Wave Energy Converter Modeling in the Time Domain: A Design Guide

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***Abstract*— As the ocean wave energy field continues to mature, developers need a generic modeling methodology to test their designs before building prototypes. A design methodology for a first-pass time-domain simulation is a goal of this work. Built on results from the frequency domain analysis, the general procedure for obtaining time domain results is presented. Wave energy researchers and developers can use this design guide as a step in the process of obtaining a cost of energy estimate for their device. Promoting the development of wave energy converters by providing a sound modeling methodology is an aim of this research.**

***Index Terms*—Ocean Wave Energy Conversion, Time Domain Modeling**

I. INTRODUCTION

As the need for alternative energy sources increases, industries such as ocean wave energy, promising utility scale power generation from a renewable source, continues to grow. Wave Energy Converter (WEC) design is still in its infancy with significant research being applied to new designs. Thus far, no topology has provided a clear benefit over others in efficiency, cost of manufacture, maintenance requirements and production, thus new devices continue to be developed [1]. This is unlike the wind industry which has established a two or three blade horizontal axis wind turbine to be the most cost effective and efficient.

A good overview of the Ocean Wave Energy field is given in [2]. Many papers on the topic of time domain WEC modeling exist, including [3],[4],[5], but lack a clear design methodology. An attempt at benchmarking devices exists in [6]. This paper provides a clear time domain modeling approach of a heaving point absorber which can be adapted to other types of devices.

A. Background

This paper assumes that a rough physical WEC design has been chosen, and frequency domain analysis has already been performed as outlined in [7]. This can be achieved using any of the industry standard hydrodynamic software packages capable of doing frequency domain analysis. The next step in the validation of the merits of the design is a time domain simulation derived from the frequency domain parameters of the WEC. A generic two body WEC is used in this paper to provide an example of the necessary steps required to do the analysis. This analysis will give more realistic results for the response of the device than the frequency domain results were capable of providing.

One benefit of time domain modeling is the introduction of nonlinearities to the model. This includes nonlinear mooring models as well as power take off (PTO) models. Such PTO nonlinearity could be realized by assuming that the linear damping has some limit of damping that can be applied. This manifests itself as saturation in the model.

Results of the time domain simulations include the motion response of the system (positions, velocities, and accelerations), the mooring and PTO forces, and a power output prediction. Throughout this paper the assumption will be made that all calculations are in heave mode only.

B. Equations of Motion

The equations of motion can be obtained by summing the forces present on each body. The total forces which act on the heave motion of the structures can be broken down to many components. The excitation wave force, $F_e(t)$ is the summation of the Froude-Krylov force and the diffraction force and is the force imparted on the device by the incoming wave. The total radiation force, $F_r(t)$ is the force on the bodies due to structure motion and can be decomposed into an added mass term and a radiation damping term. The mooring force, $F_m(t)$ can be linearized or nonlinear, and can take on many different configurations. The hydrostatic force, $F_{hs}(t)$ is the force trying

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to restore the structure to hydrostatic equilibrium. The PTO force, F_{pto} is the force absorbed by the device to be converted to usable energy, and can be either linearized or nonlinear. The general equation is as follows

$$M\ddot{z}(t) = F_e(t) + F_r(t) + F_{hs}(t) + F_v(t) + F_m(t) + F_{PTO}(t) \quad (1)$$

as first introduced in [8] where M is the mass of the body.

The excitation force, $F_e(t)$ can be computed as a convolution of the water surface elevation and impulse response function from the frequency domain results obtained in previous simulations

$$F_e(t) = \int_{-\infty}^{\infty} \eta(\tau)F_t(t-\tau)d\tau \quad (2)$$

where $\eta(t)$ is the wave surface elevation at the WEC and

$$F_t(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \quad (3)$$

is the non-causal impulse response function of the heave mode, where $F(\omega)$ is the frequency domain summation of the Froude-Krylov force and diffraction forces. This presents a challenge because the excitation force at the current time is dependent on future input values. This is partially due to the wave impacting a part of the device prior to the wave impacting the point of analysis of the device as described in [9].

The radiation force, $F_r(t)$ can be computed as a convolution of the body velocity and the radiation impulse response function combined with the contributing force of the added mass at infinity of the body.

$$F_r(t) = -\int_{-\infty}^{t} k(t-\tau)\dot{z}(\tau)d\tau - m(\infty)\ddot{z}(t) \quad (4)$$

where

$$k(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(\omega)e^{j\omega t}d\omega \quad (5)$$

as shown in [9]. The integral in (5) is guaranteed convergence by subtracting off the added mass at infinity as shown in (6)

$$K(\omega) = R(\omega) + i\omega[m(\omega) - m(\infty)] \quad (6)$$

where $R(\omega)$ is the frequency domain radiation damping coefficients and $m(\omega)$ is the added mass of the body. Reformulation of (1) by moving the $m(\infty)\ddot{z}(t)$ term to the other side and defining F_r' as

$$F_r{}'(t) = -\int_{-\infty}^{t} k(t-\tau)\dot{z}(\tau)d\tau \quad (7)$$

becomes

$$\big(M + m(\infty)\big)\ddot{z}(t) = F_e(t) + F_r'(t) + F_{hs}(t) + F_v(t) + F_m(t) + F_{PTO}(t) \quad (8)$$

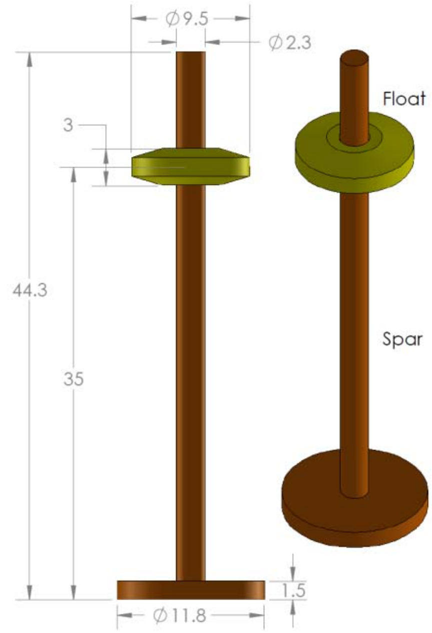

Figure 1. Generic WEC example. All Dimensions in meters.

F_{hs} can be computed as the restoring force on the body attempting to bring it back to equilibrium as follows

$$F_{hs}(t) = \rho g A\, z(t) \quad (9)$$

where ρ is the fluid density, g, is the acceleration of gravity, and A is the water plane surface area.

F_v is the viscous friction forces on the body. Typically these are considered to be proportional to the velocity of the body and are determined experimentally.

F_m is the mooring force and can take many forms. Mooring configurations can be very complicated and highly nonlinear. For this paper a simple catenary mooring is considered in the form of a spring

$$F_m(t) = -K_m z(t) \quad (10)$$

where K_m is the spring constant of the mooring.

II. Model to be Analyzed

For this research a generic two body point absorber was chosen as shown in Fig.1. It is assumed that a frequency domain analysis has already been performed as outlined in [7]. Therefore the geometry has already been chosen and results from the frequency domain analysis have already been obtained.

The pertinent frequency domain parameters needed for the time domain simulation include the following.

- The Froude Krylov plus diffraction frequency domain coefficients describing the resulting forces on a body imparted by an incoming wave.
- The radiation damping coefficients describing the motion of the body in a fluid, generating outgoing waves in phase with the body velocity thus acting as a velocity proportional damping force.

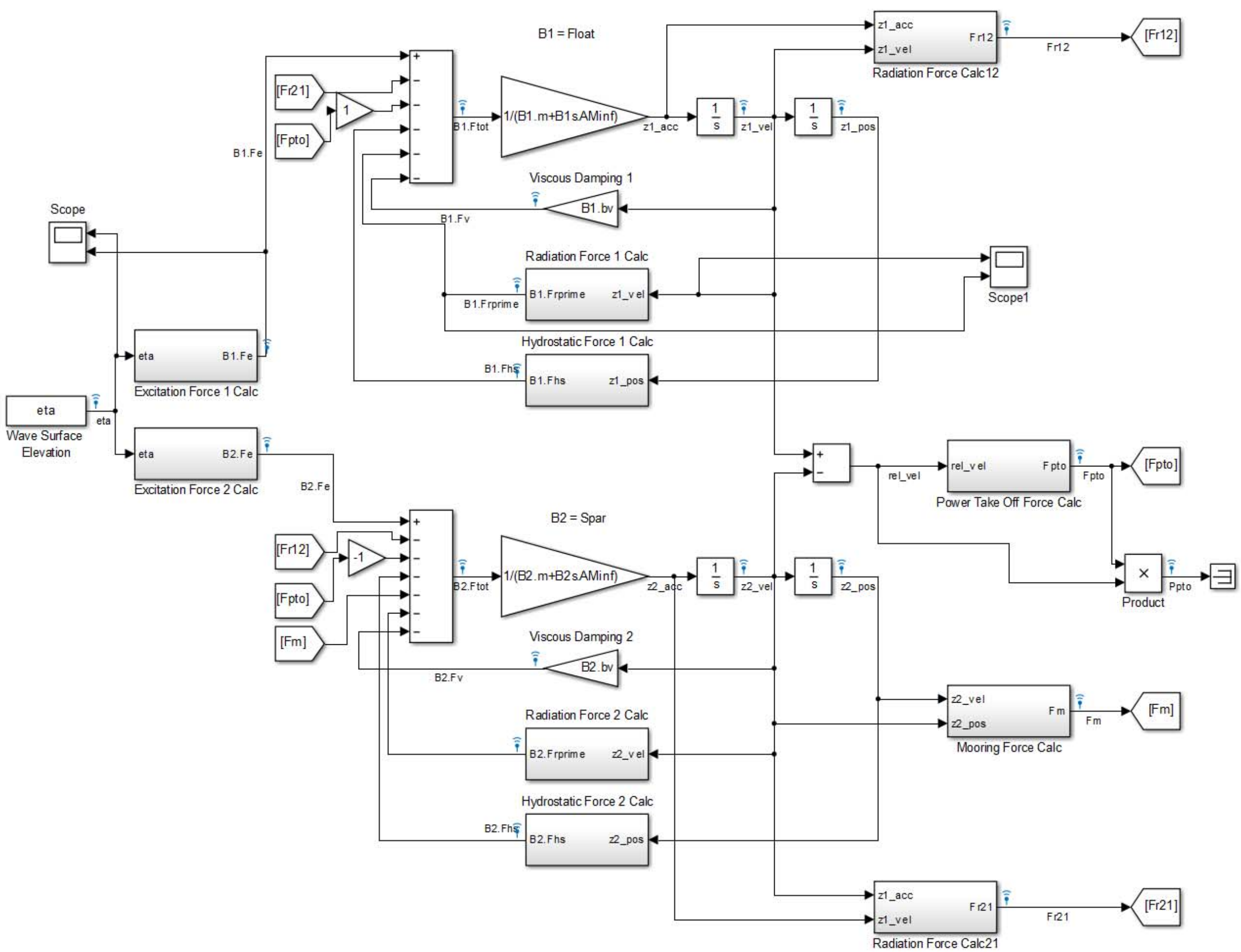

Figure 2. MATLAB/Simulink implementation of equations of motion for two body WEC.

- The added mass coefficients defined as the added inertia on a body undergoing harmonic oscillation due to the presence of the surrounding fluid.

III. Input Waves

It is possible and insightful to simulate both regular (monochromatic) and irregular (spectrum) waves in the time domain simulation. Regular waves give a controlled input that is repeatable and the response for the given input easily identified. Irregular waves provide a more realistic representation of what the WEC would face in real seas.

A. Regular Waves

For regular wave input, a sinusoidal (also called harmonic) linear wave can be defined as shown in the following linear wave input.

$$\eta(t) = Acos(\omega t) \tag{11}$$

where ω is the frequency in radians/sec, and $\eta(t)$ is the water surface elevation at the WEC in meters.

B. Irregular Waves

For irregular wave input, typical wave spectra include Jonswap, Pierson-Moskowitz, Bretschneider, and Gaussian, distribution [10]. These wave inputs are all uni-directional. A typical treatment of the spectrum is as follows. The spectrum is split into N sections of equal area. N wavelets with frequency at the centroid of the section are defined with N having a maximum of 200. The wavelets are then added together with random phase angles taking on the following form

$$\eta(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t + \phi_i) \tag{12}$$

For an example of a Pierson-Moskowitz input the following need to be specified. The wave direction, the range of frequencies to be included, the significant wave height, zero crossing period, and a seed used to define the random seed for a wave spectrum must be included. For simulation purposes the resulting time series can be input into the simulation.

IV. Time Domain Differntial Equation Solver Approach

One method for modeling a two body wave energy converter is by using a solver to solve the equations of motion and then calculate outputs from that simulation. These equations of motion describe the motion of the individual bodies due to many forces as shown in the introduction to this paper. The dominant forces which are present in an actual wave energy converter model and will be outlined here. We are considering a device with two bodies and therefore will have two equations of motion, one for each body. Subscript 1 will refer to the float and subscript 2 will refer to the spar as shown in Fig. 1.

Additional forces include a viscous damping force, a power take off force, and mooring forces. With the addition of these forces, the equations of motion in the heave direction for each body take the form

$$\begin{aligned} F_{e1}(t) - F_{r11}(t) - F_{r21}(t) - F_{hs1}(t) \\ - F_{pto}(t) - F_{v1}(t) \\ = \left(m_1 + m_1(\infty)\right)\ddot{z}(t) \end{aligned} \tag{13}$$

$$\begin{aligned} F_{e2}(t) - F_{r22}(t) - F_{r12}(t) - F_{hs2}(t) \\ + F_{pto}(t) - F_{v2}(t) \\ - F_m(t) \\ = \left(m_2 + m_2(\infty)\right)\ddot{z}(t) \end{aligned} \tag{14}$$

where $F_{e1}(t)$ is the force imparted by the incoming wave on body 1, $F_{r11}(t)$ is the radiation force imparted on body 1 as a result of the waves created by body 1, $F_{r21}(t)$ is the radiation force imparted on body 1 as a result of the wave created by body 2, $F_{hs1}(t)$ is the hydrostatic stiffness force on body 1, F_{pto}(t) is the electromechanical force on body 1 from the generator acting as the power take off, and $F_{v1}(t)$ is the force from viscous friction on the body. The second equation takes the same form as the first with the exception of the sign on the PTO force and the addition of the mooring force attached to body 2.

These equations were then input into MATLAB/Simulink as shown in Fig. 2. Note that there are two similar structures of computation, one for each body. The excitation forces, radiation forces, and hydrostatic forces are calculated in subsystems which will be detailed below.

The excitation force calculation, F_e, as shown in (3), is shown in Fig. 3. First the impulse response function is calculated by solving the integral as shown in (4) using the parameters obtained by a software package such as Ansys AQWA[11] or WAMIT[12]. The resulting non-causal impulse response function is then split into its causal part and non-causal part and convoluted with the wave surface elevation η as shown in (2). In Simulink, the convolution is implemented using the finite impulse response filter block with the impulse response function parts fed as coefficients.

Notice that the input waveform was broken into a causal and a non-causal part which are convoluted separately and the results added together. Also notice that a rate limiter was used to ease the excitation force into the simulation. This is necessary because of the non-causal nature of the excitation force. The radiation force has a similar structure, however as the impulse response function is causal in nature, it requires just one convolution.

The most common and straight forward initial model for the power take off is as a linear damping. Its implementation in the model becomes a constant damping gain that is multiplied by the relative velocities between the two bodies which provides the power take off force calculation. Although this is convenient and a relatively accurate and effective control scheme for some regions of operation, introduction of nonlinear damping parameters can both provide a more realistic model of the system and could potentially improve the power output from the device.

V. Simulation Results

Simulation of a two-body WEC was performed with both regular and irregular wave inputs. Two different power take off damping schemes were employed for regular waves. The first, linear damping produced a sinusoidal output in displacement, force, and power, as expected and shown in Fig. 4. The second, saturated linear damping, implemented limits on the damping applied and effectively clipped the force applied to the power take off and thus the power produced as shown in Fig. 5. Due to this non-linearity, this analysis cannot be done in the frequency domain.

The same set of damping conditions were then applied to an input of irregular sea data as shown in Fig. 6 for linear damping and Fig. 7 for saturated linear damping where the nonlinear clipping of the signal is clearly shown.

VI. Conclusions

In this research, a methodology for modeling a two body point absorber wave energy converter was outlined. The procedure included defining equations of motion and implementing them in a differential equation solver such as MATLAB/Simulink. A benefit of time domain simulation, namely the implementation of a nonlinear damping power take off model, was analyzed and results shown. This demonstrates that time-domain analysis, accommodating non-linearities in plant behavior or control, can be conducted starting from frequency domain analysis.

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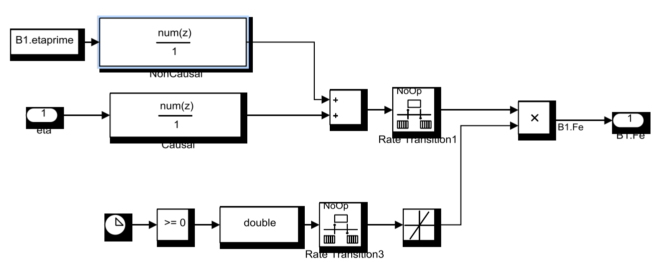

Figure 3. Excitation Force convolution.

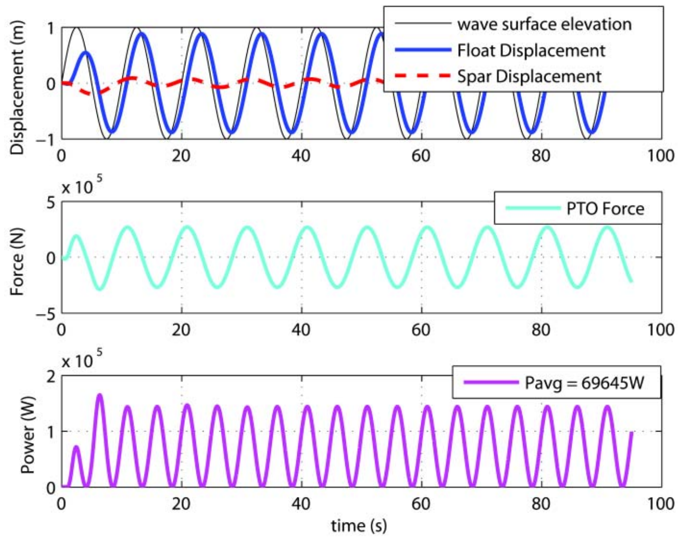

Figure 4. Linear damping regular wave input.

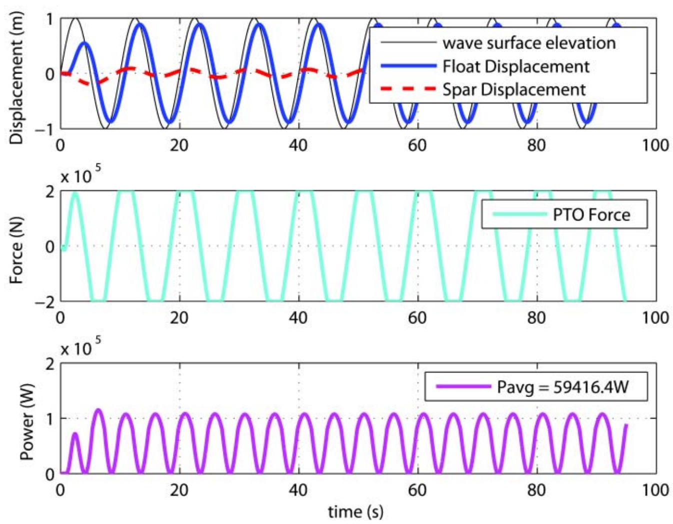

Figure 5. Linear damping regular wave input. The PTO force is clipped at 200kN to represent realistic limitations of physical equipment. Because of this nonlinearity this analysis must be done in the time domain.

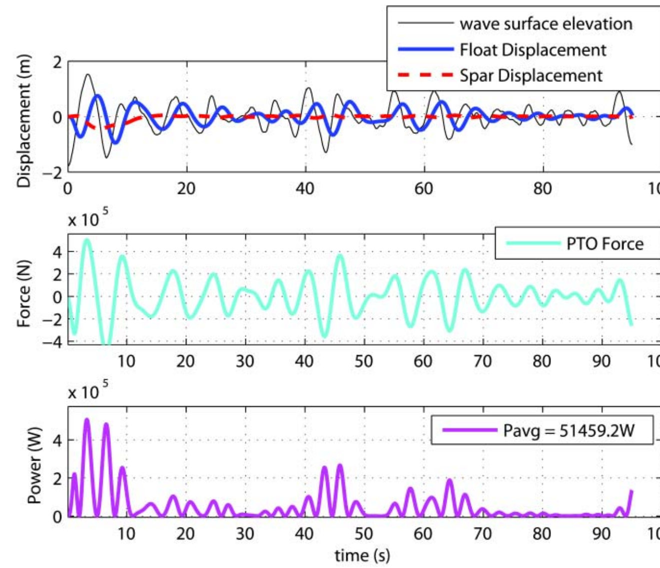

Figure 6. Linear damping irregular wave input.

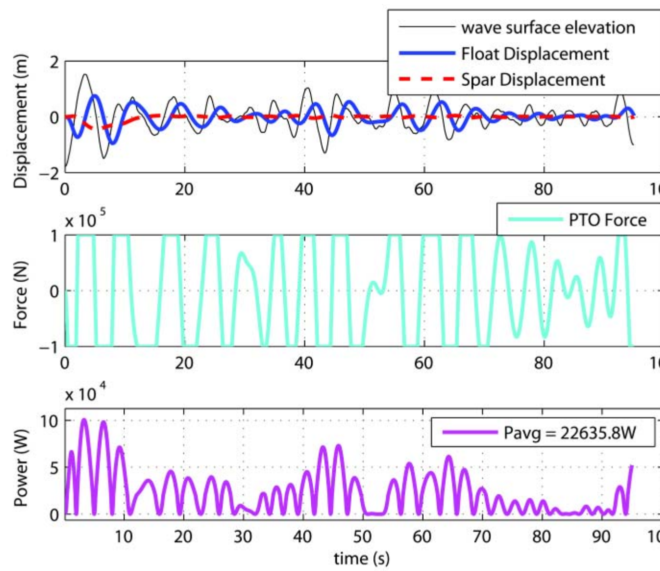

Figure 7. Linear damping irregular wave input. The PTO force is clipped at 100kN to represent realistic limitations of physical equipment. Because of this nonlinearity this analysis must be done in the time domain.

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