A method of analysis of arbitrarily shaped prestressed concrete members for biaxial bending is given. The stress-strain curve of the concrete is represented by two parabolic segments, and the effects of strain hardening of the steel are included in the analysis. An iterative numerical procedure is used to solve for the position of the neutral axis and the curvature corresponding to a given axial force and bending moment. This procedure is used as a basis for calculating the deflections of a cantilever member by Newmark's method. No consideration is given to time dependent effects nor to the possibility of failure other than in pure flexure.

Theoretical curvatures, deflections and ultimate moments are compared with results obtained from the testing of three I-shaped prestressed concrete power poles. Good correlation was obtained
between computed and measured behavior of the specimens tested. Ultimate moments were extremely well predicted.

Listings of the computer programs used in the analysis are included.
Biaxial Bending of Concrete Members with Applications to Prestressed Concrete Power Poles

by

Charles Talmon Stephens

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1. INTRODUCTION

Three fundamentally different approaches are in common use for the computation of the ultimate moment capacity of reinforced concrete members, all of which presuppose a knowledge of the reinforcement stress-strain curve. The first, referred to herein as the rational approach, is the basic approach of mechanics of materials and makes use of the following three distinct groups of relationships.

1. Equations of static equilibrium relating stress resultants to internal stresses.

2. Assumed or known kinematic relationships describing the kinematic response of the member to the given kind of loading.

3. Known or assumed stress-strain laws for the materials involved.

Stresses may be expressed in terms of unit deformations by the elimination of strains between relations two and three. These expressions for the stresses are then substituted into the equilibrium equations yielding relationships between stress resultants and unit deformations. In the case of a beam subject to an axial force and bending about a principal axis the above procedure results in two
expressions of the form

\[ P = f(e_1, e_2) \]  \hspace{1cm} (1.1)

and

\[ M = g(e_1, e_2) \]  \hspace{1cm} (1.2)

where \( P \) is the axial force, \( M \) is the bending moment and \( e_1 \) and \( e_2 \) are unit deformations, which together with relations 2 define the distribution of strains at the cross section in question. Maximization of \( g \) subject to the restraint that \( f = P_0 \) then gives the ultimate moment corresponding to \( P = P_0 \).

In the second method, which will be referred to as the approximate method, the shape of the concrete stress distribution at ultimate moment and its peak value are assumed. In the case of members with no compressive reinforcement and whose strength is governed by yielding of the tension steel, the extent of the compression zone is then determined by considering equilibrium of forces along the axis of the member. This completes the definition of the entire stress distribution and the ultimate moment is easily computed. In the case of members having compressive reinforcement or whose strength is governed by crushing of the concrete, failure is assumed to occur when the concrete reaches some limiting value of strain. Plane sections are assumed to remain plane under load, and the location of the

---

1. Symbols are defined where they first appear in the text, and are arranged alphabetically in Appendix A for reference.
neutral axis is determined from a consideration of force equilibrium. This again completes definition of the stress distribution, allowing the computation of the ultimate moment.

The third approach, which will be referred to as the empirical approach, represents the stress distribution in the concrete by a statically equivalent concentrated force which is defined by the cylinder strength $F'_c$ and the parameters $k_1$, $k_2$, and $k_3$ as defined in Figure 1.1. Inspection of the figure shows that the parameters $k_1$ through $k_3$ are functions of the cross section geometry as well as of the properties of the concrete. They have been determined empirically for rectangular sections and several concretes by Hognestad, Hanson and McHenry (1955) and by others. Computation of ultimate moments proceeds along the same lines as in the approximate method.

Figure 1.1. Conditions at ultimate load.
After Hognestad (1957).
All of the above methods are readily applied to members bent about a principal axis. Application to members in which the bending occurs about some other axis is complicated by the fact that the shape of the compression zone is unknown. The usual approach is to write separate sets of equations for each possible geometric configuration of the compressive region and to then determine the correct one for a given loading condition by trial and error. This becomes rather complicated for even the simplest cross section shapes and becomes intractable for more complicated sections. This has been no real problem in the past as the vast majority of concrete members have been either rectangular or circular. Modern precasting and prestressing methods now make feasible the manufacture of concrete members in an almost limitless variety of shapes, and a general analysis technique capable of handling this variety of shapes is highly desirable.

1.1. Goals

The goals of the study reported herein were to devise an analytical or numerical method for determining the behavior of arbitrarily shaped prestressed concrete members subjected to short term static loading producing bending about any axis, to program the method for automatic computation, and to attempt at least a partial verification of the method by the testing of some full-size prestressed concrete
power poles having an I-shaped cross section. Although the primary item of interest was the ultimate strength of the members, it seemed that the best approach would be to study the axial load-moment-curvature relationships. This seemed to be the only method which would yield results without the necessity of introducing numerous simplifying assumptions. In addition, it would enable the computation of deflections and would provide a useful tool for the study of stability and dynamic behavior.

1.2. Historical Background

The literature concerning the ultimate strength of concrete members and properties of the stress block is voluminous and dates back at least as far as 1900. Only some of the more recent studies are discussed herein, and the reader is referred to Hognestad's (1951) paper for an excellent critical review of the literature to 1950.

Of critical importance in the study of the structural behavior of concrete members is the stress-strain curve of the concrete itself. It has been universally conceded that the curve is nonlinear and is time dependent. Since the immediate goals of the present study were concerned with short term loading, the effects of creep will not be mentioned further.

Hognestad (1951) assumed a stress-strain curve as shown in Figure 1.2. This assumed stress-strain curve has been much used
by others in subsequent analytical studies. The parabolic initial portion of the curve was chosen because he felt that the initial part of the stress-strain curve in flexure was fairly similar to the relation in direct compression and his tests of cylinders were in reasonable agreement with Ritter's (as cited by Hognestad (1951)) parabola. The descending portion was assumed to be linear and the value $0.15 \frac{F''}{c}$ was chosen to give the best agreement with his column tests. The peak stress $0.85 \frac{F'}{c}$ was chosen for columns cast in a vertical position. These always appeared to be weakest at the top, which Hognestad attributes partially to poorer compaction but primarily to "water gain" in the top of the specimen.

\begin{equation}
F'' = 0.85\frac{F'}{c}
\end{equation}

\begin{equation}
S_o = \frac{2F''}{E_c} \quad S_u = 3.8\%
\end{equation}

Figure 1.2. Hognestad's stress-strain curve.

Kriz and Lee (1960) describe the stress-strain curve by the equation

\begin{equation}
F^2 + AS^2 + BFS + CF + DS = 0
\end{equation}
where $F$ is the stress, $S$ the strain, and the coefficients $A$ through $D$ are parameters based upon a regression analysis of the data given by Hognestad, Hanson and McHenry (1955). The latter authors tested rectangular plain concrete specimens loaded in such a manner as to simulate the compression zone in a beam.

Barnard (1964), who used a specially constructed testing machine, reports measuring strains as large as ten percent in the descending portion of the stress-strain curve for axially loaded specimens and concludes that concrete is capable of sustaining stress at almost unlimited strain levels. There has been much discussion of this aspect of the concrete stress-strain curve in the literature; it is now recognized that the explosive failure usually encountered in compression tests is due to release of strain energy from the testing machine and that concrete is indeed capable of sustaining load at very large strain levels in a displacement governed test.

Sturman, Shah and Winter (1965) have compared the stress-strain curves for eccentrically loaded and concentrically loaded plain concrete specimens and conclude that the peak stress in flexure is about 20 percent higher when a significant strain gradient is present. Strain gradients in their tests ranged from zero to 0.00048. These results are at variance with those reported by Hognestad, Hanson and McHenry (1955) who give peak stresses of 0.92 to 1.12 times $F'_c$. 


Karsan and Jirsa (1969) show reasonable agreement with Hognestad's curve and somewhat better agreement with an exponential stress-strain relation given by Smith and Young (1955).

There is apparently general agreement upon the qualitative nature of the stress-strain curve, but there is considerable scatter in the quantitative results reported. Some of the differences could no doubt be explained by considering differences in experimental procedure. For instance, strain measurements are considerably affected by the relationship between gage length and aggregate size and even more so by the relationship between gage length and crack spacing. Loading rates also have an effect upon the results obtained. The rather extreme measures used by some investigators to obtain information on the descending portion of the curve would suggest that extreme caution should be exercised in attempting to utilize the material in this range in actual structures. Energy release from adjacent portions of the structure may have the same effect as does the energy released from an ordinary type of testing machine.

The empirical approach for prestressed members having reinforcement on the tension side only was given in detail by Billet and Appleton (1954). They considered only the case of a rectangular compression zone with bending about the principal axis. The actual stress-strain curve for the reinforcement was used. Consequently, they were led to develop a graphical procedure for the determination
of the steel stress at ultimate moment. The common assumption that the ultimate capacity in flexure was reached when the concrete strain reached a limiting value \( S_u \) was made. It should be pointed out that this assumption is entirely different from assuming that the material fails at a limiting strain. As is the common practice, they give equations in dimensionless form and study the effect of several parameters. Unfortunately, these parameters are all dependent upon the basic geometric configuration of the cross section and it becomes impossible to interpret them in terms of other geometry. Consider for example the parameter \( p = A_{st}/bd \), where the notation is as given in Figure 1.1. Conclusions concerning \( p \) which are reached for sections such as shown in Figure 1.1 simply do not apply to a situation such as is depicted in Figure 1.3.

Design charts for unsymmetrically loaded rectangular columns were presented by Au (1958). He used the approximate approach, assuming a rectangular stress distribution for the concrete.

![Figure 1.3. Unsymmetrical cross section.](image)
Equations were written for each of the three cases shown in Figure 1.4. The method is limited in that it is applicable, as presented, only to rectangular sections and would require the derivation of new equations for any other geometry. The rather questionable assumption that the resultant tensile force is located at the centroid of the tensile bar group was made. This required the definition of a so-called average tensile stress which was used in the equilibrium equations.

![Figure 1.4. Cases studied by Au.](neutral axis)

Other authors have also studied biaxial bending of specific cross section shapes by utilizing the approximate method. Pannel (1963) and Mattock, Kriz and Hognestad (1961) used the rectangular stress distribution while Czerniak (1962) assumed the bilinear elastic-plastic distribution.

Several authors have used the rational approach in studying specific shapes. Most of them have concerned themselves solely
with the uniaxial bending of rectangular sections. Kriz and Lee (1960) used the previously described stress-strain curve (equation 1.3) which they obtained from a regression analysis. Pfrang, Siess and Sozen (1964), Pfrang and Siess (1964) and Breen (1965) have all used Hognestad's (1951) curve. Sherebourne and Parameswar (1968) claim to have used the same. Actually, they omitted the straight line portion and used instead the descending portion of the initial parabola. Aroni (1968) used Hognestad's curve with the addition of a straight line on the tension side.

Mattock and Kriz (1961) used the stress-strain curve given by Kriz and Lee (1960) in the study of two special configurations, both of which had triangular compression zones. These are shown in Figure 1.5. They were both loaded in the vertical plane, therefore the orientation of the neutral axis for the first case is obvious from inspection, while in the second case it is determined from the fact that the resultant compressive force must lie vertically above the reinforcement. Values of $k_1$ and $k_3$ were tabulated for the assumed stress-strain curve for the triangular compression zone and various values of maximum concrete strain. Moments were then computed for increasing values of strain and the ultimate moment determined by inspection of the resulting data.

Warner (1969) has presented a general rational method for computing the moment vector corresponding to given values of
curvature, axial force and inclination of the neutral axis which is based upon the partitioning of the cross section into a finite number of rectangular elements. Stresses were assumed to be uniform across each of these elements, and the resultant force and moment on the section were computed by summing the effects of the resultant force acting upon each individual element. This allowed the use of any stress-strain relation which could be expressed analytically. An iterative procedure (not described) was used to determine the position of the neutral axis required to give the correct value of the axial force. Computation of the resultant moment was then straightforward. No method was suggested for solution of the equations when the two moment components as well as the axial force are given. Results were given in the form of moment-curvature diagrams for the rectangular section studied. It was pointed out that the method theoretically is applicable to irregular shapes, however it was implied that
the iterative procedure may not work in the case of very irregular members. One hundred elements were recommended for a rectangular section. Since the method uses a rectangular array with zero elements in the appropriate locations as a means of describing irregular sections, it would appear that several hundred elements might be required for the adequate description of such sections. It was concluded that the derivation of analytical expressions for the moment and axial force is not feasible in the general case of biaxial bending.
2. METHOD OF ANALYSIS

The relationships among bending moment, curvature and axial force provide a means of determining moment capacity and they are prerequisite to the analysis of flexural members for deflection, stability and dynamic behavior. A method of determining these relationships for an arbitrarily shaped prestressed concrete member bent about an arbitrary axis is given below. The analysis includes ordinary reinforced concrete members as a special case.

2.1. Basic Equations

A cross section upon which the resultant load is a compressive force \( P \) located at the origin of coordinates and a couple \( M_r \) acting about an axis inclined at an angle \( \gamma \) from the \( x \) axis is considered, where \( x \) and \( y \) are arbitrary references axes. (It is convenient but not necessary to let \( x \) and \( y \) be principal axes of the cross section.) \( M_r \) is resolved into components \( M_u \) and \( M_v \) parallel to axes \( u \) and \( v \) which are parallel and perpendicular respectively to the neutral axis and are inclined at an angle \( \theta \) to the reference axes. The terms defined above are shown in Figure 2.1. \( M_u \) and \( M_v \) are considered to be positive when they tend to produce compression in the first quadrant.
2.1.1. Assumptions

It is assumed that plane cross sections remain plane under load and that a perfect bond exists between the steel and the concrete. While it is evident that these assumptions will not be valid after the section has begun to crack and large local variations have occurred, numerous other investigators have used them with good success in predicting overall behavior. For example, see Aroni (1968) or Baker and Amarakone (1965). This assumed kinematic response is represented by the following two equations.

![Figure 2.1. Member cross section and coordinates.](image-url)
\[ S_c = S_1 + \phi v \quad (2.1) \]
\[ S_s = \Delta_s - S_c \quad (2.2) \]

where \( S_c \) is the concrete strain, \( \phi \) is the strain gradient and curvature, \( S_1 \) is the concrete strain at the origin and \( \Delta_s \) is the difference in magnitude between the concrete and steel strains. In accordance with the usual practice compressive stresses and strains in the concrete and tensile stresses and strains in the steel are considered to be positive. The quantity \( \Delta_s \) is zero for ordinary reinforced concrete and in pretensioned members is equal to the initial prestress minus creep and shrinkage losses divided by the modulus of elasticity of the steel. These quantities are illustrated in Figures 2.2b and 2.2c.

Assumed stress-strain relationships are as follows. It is assumed that the stress-strain curve for the concrete consists of two parabolas as shown in Figure 2.3. This relationship is represented by the equations:

\[ S_c \leq 0 \quad : \quad F_c = 0 \quad (2.3) \]
\[ 0 \leq S_c \leq S_o \quad : \quad F_c = A_1 S_c + A_2 S_c^2 \quad (2.4) \]
\[ S_o \leq S_c \leq S_{cu} \quad : \quad F_c = A_3 + A_4 S_c + A_5 S_c^2 \quad (2.5) \]
\[ S_{cu} < S_c \quad : \quad F_c = 0 \quad (2.6) \]

where \( S_o \) is the strain at which the maximum stress \( F_c'' \) occurs, \( S_{cu} \) is the maximum strain for which the concrete is capable of
Figure 2.2. Assumed stresses and strains.

a. Concrete stress zones.  
b. Concrete strain.  
c. Steel strain.  
d. Assumed stress distribution.
sustaining stress and $A_1$ through $A_5$ are additional parameters of the curve. (Because of the continuity at $S_0$, only four of the A's are independent.) Note that by a proper choice of the parameters, Hognestad's curve, a rectangular distribution or the elastic-plastic distribution is directly obtainable from the equations given above.

The stress-strain curve for the steel is assumed to be as shown in Figure 2.4 and is represented by the following equations.

\begin{align}
0 < S_s &< S_{pl} \quad : \quad F_s = E_1 S_s \\
S_{pl} < S_s &< S_y \quad : \quad F_s = B_1 + B_2 S_s + B_3 S_s^2 \\
S_y < S_s &< S_{su} \quad : \quad F_s = B_4 + E_2 S_s \\
S_{su} < S_s \quad : \quad F_s = 0
\end{align}

In these equations, $S_{pl}$ is the strain at the proportional limit, $S_y$ is the strain at which the curve is again assumed to become linear, $S_{su}$ is the ultimate strain, $E_1$ is Young's modulus, $E_2$ is the slope of the second straight line portion of the curve, $B_1$ through $B_3$ are parameters defining the parabolic portion of the curve and $B_4$ is the stress intercept of the second straight line.

Since the time dependence of the stress-strain relationships is neglected, the analysis is directly applicable only to the case of short term loading. Tensile stresses in the concrete are neglected but could be included with some minor modifications. They are believed
Figure 2.3. Assumed stress-strain curve for concrete.

Figure 2.4. Assumed stress-strain curve for prestressing strand.
to have a small effect on the behavior of the member during the first
cycle of loading and essentially no effect during subsequent cycles.

2.1.2. Equilibrium Equations

The equations of equilibrium are

\[ P = \int \int_A F_c dA - \sum_{i=1}^{n_s} A_{s_i} (F_{s_i} + F_{c s_i}) \]  \hspace{1cm} (2.11)
\[ M_u = \int \int_A F_c v dA - \sum_{i=1}^{n_s} v_{s_i} A_{s_i} (F_{s_i} + F_{c s_i}) \]  \hspace{1cm} (2.12)
\[ M_v = \int \int_A F_c u dA - \sum_{i=1}^{n_s} u_{s_i} A_{s_i} (F_{s_i} + F_{c s_i}) \]  \hspace{1cm} (2.13)

where \( A_g \) is the gross cross-sectional area, \( n_s \) is the number of
strands, \( u_{s_i}, v_{s_i} \) and \( A_{s_i} \) are the coordinates and cross-sectional
area respectively of the \( i \)th strand, \( F_{s_i} \) is the stress in the \( i \)th
strand, \( F_{c s_i} \) is the concrete stress at the location of the same strand
and \( dA \) is the differential element of area.

2.1.3. Load-Moment-Curvature Relationships

The cross section is divided into four zones as shown in Figure
2.2a. Zone one is the tensile zone in which the concrete is assumed
to be cracked, in zones two and three the concrete stress is given by
equations 2.4 and 2.5 respectively, in zone four the concrete is
assumed to have failed in compression and the strands are considered to be ineffective.

Substitution of equation 2.1 into equations 2.4 and 2.5 yields

\[ F_{c_2} = \beta_{21} v^2 + \beta_{22} v + \beta_{23} \]  
\[ F_{c_3} = \beta_{31} v^2 + \beta_{32} v + \beta_{33} \]  

where \( F_{c_2} \) and \( F_{c_3} \) are the concrete stresses in zones two and three respectively and where

\[ \beta_{21} = A_2 \phi^2 \]  
\[ \beta_{22} = (2A_2 S_1 + A_1) \phi \]  
\[ \beta_{23} = A_2 S_1 + A_1 S_1 \]  
\[ \beta_{31} = A_5 \phi^2 \]  
\[ \beta_{32} = (2A_5 S_1 + A_4) \phi \]  
\[ \beta_{33} = A_5 S_1 + A_4 S_1 + A_3 \]  

Equations 2.14 and 2.15 are substituted into the equilibrium equations 2.11 through 2.13 to give the load-moment-curvature equations:

\[ P = \int \int_{\text{Zone 2}} F_{c_2} dA + \int \int_{\text{Zone 3}} F_{c_3} dA - \sum_{i=1}^{n_s} P_{s_i} \]  

(2.17)
\[ M_u = \int \int_{\text{Zone 2}} F_{c2} v \, dA + \int \int_{\text{Zone 3}} F_{c3} v \, dA - \sum_{i=1}^{n_s} P_{s_i} v_{s_i} \quad (2.18) \]

and

\[ M_v = \int \int_{\text{Zone 2}} F_{c2} u \, dA + \int \int_{\text{Zone 3}} F_{c3} u \, dA - \sum_{i=1}^{n_s} P_{s_i} u_{s_i} \quad (2.19) \]

where

\[ P_{s_i} = A_{s_i} \left( F_{s_i} + F_{cs_i} \right) \quad (2.20) \]

and is computed from equations 2.1, 2.2 and the appropriate stress-strain equations (equations 2.3 through 2.10) if the strand is located in zones one through three; \( P_{s_i} \) equals zero if the strand is in zone four.

In order to permit evaluation of the integrals in equations 2.17 through 2.19 the boundary of the cross section is approximated by a number of straight line segments--if not already so defined. The points of intersection of the line segments with each other and with the boundaries of zones two and three are then numbered in clockwise order around the perimeter of the two zones to form a closed path around each zone. In the case of hollow sections the points are numbered in counter-clockwise order around the holes. This numbering system is illustrated in Figure 2.5. The development of an algorithm facilitating the automation of the numbering described above was a key step in the analysis; without it a closed form evaluation of the integrals in equations 2.17 through 2.19 is not possible for
general geometry. The procedure is complicated by the fact that the cross section may consist of a multiply connected region and by the fact that each zone may consist of two or more unconnected regions.

The integrations indicated in equations 2.17 through 2.19 are then performed resulting in the following expressions.

\[
P = \frac{1}{2} \sum_{j=2}^{3} \left( \frac{\beta_{j1} \Sigma_{P_{1j}}}{6} + \frac{\beta_{j2} \Sigma_{P_{2j}}}{3} + \beta_{j3} \Sigma_{P_{3j}} \right) - \sum_{i=1}^{n_s} P_{s_i} \tag{2.21}
\]

\[
M_u = \frac{1}{2} \sum_{j=2}^{3} \left( \frac{\beta_{j1} \Sigma_{u_{1j}}}{10} + \frac{\beta_{j2} \Sigma_{u_{2j}}}{6} + \frac{\beta_{j3} \Sigma_{u_{3j}}}{3} \right) - \sum_{i=1}^{n_s} P_{s_i} v_{s_i} \tag{2.22}
\]

\[
M_v = \frac{1}{6} \sum_{j=2}^{3} \left( \frac{\beta_{j1} \Sigma_{v_{1j}}}{10} + \frac{\beta_{j2} \Sigma_{v_{2j}}}{4} + \beta_{j3} \Sigma_{v_{3j}} \right) - \sum_{i=1}^{n_s} P_{s_i} u_{s_i} \tag{2.23}
\]

where
\[ \Sigma_{P_{1j}} = - \sum_{i=1}^{n_j} \left[ v_i^3 (u_i - u_m) + v_i v_m (v_i + v_m)(u_i - u_m) \right] \] (2.24a)

\[ \Sigma_{P_{2j}} = - \sum_{i=1}^{n_j} \left[ v_i^2 (u_i - u_m) + v_i v_m (u_i - u_m) \right] \] (2.24b)

\[ \Sigma_{P_{3j}} = - \sum_{i=1}^{n_j} v_i (u_i - u_m) \] (2.24c)

\[ \Sigma_{u_{1j}} = - \sum_{i=1}^{n_j} \left[ v_i^4 (u_i - u_m) + 2 v_i^2 (u_i - u_m) + v_i v_m (v_i^2 + v_m^2)(u_i - u_m) \right] \] (2.25a)

\[ \Sigma_{u_{2j}} = - \sum_{i=1}^{n_j} \left[ v_i^2 (u_i - u_m) + v_i v_m (u_i - u_m) \right] \] (2.25b)

\[ \Sigma_{u_{3j}} = - \sum_{i=1}^{n_j} \left[ v_i^2 (u_i - u_m) + v_i v_m (u_i - u_m) \right] \] (2.25c)

\[ \Sigma_{v_{1j}} = \sum_{i=1}^{n_j} \left[ v_i^3 (u_i (3u_i + u_i) - u_m (3u_i + u_m)) + v_i^2 (v_i(-3u_i + u_i + 2u_i^2 + v_m(-2u_i^2 - u_i u_m + 3u_i^2))) \right] \] (2.26a)

\[ \Sigma_{v_{2j}} = \sum_{i=1}^{n_j} \left[ v_i^2 (u_i (2u_i + u_i) - u_i (2u_i + u_i)) + 2v_i v_m (u_i^2 - u_i^2) \right] \] (2.26b)

\[ \Sigma_{v_{3j}} = \sum_{i=1}^{n_j} \left[ v_i (u_i^2 - u_i^2) + u_i v_m (v_i - v_m) \right] \] (2.26c)
The subscript \( j \) is the zone number, \( u \) and \( v \) are the coordinates of the points defining the zone boundary and \( n_j \) is the number of points used to define the boundary of the \( j \)th zone. The subscript \( i \) refers to the \( i \)th point and \( \ell \) equals \( i \) minus one except that when \( i \) is equal to one, \( \ell \) equals \( n_j \). Similarly, \( m \) equals \( i \) plus one except that when \( i \) equals \( n_j \), \( m \) equals one.

As an illustration of the procedure followed in the derivation of the expressions given above consider the area \( A \), shown in Figure 2.6. It can easily be shown that

\[
\iint_A dA = \int_{x_1}^{x_2} f(x) \, dx + \int_{x_2}^{x_3} g(x) \, dx + \int_{x_3}^{x_1} h(x) \, dx
\]

Figure 2.6. Integration procedure illustration.
For a given cross section and known values of $\phi$, $S_1$ and $\theta$, the coordinates are transformed by rotation through the angle $\theta$. Equations 2.21 through 2.23 are then used to compute the resultant axial force and moments.

2.2. Numerical Solution

In the usual situation $P$, $M_r$ and $\gamma$ are known, and the response of the member is desired. This requires that the coordinates $u$ and $v$ be expressed in terms of $x$, $y$ and $\theta$, and that equations 2.21 through 2.23 be solved simultaneously for $\phi$, $S_1$ and $\theta$. No closed form solution is possible unless the problem is limited to a specific cross section having very simple geometry. Hence, resort must be made to an iterative procedure for obtaining a numerical solution. An intermediate procedure, which yields useful information, is to allow $P$, $\phi$ and $\theta$ to vary independently. Equation 2.21 can then be solved for $S_1$, again by an iterative procedure, and the remaining two equations used to develop plots of $M_u$ and $M_v$ versus $\phi$ for given values of $P$ and $\theta$.

Techniques for accomplishing both of the above procedures were developed and are described below. Computer programs for performing the computations are given in Appendix B.
2.2.1. Iterative Procedure with P, \(\theta\) and \(\phi\) Independent

The procedure adopted was essentially a straight line extrapolation, and since it worked well more sophisticated methods were not investigated. The step by step procedure is outlined below.

1. \(\Delta_s\) is computed from the prestress and estimated losses. Loss of prestress due to elastic shortening is not included at this step and is automatically included later.

2. An initial estimate of \(S_1\) is computed by assuming linearly elastic behavior.

3. The coordinates of the points defining the cross section and those of the strands are transformed from \(x\) and \(y\) to \(u\) and \(v\).

4. For eccentrically prestressed members an initial estimate of the curvature is computed for the no load condition, again assuming elastic behavior.

5. From the strain distribution defined by \(\theta\), \(\phi\) and the now assumed value of \(S_1\); \(v_{na}\), \(v_{so}\) and \(v_{su}\) are computed. The latter three terms are the \(v\) coordinates of the neutral axis, the line separating zones two and three, and the line defining the outer extremity of zone four.

6. The coordinates of the points defining the boundaries of zones two and three are determined and arranged in the proper sequence.
7. $P$ is computed from equation 2.21 and compared with the
given value. If the error in $P$ is within allowable limits, the mo-
mments are computed from equations 2.22 and 2.23. $M_x, M_y$ and $\gamma$
are then easily computed if desired. The value of $\phi$ is then increas-
ed and for the third and subsequent points the value of $S_1$ is estimated
by the equation

$$S_{1i+1} = S_{1i} + \frac{S_{1i} - S_{1i-1}}{\phi_i - \phi_{i-1}} (\phi_{i+1} - \phi_i)$$ (2.27)

where the subscript $i$ refers to the point just computed, $i-1$ refers
to the previous point and $i+1$ to the next point to be computed. The
current value of $S_1$ serves as an initial approximation in the case of
the second point. The procedure then returns to step five.

8. If the error in the computed value of $P$ is not within the
allowable limits the next approximation to $S_1$ is made in one of the
following ways.

a. For the second iteration at the current value of $\phi$, $S_1$
is merely adjusted by a small arbitrary amount in the
direction necessary to reduce the error.

b. For the third and subsequent iterations the next
approximation to $S_1$ is computed from the equation

$$S_{1i+1} = S_{1i} + \frac{S_{1i} - S_{1i-1}}{P_i - P_{i-1}} (P_{i+1} - P_i)$$ (2.28)
where the subscripts have meanings similar to those defined for equation 2.27.

9. The procedure now returns to step five.

The procedure described above was used to develop curves of moment versus curvature for given values of axial force and inclination of the neutral axis. Hence, the process was started with \( \phi \) equal to the no load value from whence \( \phi \) was gradually increased until the ultimate moment had been passed. Convergence was rapid with two to four iterations being required for most points, and only rarely did the number exceed five or six. However, the increment in curvature was small, about 25 points being used to reach the ultimate bending moment. This did not appear to impose any significant penalty in computing time however. Total computer and peripheral equipment costs for a single moment curvature curve for the I-section studied (Figure 3.3) averaged about 70 cents.

2.2.2. Iterative Procedure with \( P, M_r \) and \( \gamma \) Independent

The procedure adopted for simultaneous solution of equations 2.21 through 2.23 was similar to that described above. The straight line extrapolation worked well if the initial estimates of the unknowns were fairly close to the correct values. When they were not, the procedure sometimes tended to oscillate. In this case an interval halving procedure was resorted to for one or two iterations. The
procedure used is outlined below.

1. Initial approximations of all three of the unknowns are computed by assuming elastic behavior.

2. Upper and lower bounds are set for $\theta$ and $S_1$.

3. Equation 2.21 is solved for $S_1$ in essentially the same manner as described above. The difference being that if the computed value of $P$ is closer to the correct value than the previous value, the appropriate bound on $S_1$ is changed. A new approximation to $S_1$ is then computed by the straight line method and checked to see if it falls between the established bounds. If it does not, the next approximation is taken to be the value midway between the bounds.

4. When a value of $S_1$ is found which gives the correct value of $P$; $M_u$ and $M_v$ are computed from equations 2.22 and 2.23. These values are then used to compute $\gamma$ which is compared with the correct value. If the proper value is not obtained, $\theta$ is adjusted in much the same manner as was $S_1$ and the procedure returns to step three.

5. When convergence to the correct value of $\theta$ is obtained, $M_r$ is computed from $M_u$ and $M_v$ and $\phi$ is adjusted to obtain the correct value of $M_r$ with each iteration beginning at step three.

2.3. Deflections

The procedure given above enables one to easily compute deflections by means of any of the standard numerical methods, all of which
require a knowledge of the distribution of curvature along the length of the member in question. The program FLEX2 given in Appendix B uses the above procedure and Newmark's method (Au, 1963) to compute deflections for a cantilever member loaded as shown in Figure 2.7.

![Diagram of Cantilever Member](image)

**Figure 2.7.** Cantilever member.

The procedure converged fairly rapidly for the I-section studied (Figure 3.3); the computer charges for the computations required for all of the load deflection curves given in chapter three were about seven dollars.
2.4. Discussion

The major disadvantage of the method presented above is the fact that it is based upon a somewhat limited model of the stress-strain curve for concrete. This does not appear to be a significant disadvantage, since the method could be modified for other stress-strain curves by deriving expressions similar to equations 2.21 through 2.23 for other stress-strain functions or by approximating a new stress-strain function by a series of more than two parabolic segments. It is possible that round-off error may be a problem with some cross sections, but this seems unlikely except in the unusual case of members with very thin protrusions.
3. EXPERIMENTAL PROGRAM

A limited experimental program was carried out in order to attempt verification of the preceding method of analysis; three full-sized prestressed concrete power poles were tested to destruction. Previously, a number of cylinders made of the same concrete mix had been tested in order to investigate the stress-strain relationship for the material to be used in the poles.

3.1. Cylinder Tests

The concrete was made by a commercial ready-mix supplier using Willamette River sand and gravel, and ASTM type III Portland cement. Details of the mix, which was designed for a 28 day strength of 7000 psi, are given in Table 3.1. Specimens were six by twelve inch cylinders cast in steel molds by personnel of the prestressed concrete producer. They were consolidated by attaching the molds to steel forms, for prestressed concrete members, which were vibrated by external form vibrators. They were steam cured at 155° to 160° F for approximately twelve hours after which they were removed from the forms and exposed to the atmosphere until the time of testing. Curing for these cylinders was the same as for the full-sized members which were tested later. The specimens were capped with a high strength capping compound approximately twelve hours prior to testing.
Table 3.1. Concrete mix design.

<table>
<thead>
<tr>
<th>Material</th>
<th>Quantity lb/cu yd</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4&quot; aggregate</td>
<td>735</td>
</tr>
<tr>
<td>1/2&quot; aggregate</td>
<td>1145</td>
</tr>
<tr>
<td>sand</td>
<td>1000</td>
</tr>
<tr>
<td>cement</td>
<td>799 (8 1/2 sacks)</td>
</tr>
<tr>
<td>water</td>
<td>260 (3.7 gal/sack)</td>
</tr>
</tbody>
</table>

Strains were measured by means of an eight-inch diameter compressometer which was modified to fit the six-inch cylinders and fitted with electrical displacement transducers. The device, which is shown in Figure 3.1, consists of two brass collars which were attached to the cylinders by means of pointed set screws. Relative displacement of the two collars was measured by two cantilever beam transducers which were mounted diametrically on the upper collar. A steel rod was supported at its lower end by a micrometer screw, which was part of the original compressometer, and at its upper end by the cantilever beam. Thus, the average deflection of the two beams was equal to the average relative displacement between the two collars.

Each transducer had two resistance strain gages and the two transducers were wired as a four-arm bridge. The strain gage signals were amplified by a Tektronix model 3C-66 carrier amplifier which was used to drive one channel of a Honeywell model 580 x-y
Since the testing machine used to load the cylinders had no provision for a load signal output, a battery and push button arrangement was used to drive the second axis of the recorder and to manually place a tick mark on the trace at predetermined load values and at ultimate load. The entire strain measuring system was calibrated by means of the original micrometer screws which read directly to one ten thousandth of an inch. To serve as a check on the system six-inch SR-4 strain gages were mounted directly on the
first two cylinders tested.

A total of 24 cylinders representing four days' production were tested at ages of 28 to 30 days, but strain data were obtained for only thirteen of these. The specimens were loaded continuously to failure with the usual test duration being fifteen to twenty minutes. The testing machine was adjusted to maintain a constant rate of loading until the cylinders began to show signs of softening, after which the controls of the machine were not readjusted.

The mean strength was 7.79 ksi with a coefficient of variation of 6.2 percent; the mean strain at ultimate strength was 0.00285 with a coefficient of variation of 8.8 percent. Data for the individual specimens are given in Table C.1. Due to the characteristics of the testing machine used, it was not possible to obtain any information on the descending portion of the stress-strain curve. All of the fractures were conical and tended to be rather explosive.

The stress-strain data for each individual cylinder were normalized with respect to ultimate strength and strain at ultimate strength. Both quadratic and cubic parabolas were fitted to the normalized data by the method of least squares. Either model gives an excellent fit; both regression curves look essentially the same when plotted. The cubic model perhaps gives a very slightly better fit but the quadratic model was deemed to be adequate for the purpose of structural analysis and this judgment appears to have been
borne out by the results of the full-scale tests which were conducted later. The equation of the quadratic curve is

\[ f = 1.63504s - 0.63504s^2 \]  

(3.1)

where \( f = \frac{F}{F_c} \) and \( s = \frac{S}{S_o} \).

This curve and the normalized data are given in Figure 3.2. Note that since the curve is forced through the origin and through the point (1,1) there is actually only one parameter.

### 3.2. Power Pole Tests

The full-size specimens tested were tapered concrete power poles having an I-shaped cross section as shown in Figure 3.3. They were concentrically prestressed by ten, one-half inch, 270K grade, seven wire, stress relieved prestressing strands. The initial pre-stress of 29.9 kips per strand was released at approximately sixteen hours. Based upon information given by Lin (1963) and the test results, losses due to creep and shrinkage were estimated to be about six percent at the time of testing. In addition to the prestressing strands, the members were reinforced transversely by un prestressed steel as shown in Figure 3.4. The specimens were commercially produced and were of the manufacturer's usual design except that ladders and hardware for power line support were omitted. The concrete and curing were the same as for the cylinders previously
Figure 3.2. Normalized stress-strain data.
Figure 3.3. Cross sections of pole.

Figure 3.4. Reinforcing details.

Note: Spacing shown at ground line.
tested except that a different brand of cement was used. A limited number of cylinders was also cast from the same batches of concrete as used in the poles. Cylinders tested at 28 days had a mean strength of 6.31 ksi with a coefficient of variation of 6.8 percent and cylinders tested at ages of 33 to 35 days, which coincides approximately with the tests of the poles, had a mean strength of 6.84 ksi with a coefficient of variation of 6.2 percent. Individual cylinder strengths are given in Table C.2.

3.2.1. Stress-strain Curves used in Computations

The concrete stress-strain curve which was used in the computations for the test poles is given by equations 2.3 through 2.6 with the following values of the parameters: $S_o = 0.0025$, $S_{cu} = 0.0032$, $A_1 = 4473$, $A_2 = -695,000$, $A_3 = -16.63$, $A_4 = 18,776$ and $A_5 = -3,755,100$. These values give a peak stress equal to the mean strength of the control cylinders tested during the week of the pole tests (6.84 ksi). The value of $S_o$ was obtained from a linear regression analysis of the data from the earlier cylinder tests of the same mix. $A_1$ and $A_2$ were obtained from equation 3.1 by substituting the above values of $F'_c$ and $S_0$. $A_3$ through $A_5$ and $S_{cu}$ are approximations chosen to give reasonable agreement with previously reported values of $k_1 k_3$ and $k_2 / k_1 k_3$. The parameters used herein give $k_1 k_3 = 0.67$ and $k_2 / k_1 k_3 = 0.58$ while Billet and Appleton (1954)
report a value of 0.76 for $k_1 k_3$ and Hognestad, Hanson and McHenry (1955) give 0.64. The latter authors give 0.66 for $k_2/k_1 k_3$ while Hognestad (1951) found 0.55. Better information on the descending portion of the stress-strain curve certainly would have been desirable but could not be obtained without special testing equipment. Furthermore, it was found that the computed behavior of the test specimens was relatively insensitive to rather large changes in this portion of the curve. The stress-strain curve used is plotted in Figure 3.5.

Figure 3.5. Concrete stress-strain curve used in pole analysis.
The stress-strain curve assumed for the strand is based upon data supplied by the manufacturer (CF & I, 1969) and the Oregon State Highway Department (1969). The manufacturer's data extends from \( S = 0 \) to \( S = 0.0115 \), and the curve used in the computations is identical with it in this range. The remainder of the curve was formed by extending a straight line to the mean values of ultimate strength and total elongation given by tests performed by the Oregon State Highway Department. The values used for the parameters in equations 2.7 through 2.9 were: 

\[
E_1 = 27,910, \quad B_1 = -123.2, \quad B_2 = 64,840, \quad B_3 = -2,767,000, \quad B_4 = 249.7 \quad \text{and} \quad E_2 = 594.6.
\]

The resulting curve and the data are given in Figures 3.6a and 3.6b.

3.2.2. Instrumentation and Test Procedure

The poles were erected in 24 in. diameter holes augered 6'-10" to 7'-11" into the ground. Backfill, which was sand and gravel passing a 3/8" sieve, was compacted by flooding and vibrating with a concrete vibrator. This was accomplished four to seven days prior to test which allowed the fill and surrounding soil to drain. A heavy steel plate with strand vises attached at 22 1/2 degree increments was bolted to the top of the poles, and loading was accomplished by pulling on strands gripped by these vises. The lower end of the strand was anchored by a large lift truck and a prestressing ram used to apply the load. Loads were measured by a load cell inserted
Figure 3.6a. Stress-strain curve for prestressing strand.

Figure 3.6b. Stress-strain curve detail.
between the ram and the frame of the lift truck, and the truck was located so that the vertical angle between the strand and the ground was 45 degrees.

A metal frame was rigidly clamped to the pole at a cross section near the ground, and level and dial gage arrangements parallel to each of the principal axes of the pole were mounted thereon to measure rotation of the cross section from the horizontal. Horizontal displacement of the tip of the pole relative to this frame was indicated by two oil damped plumb lines attached to an arm at the top of the pole. Two plumb lines, two feet apart were used so that any significant twisting of the pole would be apparent. The test arrangement allowed the tip deflection due to deformation of the pole to be separated from that due to deformation of the foundation.

Strains were measured at several locations near the base of the poles by means of a ten inch Whittemore strain gage using small steel tabs cemented to the poles as gage points. The general test arrangement is shown in Figure 3.7 and Figure 3.8 shows the levels and strain gage points. The major and minor principal axes were taken as the reference axes x and y respectively. Coordinates of each of the gage points are given in Table D.1.

In order to obtain the maximum amount of information from the small number of specimens, each pole was loaded for five values of γ. For each loading direction except the last, each pole was loaded
Figure 3.7. Test arrangement.
Figure 3.8. Levels and strain gage points.
until visible cracks appeared or until existing cracks could be seen to open, with the exception that no visible cracks were developed during the first test of specimen T1. The poles were then unloaded and reloaded in a new direction. The final test of each pole was continued to failure. Loads were applied in increments of 500 lb until it seemed that failure was imminent, at which time the load increment was decreased. Each load increment was maintained while deflection and strain observations were made. The time interval between successive load increments varied from 4 to 27 minutes with the longer intervals being near failure when more time was spent in identifying, marking and photographing crack patterns. The loading sequence and maximum load applied during each test of the three poles are given in Table 3.2.

3.2.3. Data Reduction

For each value of $\gamma$, total tip deflections were plotted versus load; bending moments including the effect of the vertical load component were computed based on this plot. Changes in strain from zero load were computed at each load increment for each test, i.e. residual strains accumulated from previous tests of the same pole were not included. The values of $\phi$ and $\theta$, at the two cross sections where strains were measured, were then estimated by the method of least squares, using the assumption that strains could be expressed
Table 3.2. Loading Sequence

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Test No.</th>
<th>( \gamma )-Degrees</th>
<th>( Q_{\text{max}} ) Kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>0</td>
<td>3.50</td>
</tr>
<tr>
<td>T1</td>
<td>2</td>
<td>22 ( \frac{1}{2} )</td>
<td>3.00</td>
</tr>
<tr>
<td>T1</td>
<td>3</td>
<td>45</td>
<td>2.00</td>
</tr>
<tr>
<td>T1</td>
<td>4</td>
<td>67 ( \frac{1}{2} )</td>
<td>2.00</td>
</tr>
<tr>
<td>T1</td>
<td>5</td>
<td>90</td>
<td>3.90 Failure</td>
</tr>
<tr>
<td>T2</td>
<td>1</td>
<td>90</td>
<td>2.50</td>
</tr>
<tr>
<td>T2</td>
<td>2</td>
<td>67 ( \frac{1}{2} )</td>
<td>2.00</td>
</tr>
<tr>
<td>T2</td>
<td>3</td>
<td>22 ( \frac{1}{2} )</td>
<td>3.00</td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
<td>0</td>
<td>3.50</td>
</tr>
<tr>
<td>T2</td>
<td>5</td>
<td>45</td>
<td>4.35 Failure</td>
</tr>
<tr>
<td>T3</td>
<td>1</td>
<td>90</td>
<td>2.00</td>
</tr>
<tr>
<td>T3</td>
<td>2</td>
<td>67 ( \frac{1}{2} )</td>
<td>2.00</td>
</tr>
<tr>
<td>T3</td>
<td>3</td>
<td>45</td>
<td>2.50</td>
</tr>
<tr>
<td>T3</td>
<td>4</td>
<td>0</td>
<td>4.00</td>
</tr>
<tr>
<td>T3</td>
<td>5</td>
<td>22 ( \frac{1}{2} )</td>
<td>5.35 Failure</td>
</tr>
</tbody>
</table>

1 Failure load for specimen T1 is estimated but it is believed that the value given is within 100 lb of the true load. The load is known to have been well above 3.5 kips and slightly under 4 kips.

as a linear function of \( x \) and \( y \) at a particular cross section.

3.2.4. Results

All of the specimens exhibited extremely uniformly spaced tension cracks which did not begin to appear until considerably above the computed cracking moments. These cracks first appeared near the base of the pole and progressed rapidly upward as the load was slightly increased. Crack spacing was about twelve to fourteen
inches and maximum crack width was approximately one tenth inch. 
Upon unloading, all of the cracks closed completely with the excep-
tion of those tests which were continued to failure. Figure 3.9 shows 
the crack pattern in pole T1 with $\gamma = 90$ degrees and $Q = 3.0$ kips. 
All of the failures appeared to be pure flexural failures with no indi-
cation of distress caused by torsional or transverse shear. As loads 
were increased there was a continuing widening of the tension cracks 
and no indication of failure in the compression region until immedi-
ately prior to failure when there was some extremely minor spalling 
followed by a sudden compressive failure. Figures 3.10 through 3.12 
show the nature of the failures. Note that in specimen T3 compres-
sion failures occurred in both flanges. The theory predicts tension 
in what appears as the far flange in the figure, and examination of the 
strain data shows that this was the case at 98 percent of ultimate load 
when the last measurements were made. It is therefore concluded 
that the failure was progressive, and that the neutral axis rotated 
drastically after failure of the near flange.

Moment-curvature relations for $P = 0$ and various values of $\theta$ 
were computed and plotted for the cross section at the ground line. 
These are shown as Figures 3.13 and 3.14 which also show computed 
values of $\gamma$ versus curvature. The maximum moment values obtained 
from these curves were used to plot the interaction diagram for ulti-
mate moments shown in Figure 3.15 which also shows the ultimate
Figure 3.9. Specimen T1 under load.

Figure 3.10. Failure of specimen T1.

Figure 3.11. Failure of specimen T2.

Figure 3.12. Failure of specimen T3.
Figure 3.13. Computed moment curvature curves for $\theta = 0$, $22\frac{1}{2}$ and 45 degrees.
Figure 3.14. Computed moment curvature curves for $\theta = 67\frac{1}{2}^\circ$, 80 and 90 degrees.
Figure 3.15. Ultimate moment interaction curve.
moments for the three poles tested. Recomputation of the moment-curvature relationships for \( P = 10 \) kips indicated that the error introduced by neglecting the vertical load component in the above computations was negligible.

Figures 3.16 through 3.20 show computed values of curvature and inclination of the neutral axis plotted versus load (tension in loading strand) as well as those values estimated from the strain data by the least squares procedure. The plotted experimental values are the average of the two cross sections where strain readings were made as long as there was a good fit at both locations, otherwise the data from only one section is shown. The fit was excellent for all tests until cracking began to affect the strain readings. Often only one set of readings appeared to be effected and it was apparent from studying the photographs of the cracks that this was due to the relative location of the cracks and the gage points. The strain data are given in Appendix D.

Computed tip deflections and that part of the measured deflections attributable to bending of the poles are shown in Figures 3.21 through 3.25. No twisting of the poles was observed, but the twist would have had to have been large to be detected.
Figure 3.16. Computed and measured curvatures for $\gamma = 0^\circ$.

Figure 3.17. Computed and measured curvatures for $\gamma = 22\frac{1}{2}^\circ$. 
Figure 3.18. Computed and measured curvatures for $\gamma = 45^\circ$.

Figure 3.19. Computed and measured curvatures for $\gamma = 67\frac{1}{2}^\circ$. 
Figure 3.20. Computed and measured curvatures for $\gamma = 90^\circ$.

Figure 3.21. Computed and measured curvatures for $\gamma = 0^\circ$. 
Figure 3.22. Computed and measured deflections for $\gamma = 22\frac{1}{2}^\circ$.

Figure 3.23. Computed and measured deflections for $\gamma = 45^\circ$. 
Figure 3.24. Computed and measured deflections for $\gamma = 67\frac{1}{2}^\circ$.

Figure 3.25. Computed and measured deflections for $\gamma = 90^\circ$. 
3.2.5. Discussion

The method of testing presented certain problems, however it was felt that vertical erection and testing of the poles was superior to a horizontal arrangement primarily for two reasons. First, due to the flexibility of the pole it would have been necessary to provide intermediate supports had the pole been placed in a horizontal position. This would have made the problem statically indeterminate and would have required much more complicated measurements. Secondly, additional complications would have arisen due to the fact that the direction of the load and the direction in which the pole deflects are the same only in the case of bending about a principal axis.

Burial depths were adequate for the first two specimens tested but considerable apprehension on the part of those involved in the testing was caused by the behavior of the foundation for specimen T3, which was mistakenly placed one foot too shallow. Displacement of this specimen at the ground line was estimated to be about four inches at the time of failure.

Coordination of the experimental phases of the project was slightly complicated by the fact that the prestressing plant was located in a town several miles from the university. It was planned to install some of the strain gage points prior to release of the
prestressing strands so that losses could be estimated more accurately, however the poles were fabricated ahead of schedule without the knowledge of the investigator.

Continuous indication of strains would have been desirable to enable measurement of strains at ultimate moment but due to the sequence of testing, which produced tension cracks in three corners of the poles prior to failure, and the uncertainties involved in attempting to use electrical strain gages out of doors in a wet climate it was decided to use a mechanical strain measuring device. As was to be expected, there was much scatter in the strain data in the tension regions subsequent to cracking. Owing to the crack spacing, the gage length was too short to give a reasonable average and too long to indicate local strains in the cracked region after the cracks had opened significantly.

Deflection measurements for specimen T3 were severely hampered by gusty winds which caused a large amount of scatter in the data. There appears to be an appreciable zero error in both the deflection and strain data for specimen T1 which is attributed to the fact that the loading strand was attached to the ram prior to making zero load observations, and although there was considerable slack in the strand there seems to have been enough tension to significantly affect the pole. In subsequent tests zero readings were made prior to attachment of the strand.
Ultimate moments for all three specimens were well predicted by the analysis with the maximum error being six percent. Curvatures, deflections and the inclination of the neutral axis all appear to have been predicted within reasonable limits considering the measurement methods and the variability of the properties of the concrete.
4. CONCLUSIONS

It has been shown that the derivation of analytical expressions for the general case of biaxial bending of concrete members is feasible. The analysis is based upon reasonable representations of the stress-strain relationships for both the concrete and the reinforcement. The representation of cross section geometry is exact for any section bounded by straight line segments; cross sections having curvilinear boundaries can easily be approximated to any reasonable degree of accuracy.

Simultaneous solution of all three of the equations relating loads to strain distribution has been accomplished by means of an iterative procedure. This is significant because the only alternative available in the analysis of framed structures involving biaxial bending of concrete members is the storage and subsequent retrieval of massive amounts of tabular data.

The experimental evidence presented, although not conclusive, strongly supports the theoretical analysis. All of the experimental data appear to be in reasonable agreement with predicted values, and unusually good agreement was obtained between calculated and measured values of ultimate moment. The use of cylinder tests as a basis for predicting flexural behaviour has been challenged (Sturman, Shah and Winter, 1965), but in this instance it has been shown to give
reasonable results. Arguments against the assumptions concerning bond and strain distribution were discussed in chapter two.

No consideration was given to possible modes of failure other than pure flexure nor was the time dependence of the material behavior considered. These items were considered to be outside of the scope of the present study.

The nature of the failures in the three specimens tested very strongly suggests that the extreme ductility of concrete, which has been observed by some investigators using carefully controlled displacement governed tests, may be of little or no significance in some applications of structural concrete.

The inability to predict the stress-strain curve of concrete with any degree of precision is still one of the most significant obstacles to accurate prediction of the behavior of concrete structures. It appears that much further work is needed in this area, and it is suggested that progress could be speeded by more complete testing and reporting of the properties of the constituent materials used in concrete which is to be tested for structural properties.

It is apparent from the deflection data given in the preceding chapter that the method of measurement employed leaves much to be desired. It is suggested that in future work of this nature some other method, such as triangulation, be employed unless adequate protection from the wind is ensured.
BIBLIOGRAPHY


Pannell, F. N. 1963. Failure surfaces for members in compression and biaxial bending. Proceedings, American Concrete Institute 60:129-140.


APPENDIX A

NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area, parameter in equation 1.3.</td>
</tr>
<tr>
<td>A₁...A₅</td>
<td>parameters of assumed concrete stress-strain function.</td>
</tr>
<tr>
<td>A₇</td>
<td>gross cross-sectional area.</td>
</tr>
<tr>
<td>Aₛᵢ</td>
<td>cross-sectional area of i th strand.</td>
</tr>
<tr>
<td>Aₛᵗ</td>
<td>total area of steel.</td>
</tr>
<tr>
<td>B</td>
<td>parameter in equation 1.3.</td>
</tr>
<tr>
<td>B₁...B₄</td>
<td>parameters of assumed steel stress-strain function.</td>
</tr>
<tr>
<td>b</td>
<td>width.</td>
</tr>
<tr>
<td>C</td>
<td>resultant compressive force, parameter in equation 1.3.</td>
</tr>
<tr>
<td>c</td>
<td>depth to neutral axis.</td>
</tr>
<tr>
<td>Eₖ</td>
<td>Young's modulus for concrete.</td>
</tr>
<tr>
<td>E₁</td>
<td>Young's modulus for strand.</td>
</tr>
<tr>
<td>E₂</td>
<td>slope of stress-strain curve for strand in strain hardening range.</td>
</tr>
<tr>
<td>e₁, e₂</td>
<td>general unit deformations.</td>
</tr>
<tr>
<td>F</td>
<td>stress.</td>
</tr>
<tr>
<td>Fₖ</td>
<td>concrete stress.</td>
</tr>
<tr>
<td>Fₑ₂, Fₑ₃</td>
<td>concrete stress in zones two and three.</td>
</tr>
</tbody>
</table>
\( F'_c \)  
concrete cylinder strength

\( F''_c \)  
peak concrete stress in flexure.

\( F'''_c \)  
concrete stress at material failure.

\( F_{cs_i} \)  
concrete stress at location of \( i \) th strand.

\( F_{pl} \)  
steel stress at proportional limit.

\( F_s \)  
steel stress.

\( F_{si} \)  
stress in \( i \) th strand.

\( F_{su} \)  
steel stress at material failure.

\( F_y \)  
yield stress of steel.

\( f \)  
normalized stress, function.

\( g, h \)  
functions.

\( i, j, l \)  
indexes.

\( k_1, k_2, k_3 \)  
parameters of concrete stress distribution defined in Figure 1.1.

\( M \)  
bending moment.

\( M_r \)  
resultant bending moment.

\( M_u, M_v, M_x, M_y \)  
components of bending moment acting about axis indicated by subscript.

\( m \)  
index.

\( n_j \)  
number of boundary points defining \( j \) th zone.

\( n_s \)  
number of strands.

\( P, P_o \)  
axial compressive force.

\( P_{si} \)  
tensile force in \( i \) th strand.

\( Q \)  
inclined force at tip of cantilever member.
S  strain.
S_1  concrete strain at origin of coordinates.
S_c  concrete strain.
S_{cu}  concrete strain at material failure.
S_o  concrete strain at peak stress.
S_{pl}  steel strain at proportional limit.
S_s  steel strain.
S_{su}  maximum concrete strain at member failure.
S_y  steel yield strain.
s  normalized strain.
T  resultant force in tensile reinforcing group.
u, v  axes parallel and perpendicular respectively to neutral axis.
x, y  reference axes.
\beta_{ij}  coefficients defined in equations 2.16
\gamma  inclination of M_r from u axis.
\Delta_s  difference in strain between steel and concrete.
\theta  angle between u and x axes.
\Sigma_{ij}  coefficients defined in equations 2.24-2.26.
\phi  curvature.
\psi  inclination of Q from normal to member axis.
APPENDIX B

COMPUTER PROGRAMS

Programs implementing the analysis described in chapter two were written in FORTRAN IV for the CDC 3300 computer operating under the Oregon State Open Shop Operating System (OS-3) and are presented herein. Extensive use was made of subroutines utilizing common storage and two slightly different versions of many of them were used by the two main programs FLEX and FLEX2. The essential differences between the two versions are in the allocation of common and the fact that a few variables which are simple in one version are subscripted in the other. The program and subroutine names together with the function of each are listed below. Figure B.1 gives listings of the programs and subroutines with the exception of those such as DATE, TIME and FFIN which are a part of the system library. Those subroutines indicated by an asterisk are ones which were modified as indicated above and only the version used by FLEX is given in Figure B.1.

**FLEX** - Used to generate moment-curvature curves for constant values of $P$ and $\theta$. Designed to do computations for each of several stations along the length of a tapered member but will also work for a single cross-section without reference to a particular member.
Reads job control variables. Performs various I/O housekeeping functions and calls subroutines to perform the computations.

MPHI - Solves equation 2.21 for \( S_1 \) with \( P, \phi, \) and \( \theta \) known.

SECIN * - Inputs problem number, heading, problem control variables, material properties, steel geometry, pre-stress and losses. Generates geometry at stations along the length of a tapered member.

CONCIN* - Inputs concrete geometry of arbitrarily shaped section.

ISECIN * - Generates coordinates of points defining the I section used in the experimental study from depth, width, flange thickness and web thickness.

SYMSEC*-Computes and prints section properties for a cross section symmetrical about the y axis. Entry TRSEC is used for new reinforcing geometry and previous concrete geometry. The method used is similar to that given by Brandt (1962).

ZONES * - Constructs IZ array which indicates location of points relative to zone boundaries. IZ(I) is equal to:

1 if ith point is in zone 1

2 if ith point is on boundary between zones 2 and 3
if ith point is in zone 2

.............

7 if ith point is in zone 4.

Constructs arrays U2, V2, U3, and V3 which are points defining the boundaries of zones two and three arranged in clockwise order.

SUMCP *-Computes equations 2.24.

SUMCM *-Computes equations 2.25 and 2.26.

FLEX2 -Computes deflections of cantilever loaded at end.

MPHI2 -Solves equations 2.21, 2.22 and 2.23 simultaneously for \( S_1, \phi \) and \( \theta \).

Much of the input for programs FLEX and FLEX2 is identical, this common input is given in Table B.1. Tables B.2 and B.3 give additional input required by FLEX and FLEX2 respectively while Figures B.2 and B.3 give sample input for the two programs. All input is from logical unit (LUN) one which is rewound by the program at the beginning of execution. Sample output from the two programs is given in Figures B.4 and B.5.
DEFINE DECLAR

C*****THIS GROUP OF STATEMENTS TO BE INCLUDED WHEN COMPILING
C*****FLEX AND ASSOCIATED SUBROUTINES
REAL IX,IXO,IY,ITYO,IXO,LOSS
COMMON X(15,11),Y(15,11),U(15,11),V(15,11),XS(40,11),YS(40,11)
COMMON AS(40,11),US(40,11),VS(40,11),AT(11),AST(11),ECC(11)
COMMON DELAY(11),YBART(11),ITX0(11),YBALL(11),AX(11),IT(11)
COMMON A(11),Z(11)
COMMON CPROP(7),SPROP(9)
COMMON NS,NP,IPNOLD,IPRT,PRINT,NALLOW,OUT
COMMON XSU,NVATNXAT,P(11),ITX0(11),ITY0(11),ITX0(11),IY(11)
COMMON A(11),2(11)
COMMON CPROP(7),SPROP(9)
COMMON NS,NP,IPNOLD,IPRT,PRINT,NALLOW,OUT
COMMON VSU,VNAINSO,S1NEW
COMMON NEWSP,NEWCP,NEWCG,NEWCS,IPN
COMMON OAL,P,O,GAM,TAN,PH1(11),SGSO,CGSO,ITY0(11),FSE,DS
COMMON ISTART,ISTOP
END

PROGRAM FLEX

CUSES VERSION 1 SUBROUTINES
INCLUDE DECLAR
CALL IN

COMMON VARIABLES
REAL IX,IXO,IY,ITYO,IXO,LOSS
COMMON X(15,11),Y(15,11),U(15,11),V(15,11),XS(40,11),YS(40,11)
COMMON AS(40,11),US(40,11),VS(40,11),AT(11),AST(11),ECC(11)
COMMON DELAY(11),YBART(11),ITX0(11),YBALL(11),AX(11),IT(11)
COMMON A(11),Z(11)
COMMON CPROP(7),SPROP(9)
COMMON NS,NP,IPNOLD,IPRT,PRINT,NALLOW,OUT
COMMON VSU,VNAINSO,S1NEW
COMMON NEWSP,NEWCP,NEWCG,NEWCS,IPN
COMMON OAL,P,O,GAM,TAN,PH1(11),SGSO,CGSO,ITY0(11),FSE,DS
COMMON ISTART,ISTOP
END

Figure B.1. Program and subroutine listings.
906 VNA=-SINEW/PHI(K)  
V50=(CPROP(7)*SINEW)/PHI(K)  
VSU=(CPROP(6)*SINEW)/PHI(K)  
PC=0.  
CALL ZONES(K,N2,U2,V2,V3)  
IF(N2.EQ.0)1001,56  
1001 IF(N3.NE.0)GO TO 58  
IF(SINEW.LE.0)1002,1003  
1002 SINEW=0.00001  
IPASS=IPASS+1  
GO TO 901  
1003 SINEW=CPROP(6)+0.00001  
IPASS=IPASS+1  
GO TO 901  
C.....SUM ZONES 2 CONTRIBUTION  
56 CALL SUMCP(U2*V2,N2,SP12,SP22,SP32)  
B11=CPROP(2)*PHI(K)*PHI(K)  
B12=2.*CPROP(2)  
B13=(CPROP(2)+SINEW*CPROP(1))*SINEW  
PC=SP12*B11+SP22*B12+SP32*B13  
IF(N3.EQ.0)59,58  
C.....SUM ZONES 3 CONTRIBUTION  
58 CALL SUMCP(U3,V3,N3,SP13,SP23,SP33)  
B21=CPROP(5)*PHI(K)*PHI(K)  
B22=2.*CPROP(5)  
B23=(CPROP(5)+SINEW*CPROP(4))*SINEW  
PC=PC+SP13*B21+SP23*B22+SP33*B23  
C.....STEEL CONTRIBUTION  
59 DO 1004 I=1,NS  
SC=SINEW+PHI(K)*VS(I,K)  
SS=DCSC  
IF(SC.GE.CPROP(6))GO TO 1005  
IF(SS.LE.SPROP(2))60,61  
1005 PCS(I)=0.  
GO TO 1004  
60 PCS(I)=CPROP(1)+SS*AS(I,K)  
GO TO 65  
61 PCS(I)=SS*PROP(8)+87,62  
87 PCS(I)=0.  
GO TO 65  
62 PCS(I)=SS*PROP(6)+64  
PC(I)=SS*PROP(3)+CPROP(4)+PROP(5)+SS*AS(I,K)  
GO TO 66  
64 PCS(I)=AS(I,K)*CPROP(7)+SS*PROP(8)  
65 IF(SC+LE.0)GO TO 69  
IF(SC+LE.0)GO TO 69  
67 PCS(I)+CPROP(2)+SC+AS(I,K)  
GO TO 68  
68 PCS(I)+CPROP(4)+CPROP(5)+SC*CSC+CPROP(3)+AS(I,K)  
69 PC=PC+PCS(I)  
1004 CONTINUE  
C.....P IS NOT BALANCED  
IF(IPASS+NE.0)GO TO 73  
IF(IPASS+NE.0)GO TO 72  
IPASS=IPASS+1  
S1OLD=SINEW  
IF(PCLT+P16GO TO 75  
SINEW+SIOLD+0.00001  
PLAST+PC  
GO TO 76  
75 SIOLD+SIOLD+0.00001  
PLAST+PC  
GO TO 76  
76 SIOLD+SINEW  
SINESSIOLD+DSIDP*PDIFF  
PLAST+PC  
IPASS=IPASS+1  
GO TO 77  
77 WRITE(2,78)IPASS+SIOLD,PC  
78 FORMAT(1X,ITERATION=1,E4.1,COMPUTED P=1.,1E11,41  
C.....TRY AGAIN  
GO TO 901  
C.....SUM STEEL MOMENTS  
79 MU=MV=0.  
SIOLD+SINEW  
IPASS=0  
IF(N2.EQ.0)80,81  
80 CALL SUMCM(U2*V2,N2,SMU12,SMU22,SMU32,SMV12,SMV22,SMV32)  
MU=SMU12+SMU22+SMU32+SMV12+SMV22+SMV32  
MV=SMU12+SMU22+SMV12+SMV22+SMV32  
81 IF(N3.EQ.0)83,82  
82 CALL SUMCM(U3*V3,N3,SMU13,SMU23,SMV13,SMV23,SMV33)  
MU=MU+SMU13+SMU23+SMV13+SMV23+SMV33  
MV=MV+SMU13+SMU23+SMV13+SMV23+SMV33  
83 DO 84 I=1,NS  
MU=MU+PCS(I)*VS(I,K)  
84 MV=MV+PCS(I)*US(I,K)  
MU=MU/12.  
MV=MV/12.  
FM=SORT(MU*MU+MV*MV)  
IF(THETAD.LT.1.)88,89  
88 MX=MU  
MY=MV  
GO TO 90  
89 MX=MU*CI*NVS(I,K)  
MY=MV*CI*NUS(I,K)  
IF(THERAO+LT.1+180+89  
88 MX=MV  
MY=MV  
GO TO 90  
89 MX=MU+MF=SQRT(MU+MV)  
88 MX=MV  
MY=MV  
GO TO 90  
87 MU+MV+MC+MV=  
MY=MU+MV+MC  
90 T=PHI(K)+1,E=0  
GAM=ATAN(MY/MX)+57.2958  
SCMAX+SIOLD+1,E=0+TVMAX  
SCMIN+SIOLD+1,E=0+TVMIN  
SCTX+DSIDP+PHI(K)*VSMIN+1,E=0  
WRITE(2,86)PHI(K)+FM,MX,MY+GAM+VNA+S MAX+SCMIN+SCMAX  
TEMP=0.95FMAX  
PHIOLD+PHI(K)
SUBROUTINE SECIN

C VERSION I
C I/O ROUTINE
C 1. GENERATES GEOM. ALONG LENGTH OF TAPERED MEMBER FROM
C 2. GEOM. AT ENDS.

INCLUDE DECLARS
DIMENSION DX(30),DY(30)
DIMENSION NSCO(15)
READ(1,112)IPN
IF(E0F(111)CALL EXIT
WRITE(2,91)
91 FORMAT('ANALYSIS OF PRESTRESSED MEMBERS FOR',1X,A8,2X,92A8)
READ(1,94)
94 FORMAT(6I1)
IF(NEWCP.E0.111,3
1 CPROP(I)=FFIN(1)
IF(NEWSP.E0.0)694
3 IF(NEWSP.E0.11G0 TO 4
WRITE(2,96)IPNOLD
96 FORMAT('MATERIAL PROPERTIES SAME AS PROB NO',1X,6I1)
GO TO 7
4 DO 5 I=1,9
5 SPROP(I)=FFIN(I)
6 EC=CPROP(1)+CPROP(2)*CPROP(7)**.5
MR=SPROP(1)/EC-1.
IF(CPROP(1).LT..1)MR1.5.
WRITE(2,97)
97 FORMAT('CONCRETE****
WRITE(2,98)CPROP(7)
98 FORMAT('STRAIN BETWEEN 0 AND',F7.5,92X,EC=',E10.3,'\n99 FORMAT(4X,'+',E10.30*S',E11.30,10**2)
WRITE(2,100)CPROP(6)
100 FORMAT('STEEL****
WRITE(2,103)SPROP(2)
103 FORMAT('STRAIN BELOW',F8.6)
104 FORMAT(4X,'+',E10.30*S')
105 FORMAT('STEEL****
WRITE(2,106)IPN
106 FORMAT(4F10.0)
IF(TAPER.LE.1.0E-10)12,13
12 Z(1)=0.
B=BB
H=HH
CALL ISECIN(1)
GO TO 25
13 TAPER=TAPER*2.
DO 17 I=1,NSTA
17 IF(I.E0.1)Z(1)=GL
IF (I.E0.1)14,15
14 Z(1)=GL
15 T=ABS(PHI(K))
IF(T.LE.1.E-05)200,201
200 PHI(K)=PHI(K)+.0002/VMAX
GO TO 202
201 PHI(K)=PHI(K)+.0002/ABS(VMAX*VNA)
202 IF(IPOINT.E0.0)G0 TO 203
TEMP=S1NEW
SlNEW=S1NEW+(S1NEWS1OLDP)*(PHI(K)PHIOLD)/(PHI(K)PHOLDP)
PHOLDP=PHIOLD
SlOLDP=TEMP
GO TO 906
203 IPOINT=1
SIOLDP=S1NEW
PHOLDP=PHIOLD
GO TO 906
END
AYBAR = A(K) = IX = IY(K) = 0.0
T = Y(1.1(K))*Y(1.K)
T1 = X(1.K)*X(1.K)
DO 10 I = 1, NP
L = I - 1
M = I + 1
IF (L.LT.1) L = NP
IF (M.GT.NP) M = 1
A(K) = A(K) + (X(I,K)*Y(L,K)*Y(M.K) + Y(L,K)*Y(I,K) + Y(M.K))*T1 + T3)
Y(1.K) = X(M,K)*X(M,K)
T2 = Y(M,K)*Y(M,K)
T3 = X(I,K)*X(I,K)
IX = T1 + T3
IY(K) = T2 + T4
10 T1 = T3
AYBAR = AYBAR/6.
IX = IX/12.
IY(K) = IY(K)/12.
A(K) = A(K)/2.
YBARG(K) = AYBAR/A(K)
ENTRY TRSEC
C***NEW STEEL, SAME CONC. AS PREVIOUS PROB.
AST(K) = 0.0
T = 0.0
T3 = 0.0
DO 20 I = 1, NS
AST(K) = AST(K) + AS(I,K)
T = T + XS(I,K)*AS(I,K)*XS(I,K)
20 T3 = T3 + T1
ECC(K) = T3/AST(K)
AT(K) = AST(K)*MR + A(K)
T = T*MR
T2 = T3 + T1
T3 = T3 + T1
END
C***THE FOLLOWING DEFINITIONS TO BE INCLUDED WITH ZONES
N2 = N2 + 1
U2(N2) = U(I,K)
V2(N2) = V(I,K)
END
DEFINE IN2
N2 = N2 + 1
U2(N2) = VNA
V2(N2) = U(I,K) + VNA - U(I,K) + V(I,K)
END
DEFINE INS02
N3 = N3 + 1
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
DEFINE INS023
N3 = N3 + 1
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
DEFINE INS03
N3 = N3 + 1
V3(N3) = VSO
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
DEFINE INSU3
N2 = N2 + 1
V2(N2) = V3(N3) + VSO - V(I,K)
END
DEFINE INS03
N3 = N3 + 1
V3(N3) = VSO
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
DEFINE INS03
N3 = N3 + 1
V3(N3) = VSO
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
SUBROUTINE ZONES(K, N2, U2, V2, N3, U3, V3)
C***VERSION 1
C***DETERMINES POINTS DEFINING ZONE 2 AND 3 BOUNDARIES
DIMENSION I2(30), U2(30), V2(30), U3(30), V3(30)
INCLUDE DECLAR
N2 = N2 + 1
V2(N2) = VNA
U2(N2) = U(I,K)
V2(N2) = V3(N3) + VSO - V(I,K)
END
DEFINE INS03
N3 = N3 + 1
V3(N3) = VSO
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
DEFINE INS03
N3 = N3 + 1
V3(N3) = VSO
U3(N3) = U(I,K) + VSO - U(I,K) + VSO - V(I,K)
END
END
GO TO 17
44 CONTINUE
INCLUDE INS02
INCLUDE INS02
INCLUDE INSU3
INCLUDE INS03
GO TO 17
45 CONTINUE
INCLUDE INS02
INCLUDE INSU3
INCLUDE INS03
GO TO 17
46 CONTINUE
INCLUDE INSU3
17 CONTINUE
GO TO 52
47 IF(S1NEW.LE.CPROP(6).AND.S1NEW.GE.CPROP(7))48,49
48 N3=NP
N2=0
DO 50 I=1,NP
U3(I)=U(I,K)
50 V3(I)=V(I,K)
GO TO 52
49 N2=NP
N3=0
DO 51 I=1,NP
U2(I)=U(I.K)
51 V2(I)=V(I.K)
C OPTIONAL OUTPUT OF IZ
52 IF(I2PRT.EQ.1)WRITE(2,57)(IIZ(I)),I=1.NP)
57 FORMAT(.71Z.,3012)
IF(NZPRT.EQ.1)56,58
C OPTIONAL OUTPUT OF ZONE 2/3 BOUNDARIES
56 WRITE(61,53)
53 FORMAT(/X'ZONE 2'/
WRITE(61,54)((I,U2(I),V2(I)),I=1,N2)
54 FORMAT(X12,2F10.3)
WRITE(61,55)
55 FORMAT(/X'ZONE 3'/
WRITE(61,541(1I,U3(I)1N3(I)),I=1,N3)
58 RETURN
SUBROUTINE SUMCM(U,V,N,SU1,SU2,SU3,SV1,SV2,SV3)
C VERSION 1
C USED IN DETERMINING CONC. CONTRIBUTION TO MU AND MV
DIMENSION U(20),V(30)
SU1=SU2=SU3=SV1=SV2=SV3=0.
DO 6 1=1.N
IF(I.EQ.1)6
2 L=N
GO TO 9
3 L=1-1
9 IF(I.EQ.N+4)5
4 M=1
GO TO 10
5 M=L+1
10 T1=U(L)-U(M)
T2=U(I)-U(M)
T3=V(I)*V(M)
T7=V(I)*V(I)
T5=T7*T1
76=T3*T2
SU3=SU3+75+1.6
T5=T5*V(I)
SU2=SU2+T5+(V(I)+V(M))*T6
SU1=SU1+V(I)*T5+73*(76+(1.7+V(M)*V(M))*T2)
T1=U(L)*U(L)
T2=U(I)*U(I)
T4=U(M)*U(M)
T6=U(L)*U(I)
SV3=SV3+V(I)*(74-1.1)+75*(V(I)+V(M))
SV2=SV2+17*(2.*(T4-U(L)*U(I))+T4-71)+T3*2.*(T4-72)
6 SV1=SV1+1.7*(V(I)*(3.*(1.6-75)+T1-74)+V(L)*(T6-3.*T2+2.*T1))
+V(M)*(-2.*T4-75+3.*T2))
SU1=SU1/20.
SU2=SU2/12.
SU3=SU3/6.
SV1=SV1/60.
SV2=SV2/24.
SV3=SV3/6.
RETURN
END
DEFINE DECLARE
C THIS GROUP OF STATEMENTS TO BE INCLUDED WHEN COMPILING FLEX 2
C*****AND ASSOCIATED SUBROUTINES
REAL IX, Y, IXO, M, ITO, ITXO, LOSS
COMMON X(15, 11), Y(15, 11), U(15, 11), V(15, 11), XS(40, 11), YS(40, 11)
COMMON DELTAY(11), YBART(11), ITO(11), YBARG(11), IXO(11), ITI(11), YI(11)
COMMON A(11), Z(11)
COMMON CP(11), SP(11)
COMMON NS(40, 11), US(40, 11), VS(40, 11), A7(11), AST(11), ECC(11)
COMMON NSO, IPNOLD, MR, PRESTR, LOSS, ECTB, HIFTW, NSTAWRITE(2, 12)
COMMON IZPRT, NZPRT, NPRT, NALLOW, IOUT
COMMON FMOLD(11), FM(11), THETA(11), DS(11)
COMMON DPHIDM(11), S(11)
COMMON ERR, PHINEW
GAM(I) = GAMMA
END
FM(I) = 0.5 * ARM(I)
CALL MPHI2(I)

PROGRAM FLEX2
C*****USES VERSION 2 OF SUBROUTINES
C*****COMPUTES DEFLECTIONS FOR TAPERED CANTILEVER LOAD AT TIP
INCLUDE DECLAR2
DIMENSION DEF(11), YDEF(11), PHIX(11), PHIY(11)
DIMENSION ARM(11), RX(11), RY(11)
REWIND 1
READ (1, 90) IOUT, NALLOW, NPRT, NOUT, ISTOP
15 FORMAT (XI 3 9F8.2, 9E11.3, 2F6.3)
2 CALL EQUIP(2, 1000000007)
3 CALL SECIN
4 DO 5 I = 2, J
ARM(I) = Z(NSTA) Z(I)
RX(I) = PHIX(I-1) + 10. * PHIX(I) + PHIX(I+1)
5 CALL SYMSEC(I)
RY(I) = PHIY(I-1) + 10. * PHIY(I) + PHIY(I+1)
6 IF (NEWSP .EQ. 0.0 .OR. NEWCP .EQ. 0.0 .OR. NEWCG .EQ. 0.0) 7, 9
7 DO 8 I = 1, NSTA
TM = FM(I) / 12.
TT = THETA(I) * 57.29578
WRITE (2, 15) I, TM, PHI(I), TT, TG, XDEF(I), YDEF(I), DEF(I)
8 CALL TRSEC(I)
9 WRITE (2, 91) IPNOLD
10 GAMMD = FFIN(1)
IF (GAMMD .LT. 0.0 .GO TO 3
GAMMA = GAMMD * 0.01745

C*****FORCE PARALLEL TO MEMBER AXIS
PP = FFIN(1)
ERR = FFIN(1)

IQ = 0
C*****INCLINED FORCE
11 Q = FFIN(1)
IF (IQ .LT. 0.0) GO TO 10
BETAD = FFIN(1)
BETAD = BETAD * 10.17453
P = PP + Q * ERR
GAMMD = BETAD
H = Q * COS(BETA)
DO 18 I=1,J
   XM=OHY*ARM(I)+P*(YDEF(NSTA)-YDEF(I))
   YM=OHX*ARM(I)+P*(XDEF(NSTA)-XDEF(I))
   GAM(I)=ATAN(YM/XM)
   FM(1)=SORT(XM*XM+YM*YM)
   CALL AGAIN(I)
   PHIX(I)=PHI(I)*SIN(THETA(I))
   PHIY(I)=PHI(I)*COS(THETA(I))
   RX(1)=3.5*PHIX(1)+3.*PHIX(2)+3.5*PHIX(3)
   RY(1)=3.5*PHIY(1)+3.*PHIY(2)+3.5*PHIY(3)
   TT=THETA(I)*57.29578
   TG=GAM(I)*57.29578
18   WRITE(2,14)
   WRITE(2.15)1,TM,PHI(1),TT,TG,DEF(1)
   TX=XDEF(2)-RX(1)
   TY=YDEF(2)-RY(1)
   DO 19 I=2.J
      L=I-1
      M=I+1
      RX(I)=PHIX(L)+10.*PHIX(I)+PHIX(M)
      RY(I)=PHIY(L)+10.*PHIY(I)+PHIY(M)
      TX=TX+RX(I)
      TY=TY+RY(I)
      XDEF(M)=XDEF(I)+TX
      YDEF(M)=YDEF(I)+TY
      XDEF(I)=XDEF(I)*FACT
      YDEF(I)=YDEF(I)*FACT
      DEF(I)=SGRT(XDEF(I)*XDEF(I)+YDEF(I)*YDEF(I))
      TM=FM(I)/12.
      TT=THETA(1)*57.29578
      TG=GAM(I)*57.29578
51   WRITE(2,15)I,TM,PHI(I),TT,TG,XDEF(I),YDEF(I),DEF(I)
   XDEF(NSTA)=XDEF(NSTA)*FACT
   YDEF(NSTA)=YDEF(NSTA)*FACT
   DEF(NSTA)=SCAT(XDEF(NSTA)*XDEF(NSTA)+YDEF(NSTA)*YDEF(NSTA))
   TM = O.
   WRITE(2,15)NSTA,TM,TM,TT,TG, XDEF(NSTA),YDEF(NSTA),DEF(NSTA)
   GO TO 11
END
SUBROUTINE MPH12(K)
C DETERMINES S1,PHI AND THETA GIVEN P.MR, AND GAMMA
DIMENSION U2(30),V2(30),U3(30),V3(30)
DIMENSION PCS(40)
INCLUDE DECLAR2
REAL MU,MV,MX,MY
IALLOW=20
ILOAD=0
IF(NALLOW.NE.0)IALLOW=NALLOW
C COMPUTE NOLOAD COND BASED ON ELAST BEHAVIOR.
FSE=PRESTR*(1.LOSS/100.)
FE=FSE*PROP(I)
DS=FSE/PROP(I)
XHOM=FE+EC(I)-DELAY(K)*COS(GAM(I))
YHOM=FM(K)*SIN(GAM(I))
THETA(K)=ATAN(YHOM/XHOM)
XPHI=XHOM/ITX0(K)
YPHI=YHOM/ITY0(K)
PHINew=SORT(XPHI*PHI+YPHI*YPHI)/EC
SINEW=FE+P)/IT(K)*EC
GO TO 1001
ENTRY AGAIN
DM=FDM(K)-FMOLD(K)
DPM=FPM(K)-FPDM(K)
PHINew=PHI(K)+DPM
SINEW=S1(K)+DSPM*(PHI*K)*DPM
ILOAD=1
1001 IF(I==0)IPHAS=0
   IFABS(SINEW GT 1.)SINEW=S1(K)
   IF(INIT(K).LT.0.1745)GO TO 4
C ROTATE COORDINATES
C=COS(INIT(K))
S=SIN(INIT(K))
DO 902 I=1,NS
   US(I,K)=C*XS(I,K)+S*YS(I,K)
902   V5(I,K) =XS(I,K) *S +C *YS(I,K)
DO 3 I=1,NP
   U(I,K)=C*X(I,K)+S*Y(I,K)
   V(I,K)=S*X(I,K)+C*Y(I,K)
3 CONTINUE
C THETA IS ZERO
4 DO 5 I=1,NP
   UI(I,K)=X(I,K)
5 CONTINUE
C COMPUTE P
C ZONE 2 CONTRIBUTION
56 CALL SUMP1(U2+V2,N2,SP12,SP22,SP32)
B1+CPROP(2)*PHI+PHINEW
T1=2.*CPROP(2)
B21=(CPROP(2)*S1NEW)*PHINEW
S1OLD=S1NEW
S1NEW=S1OLD+DS1DP*PDIFF
PLAST=PC
1312=(T1*S1NEW+CPROP(11)*PHINEW)
1323=(CPROP(2)*S1NEW+CPROP(1))*S1NEW
51OLD=S1NEW
51NEW=S1OLD+DS1DP*PDIFF
PLAST=PC
P0=SP12*B11+SP22*B12 +SP32*B13
IF(S1NEW.GT.S1U.OR.S1NEW.LT.S1L)S1NEW=(S1U+S1L)/2.
IF(N3.EQ.0)GO TO 59
C
ZONE 3 CONTRIBUTION
CALL SUMCP(U30/39N39SP13,SP23,SP33)
IF(NPRT.NE.1)GO TO 906
C
STEEL CONTRIBUTION
C
BAIL OUT
DO 1004 I=1,NS
SC=S1NEW+PHINEW*VS(I,K)
77 FORMAT(X,K,IPASSeIGPASS,IPHPAS,PC),/413.3E10.3)
SS=DSSC
CALL EXIT
IF(SC.GE.CPROP(6))G0 TO 1005
C
COMPUTE MOMENTS
IF(SS.LE.SPROP(2))60,61
79 MU=MV=0.
60 PCS(I)=SPROP(1)*SS*AS(I,K)
CALL SUMCM(U2,V2.N2,SMU12,SMU22.SMU32,SMV12.SMV22.SMV32)
MU=MU+SMU13,1611+SMU23*B22+SMU33*B23
61 IF(SS.GT.SPROP(9))60 TO 64
MV=SMV12*B11+SMV22*B12+SMV32*B13
PCS(I)=0.
62 IF(SS.GT.SPROP(6))60 TO 64
MV=MV+SMV131,821+SMV23*B22+SMV33*823GO TO 65
64 PCS(I)=(SPROP(3)+SS*SPROP(4)+SPROP(5)*SS)*AS(I,K)
MV=MV+SMV131,821+SMV23*B22+SMV33*823GO TO 65
65 IF(SC=LE.SPROP(6))GO TO 69
66 PCS(I)=(SPROP(3)+SS*SPROP(4)+SS*SPROP(5)*SS)*AS(I,K)
67 IF(SS.LE.SPROP(2))60,61
71 IF(SS.LE.CPROP(7))67,68
67 PCS(I)=PCS(I)+(CPROP(1)+CPROP(2)*SC)*SC*AS(I+K)
GO TO 69
68 PCS(I)=PCS(I)+(CPROP(4)+CPROP(5)*SC)*SC*AS(I+K)
69 PC=PCPCS(1)
1004 CONTINUE
C
P IS NOT BALANCED
IF(IPASS.EO.IALLOW)G0 TO 901
IF(IPASS.NE.0)GO TO 73
IF(IPASS.EO.IALLOW)G0 TO 901
IF(IPASS.EO.1)G0 TO 1015
DTHETA=THETA(K)THOLD
DG=GAM(K)GAMOLD
GAMOLD=GAMC
THOLD=THETA(K)
THETA(K)=THETA(X)+GDIFF
IF(THETA(K).GT.THETU.OR.THETA(K).LT.THETL)THETA(K)=(THETU+THETL)/2.
1012 THETU=THOLD
GO TO 1006
1013 IF(GDIFF.LT.0..AND.THETL.LT.THETU)1012
1014 IF(GDIFF.GT.0..AND.THETL.LT.THETU)1012
1015 THETU=THOLD
GO TO 1006
}

C
TRY AGAIN
GO TO 906
C
BAIL OUT
WRITE(2,77)IPASS.IDIGPASSolPHPAS.PC,THETAIK).PMC
77 FORMAT(X,K,IPASSeIGPASS,IPHPAS,PC),/413.3E10.3)
SS=DSSC
CALL EXIT
IF(SC.GE.CPROP(6))G0 TO 1005
C
COMPUTE MOMENTS
IF(SS.LE.SPROP(2))60,61
79 MU=MV=0.
60 PCS(I)=SPROP(1)*SS*AS(I,K)
CALL SUMCM(U2,V2.N2,SMU12,SMU22.SMU32,SMV12.SMV22.SMV32)
MU=MU+SMU13,1611+SMU23*B22+SMU33*B23
61 IF(SS.GT.SPROP(9))60 TO 64
MV=SMV12*B11+SMV22*B12+SMV32*B13
PCS(I)=0.
62 IF(SS.GT.SPROP(6))60 TO 64
MV=MV+SMV131,821+SMV23*B22+SMV33*823GO TO 65
64 PCS(I)=(SPROP(3)+SS*SPROP(4)+SS*SPROP(5)*SS)*AS(I,K)
MV=MV+SMV131,821+SMV23*B22+SMV33*823GO TO 65
65 IF(SC=LE.SPROP(6))GO TO 69
66 PCS(I)=(SPROP(3)+SS*SPROP(4)+SS*SPROP(5)*SS)*AS(I,K)
67 IF(SS.LE.SPROP(2))60,61
71 IF(SS.LE.CPROP(7))67,68
67 PCS(I)=PCS(I)+(CPROP(1)+CPROP(2)*SC)*SC*AS(I+K)
GO TO 69
68 PCS(I)=PCS(I)+(CPROP(4)+CPROP(5)*SC)*SC*AS(I+K)
69 PC=PCPCS(1)
1004 CONTINUE
C
P IS NOT BALANCED
IF(IPASS.EO.IALLOW)G0 TO 901
IF(IPASS.NE.0)GO TO 73
IF(IPASS.EO.IALLOW)G0 TO 901
IF(IPASS.EO.1)G0 TO 1015
DTHETA=THETA(K)THOLD
DG=GAM(K)GAMOLD
GAMOLD=GAMC
THOLD=THETA(K)
THETA(K)=THETA(X)+GDIFF
IF(THETA(K).GT.THETU.OR.THETA(K).LT.THETL)THETA(K)=(THETU+THETL)/2.
1012 THETU=THOLD
GO TO 1006
1013 IF(GDIFF.LT.0..AND.THETL.LT.THETU)1012
1014 IF(GDIFF.GT.0..AND.THETL.LT.THETU)1012
1015 THETU=THOLD
GO TO 1006
1006 IF(NPRT.NE.1)GO TO 903
WRITE(2,77)K,IPASS,IGPASS,PC,THETA(K),FMC
GO TO 903
1007 FMC=SQR(MX*MX+MY*MY)
FMDIF=FM(K)-FMC
T=+0.1*FM(K)
IF(FM(1).LT.1.)T=.1
IF(ABS(FMDIF).LE.T)GO TO 1010
IF(IPHAS.EQ.1 ALLOW)GO TO 901
IF(IPOINT.EQ.0)GO TO 1008
DPHI=PHINEW-PHI(K)
DM=FMC-FMOLD(K)
FMOLD(K)=FMC
PHI(K)=PHINEW
PHINEW=PHI(K)+DPHI*FMDIF/DM
GO TO 1009
1008 PHI(K)=PHINEW
PHINEW=PHINEW*FM(K)/FMC
1009 IGPASS=0
GO TO 1006
1010 IF(ILOAD.EQ.1)GO TO 1011
DPHIDM(K)=PHINEW/FM(K)
DSPHI(K)=S1NEW/PHINEW
PHI(K)=PHINEW
S1(K)=S1NEW
FMOLD(K)=FM(K)
RETURN
1011 T=A8S(FM(K)-FMOLD(K))
IF(T.GT.0)GO TO 2000
FMOLD(K)=FM(K)
PHI(K)=PHINEW
S1(K)=S1NEW
RETURN
2000 DPHIDM(K)=(PHINEW-PHI(K))/(FM(K)-FMOLD(K))
DSPHI(K)=(S1NEW-S1(K))/(PHINEW-PHI(K))
PHI(K)=PHINEW
S1(K)=S1NEW
FMOLD(K)=FM(K)
RETURN
END
Table B.1. Input Common to FLEX and FLEX2.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Format</th>
<th>Number of Records</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>IOUT, NALLOW,</td>
<td>(I1, I2, 3I3)</td>
<td>1/job</td>
<td>Output is to LUN 2, IOUT=0 or blank equips 2=file, otherwise 2 equipped = 61. NALLOW = no. of iterations allowed, assumes 20 if 0 or blank, execution is terminated if input unit is at end of file. NPRT = 1 prints iteration no., S1, Computed P and θ for each iteration, NZPRT = 1 prints U2, V2, U3 and V3 for each iteration. IZPRT = 1 prints IZ for each iteration.</td>
</tr>
<tr>
<td>NPRT, NZPRT,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IZPRT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPN</td>
<td>(X, I4)</td>
<td>1</td>
<td>Problem number.</td>
</tr>
<tr>
<td></td>
<td>(X, 39H)</td>
<td>1</td>
<td>Problem heading.</td>
</tr>
<tr>
<td>NEWCP, NEWSP,</td>
<td>(6I1)</td>
<td>1</td>
<td>A value of 1 for the variables indicates: new concrete properties, new steel properties, new concrete geometry, new steel geometry, new cutoff points, new prestress. All except NEWCO must be 1 for first problem of batch.</td>
</tr>
<tr>
<td>NEWCG, NEWSG,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NEWCO, NEWPS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPROP</td>
<td>Free Form</td>
<td>Immaterial</td>
<td>A1 through A5, Scu, So</td>
</tr>
<tr>
<td>SPROP</td>
<td>Free Form</td>
<td>Immaterial</td>
<td>E1, Sp1, B1 through B3, Sγ, B4, E2, Su</td>
</tr>
<tr>
<td>ITYPE, TAPER,</td>
<td>(I1, F9, 2F10, I3, 2I2)</td>
<td>1</td>
<td>ITYPE = 1 causes ISECIN to be called, other values cause CONCIN to be called, TAPER is dimensionless. OAL=overall length-in. GL =</td>
</tr>
<tr>
<td>ISTART, ISTOP</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variables</td>
<td>Format</td>
<td>Number of Records</td>
<td>Remarks</td>
</tr>
<tr>
<td>--------------------</td>
<td>----------</td>
<td>-------------------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>BB, HH</td>
<td>(4F10)</td>
<td>1</td>
<td>Butt dimensions of I section. Use only if ITYPE = 1. Skip to NRS.</td>
</tr>
<tr>
<td>TF, TW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP</td>
<td>(I5)</td>
<td>1</td>
<td>Number of points defining section. Use only if ITYPE≠1.</td>
</tr>
<tr>
<td>X, Y</td>
<td>(2F10)</td>
<td>NP</td>
<td>Coordinates of points at butt of member.</td>
</tr>
<tr>
<td>NRS</td>
<td>(I5)</td>
<td>1</td>
<td>Number of rows of strands.</td>
</tr>
<tr>
<td>NSR, YRB, YRT</td>
<td>(I5, 5X, 2F10)</td>
<td>1</td>
<td>Number of strands this row, Y coordinate at butt, Y coordinate at tip.</td>
</tr>
<tr>
<td>XSB, AS(I), XST</td>
<td>(3F10)</td>
<td>NSR</td>
<td>X coordinate at butt, area of strand (if blank AS(I-1) is used), X coordinate at tip. Repeat NRS times.</td>
</tr>
<tr>
<td>NCOP</td>
<td>(I5)</td>
<td>1</td>
<td>Number of cutoff points, use only if NEWC ≠ 0.</td>
</tr>
<tr>
<td>ISTA, NCO</td>
<td>(215)</td>
<td>1</td>
<td>Station number, no. of strands cut off at this station.</td>
</tr>
<tr>
<td>NSCO</td>
<td>(12I5)</td>
<td>1</td>
<td>Index numbers of strands cut off. Repeat NCOP times.</td>
</tr>
<tr>
<td>PRESTRESS, LOSS</td>
<td>(2F10)</td>
<td>1</td>
<td>Prestress ksi, loss percent. Note: the effect of elastic shortening is computed and should not be incl. in LOSS.</td>
</tr>
</tbody>
</table>

dist from butt to ground line-in, NSEG = no. of segments (0 if member has zero length i.e. cross-section only). ISTART = no. of first station where computations are desired. ISTOP = no. of last station where computations are desired. (ISTOP and ISTART are ignored by FLEX2)
Table B.2. Additional input for FLEX.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Format</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Free Form</td>
<td>External axial force kips. If value is less than 0 returns for a new problem.</td>
</tr>
<tr>
<td>ERR</td>
<td>Free Form</td>
<td>Allowable error in computed value of P-kips.</td>
</tr>
<tr>
<td>THETAD</td>
<td>Free Form</td>
<td>Value of θ-degrees. If value is less than 0 returns for a new value of P.</td>
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</tbody>
</table>

Table B.3. Additional input for FLEX2

<table>
<thead>
<tr>
<th>Variables</th>
<th>Format</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAMMD</td>
<td>Free Form</td>
<td>Value of γ-degrees. If value is less than 0 returns for a new problem.</td>
</tr>
<tr>
<td>P, ERR</td>
<td>Free Form</td>
<td>Same as for FLEX.</td>
</tr>
<tr>
<td>Q</td>
<td>Free Form</td>
<td>External transverse force at tip of member. If less than 0 returns for a new value of GAMMD.</td>
</tr>
<tr>
<td>BETAD</td>
<td>Free Form</td>
<td>Angle between Q and normal to member axis-degrees.</td>
</tr>
</tbody>
</table>
Figure B.2. Sample input for FLEX.

Figure B.3. Sample input for FLEX2.
Figure B.4. Sample output from FLEX.
Figure B.5. Sample output from FLEX 2.
### APPENDIX C

**CYLINDER TEST DATA**

**Table C.1. Cylinder Test Results**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Age (days)</th>
<th>Strength (kips/sq. in.)</th>
<th>Strain at Ultimate Strength</th>
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* Not measured.

**Strain measurements not obtained due to equipment malfunction.**
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<th>Specimen</th>
<th>Age (days)</th>
<th>Strength (kips/sq. in.)</th>
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<tbody>
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APPENDIX D

POLE TEST DATA

Table D.1. Strain gage point coordinates.

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1 All suffix A gage points were 39'-5" from tip of pole; all suffix B points were 41'-5" from tip of pole.
Table D.2, Strain data--specimen Ti.

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<th>2B</th>
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<th>3B</th>
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