BUCKLING OF FLAT PLYWOOD PLATES IN COMPRESSION, SHEAR, OR COMBINED COMPRESSION AND SHEAR

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BUCKLING OF FLAT PLYWOOD PLATES IN COMPRESSION, SHEAR, OR COMBINED COMPRESSION AND SHEAR

By

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Introduction

In this report are presented the results of mathematical analyses of several buckling problems for flat plates under various types of loading. The critical stresses given were frequently obtained by the use of approximate methods. Although they must be considered as tentative until confirmed by tests, they are given at this time in the belief they will be useful. In a number of cases, the plates are taken to be infinitely long to simplify the mathematics. Shown in such cases are the effects of differences in orientation of the grain of the face plies with respect to the direction of loading and the effects of differences in the structure of the plywood.

For finite plates, the critical stress will not be less and will usually be larger than that for the corresponding infinitely long plate. When the critical stress is known only for an infinitely long plate, it should be possible to make a satisfactory estimate of the critical stress for

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1 This is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft. Results here reported are preliminary and may be revised as additional data become available. Original report published April 1942.

2 E. W. Kuenzi, Jr., Junior Engineer, attended to many details in the preparation of this report and checked the integrations and mathematical manipulations leading to the final formulas.
a finite plate of the same construction by considering the effect of the
length-breadth ratio in cases that have been completely worked out.

The approximate methods lead to critical stresses that are too large.
In cases where a comparison with exact values was possible, the errors
were found to be less than 8 percent. It is believed that in other cases
the errors may be considered to be of this order of magnitude.

In all cases, the angle between the grain of the wood in adjacent plies
is 90°. The grain of the face plies makes an angle of 0°, 45°, or 90°
with the edges of the plates, which are taken to be simply supported.
The state of stress is uniform compression, uniform shear, or com-
bined uniform compression (or tension) and uniform shear.

In all calculations, the values of the elastic constants of Douglas-fir
were used, as given in the appendix, and the plies were assumed to be
rotary cut. But the results for plywood with sliced veneer that is ap-
proximately edge-grain should not be greatly different from those ob-
tained for plywood of rotary-cut veneers. The critical stresses are
expressed in a form containing the modulus of elasticity along the grain
\( E_L \) as a factor. It is to be expected that differences in species, den-
sity, and moisture content are taken into account to a considerable ex-
tent in the choice of this factor.

In the case of plates under uniform shear and having the grain of the
face plies either parallel or perpendicular to the edges, the results
obtained by E. Seydel3 for plates of orthotropic material are used. It
is shown how to calculate the constants for plywood that are needed to
utilize the family of curves given in his paper and reproduced here.

**Notation**

The choice of axes is shown in figure 1. The direction of loading is
parallel to the \( Y \) axis as shown in figure 4.

\(^3\)Zeitsch. für Flugtech. und Motorluftschifffahrt 24, 78-83, 1933.

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a = width of plate measured parallel to OX.

b = half wave length of the buckled surface in the case of an infinitely long plate; or the length of a finite plate.

h = thickness of plate.

p = uniform compressive stress parallel to OY. A compressive stress is considered to be positive.

$p_{cr}$ = value of $p$ for which buckling occurs.

q = uniform shearing stress. q is positive when directed as shown in figure 4.

$q_{cr}$ = value of $q$ for which buckling occurs.

$\theta$ = angle between grain of face plies and OX.

$\phi$ = angle between wrinkles and OX.

$\gamma = \tan \phi =$ slope of wrinkles.

$z = \frac{b}{a}$

$k_c = \text{coefficient in formula } p_{cr} = k_c \frac{E}{L} \frac{h^2}{a^2}$

$k_s = \text{coefficient in formula } q_{cr} = k_s \frac{E}{L} \frac{h^2}{a}$

$f = \text{coefficient in formula } q = fp$

Other symbols are defined in the Appendix.
1. Rectangular Plywood Plates Under Uniform Compression in a Direction Parallel to One Pair of Edges

Case 1: \( \theta = 0^\circ \) or \( \theta = 90^\circ \). Uniform compressive stress parallel to \( OY \).

The critical buckling stress for a plate buckling in \( n \) half waves is given by the formula \(^4\)

\[
P_{cr} = \frac{\pi^2}{12\lambda} \left[ \frac{E_1 b^2}{n^2 a^2} + 2A + \frac{E_2}{b^2} \frac{n^2 a^2}{b^2} \right] \left( \frac{h}{a^2} \right)
\]

(1)

The constants that occur in this formula are defined in the appendix.

For a plate of given width and thickness, \( P_{cr} \) will be least when

\[
b = na \left( \frac{E_2}{E_1} \right)^{\frac{1}{4}}
\]

(2)

In this case

\[
P_{cr} = \frac{\pi^2}{6\lambda} \left[ \frac{1}{(E_1 E_2)^{\frac{1}{2}}} + A \right] \left( \frac{h}{a^2} \right)
\]

(3)

When \( n = 1 \), the least critical stress (3) will be found for a plate for which

\(^4\)See for example S. Timoshenko, Theory of Elastic Stability, (McGraw-Hill Book Co. 1936) p. 381. The formula there given for a plate of homogeneous orthotropic material can be used for plywood which is nonhomogeneous from the standpoint of its elastic properties, if the constants occurring in it are given the values that are found in the differential equation for the bending of a plywood plate. See U. S. Forest Products Laboratory Report 1312, "Flat Plates of Plywood Under Uniform and Concentrated Loads."
\[ b = a \left( \frac{E_2}{E_1} \right)^{\frac{1}{4}} \]  

(4)

The buckled form of the plate will change from \( n \) half waves to \( n + 1 \) half waves when

\[ b = a \left[ \frac{n(n+1)}{2} \right]^{\frac{1}{2}} \left( \frac{E_2}{E_1} \right)^{\frac{1}{4}} \]  

(5)

It will be convenient for the purpose of comparison with other results to write (1) in the form

\[ p_{cr} = k \frac{E}{L} \frac{h}{a^2} \]  

(6)

where

\[ k = \frac{\pi^2}{12\lambda} \left[ \frac{E_1}{E_L} \frac{b}{n^2a^2} + 2 \frac{A}{E_L} + \frac{E_2}{E_L} \frac{n^2a^2}{b^2} \right] \]  

(7)

In tables 2 and 3 are given the values of \( k \) for certain ratios of \( \frac{b}{a} \) for 3-ply plates having the thickness of the face plies equal to one-half that of the center ply and having \( \theta = 0^\circ \) and \( \theta = 90^\circ \). Curves for the factor \( k_c \) as a function of the ratio \( \frac{b}{a} \) are shown for these two types of plates in figures 5 and 6.

When a plate is very long, it buckles into a large number of waves.

The half wave length approaches asymptotically the value \( a \left( \frac{E_2}{E_1} \right)^{\frac{1}{4}} \) as the length of the plate is increased. This may be called the ideal half wave length. It corresponds to the minimum critical load. When the length of the plate is greater than three times the ideal half wave length.
length, the critical load differs but little from the minimum critical load. In the curves of figures 5 and 6 the greatest difference between the critical stress for a given plate and the minimum critical stress is about 3 percent when the length of the plate is greater than three times the ideal half wave length. 

Case 2: Infinite strip. (Long narrow plate.) $\theta = 45^\circ$.

In this case, the differential equation governing deflection becomes more complicated than the corresponding equation for the cases in which the direction of the grain of the face plies is parallel or perpendicular to the edges. Its solution requires different and apparently more complicated analysis. Accordingly, an approximate solution was obtained by the energy method. In order to obtain some information quickly by avoiding long numerical calculations, the plate was assumed to be very long in the direction of loading. When the results are compared with those obtained in case 1 for long plates having the grain of the face plies either parallel or perpendicular to the direction of loading, an estimate can be formed of the effect of inclination of the grain to the direction of loading that will be useful in the case of plates of finite length.

Let the deflection of the buckled plate be represented by the equation

$$w = H \sin \frac{\pi x}{a} \sin \frac{\pi}{b} (y - \gamma x)$$  

(8)
where

\[ b = \text{the half wave length of the buckled surface, that is, one-half the distance within which the phenomenon repeats itself.} \]

\[ \gamma = \tan \phi, \text{ where } \phi \text{ is the inclination of the wrinkles to the } X\text{-axis.} \]

See figure 7.

The strain energy of bending due to the buckling over a half wave length \( b \), a length which is to be determined, was equated to the work done by the load in producing the shortening resulting from the buckling of this portion of the plate. The resulting expression can be solved for the critical stress \( p \) as a function of \( b \) and \( \gamma \). These two quantities, \( b \) and \( \gamma \), were determined by requiring that \( p \) should be a minimum as a function of these variables, that is, that

\[
\frac{\partial p}{\partial b} = 0 \quad \text{and} \quad \frac{\partial p}{\partial \gamma} = 0
\] (9)

The integral of the expression for the strain energy in bending \(^7\) taken over one half wave length of the buckled surface is first written down, the axes of reference being parallel and perpendicular to the grain of the face plies. In this integral a transformation is then made which rotates the axes to the position \( OX \) and \( OY \) of figure 7.

Using the abbreviation \( z = \frac{b}{a} \), it is found that

\[
p_{cr} = \frac{\pi^2}{12\lambda} \left[ G \left( z^2 + 6\gamma^2 + \frac{\gamma^4 + 1}{z^2} \right) + R \left( 1 + \frac{\gamma^2}{z^2} \right) \right]
\]

\[ - M \left( 3\gamma + \frac{\gamma^3 + \gamma}{z^2} \right) \frac{h^2}{a^2} \] (10)

\(^7\)For the integrand, see equation (3.21) of appendix 3 of Forest Products Laboratory Report No. 1312.
where \( z \) and \( y \) satisfy the simultaneous equations

\[
z^4 = 1 + y^4 + \frac{Ry^2 - M(y^3 + y)}{G}
\]  

\[
z^2 = \frac{3My^2 + M - 4Gy^3 - 2Ry}{12Gy - 3M}
\]

and

\[
G = \frac{E_1 + E_2 + 2A}{4}
\]

\[
R = \left( \frac{3}{2} \right) (E_1 + E_2) - A
\]

\[
M = E_1 - E_2
\]

The symbols \( E_1 \) and \( E_2 \) are defined in the appendix. \( E_1 \) and \( E_2 \) are to be taken for strips parallel and perpendicular, respectively, to the grain of the face plies.

Equation (10) will be written in the form

\[
p_{cr} = k_c \frac{E}{L} \frac{h^2}{a^2}
\]

where

\[
k_c = \frac{\pi^2}{12\lambda} \frac{1}{E_L} \left[ G \left( z^2 + 6y^2 + \frac{y^4}{z^2} \right) 
+ R \left( 1 + \frac{y^2}{z^2} \right) - M \left( 3y + \frac{y^3 + y}{z^2} \right) \right]
\]
The values of \( z \) and \( y \) to be used in (17) are the solutions of the simultaneous equations (11) and (12).

The values of \( k \) are given for certain types of plates in tables 5 and 6 and at points on the horizontal axis in figures 13 to 26.

2. Rectangular Plywood Plates Under Uniform Shear

Case 1: \( \theta = 0^\circ \) or \( \theta = 90^\circ \).

The results for this case can be obtained from formulas and a family of curves contained in a paper by Seydel. These curves are reproduced in figure 8. A certain amount of approximation was involved in drawing these curves, but it appears that they can be considered sufficiently accurate for practical purposes.

In order to make use of the family of curves of figure 8 to determine the critical stress for a given plate, it is necessary to know the values of two quantities, \( \alpha \) and \( \beta_a \), which are defined in terms of the constants \( E_1 \), \( E_2 \), and \( A \) and the dimensions \( a \) and \( b \) by the following equations:

\[
a = \frac{A}{(E_1 E_2)^{\frac{1}{2}}} \tag{18}
\]

\[
\beta_a = \frac{a}{b} \left( \frac{E_2}{E_1} \right)^{\frac{1}{4}} \tag{19}
\]

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Edgar Seydel, Zeitschrift fur Flugtechnik und Motorluftschifffahrt 24, 78-83, 1933. A translation of this paper was published by the National Advisory Committee for Aeronautics as Technical Memorandum No. 705.
It is to be observed that, in accordance with the definitions given in the appendix, the mean moduli in bending, $E_1$ and $E_2$, are measured parallel to the sides $a$ and $b$, respectively.

The quantities $\alpha$ and $\beta$ being known, the quantity $c_a$ can be read from the curves of figure 8.

The critical shearing stress $q_{cr}$ can then be calculated from the formula

$$q_{cr} = \frac{c_a h^2}{3\lambda a^2} \left(\frac{E_1}{E_2}\right)^{\frac{1}{4}}$$

Equation (20) can be written in the form

$$q_{cr} = k_s E_L h^2$$

where, after introducing for convenience the ratios $\frac{E_1}{E_L}$ and $\frac{E_2}{E_L}$,

$$k_s = \frac{c_a}{3\lambda} \left[\left(\frac{E_1}{E_L}\right)^3 \frac{E_2}{E_L}\right]^{\frac{1}{4}}$$

To illustrate the procedure, the critical shearing stress for a 3-ply panel of Douglas-fir plywood will be calculated. All the plies are taken to be of the same thickness. The grain of the face plies is assumed to be parallel to the side $b$, which is twice as long as the side $a$. See figure 1.
\[
E_1 = 0.0927 \frac{E}{L} \quad E_2 = 0.9651 \frac{E}{L}
\]

\[A = 0.1451 \frac{E}{L}\]

\[a = \frac{0.1451}{(0.9651 \times 0.0927)^2} = 0.485 \]

\[\beta_a = \left(\frac{1}{2}\right) \left(\frac{0.9651}{0.0927}\right)^\frac{1}{4} = 0.898 \]

\[c_a = 15.6 \]

\[k_s = \frac{15.6}{3a} \left[ (0.0927)^3 \quad 0.9651 \right]^{\frac{1}{4}} = 0.874 \]

Hence

\[q_{cr} = 0.874 \frac{E}{L} \frac{h^2}{a^2} \quad (23)\]

A difficulty will be met in this type of plywood if the side \(b\) in figure 1 is less than \(1.797a\).

To illustrate the procedure in this case, let \(b = 1.5a\). It will be found that \(\beta_a\) is greater than 1 and \(c\) cannot be found from figure 8. It is only necessary to interchange the sides and the corresponding constants \(E_1\) and \(E_2\) as in figure 9.

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Then
\[ b' = \frac{2}{3} a' \]

\[ E_1' = 0.9651 E_L \]

\[ E_2' = 0.0927 E_L \]

\[ A = 0.1451 E_L \]

\[ a = 0.485 \]

\[ \beta_a = \frac{a'}{b'} \left( \frac{0.0927}{0.9651} \right)^{\frac{1}{4}} \]

\[ = 0.5566 \frac{a'}{b'} = 0.835 \]

From figure 8,
\[ c_a = 14.7 \]

Then
\[ \frac{k_s}{s} = \frac{14.7}{3\lambda} \left[ (0.9651)^3 0.0927 \right]^{\frac{1}{4}} = 2.659 \]

and
\[ \frac{q_{cr}}{E} = 2.659 \frac{h^2}{L_a'^2} \]

But
\[ a' = b = 1.5a \]

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Hence, in terms of the side $a$ of the given plate

\[
q_{cr} = 2.659 \ E \ \frac{h^2}{L} \ \frac{1}{2.25a^2} = 1.182 \ E \ \frac{h^2}{L} \ \frac{1}{a^2}
\]  

(24)

For an infinitely long plate $\beta_a = 0$. The corresponding value $c_a$ is the intercept on the axis $\beta_a = 0$ for the proper value of $a$ in figure 8.

Thus, for an infinitely long plate of 3-ply, Douglas-fir plywood with all plies of the same thickness and $\theta = 90^\circ$, 

\[
a = 0.485 \\
c_a = 10.7 \\
k_s = \frac{10.7}{3\lambda} \ \left[ (0.0927)^3 \ \frac{1}{0.9651} \right]^{\frac{1}{4}} = 0.600
\]

Then

\[
q_{cr} = 0.600 \ E \ \frac{h^2}{L} \ \frac{1}{a^2}
\]  

(25)

If $\theta = 0$

\[
a = 0.485 \\
c_a = 10.7 \\
k_s = \frac{10.7}{3\lambda} \ \left[ (0.9651)^3 \ \frac{1}{0.0927} \right]^{\frac{1}{4}} = 1.935
\]

Then

\[
q_{cr} = 1.935 \ E \ \frac{h^2}{L} \ \frac{1}{a^2}
\]  

(26)
Case 2: Infinite strip (long narrow plate). \( \theta = 45^\circ \).

In this case, the mathematical analysis leading to the results of case 1 of this section does not apply. Exactly as in section 1, the energy method can be used to obtain an approximate solution for an infinitely long plate. The form of the deflected surface was taken to be given by (8) of section 1. The same reasoning that was used in that section (except that the strain energy of bending is equated to the work done by the load in producing the shearing deformation associated with the bending) leads to the following expression for the critical shearing stress:

\[
q_{cr} = k_s E \frac{h}{L} \frac{h^2}{a^2}
\]  

(27)

where

\[
k_s = \frac{\pi^2}{24\lambda} \frac{1}{E_L} \frac{1}{\gamma} \left[ G\left(z^2 + 6\gamma^2 + \frac{\gamma^4 + 1}{z^2}\right) + R\left(1 + \frac{\gamma^2}{z^2}\right) \right] - M\left(3\gamma + \frac{\gamma^3 + \gamma}{z^2}\right)
\]  

(28)

In this equation \( z \) and \( \gamma \) satisfy the simultaneous equations:

\[
z^4 = 1 + \gamma^4 + \frac{R\gamma^2}{G} - M\left(\gamma^3 + \gamma\right)
\]  

(29)

\[
z^2 = \frac{2G\left(1 - \gamma^4\right) - M\left(\gamma - \gamma^3\right)}{6G\gamma^2 - R}
\]  

(30)

The quantities \( G, M, \) and \( R \) are defined by equations (13), (14), and (15) of section 1. It is to be observed that here, as in case 2 of section 1, \( E_1 \) and \( E_2 \) are associated with strips parallel and perpendicular, respectively, to the grain of the face plies.

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\footnote{See for example S. Timoshenko, Theory of Elastic Stability. p. 357.}
It will be found that equations (29) and (30) yield two values of \( \gamma \), one positive and the other negative. The positive value corresponds to the situation of figure 10, while the negative value corresponds to that of figure 11. Evidently \( q_{cr} \) should be expected to be much greater, numerically, in the second case. This is found to be true in table 6 and in figures 14, 17, 20, 23, and 26.

As a check on the probable accuracy of the energy method that was used in obtaining formula (27), this method was used to find the critical shearing stress for infinitely long plywood plates in which the grain of the face plies is either parallel or perpendicular to the edges of the plate.

Using the energy method together with equation (8) for the buckled surface, the following formula was found for the critical shearing stress:

\[
q_{cr} = \frac{k}{s} \frac{h^2}{E_L a^2}
\]

where

\[
k = \frac{\pi^2}{24} \frac{1}{\gamma} \left[ \frac{E_1}{E_L} z^2 + \frac{6}{E_L} \gamma^2 + \frac{2A}{E_L} + \frac{1}{z^2} \left( \frac{E_2}{E_L} \right) \right]
\]

and \( z \) and \( \gamma \) satisfy the simultaneous equations

\[
z^4 = \frac{E_2 + 2A \gamma^2 + E_1 \gamma^4}{E_1}
\]

\[
z^2 = \frac{E_2 - E_1 \gamma^4}{3E_1 \gamma^2 - A}
\]
In these formulas $E_1$ and $E_2$ are to be taken as corresponding to strips parallel to the $X$ and $Y$ axes, respectively. A comparison of the values of $k_s$ as calculated by formula (32) and as calculated from Seydel's curves is shown in table 4 for a number of types of plates.

It appears that the energy method leads to results for the cases $\theta = 0^\circ$ and $\theta = 90^\circ$ that are from 6 to 8 percent too high. In the case of the isotropic infinite strip it also leads to an approximate value of the critical shearing stress that is about 7 percent too high. The fact that a useful approximation is attained in these cases by the energy method lends support to the belief that the same is true for the infinite plywood strip with $\theta = 45^\circ$.

An estimate of the variation of critical shearing stress with length that may be expected to be useful for the case $\theta = 45^\circ$ can be made by studying the curve of figure 12. In this figure the ratio of the value of the factor $k_s$ of formula (21) for a plate of length $b$ to the value of $k_s$ for an infinitely long plate of the same construction is plotted as a function of the ratio of the length $b$ of the finite plate to the half wave length $b'$ of the buckled surface of the infinitely long plate. Points are plotted for several types of plywood with $\theta = 0^\circ$ or $\theta = 90^\circ$. They are seen to lie almost exactly on a common curve. The values of $k_s$ were calculated with the aid of the curves of figure 8, which was taken from Seydel's paper, to which reference has been made. The fact that maxima and minima similar to those in figures 5 and 6 do not appear in figure 12 is a consequence of the approximations involved in Seydel's curves in figure 8, which are intended to lead to curves for $k_s$ as a function of $b/a$ which pass through the minima of the exact curves, which would be similar to those of figures 5 and 6, except that the ordinates of the minimum points decrease to a limiting value as the ratio $b/a$ increases.

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See for example S. Timoshenko, Theory of Elastic Stability, p. 361.
3. Buckling of Infinitely Long Plywood Plates Under Combined Uniform Compression and Uniform Shear

Case 1: \( \theta = 0^\circ \) or \( \theta = 90^\circ \).

Let \( q = fp \) \hspace{1cm} (35)

where \( f \) is either positive or negative. A positive value of \( p \) denotes a compression. The sense of \( q \) positive is indicated by the arrows in figure 4.

The energy method was used and the form of the buckled surface was taken to be that given by (8). The strain energy of bending of the plate in buckling to this form was equated to the work done by \( p \) and \( q \) in producing the deformation associated with the bending, that is, the shortening of the plate and the shearing strain. This equation was solved for \( p \) after making use of equation (35). Then, \( p_{cr} \) is the minimum value of \( p \) as a function of \( z = \frac{b}{a} \) and \( \gamma \). It was found that

\[
p_{cr} = k \frac{E}{c} \frac{h}{L} \frac{a}{2} \]

where

\[
k = \frac{\pi^2}{12\lambda} \frac{1}{E_L} \frac{1}{1 + 2\gamma^f} \left[ E_1 \left( z^2 + 6\gamma^2 \right) + 2A + \frac{1}{z^2} \left( E_2 + 2A\gamma^2 + E_1 \gamma^4 \right) \right] \]

\hspace{1cm} (37)

and \( z = \frac{b}{a} \) and \( \gamma \) satisfy the simultaneous equations:

\[
z^4 = \frac{E_2 + 2A\gamma^2 + E_1 \gamma^4}{E_1} \]

\hspace{1cm} (38)
\[
2z = \frac{A\gamma + E_1 \gamma^3 + E_1 \gamma^4 f - E_2 f}{Af - 3E_1 \gamma - 3E_1 \gamma^2 f}
\] (39)

After \( p_{cr} \) has been found, the associated critical shearing stress is given by \( q_{cr} = fp_{cr} \).

It will be convenient to write

\[
q_{cr} = f k_c E_L \frac{h^2}{a^2} = k_s E_L \frac{h^2}{a^2}
\] (40)

where

\[
k_s = f k_c
\] (41)

The values of \( k_c \) and \( k_s \) are listed in table 5 for several types of plywood and for several values of \( f \).

**Case 2: Infinite strip. \( \theta = 45^\circ \).**

As in case 1, let \( q = fp \) (35)

and let the form of the buckled surface be given by (8).

The energy method leads to the formula

\[
p_{cr} = k_c E_L \frac{h^2}{a^2}
\] (42)
where
\[
 k_c = \frac{\pi^2}{12\lambda} \frac{1}{E_L (1 + 2\gamma f)} \left[ G (z^2 + 6\gamma^2 + \frac{1 + \gamma^4}{z^2}) \\
 + R \left( 1 + \frac{\gamma^2}{z^2} \right) - M \left( 3\gamma + \frac{\gamma^3 + \gamma}{z^2} \right) \right]
\]  
(43)

and \( z = \frac{b}{a} \) and \( \gamma \) satisfy the equations
\[
z^4 = 1 + \gamma^4 + \frac{R \gamma^2 - M (\gamma^3 + \gamma)}{G}
\]  
(44)
\[
z^2 = \frac{4G \left[ f (1 - \gamma^4) - \gamma^3 \right] - 2R \gamma + M \left[ 3\gamma^2 + 1 - 2 (\gamma - \gamma^3) f \right]}{12G (\gamma + \gamma^2 f) - 3M - 2Rf}
\]  
(45)

Two values of \( \gamma \) may be expected corresponding to positive or negative values of \( f \). The positive sense of \( f \) is shown in figure 4.

After \( p \) has been found, \( q \) can be found from (35).

Then
\[
 q_{cr} = f p_{cr} = f k_c \frac{E}{a^2} = k_s \frac{E}{a^2}
\]

Values of \( k_c \) and \( k_s \) for several types of plywood and several values of \( f \) are given in table 6.

The values of \( k_c \) and \( k_s \) given in table 6 are for an infinitely long strip.

As in other cases, it may be expected that they hold approximately for plates whose lengths are three or more times their widths except possibly in a few cases where the wave lengths are extremely large. In the case of shorter plates a rough estimate of the values of \( k_c \) and \( k_s \) can be made by consideration of the curve of figure 12 in connection with the values for the infinitely long plate.
In many instances of combined stresses, the formulas for the calculation of \( k_c, \ z = \frac{b}{a} \), and \( \gamma \) have two solutions and one of these solutions may be found to lead to a negative value of \( p \), or \( q \), or both. A negative value of \( p \) indicates that the direct stress is a tension, since the convention has been adopted here that a direct compressive stress is positive. In such a case, the buckling takes place under tension combined with a positive or negative shearing stress. The direction of a positive shearing stress is shown in figure 4.

In addition to tables 5 and 6, the results of this report are shown in the curves of figures 13 to 26 for several types of plywood. Associated values of \( k \) and \( k_s \) are plotted as the rectangular coordinates of a point. Along a line \( k = \frac{f}{k} \), drawn from the origin with slope \( f \), points nearer the origin than the point of intersection with the curve correspond to states of combined stress for which buckling does not occur. Curves showing the ratio of the half wave length \( b \) to the width \( a \), and the slope \( \gamma \) of the wrinkles as functions of \( k_s \), are plotted to the left in each figure. As previously stated, the values of the constants of Douglas-fir were used in the calculations.

Conclusion

In conclusion, attention is again called to the fact that the results of this report have been derived by theoretical treatment, much of it approximate, and that they have not yet been confirmed by Laboratory tests. The calculated approximate critical stresses will be too high, since they were obtained by the energy method. However, these errors may be expected to be at least partially offset in practical applications by the effect of restraints at the edges.

A limited series of tests appears to confirm formula (1) for plates having the grain of the face plies parallel or perpendicular to the edges and under uniform compressive stress parallel to one pair of edges, if care is taken to avoid the effects of initial curvature. Because of initial lack of perfect flatness many, probably most, plates do not buckle sharply under any of the types of loading considered in this report. Instead, a characteristic behavior, as is well known, is a gradual increase in
lateral deflection with increasing load, then a more rapid increase in
the lateral deflection as the load nears the critical load. When the
lateral deflections are of the order of magnitude of the thickness of the
plate -- greater than one-half the thickness -- the effect of membrane
stresses becomes appreciable. The apparent stiffness of the plate in-
creases and prevents the occurrence of sudden buckling. Frequently,
in this stage of loading, the load-lateral deflection curve is observed
to be nearly a straight line inclined to the axes and not asymptotic to
a horizontal line.

For very small initial curvatures, the methods proposed by Southwell\textsuperscript{11}
and modified by Lundquist\textsuperscript{12} may be used to calculate the theoretical
critical load from the observed load deflection curve. The method ap-
ppears to fail when the initial curvature is so great that appreciable
membrane stresses develop long before the critical load is reached.
The method when applied to plates assumes that the departures of the
middle surface from a plane are small in comparison with the thickness,
in order that the behavior of the plate can be described by a linear dif-
ferential equation.

Tests are under way to check the formulas presented in this report, and
attention is being given to the effects of initial curvature.

\textsuperscript{12} Lundquist, E. E., N. A. C. A. Technical Note No. 658, 1938.
APPENDIX: Elastic Constants of Wood and Plywood

Wood

Wood will be considered to be an orthotropic material, that is, a material which has three mutually perpendicular planes of elastic symmetry. These planes are determined by the three principal directions, the longitudinal, radial, and tangential directions, as shown in figure 3. There are three Young's moduli, $E_L$, $E_R$, and $E_T$, associated with extensions or contractions parallel to the longitudinal, radial, and tangential directions, respectively. There are three shearing moduli $\mu_{LT}$, $\mu_{LR}$, and $\mu_{RT}$. For example, $\mu_{LT}$ is the modulus associated with a shearing strain with respect to the axes $OL$ and $OT$. In such a strain a square whose sides are parallel to $OL$ and $OT$, respectively, becomes a rhombus.

There are six Poisson's ratios, $\nu_{LT}$, $\nu_{TL}$, $\nu_{LR}$, $\nu_{RL}$, $\nu_{RT}$, and $\nu_{TR}$. Thus $\nu_{LT}$ is the ratio of the contraction parallel to $OT$ to the extension parallel to $OL$ associated with a tension parallel to $OL$.

Under the assumption that wood has three planes of elastic symmetry, the three following relations hold among the elastic constants:

\[
\frac{\nu_{LT}}{E_L} = \frac{\nu_{TL}}{E_T}; \quad \frac{\nu_{LR}}{E_L} = \frac{\nu_{RL}}{E_R}; \quad \frac{\nu_{RT}}{E_R} = \frac{\nu_{TR}}{E_T}
\]  \hspace{1cm} (1)

Experimentally determined values of the constants for some species are given in table 1.

The values given below of the elastic moduli of Douglas-fir at 10 percent moisture content were obtained from a limited number of tests. Because of the small number of tests, these values are considered to be tentative.
\[ E_L = 1,960,000 \text{ lb. per sq. in.} \quad E_T = 113,200 \text{ lb. per sq. in.} \]
\[ E_R = 155,800 \text{ lb. per sq. in.} \quad \mu_{LT} = 123,800 \text{ lb. per sq. in.} \]
\[ \mu_{LR} = 110,600 \text{ lb. per sq. in.} \quad \mu_{RT} = 7,100 \text{ lb. per sq. in.} \]

The values of Poisson's ratios for spruce were used in the calculations, since those for Douglas-fir have not been determined. The influence of considerable variations of these ratios on the buckling stress is slight.

**Plywood Plates**

In this appendix rectangular plates of plywood will be first considered in which the directions of the grain of the wood in adjacent plies are mutually perpendicular, and perpendicular or parallel to the edges of the plate. Later it will be shown how to use the constants defined for plates of this type in treating plates in which the grain of the wood in the respective plies is inclined to the edges of the plate. The dimensions and choice of axes are shown in figure 2. The thickness of a plate is denoted by \( h \).

The effect of the glue, other than that of securing adherence of adjacent plies, was assumed to be negligible in obtaining the formulas for calculating the mean moduli in bending \( E_1 \) and \( E_2 \), to be defined below. These formulas for \( E_1 \) and \( E_2 \) cannot be expected to apply to partially or completely impregnated plywood nor to compregnated wood. Further, the relevant shearing modulus cannot in these cases be regarded as independent of the glue. The quantities \( E_1 \) and \( E_2 \), however, can be determined by static bending tests and the shearing modulus \( \mu_{XY} \) by a suitable shear test.

To define \( E_1 \), consider a strip of unit width, with its edges parallel to the \( X \) axis, to be cut from the plate of figure 1. The grain of the wood in the various plies is assumed to be either parallel or perpendicular to the edges of the plate. The stiffness of the strip is determined by a modulus \( E_1 \) defined by the equation

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where the summation is extended over all of the plies numbered, for example, as in figure 2; \( (E_x)_i \) is the Young's modulus of the \( i^{th} \) ply measured in a direction parallel to the length of the strip; \( I_i \) is the moment of inertia, with respect to the neutral axis, of the area of the cross section of the \( i^{th} \) ply made by a plane perpendicular to the length of the strip; and \( I \) is the moment of the inertia of the entire cross section of the strip with respect to its central line, that is, 
\[
I = \frac{h^3}{12} \text{ for a strip of unit width.}
\]

An approximate formula, in which the error is very slight, is obtained for \( E_1 I \) by taking the sum of the products \( (E_x)_i I_i \) formed for only those plies in which the grain is parallel to the length of the strip. Exception is to be made of a three-ply strip having the grain of the face plies perpendicular to the length of the strip.

In the case of a rectangular plate with sides \( a \) and \( b \), the modulus \( E_1 \) would determine the stiffness of a strip cut from the plate with its edges parallel to the side \( a \), as in figure 1. The modulus \( E_2 \) similarly defined, namely,

\[
E_2 I = \sum (E_y)_i I_i \tag{3}
\]
determines the stiffness of strips parallel to the side \( b \).

As in the case of \( E_1 \), the calculation of \( E_2 \) can be based on the parallel plies only, except in the case of a three-ply strip having the grain of the face plies perpendicular to the length of the strip.

The reasoning\(^\text{13}\) leading to the definitions of \( E_1 \) and \( E_2 \) neglects the irregularities that exist in the state of stress at the junctions of plies in

\(^{13}\)March, H. W., "Flat Plates of Plywood Under Uniform or Concentrated Loads." U. S. Forest Products Laboratory Report No. 1312, 1942.

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a strip of plywood. A long series of static bending tests at the Forest Products Laboratory shows that this procedure is justified.

If the plies in a plate are all of identical material and are all either flat-grained or edge-grained, the shearing modulus $\mu_{xy}$ that is needed in the theory of plates is the same in all plies. If the plies are of wood of different species, or if some of the plies are rotary cut and some are sliced, a mean modulus $\mu_{12}$ is needed that is defined in the same way as $E_1$ and $E_2$, viz.

$$\mu_{12} = \Sigma (\mu_{xy})_i I_i$$

(4)

The factor $1 - \theta^2$, where $\theta$ denotes Poisson's ratio as it occurs in the theory of isotropic plates, is replaced by the factor

$$\lambda = 1 - \frac{\theta}{xy} \frac{\theta}{yx}$$

(5)

in plywood plates having the grain of the wood parallel or perpendicular to the axes. The definition (5) strictly applies to a single ply. But $\lambda$ is not greatly different from unity if the axes of reference, $x$ and $y$, are parallel and perpendicular to the grain of the wood and the annual rings are either parallel or perpendicular to the $XY$ plane. Without serious error, its value may be taken to be the same in all plies of the type specified. As in other reports of this series, its value will be taken to be 0.99, which is approximately correct for spruce. This value is taken instead of unity chiefly to emphasize the fact that this factor, which is about 0.91 for metals, has been taken into account.

Frequently occurring in the discussion of plywood plates is a constant, $A$, which is defined as follows:

$$A = (E_x \theta_yx) + 2\lambda \mu_{12}$$

(6)

where $(E_x \theta_yx)$ is defined by an equation like (4).
For a plate with flat-grained plies of the same species

\[ A = E_L \theta_{TL} + 2 (1 - \theta_{LT} \theta_{TL}) \mu_{LT} \]  \hspace{1cm} (7)

When the grain of the face plies, as in figure 4, is not perpendicular or parallel to the edges of the plate, the constants \( E_1', \) \( E_2', \) and \( A \) that appear in the formulas below are to be calculated with respect to temporary axes in a face ply, the \( X \) axis being taken to be in the direction of the grain and the \( Y \) axis perpendicular to it. After these constants have been determined in this way, the axes are rotated so that the \( X \)-axis is perpendicular to the edges of the plate. In this report the face grain is taken to make an angle of 45° with the edges of the plate when it is not parallel or perpendicular to the edges. As a consequence of the rotation of the axes through an angle of 45°, the following new constants, which are a combination of \( E_1', E_2', \) and \( A, \) are introduced:

\[ G = \frac{E_1' + E_2' + 2A}{4} \]

\[ R = \left( \frac{3}{2} \right) (E_1' + E_2') - A \]

\[ M = E_1' - E_2' \]

The new constants can be readily calculated as soon as the structure of the plywood in the plate is known.
Table 1.—Elastic constants of timber

<table>
<thead>
<tr>
<th></th>
<th>Compression</th>
<th>Tension</th>
<th>Shear</th>
<th>Poisson's Ratio</th>
<th>Constants</th>
<th>Shear Moduli ( \times 10^4 ) (from shear tests)</th>
<th>Shear Moduli ( \times 10^4 ) (calculated from ( 45^\circ ) comp. test)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L. R. T.</td>
<td>L. R. T.</td>
<td>LT. LR. RT.</td>
<td>( \sigma_{LR} )</td>
<td>( \sigma_{LT} )</td>
<td>( \sigma_{RT} )</td>
<td>( \mu_1 )</td>
</tr>
<tr>
<td>Spruce</td>
<td>E ( \times 10^6 )</td>
<td>0.895</td>
<td>0.13</td>
<td>0.07</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Density 27 lbs, per c.ft.</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Moisture 12.2%</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Light Mahogany</td>
<td>E ( \times 10^6 )</td>
<td>1.8</td>
<td>0.14</td>
<td>0.07</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Density 33 lbs, per c.ft.</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Moisture 13.4%</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Ash</td>
<td>E ( \times 10^6 )</td>
<td>2.18</td>
<td>0.238</td>
<td>0.14</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Density 50 lbs, per c.ft.</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Moisture 13.6%</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Walnut</td>
<td>E ( \times 10^6 )</td>
<td>1.68</td>
<td>0.172</td>
<td>0.072</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Density 37 lbs, per c.ft.</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Moisture 10.9%</td>
<td>lbs/sq.in.</td>
<td>18000</td>
<td>3200</td>
<td>2900</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

### Table 2. Buckling of flat rectangular plates. Three-ply (1:2:1). $\theta = 0$. Uniform compression. Simply supported edges. Equation (6).

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.68</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>0.945</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.336</th>
<th>1.5</th>
<th>1.636</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_c$</td>
<td>1.927</td>
<td>1.270</td>
<td>1.008</td>
<td>0.910</td>
<td>0.895</td>
<td>0.898</td>
<td>0.895</td>
<td>1.015</td>
<td>1.058</td>
<td>1.008</td>
<td>0.945</td>
<td>0.910</td>
<td>0.895</td>
<td>0.913</td>
<td>0.949</td>
</tr>
</tbody>
</table>

### Table 3. Buckling of flat rectangular plates. Three-ply (1:2:1). $\theta = 90^\circ$. Uniform compression. Simply supported edges. Equation (6).

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.497</th>
<th>1.8</th>
<th>2.117</th>
<th>2.5</th>
<th>2.995</th>
<th>3.3</th>
<th>3.668</th>
<th>4.0</th>
<th>4.492</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_c$</td>
<td>3.209</td>
<td>2.329</td>
<td>1.808</td>
<td>1.480</td>
<td>1.120</td>
<td>0.960</td>
<td>0.901</td>
<td>0.895</td>
<td>0.940</td>
<td>1.058</td>
<td>0.938</td>
<td>0.895</td>
<td>0.907</td>
<td>0.949</td>
<td>0.913</td>
<td>0.895</td>
</tr>
<tr>
<td>$b/a$</td>
<td>4.8</td>
<td>5.187</td>
<td>5.989</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_c$</td>
<td>0.901</td>
<td>0.922</td>
<td>0.895</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Table 4.--Comparison of values of $k_s$ in formula (31) obtained by the energy method with those obtained from Seydel's curves in figure 8. Long narrow plates.

<table>
<thead>
<tr>
<th>Type</th>
<th>$\theta$</th>
<th>$k_s$ (Energy method)</th>
<th>$k_s$ (Seydel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-ply (1:1:1)</td>
<td>0</td>
<td>2.067</td>
<td>1.935</td>
</tr>
<tr>
<td>3-ply (1:1:1)</td>
<td>90°</td>
<td>0.640</td>
<td>0.600</td>
</tr>
<tr>
<td>3-ply (1:2:1)</td>
<td>0</td>
<td>2.145</td>
<td>2.004</td>
</tr>
<tr>
<td>3-ply (1:2:1)</td>
<td>90°</td>
<td>0.957</td>
<td>0.894</td>
</tr>
<tr>
<td>5-ply (1:1:1:1:1)</td>
<td>0</td>
<td>2.142</td>
<td>1.988</td>
</tr>
<tr>
<td>5-ply (1:1:1:1:1)</td>
<td>90°</td>
<td>1.204</td>
<td>1.117</td>
</tr>
<tr>
<td>5-ply (1:2:2:2:1)</td>
<td>0</td>
<td>1.972</td>
<td>1.834</td>
</tr>
<tr>
<td>5-ply (1:2:2:2:1)</td>
<td>90°</td>
<td>1.666</td>
<td>1.549</td>
</tr>
</tbody>
</table>
Table 5.--Buckling of long plywood plates under combined compression or tension and shear. Edges simply supported. For compression

\[
p_{cr} = \frac{k E h^2}{a^2}
\]

Negative \( k \) denotes tension. For shear

\[
q_{cr} = \frac{k E h^2}{a^2}
\]

For combined stress the shear stress is equal to \( f \) times the compressive (or tensile) stress.

<table>
<thead>
<tr>
<th>Loading</th>
<th>( \theta )</th>
<th>( f )</th>
<th>( k )</th>
<th>2 ply</th>
<th>3 ply</th>
<th>3 ply</th>
<th>5 ply</th>
<th>5 ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td>0</td>
<td>( k_0 )</td>
<td>0.60</td>
<td>0.74</td>
<td>0.90</td>
<td>0.99</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>0</td>
<td>( k_0 )</td>
<td>0.69</td>
<td>2.07</td>
<td>2.14</td>
<td>2.14</td>
<td>1.97</td>
<td></td>
</tr>
<tr>
<td>Compression and shear</td>
<td>0</td>
<td>( k_0 )</td>
<td>0.54</td>
<td>0.72</td>
<td>0.86</td>
<td>0.94</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>( k_0 )</td>
<td>0.45</td>
<td>0.66</td>
<td>0.77</td>
<td>0.84</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>( k_0 )</td>
<td>0.37</td>
<td>0.60</td>
<td>0.68</td>
<td>0.73</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>( k_0 )</td>
<td>0.18</td>
<td>0.36</td>
<td>0.40</td>
<td>0.41</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>Tension and shear</td>
<td>0</td>
<td>( k_0 )</td>
<td>-6.16</td>
<td>-26.14</td>
<td>-23.95</td>
<td>-21.89</td>
<td>-16.95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>( k_0 )</td>
<td>-3.08</td>
<td>-13.07</td>
<td>-11.98</td>
<td>-10.94</td>
<td>-8.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>( k_0 )</td>
<td>-2.83</td>
<td>-6.39</td>
<td>-6.39</td>
<td>-5.90</td>
<td>-4.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>( k_0 )</td>
<td>-1.47</td>
<td>-4.99</td>
<td>-4.69</td>
<td>-4.39</td>
<td>-3.61</td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td>90</td>
<td>( k_0 )</td>
<td>0.60</td>
<td>0.74</td>
<td>0.90</td>
<td>0.99</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>90</td>
<td>( k_0 )</td>
<td>0.69</td>
<td>2.07</td>
<td>2.14</td>
<td>2.14</td>
<td>1.97</td>
<td></td>
</tr>
<tr>
<td>Compression and shear</td>
<td>90</td>
<td>( k_0 )</td>
<td>0.54</td>
<td>0.75</td>
<td>0.86</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>( k_0 )</td>
<td>0.45</td>
<td>0.57</td>
<td>0.67</td>
<td>0.83</td>
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<tr>
<td></td>
<td>1.5</td>
<td>( k_0 )</td>
<td>0.37</td>
<td>0.45</td>
<td>0.54</td>
<td>0.68</td>
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<tr>
<td></td>
<td>4.0</td>
<td>( k_0 )</td>
<td>0.18</td>
<td>0.21</td>
<td>0.26</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension and shear</td>
<td>90</td>
<td>( k_0 )</td>
<td>-6.16</td>
<td>-26.14</td>
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<td>-7.29</td>
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<td></td>
<td>1.0</td>
<td>( k_0 )</td>
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<td>-1.47</td>
<td>-2.60</td>
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<td>-6.13</td>
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<td>1.5</td>
<td>( k_0 )</td>
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<td>-2.22</td>
<td>-3.52</td>
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<tr>
<td></td>
<td>4.0</td>
<td>( k_0 )</td>
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<td>-1.38</td>
<td>-1.82</td>
<td>-2.79</td>
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Table 5.--Bi-axial loading of long plywood plates under combined compression or tension and shear. Edges simply supported. For compression

\[ p = k_L \frac{h^2}{a^2} \]

Negative \( k \) denotes tension. For shear

\[ q = k_S \frac{h^2}{a^2} \]

For combined stress the shear stress is equal to \( f \) times the compressive (or tensile) stress.

<table>
<thead>
<tr>
<th>Loading</th>
<th>Degrees</th>
<th>( k_L )</th>
<th>( k_S )</th>
<th>( k_T )</th>
<th>( k_S )</th>
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<tbody>
<tr>
<td>( \theta )</td>
<td>( f )</td>
<td>2 ply</td>
<td>3 ply</td>
<td>3 ply</td>
<td>5 ply</td>
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<td>Compression</td>
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<td>0.71</td>
<td>0.92</td>
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<td>0.95</td>
<td>0.63</td>
<td>0.95</td>
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<tr>
<td>Shear</td>
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<td>( k_T )</td>
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<td>-3.21</td>
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<td>Compression and shear</td>
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<td>( k_L )</td>
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<td>0.56</td>
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<td>( k_T )</td>
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<td>-5.50</td>
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Mimeo. No. 1316

Z M 41124 F
FIG. 1
SURFACE OF A RECTANGULAR PLYWOOD PLATE SHOWING DIMENSIONS OF SIDES AND CHOICE OF AXES.

FIG. 2
SECTION OF A PLYWOOD STRIP SHOWING NUMBERING OF PLIES
(a) LOG

(b) ROTARY CUT VENEER

(c) SLICED VENEER, EDGE GRAIN

FIG. 3
PRINCIPAL DIRECTIONS IN WOOD AND PLYWOOD
FIG. 4
LONG PLATE UNDER COMPRESSION AND SHEAR.
Fig. 5
Buckling under uniform compressive stress
3-Ply (1:2:1) \( \theta = 0^\circ \)
Values of \( k_e \) in the formula \( \sigma_{cr} = k_e \frac{E}{\rho} \)

Fig. 6
Buckling under uniform compressive stress
3-Ply (1:2:1) \( \theta = 90^\circ \)
Values of \( k_e \) in the formula \( \sigma_{cr} = k_e \frac{E}{\rho} \)
FIG. 7
LONG PLATE UNDER COMPRESSION
\( \theta = 45^\circ \)

FIG. 8
CURVES FOR CALCULATING THE BUCKLING SHEARING STRESS IN ORTHOTROPIC RECTANGULAR PLATES WHOSE AXES OF ELASTIC SYMMETRY ARE PARALLEL TO THE EDGES.
(TAKEN FROM PAPER BY E. Seydel, Zeitschrift für Flugtechnik und Luftschifffahrt 24,78-83, 1933)
FIG. 9
NOTATION TO BE USED IN CONNECTION WITH THE CURVES OF FIG. 8

FIG. 10
INCLINATION OF WRINKLES.
POSITIVE SHEARING STRESS.

FIG. 11
INCLINATION OF WRINKLES.
NEGATIVE SHEARING STRESS.
LEGEND

- 3-PLY (1:1:1) \(\theta = 0^\circ\)
- 3-PLY (1:1:1) \(\theta = 90^\circ\)
- 3-PLY (1:2:1) \(\theta = 0^\circ\)
- 3-PLY (1:2:1) \(\theta = 90^\circ\)

**FIG. 12**

The ratio \(k_s/(k_s)_\infty\) plotted against the ratio \(b'/b\). 

\((k_s)_\infty\) denotes the value of the factor \(k_s\) and \(b'\) the half wave length for an infinitely long plate buckling under a uniform shearing stress. Seydel's curves (Fig. 8) were used in calculating \(k_s\).
EXPLANATION OF FIGURES 13 TO 26

The following figures show associated values of $k_c$ and $k_s$, the constants in equations (36) and 40, for the buckling of long rectangular plates of plywood under combined uniform compression (or tension) and shear. The edges were taken to be simply supported. The constants for rotary-cut Douglas-fir plywood were used in the calculations of the coordinates of the points on these curves. Curves showing the ratio of the half wave length $b$ to the width $a$ and the slope $\gamma$ of the wrinkles as functions of $k_s$ are plotted to the left in each figure.

FIG. 13
2-PLY (1:1) $\theta = 0^\circ$ AND $\theta = 90^\circ$

FIG. 14
2-PLY (1:1) $\theta = 45^\circ$
*Further description on page containing figures 13 and 14.*