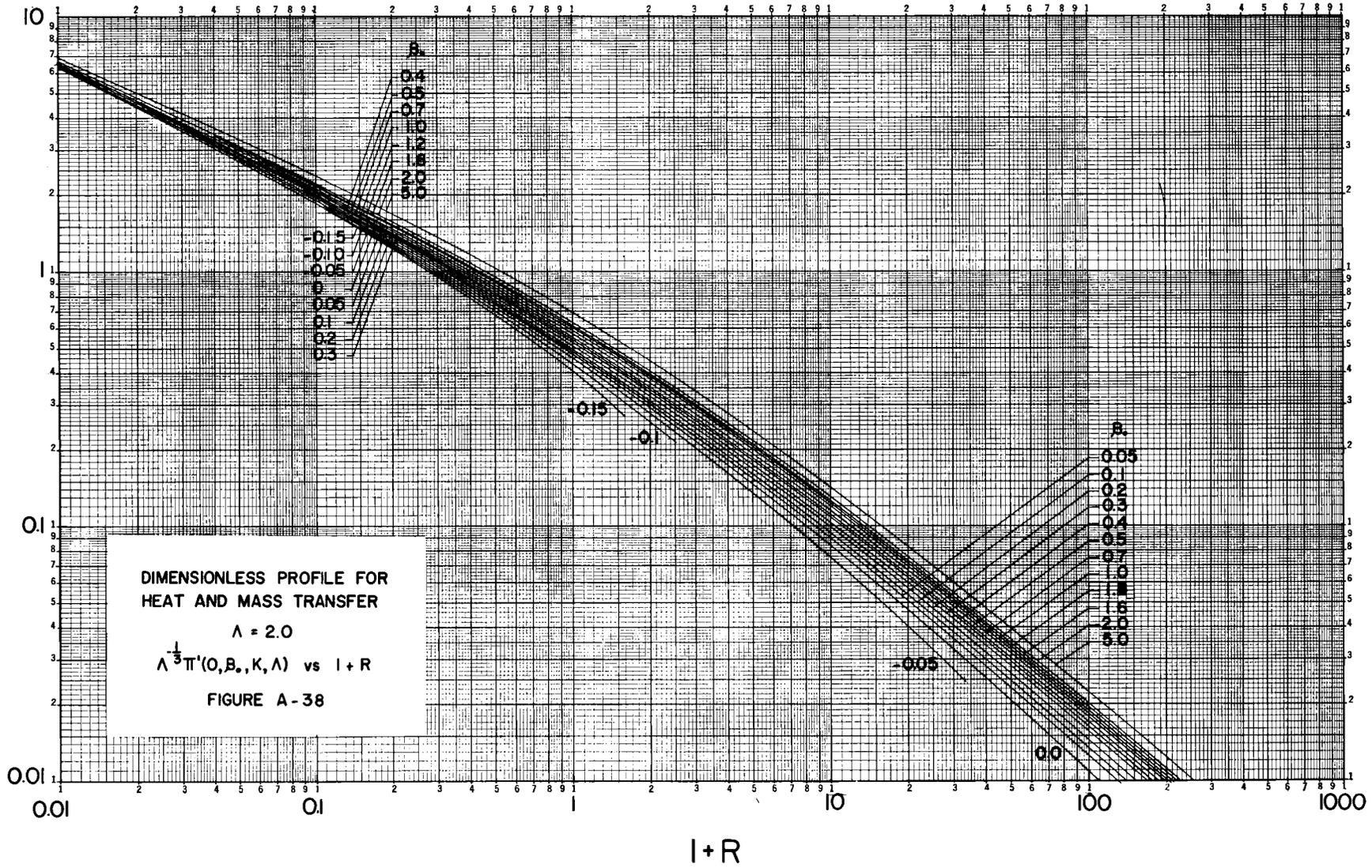


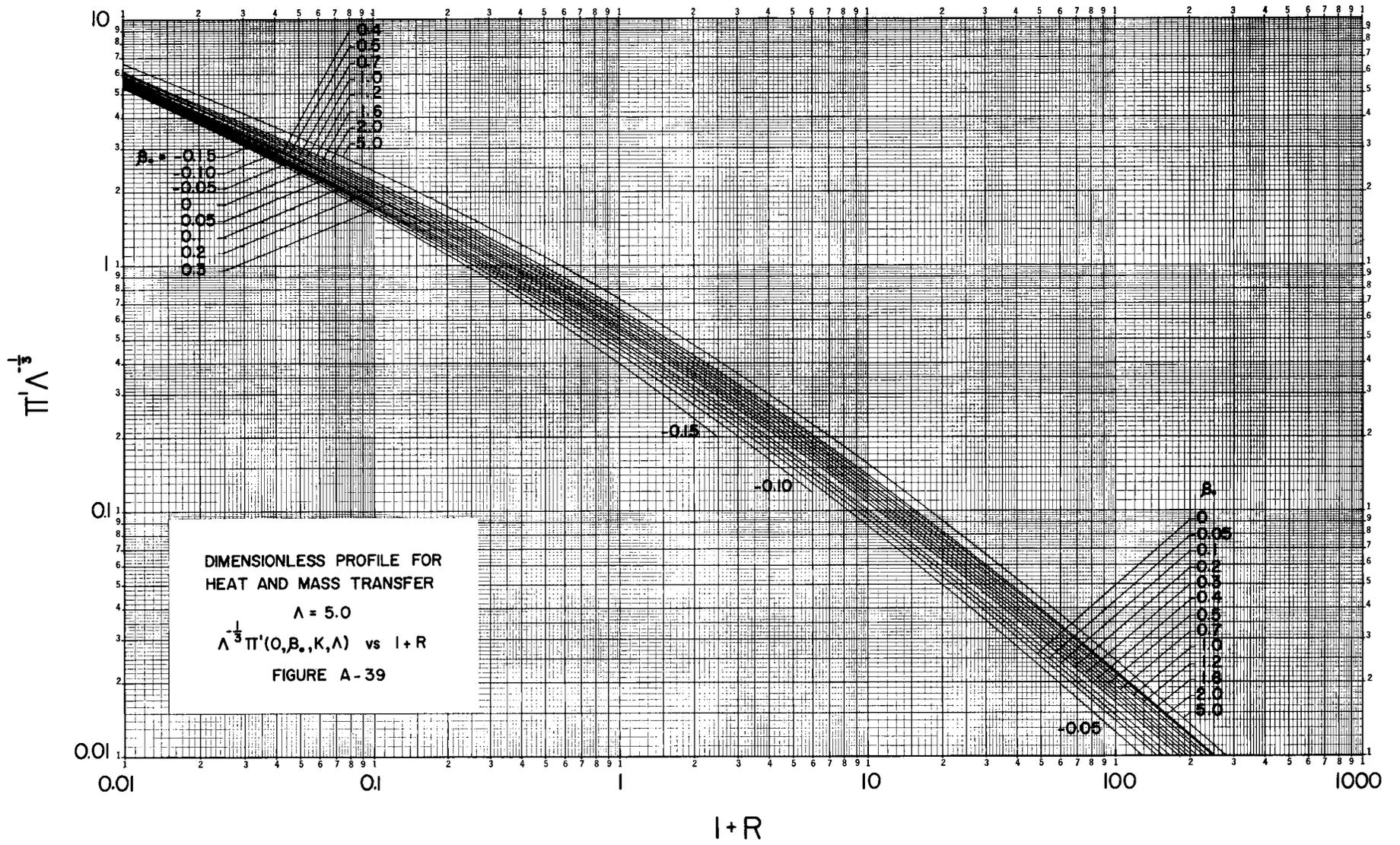


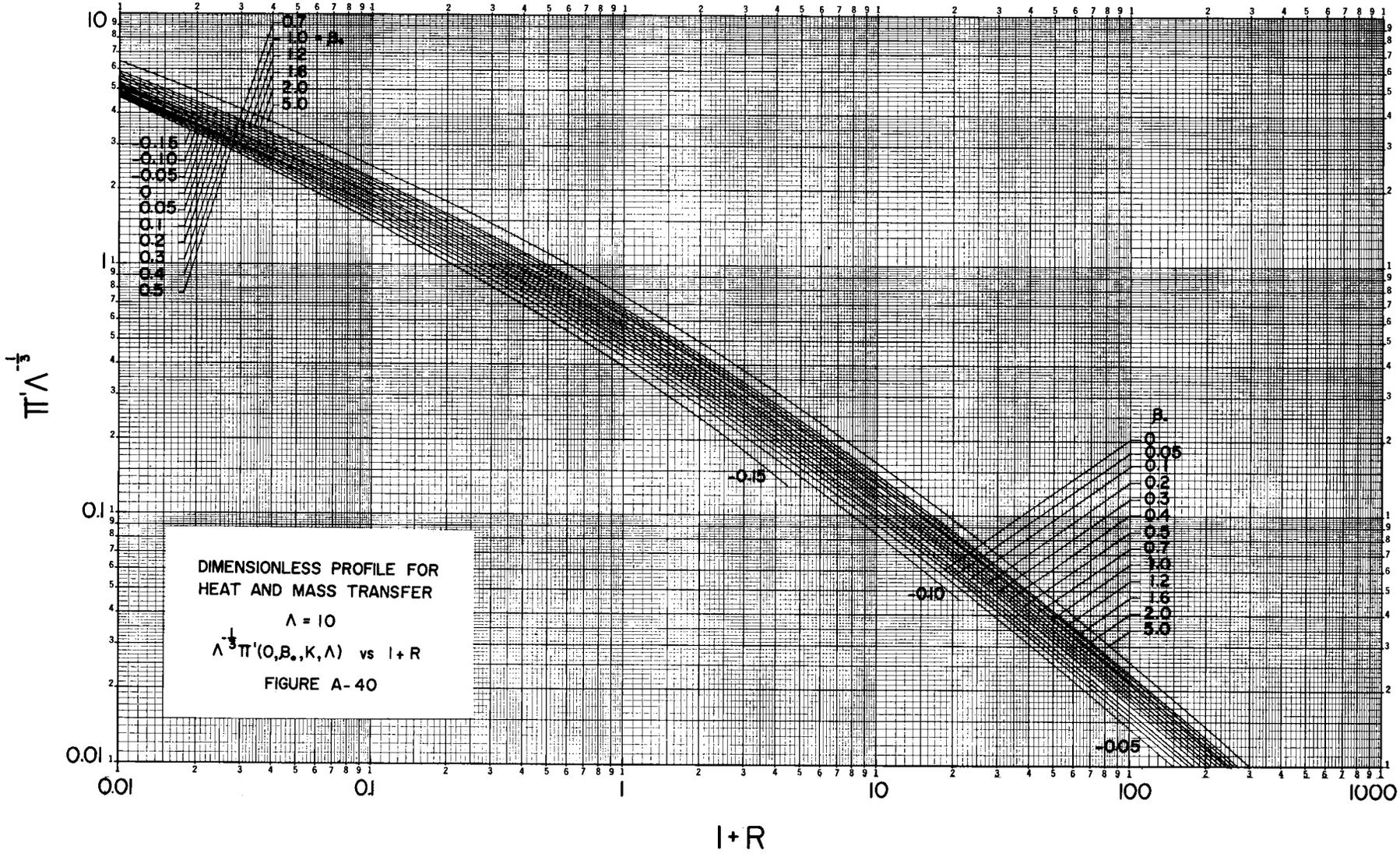




$\Lambda^{-\frac{1}{3}} \Pi'$









APPENDIX B

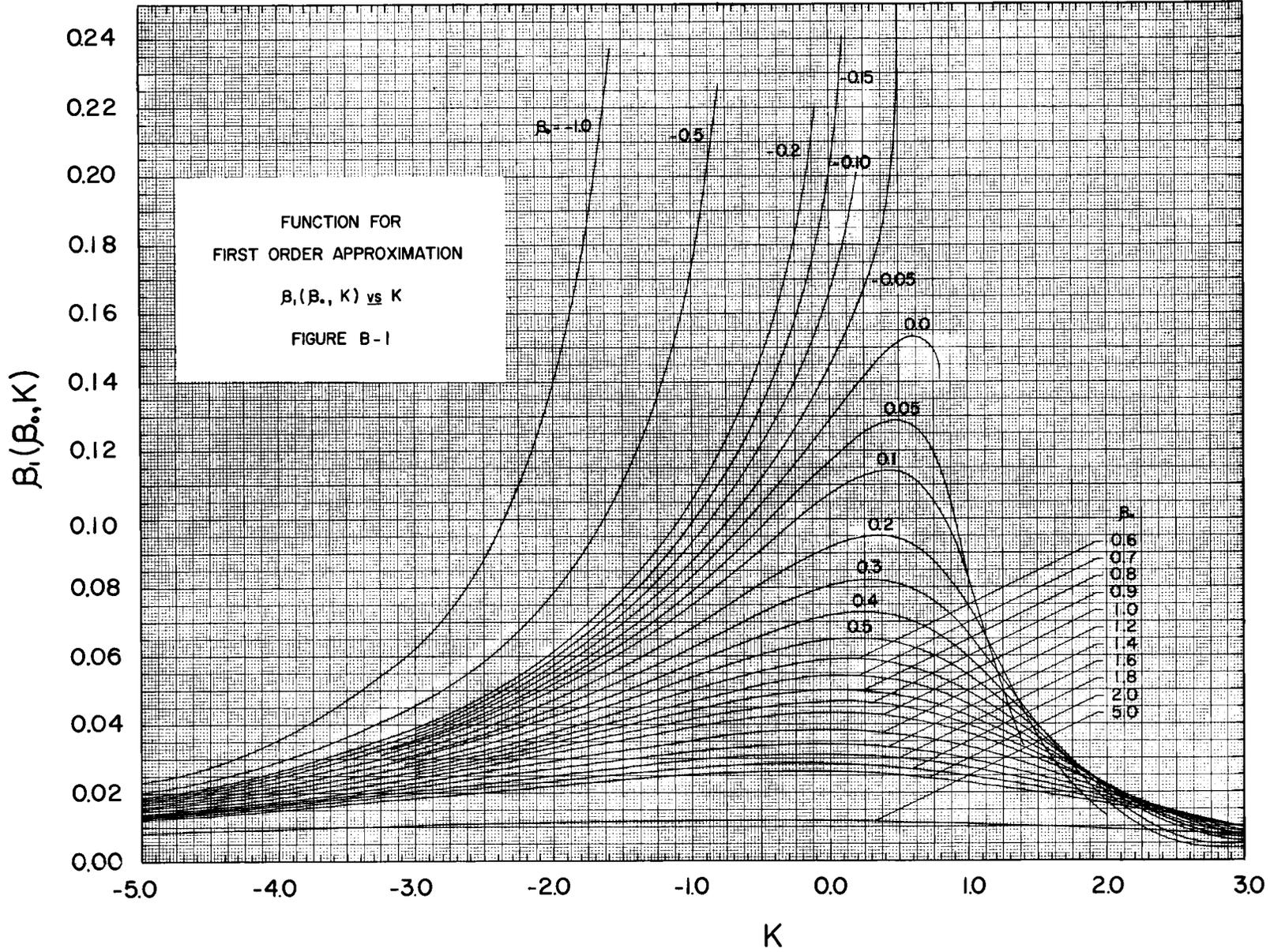


TABLE B-1a. THE FUNCTION  $\beta_1$  ( $\beta_0, K$ )

$\beta_0$	-1.	-0.5	-0.2	-0.15	-0.1	-0.05
-5. K	0.233256-1	0.203698-1	0.189121-1	0.186881-1	0.184689-1	0.182546-1
-4.	0.357854-1	0.294184-1	0.265207-1	0.260887-1	0.256696-1	0.252628-1
-3.	0.619959-1	0.456076-1	0.391344-1	0.382170-1	0.373384-1	0.364963-1
-2.	0.138539	0.787118-1	0.615372-1	0.593264-1	0.572567-1	0.553157-1
-1.6	0.237153	0.102821	0.753552-1	0.720556-1	0.690122-1	0.661975-1
-1.2		0.141754	0.937507-1	0.886192-1	0.839889-1	0.797909-1
-1.0		0.173171	0.105363	0.988205-1	0.930090-1	0.878128-1
-0.8		0.226494	0.119229	0.110700	0.103286	0.967779-1
-0.6			0.136262	0.124758	0.115076	0.106792
-0.5			0.146486	0.132856	0.121651	0.112231
-0.4			0.158386	0.141896	0.128769	0.117980
-0.3			0.172783	0.152176	0.136534	0.124061
-0.2			0.191479	0.164225	0.145105	0.130504
-0.1			0.219983	0.179138	0.154755	0.137349
0.0				0.199812	0.166005	0.144667
0.1				0.240398	0.180071	0.152584
0.2					0.200886	0.161377
0.3						0.171768
0.4						0.186396
0.5						0.252467
0.6						
0.8						
1.0						
1.4						
2.						
3.						

VALUES IN TABLE MUST BE MULTIPLIED BY THE POWER OF TEN INDICATED AFTER THE NUMBER

TABLE B-1b. THE FUNCTION  $\beta_1$  ( $\beta_0, K$ )

$K$	$\beta_0$	0.0	0.05	0.1	0.2	0.3	0.4
-5.		0.180448-1	0.178396-1	0.176388-1	0.172495-1	0.168762-1	0.165179-1
-4.		0.248679-1	0.244844-1	0.241117-1	0.233973-1	0.227215-1	0.220814-1
-3.		0.356885-1	0.349131-1	0.341683-1	0.327638-1	0.314627-1	0.302546-1
-2.		0.534921-1	0.517762-1	0.501590-1	0.471899-1	0.445305-1	0.421366-1
-1.6		0.635873-1	0.611610-1	0.589005-1	0.548155-1	0.512269-1	0.480525-1
-1.2		0.759685-1	0.724744-1	0.692691-1	0.635955-1	0.587341-1	0.545261-1
-1.0		0.831394-1	0.789144-1	0.750770-1	0.683725-1	0.627146-1	0.578803-1
-0.8		0.910157-1	0.858777-1	0.812674-1	0.733363-1	0.667606-1	0.612238-1
-0.6		0.996132-1	0.933246-1	0.877673-1	0.783841-1	0.707641-1	0.644535-1
-0.5		0.104177	0.972016-1	0.910931-1	0.808911-1	0.727028-1	0.659832-1
-0.4		0.108910	0.101155	0.944365-1	0.833511-1	0.745669-1	0.674279-1
-0.3		0.113796	0.105156	0.977626-1	0.857293-1	0.763260-1	0.687622-1
-0.2		0.118814	0.109165	0.101025	0.879827-1	0.779446-1	0.699576-1
-0.1		0.123933	0.113126	0.104166	0.900592-1	0.793813-1	0.709818-1
0.0		0.129105	0.116964	0.107107	0.918956-1	0.805883-1	0.717990-1
0.1		0.134260	0.120574	0.109746	0.934157-1	0.815113-1	0.723700-1
0.2		0.139297	0.123811	0.111952	0.945292-1	0.820894-1	0.726526-1
0.3		0.144058	0.126469	0.113552	0.951302-1	0.822559-1	0.726030-1
0.4		0.148299	0.128255	0.114326	0.950981-1	0.819402-1	0.721768-1
0.5		0.151607	0.128747	0.113991	0.942996-1	0.810704-1	0.713314-1
0.6		0.153217	0.127329	0.112200	0.925961-1	0.795784-1	0.700292-1
0.8		0.140773	0.115014	0.102653	0.859796-1	0.745180-1	0.659507-1
1.0			0.836340-1	0.833402-1	0.747354-1	0.666022-1	0.598948-1
1.4				0.336160-1	0.438260-1	0.448593-1	0.434322-1
2.					0.130303-1	0.174121-1	0.199968-1
3.						0.367946-2	0.477801-2

VALUES IN TABLE MUST BE MULTIPLIED BY THE POWER OF TEN INDICATED AFTER THE NUMBER

TABLE B-1c. THE FUNCTION  $\beta_1$  ( $\beta_0, K$ )

$K$	$\beta_0$	0.5	0.6	0.7	0.8	0.9	1.0
-5.		0.161737-1	0.158427-1	0.155244-1	0.152180-1	0.149229-1	0.146385-1
-4.		0.214743-1	0.208978-1	0.203498-1	0.198282-1	0.193313-1	0.188574-1
-3.		0.291301-1	0.280812-1	0.271008-1	0.261826-1	0.253212-1	0.245116-1
-2.		0.399717-1	0.380055-1	0.362130-1	0.345727-1	0.330669-1	0.316801-1
-1.6		0.452266-1	0.426967-1	0.404200-1	0.383615-1	0.364922-1	0.347880-1
-1.2		0.508515-1	0.476173-1	0.447510-1	0.421946-1	0.399019-1	0.378349-1
-1.0		0.537055-1	0.500667-1	0.468690-1	0.440387-1	0.415173-1	0.392578-1
-0.8		0.565009-1	0.524277-1	0.488809-1	0.457665-1	0.430114-1	0.405580-1
-0.6		0.591437-1	0.546161-1	0.507119-1	0.473121-1	0.443263-1	0.416844-1
-0.5		0.603704-1	0.556133-1	0.515316-1	0.479924-1	0.448957-1	0.421643-1
-0.4		0.615101-1	0.565254-1	0.522704-1	0.485968-1	0.453943-1	0.425784-1
-0.3		0.625420-1	0.573356-1	0.529142-1	0.491135-1	0.458122-1	0.429185-1
-0.2		0.634430-1	0.580253-1	0.534482-1	0.495304-1	0.461394-1	0.431761-1
-0.1		0.641881-1	0.585746-1	0.538564-1	0.498346-1	0.463655-1	0.433429-1
0.0		0.647501-1	0.589628-1	0.541227-1	0.500134-1	0.464804-1	0.434103-1
0.1		0.651001-1	0.591682-1	0.542304-1	0.500537-1	0.464737-1	0.433704-1
0.2		0.652083-1	0.591688-1	0.541632-1	0.499433-1	0.463358-1	0.432155-1
0.3		0.650443-1	0.589432-1	0.539053-1	0.496703-1	0.460578-1	0.429386-1
0.4		0.645788-1	0.584712-1	0.534426-1	0.492243-1	0.456319-1	0.425339-1
0.5		0.637851-1	0.577351-1	0.527627-1	0.485966-1	0.450518-1	0.419968-1
0.6		0.626404-1	0.567205-1	0.518563-1	0.477810-1	0.443133-1	0.413242-1
0.8		0.592433-1	0.538255-1	0.493468-1	0.455766-1	0.423559-1	0.395707-1
1.0		0.543824-1	0.497976-1	0.459319-1	0.426308-1	0.397796-1	0.372922-1
1.4		0.413202-1	0.390919-1	0.369455-1	0.349474-1	0.331132-1	0.314384-1
2.		0.213745-1	0.219971-1	0.221545-1	0.220264-1	0.217242-1	0.213180-1
3.		0.575461-2	0.659711-2	0.730684-2	0.789331-2	0.836983-2	0.875079-2

VALUES IN TABLE MUST BE MULTIPLIED BY THE POWER OF TEN INDICATED AFTER THE NUMBER

TABLE B-1d. THE FUNCTION  $\beta_1$  ( $\beta_0, K$ )

$K$	$\beta_0$ 1.2	1.4	1.6	1.8	2.0	5.0
-5.	0.140995-1	0.135970-1	0.131276-1	0.126882-1	0.122761-1	0.820571-2
-4.	0.179726-1	0.171634-1	0.164206-1	0.157367-1	0.151051-1	0.934832-2
-3.	0.230307-1	0.217102-1	0.205262-1	0.194591-1	0.184928-1	0.104769-1
-2.	0.292128-1	0.270861-1	0.252358-1	0.236127-1	0.221782-1	0.114261-1
-1.6	0.317966-1	0.292591-1	0.270820-1	0.251953-1	0.235458-1	0.117305-1
-1.2	0.342601-1	0.312799-1	0.287603-1	0.266042-1	0.247398-1	0.118989-1
-1.0	0.353803-1	0.321768-1	0.294887-1	0.272030-1	0.252372-1	0.119597-1
-0.8	0.363810-1	0.329612-1	0.301130-1	0.277063-1	0.256473-1	0.119934-1
-0.6	0.372221-1	0.336016-1	0.306082-1	0.280940-1	0.259539-1	0.119983-1
-0.5	0.375692-1	0.338573-1	0.307992-1	0.282381-1	0.260631-1	0.119893-1
-0.4	0.378596-1	0.340643-1	0.309481-1	0.283455-1	0.261403-1	0.119726-1
-0.3	0.380876-1	0.342182-1	0.310516-1	0.284137-1	0.261834-1	0.119480-1
-0.2	0.382471-1	0.343148-1	0.311064-1	0.284403-1	0.261906-1	0.119152-1
-0.1	0.383323-1	0.343497-1	0.311096-1	0.284231-1	0.261601-1	0.118742-1
0.0	0.383375-1	0.343191-1	0.310582-1	0.283598-1	0.260903-1	0.118249-1
0.1	0.382574-1	0.342193-1	0.309497-1	0.282486-1	0.259798-1	0.117672-1
0.2	0.380870-1	0.340470-1	0.307817-1	0.280878-1	0.258274-1	0.117010-1
0.3	0.378221-1	0.337992-1	0.305523-1	0.278761-1	0.256322-1	0.116265-1
0.4	0.374590-1	0.334739-1	0.302601-1	0.276125-1	0.253934-1	0.115435-1
0.5	0.369952-1	0.330695-1	0.299041-1	0.272965-1	0.251107-1	0.114521-1
0.6	0.364292-1	0.325852-1	0.294840-1	0.269280-1	0.247842-1	0.113524-1
0.8	0.349912-1	0.313788-1	0.284537-1	0.260353-1	0.240014-1	0.111284-1
1.0	0.331612-1	0.298681-1	0.271802-1	0.249436-1	0.230528-1	0.108727-1
1.4	0.285167-1	0.260727-1	0.240076-1	0.222439-1	0.207217-1	0.102732-1
2.	0.203547-1	0.193335-1	0.183363-1	0.173956-1	0.165223-1	0.919167-2
3.	0.928026-2	0.957621-2	0.970946-2	0.973059-2	0.967508-2	0.712688-2

VALUES IN TABLE MUST BE MULTIPLIED BY THE POWER OF TEN INDICATED AFTER THE NUMBER

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- No. 6. A Sanitary Survey of the Willamette River from Sellwood Bridge to the Columbia, by G. W. Gleeson. 1936. 25¢.
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- ROBERT RAY MICHAEL, M.S., Electrical Materials and Instrumentation.
- JOHN GLENN MINGLE, M.S., Automotive Engineering.
- ROGER DEAN OLLEMAN, Ph.D., Metallurgical Engineering.
- OLAF GUSTAV PAASCHE, M.S., Metallurgical Engineering.
- DONALD CHARLES PHILLIPS, Ph.D., Sanitary Engineering.
- JAMES LEAR RIGGS, Ph.D., Industrial Engineering.
- JOHN CLAYTON RINGLE, Ph.D., Nuclear Engineering.
- JEFFERSON BELTON RODGERS, A.E., Agricultural Engineering.
- JOHN LOUIS SAUGEN, Ph.D., Control Systems.
- ROBERT JAMES SCHULTZ, M.S., Surveying and Photogrammetry.
- MILTON CONWELL SHEELY, B.S., Manufacturing Processes.
- LOUIS SLEGEL, Ph.D., Mechanical Engineering.
- CHARLES EDWARD SMITH, Ph.D., Applied Mechanics.
- LOUIS NELSON STONE, B.S., High Voltage and Computers.
- JESSE SEBURN WALTON, B.S., Chemical and Metallurgical Engineering.
- LEONARD JOSEPH WEBER, M.S., Communications Engineering.
- JAMES RICHARD WELTY, Ph.D., Transport Phenomena.
- CHARLES EDWARD WICKS, Ph.D., Chemical Engineering.

# Oregon State University

CORVALLIS

## RESIDENT INSTRUCTION

### Undergraduate Liberal Arts and Sciences

School of Humanities and Social Sciences (B.A., B.S. degrees)  
School of Science (B.A., B.S. degrees)

### Undergraduate Professional Schools

School of Agriculture (B.S., B.Agr. degrees)  
School of Business and Technology (B.A., B.S. degrees)  
School of Education (B.A., B.S. degrees)  
School of Engineering (B.A., B.S. degrees)  
School of Forestry (B.S., B.F. degrees)  
School of Home Economics (B.A., B.S. degrees)  
School of Pharmacy (B.A., B.S. degrees)

### Graduate School Fields of Study

Agriculture (M.S., M.Agr., Ph.D. degrees)  
Biological and Physical Sciences (M.A., M.S., Ph.D. degrees)  
Business and Technology (M.B.A., M.S. degrees)  
Education (M.A., M.S., Ed.M., Ed.D., Ph.D. degrees)  
Engineering (M.A., M.S., M.Bioeng., M.Eng., M.Mat.Sc., A.E.,  
Ch.E., C.E., E.E., I.E., M.E., Met.E., Min.E., Ph.D. degrees)  
Forestry (M.S., M.F., Ph.D. degrees)  
Home Economics (M.A., M.S., M.H.Ec., Ph.D. degrees)  
Pharmacy (M.A., M.S., M.Pharm., Ph.D. degrees)

### Summer Term (four, eight, and eleven week sessions)

### Short Courses, Institutes, Workshops

## RESEARCH AND EXPERIMENTATION

(Date indicates year established)

### General Research (1932)

### Agricultural Experiment Station (1888)

Branch stations at Astoria, Union, Klamath Falls, Ontario, Hood River, Aurora,  
Pendleton, Moro, Medford, Burns, Hermiston, and Redmond.

### Computer Center (1965)

### Engineering Experiment Station (1927)

### Forest Research Laboratory (1941)

### Genetics Institute (1963)

### Marine Science Center at Newport (1965)

### Nuclear Science and Engineering Institute (1966)

### Nutrition Research Institute (1964)

### Radiation Center (1964)

### Science Research Institute (1952)

### Transportation Research Institute (1960)

### Water Resources Research Institute (1960)

## EXTENSION

### Federal Cooperative Extension (Agriculture and Home Economics)

### Division of Continuing Education, State System of Higher Education