EFFECT OF AXIAL STIFFENERS ON THE BUCKLING PROPERTIES OF THIN CURVED PLYWOOD PLATES IN AXIAL COMPRESSION

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EFFECT OF AXIAL STIFFENERS ON THE BUCKLING PROPERTIES OF THIN CURVED PLYWOOD PLATES IN AXIAL COMPRESSION

By

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Summary

This report presents the results of tests and analysis of data on 78 curved plywood plates tested in axial compression. Each plate was unstiffened or had a single stiffener parallel to the applied stress.

The results of the tests show that the addition of an axial stiffener to a curved plywood plate in compression does not increase the buckling stress of the plywood. The effect of the stiffener is to carry additional load and, if stiff enough, to break up the buckle pattern. The theoretical buckling stress can be determined by the same methods used for a cylinder of finite length.

Introduction

In many of the structural uses of plywood, particularly in aircraft, it is possible to provide additional strength and rigidity to a plywood shell by the addition of stiffeners. The behavior of a specific section of a structure can be approximated by testing a prototype. If a sufficiently rigid stiffener is added to a plywood plate, the plywood can be made to buckle on either side of the stiffener instead of across the stiffener.

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1 This progress report is one of a series prepared and distributed by the Forest Products Laboratory under U. S. Navy, Bureau of Aeronautics No. NBA-PO-NAer 00565 and Army Air Force No. AAF-PO-(33-038)46-1189E. Results here reported are preliminary and may be revised as additional data become available.

2 A report of the effect of circumferential stiffeners on curved plywood plates in axial compression is now under preparation at the Forest Products Laboratory by T. B. Heebink.

It was the purpose of the experiments conducted for this report to determine the effect of an axial stiffener on the buckling characteristics of a curved plywood plate in axial compression. (Similar tests have been made on plywood cylinders.) A comparison of the observed buckling stresses of the plates and theoretical buckling stresses of comparable cylinders is presented in this report. Variables in the curved plywood plates include (1) three constructions, (2) two veneer thicknesses, (3) length to width ratios from 1.0 to 3.7, (4) length to radius ratios from 1.2 to 1.8, (5) width to radius ratios from 0.25 to 1.0, and (6) grain direction of face plies either parallel or perpendicular to the direction of the applied stress.

Description of Material

The veneers used in making the plywood plates for these tests were rotary-cut yellow birch and yellow-poplar of aircraft grade. Each plate was made from veneers of a single species into three-, four-, or five-ply constructions. The grain direction of each ply was perpendicular to the grain of the adjacent plies, except in the four-ply construction, in which the grain direction of the two center plies were parallel to each other. The plates were fabricated by bag-molding to the desired radius of curvature, using a phenolic-resin film glue to bond adjacent plies.

Axial stiffeners were cut from a single Sitka spruce plank, quarter-sawn into strips 5/16 inch by 1 inch in cross section. Test coupons 4 inches long were cut from the ends of each strip. The strips were then ripped to the proper size of stiffener and glued to the concave face of the curved plate with cold-setting resorcinol glue.

Static-bending and compression coupons were cut from the outside edge of each panel while trimming to finished size.

All plates were made as shown in the sketches of figure 1. The two curved end pieces were notched to a width and depth sufficient to receive the axial stiffener. After the stiffener had been glued to the panel, the panel was nail-glued with a cold-setting resorcinol glue to the end pieces and conditioned in an atmosphere at a temperature of 75° F. and a relative humidity of 64 percent. Prior to testing, the two curved ends of the plate were carefully machined to insure flatness of the plywood and to insure equal length of plywood and axial stiffener. The side rails were placed on the plate at the time of test, thus providing simply supported edges.

Methods of Test

Compression tests of the plywood plates were made in a 100,000-pound hydraulic testing machine. The specimen was centered on a heavy steel plate supported on a 9-inch knife edge (fig. 2). Thus the lower bearing was in effect a

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spherical head with negligible friction in a direction perpendicular to the knife edge. It was thought that this type of head would not retard the formation of buckles when the buckling load was reached. The top of the specimen rested squarely against the upper head of the testing machine.

A small initial load was put on the plywood plate and the plate carefully centered and adjusted to obtain uniform bearing on the ends. The load was then increased to about one-fourth or one-half of the estimated buckling load, and the lateral deflection measured by a dial reading to 0.001 inch. If the dial reading changed appreciably, the load was reduced and the plate recentered and readjusted. The load was then increased and lateral deflections again observed. Corrections were made as necessary until the applied load produced negligible deflection (of the order of 0 to 0.002 inch). Thus bending due to eccentricity of loading was reduced to a minimum amount.

Two metallectric strain gages were attached at the center of the panel, the inner gage being glued to the stiffener and the outer gage glued directly opposite on the convex face of the plate. Strain and lateral deflection readings were taken at regular increments of load.

In all tests the plywood buckling was rapid and was accompanied by a drop in load. Movement of the testing machine head was stopped when the plywood buckled and the buckle pattern observed. If the stiffener had not buckled, loading was resumed until the stiffener buckled or failed. In several instances, the load required to buckle or cause failure of the stiffener was greater than the load required to buckle the plywood. The highest load supported by the specimen was recorded as the maximum load.

The static bending and compression coupons of plywood were tested in accordance with the A.S.T.M. tentative methods. The properties of the stiffeners were obtained from 1- by 4-inch coupons using the same apparatus and techniques applied to the plywood compression coupons.

Computation of Results

The theoretical buckling stress of the cylinder comparable to each curved plate was computed by the formula

\[ p = kE_L h \]

where:

- \( p \) = buckling stress, pounds per square inch.
- \( k \) = a constant, obtained as indicated below.
- \( E_L \) = modulus of elasticity parallel to the grain of the wood in the veneers of the plywood, pounds per square inch.

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2"Tentative Methods of Testing Plywood, Veneer, and Other Wood and Wood-base Materials." A.S.T.M. Designation D805-4ST.

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h = thickness of plywood, inches.

r = mean radius of curvature, inches.

The observed buckling stress of the plywood plate was calculated by dividing the load carried by the plywood at buckling by the loaded area. The load carried by an axial stiffener when the plywood buckled was calculated from the size, modulus of elasticity, and average strain of the stiffener. The total load supported by the stiffened plate is the sum of the load supported by the plywood and that supported by the stiffener.

The following notation applies in addition to that above:

- $E_1$ = modulus of elasticity in bending of the plywood in the axial direction of the curved plate, pounds per square inch.
- $E_2$ = modulus of elasticity in bending of the plywood in the circumferential direction of the curved plate, pounds per square inch.
- $E_b$ = modulus of elasticity in compression of the plywood in the axial direction of the curved plate, pounds per square inch.
- $E_a$ = modulus of elasticity in compression of the plywood in the circumferential direction of the curved plate, pounds per square inch.
- $E_T$ = modulus of elasticity tangential to the annual growth rings of the wood in the veneers of the plywood, pounds per square inch.

$E_L$ can be calculated from the relations

$$E_1 + E_2 = E_a + E_b = E_L + E_T = E_L (1 + C).$$

$$E_L = \frac{E_1 + E_2 + E_a + E_b}{2(1 + C)}$$

The ratio $\frac{E_T}{E_L}$ or $(C)$ is taken as 0.050 for yellow birch and 0.045 for yellow-poplar, based on recent values obtained at the Forest Products Laboratory.

Theoretical values for the buckling constant, $k_\omega$, can be obtained by entering the curve of figure 3 with the proper $\frac{E_1}{E_1 + E_2}$ ratio. The value of $k/k_\omega$ is plotted against the parameter $\frac{l}{\sqrt{hr}} \left(\frac{E_2}{E_1}\right)^{1/4}$ in figure 4. The value of $k$ can then be determined from the above relationships. These curves are a reproduction of those used in a previous report.2
Table 1 presents the data used in determining the buckling stresses of the plywood plates. The plates are arranged in 13 groups of six plates each, with the plates of each group being of similar dimensions and plywood construction. The first plate listed in a group was tested without a stiffener, the second with a light stiffener, and the others with increasingly heavier stiffeners. Examination of the observed buckling stresses of the plates within a group showed no tendency of an increase in critical stress due to the addition of the stiffeners. Variation between the buckling stresses within a group of plates can be attributed largely to (1) initial imperfections in the plates, (2) dimensional and physical differences between the minor specimens and the plates, and (3) inevitable experimental errors.

A curved plate may be considered as a segment of a cylinder, and it would seem likely that a relationship might exist between their critical stresses. In figure 5 the observed buckling stress of each plate is plotted against the theoretical buckling stress, corrected for length, of a comparable cylinder. This figure shows that the observed values for the plates are only slightly greater, on the average, than those computed for comparable cylinders. The average observed stresses for the plates of groups "d" and "m" are about 35 percent greater than those for comparable cylinders.

If the length and/or width of a plate is sufficiently reduced to prevent the formation of a full-size natural buckle, it would seem that the critical stress would be increased. The effect of a decrease in length upon the critical stress for a complete cylinder is illustrated in figure 4. The stiffeners of a stiffened cylinder or of a stiffened curved plate and the restraints at the edges of a curved plate impose a maximum limitation on the circumferential size of the buckle formed and therefore may be expected to increase the critical stress. The width-to-radius ratios of the plates tested for this report varied from 0.25 to 1.0; thus they were segments of comparable cylinders varying from 0.04 to 0.16 of the complete cylinders. In spite of this fact, as has been previously noted, the average observed critical values for the plates were found to be only slightly higher than those computed for the cylinders, indicating little increase due to the width limitation. This failure in obtaining a consistent increase in critical stress over that of a comparable cylinder may be understood from the theory developed in Forest Products Laboratory Report No. 1322-A, upon which the following discussion is based.

The buckling phenomenon in a cylinder subjected to axial compression is greatly influenced by the presence of initial imperfections in the shape or material of the cylinder and upon the circumferential width of the buckle. Such initial imperfections are likewise found in curved plates, which may be considered as sections of cylinders. The width of the buckle might be limited by sufficiently reducing the width of the curved plate. Figure 6, which is a reproduction of figures 10 to 13, inclusive, of Report No. 1322-A, shows

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5March, H. W., "Buckling of Long, Thin Plywood Cylinders in Axial Compression (Mathematical Treatment)," 1943.

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the relation between the compressive stress, \( p \), and \( \xi \), which is proportional to the amplitude of the buckle. A family of curves for varying values of \( \eta \) is drawn for four values of initial buckle amplitude, \( \xi_0 \). The value of \( \eta \) is equal to \( 4\pi \frac{hr}{b^2} \), where \( h \) is the plywood thickness, \( r \) is the radius of curvature, and \( b \) is the width of the buckle. The critical stress associated with each value of \( \eta \) is the minimum value of the stress indicated by the corresponding curve.

It will be noted from the curves that the minimum value of \( pr/E_L h \), regardless of the value of \( \xi_0 \), is 0.095 at a value of \( \eta \) of 0.350. This indicates that the width of the natural buckle of a cylinder of plywood, of the construction for which the curves were drawn, can be obtained from the relationship \( \eta = 0.350 = 4\pi \frac{hr}{b^2} \). If the full width of this natural buckle is not permitted to form, either by the use of heavy axial stiffeners or reduced panel width, the critical stress may be associated with some higher value of \( \eta \) and may, therefore, be increased. For low values of \( \xi_0 \), it may be noted that there is a sharp increase in critical stress with an increase in \( \eta \), but that for large values of \( \xi_0 \), such as 0.5, the critical stress remains about the same for different values of \( \eta \).

Since the addition of axial stiffeners to the panels tested for this report did not greatly increase the buckling stress, it is evident that the values of \( \xi_0 \) were large. This is to be expected, particularly for thin cylindrical forms, because \( \xi_0 \) is the ratio of the amplitude of the original buckle to the thickness. Although the above discussion is based on curves for three-ply (1:2:1) plywood with axial face grain, a similar argument can be made for other constructions.

It would appear, from the above, that the buckling stress could be increased by the addition of closely spaced heavy stiffeners, and tests on such stiffened plywood cylinders have been made. These tests show that an increase in the buckling stress of the plywood is obtained from such stiffening if the stiffeners are very closely spaced.

Previous work on the buckling of unstiffened curved plywood plates in compression indicated a slight increase of the critical stress when the width was decreased to about one-half radian. These plates were of smaller curvature than those tested for this report, and it is therefore probable that the initial deflections, \( \xi_0 \), were smaller.

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Forest Products Laboratory Report No. 1508, "Buckling of Thin, Curved, Plywood Plates in Axial Compression," by E. W. Kuenzi, 1944.

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Conclusions

The following conclusions may be drawn from the results of the tests on thin curved plywood plates simply supported at the edges and loaded in axial compression:

(1) An axial stiffener does not increase the buckling stress of the plywood. The stiffener carries additional load and, if stiff enough, breaks up the buckle pattern.

(2) The buckling stress is approximately equal to or slightly greater than that of a thin-walled plywood cylinder, and may be calculated as such.

(3) If local imperfections could be reduced and the curved plywood plates made more perfectly, increases in critical stress might be attained by the addition of a stiffener that would reduce the width of the buckle. The plates tested were carefully made, however, and are thought to be as perfect as those made commercially; yet no appreciable increases were obtained.

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<th>Shear Modulus of Plasticity</th>
<th>Moment of Plastic Resistance</th>
<th>Yielding Load</th>
<th>Load Carried by Plate</th>
<th>Remaining Loading Capacity</th>
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Note: The values in the table are hypothetical and not intended for actual use.
Figure 1.—Sketch showing the construction and dimensions of an axially stiffened curved plywood panel to be tested in axial compression. Unstiffened panel similar, but without stiffener.
Figure 2.--Axial-compression test of a curved plywood panel having axial stiffener.
Figure 3.—Theoretical curve for long, thin, plywood cylinders in axial compression.
The ordinates represent $k$ in the formula $\bar{\rho} = kE_i h/r$, where $\bar{\rho}$ is the buckling stress. The abscissas represent the ratio $E_i/(E_1 + E_2)$ where $E_1$ and $E_2$ are proportional to the stiffness of the plywood in the axial and circumferential directions, respectively.
Figure 4.—The effect of length of specimen on the buckling constant for thin plywood cylinders in axial compression.
Figure 5.—A comparison of the observed buckling stress of a curved plywood plate with the theoretical buckling stress of a comparable plywood cylinder. Plates are either unstiffened or have an axial stiffener. Cylinder stress calculated on basis of a plywood cylinder having the same length, thickness, radius of curvature, and mechanical properties as the plywood plate. Values are plotted from the data of table 1.
Figure 6.—Variation of applied compressive stress \( \sigma \) to \( \xi \), which is proportional to the deflection at the center of a depression, and \( \eta \), which is determined for a given cylinder by the width of a depression in the circumferential direction. These curves are a reproduction of Figures 10 to 13, inclusive, of Forest Products Laboratory Report No. 1322-A.²