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September 1952

INFORMATION REVIEWED AND REAFFIRMED 1958

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No. 1834

UNITED STATES DEPARTMENT OF AGRICULTURE FOREST SERVICE FOREST PRODUCTS LABORATORY Madison 5, Wisconsin In Cooperation with the University of Wisconsin
BEHAVIOR OF A RECTANGULAR SANDWICH PANEL UNDER A UNIFORM LATERAL LOAD
AND COMpressive EDGE LOADS

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Introduction

In this report, formulas are developed for the deflections, bending moments, and reactions of the supports of a rectangular sandwich panel with simply supported edges that is subjected to a combination of a uniform lateral load and compressive edge loads. The facings and core of the panel are assumed to be composed of isotropic materials. Formulas that include the effect of shear deformation in the core are developed for a general rectangular panel whose sides may have any prescribed ratio. The results of computations are presented for square panels having certain stiffnesses in shear. For rectangular panels that are not square, computations can be made by using the general formulas.

The basic differential equation for the deflection of the panel is derived by using a small-deflection theory and a method suggested by Donnell (2) for taking into account the effects of transverse shear deformation. Because the boundary conditions are those of simple support, this method leads to results, for the problem under consideration, that would be obtained by the methods of Reissner (6, 7, 6) and of Libove and Batdorf (5).

The formulas that are obtained are applicable to panels with facings of either equal or unequal thicknesses. Although the facings and core are assumed to be isotropic, the results obtained should form the basis for useful estimates of the behavior of a sandwich panel with isotropic facings and an orthotropic core, provided that the two transverse moduli of rigidity of the core do not differ greatly.

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1 This progress report is one of a series prepared and distributed by the Forest Products Laboratory under U. S. Navy, Bureau of Aeronautics Order No. NAer 01319 and U. S. Air Force No. USAF-18(600)-70. Results here reported are preliminary and may be revised as additional data become available.

2 Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

3 Underlined numbers in parentheses refer to Literature Cited at end of report.

Rept. No. 1834 -1- Agriculture-Madison
Derivation of the Differential Equation

In the derivation of the differential equation governing the deflection of a rectangular sandwich panel under a uniform lateral load and compressive edge loads, the following assumptions are made: (1) The facings are so thin that their flexural rigidities can be neglected, and, thus, they can be treated as membranes. (2) The components of transverse shear stress in the core are uniform across its thickness, and all other components of stress in the core can be neglected. (3) Deflections of the panel are such that a small-deflection theory can be employed.

The derivation of the differential equation is based on a method suggested by Donnell (2). The methods of Reissner (6, 7, 8) and of Libove and Batdorf (5) are applicable to a wider range of boundary conditions than just simply supported edges, the conditions in the problem under consideration.

As shown in figure 1, the panel is acted upon by a uniform lateral load, q, per unit area and by compressive edge forces, P_X and P_Y, per unit length of edge. According to Donnell's method (2) the deflection w is the result of superposing w_S, the deflection due to shear, on w_B, the deflection due to bending. The bending moments M_X and M_Y and the twisting moment M_XY, whose positive senses are indicated by the vectors in figure 2, are connected with the deflection w by the usual relations:

\[ M_X = -D \left( \frac{\partial^2 w_B}{\partial x^2} + \sigma \frac{\partial^2 w_B}{\partial y^2} \right) \]
\[ M_Y = -D \left( \frac{\partial^2 w_B}{\partial y^2} + \sigma \frac{\partial^2 w_B}{\partial x^2} \right) \]
\[ M_{XY} = -D \left( 1 - \sigma \right) \frac{\partial^2 w_B}{\partial x \partial y} \]

(1)

where \( \sigma \) denotes the Poisson's ratio of the facings and D the flexural rigidity of the panel.

For a panel with facings of thicknesses \( f_1 \) and \( f_2 \) and a core of thickness \( c \),

\[ D = \frac{E_1 f_1 f_2 (c + \frac{f_1 + f_2}{2})^2}{\lambda (f_1 + f_2)} \]

(2)

\(^4\text{The positive sense of } M_{XY} \text{ is taken opposite to that chosen by Libove and Batdorf (5).} \)
where $E_f$ denotes the Young's modulus of the facings and

$$\lambda = 1 - \sigma^2$$

(3)

In deriving expression (2) for $D$, it is assumed that the bending stresses in the core can be neglected. For a panel with facings of equal thickness $f$, equation (2) becomes:

$$D = \frac{E_f f (c + f)^2}{2\lambda}$$

(4)

Let $Q_x$ and $Q_y$ represent the transverse shearing forces per unit length of edge of the edges $dy$ and $dx$, respectively, of an element of the panel (figure 2). The conditions of equilibrium of the moments acting on the element require that

$$q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = -D \frac{\partial}{\partial x} \left( \nabla^2 w_b \right)$$

(5)

$$q_y = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = -D \frac{\partial}{\partial y} \left( \nabla^2 w_b \right)$$

Under the assumptions that have been made, the shearing strains $e_{zx}$ and $e_{yz}$ are uniform across the thickness of the core. These strains correspond to the additional slopes due to shear, $\frac{\partial w_s}{\partial x}$ and $\frac{\partial w_s}{\partial y}$, of the deflection surface. They will be proportional to the shear forces $Q_x$ and $Q_y$. Consequently,

$$\frac{\partial w_s}{\partial x} = \frac{Q_x}{K}$$

(6)

$$\frac{\partial w_s}{\partial y} = \frac{Q_y}{K}$$

where $K$ is a constant that will be called the stiffness of the sandwich panel in shear.

For comparison with Libove and Batdorf (5), it is noted that $\frac{\partial w_s}{\partial x}$ and $\frac{\partial w_s}{\partial y}$ correspond to their $\gamma_x$ and $\gamma_y$, respectively, and $K$ to their $D_{xx}$ and $D_{yy}$, which are
equal for the panel considered in this report because its facings and core are isotropic. The quantities \( \frac{\partial w_s}{\partial x} \) and \( \frac{\partial w_s}{\partial y} \), or \( \gamma_x \) and \( \gamma_y \), describe the inclination to the normal of the deflected surface of lines that were initially normal to the undeformed surface.

In an analysis made at the Forest Products Laboratory (2) by the "tilting" method of Williams, Leggett, and Hoppins (4, 5), it was found that

\[
K = \left( \frac{c + \frac{f_1 + f_2}{2}}{c} \right)^2 \mu_c
\]

where \( \mu_c \) is the modulus of rigidity of the core. More precisely, it was found that, for the compressive buckling of a sandwich panel with simply supported edges and with the facings treated as membranes, the tilting method and the method of Libove and Batdorf (5) lead to the same result if their \( DQ \) is defined by equation (7). For panels with facings of equal thickness, equation (7) reduces to a form obtained by Bijlaard (1).

From equations (5) and (6),

\[
\frac{\partial w_s}{\partial x} = -D \frac{\partial}{\partial x} \left( \nabla^2 w_b \right) \\
\frac{\partial w_s}{\partial y} = -D \frac{\partial}{\partial y} \left( \nabla^2 w_b \right)
\]

Because of the assumed conditions of simple support, \( w_s = 0 \) on the edges and \( \nabla^2 w_b \) also vanishes on the edges. It follows that

\[
w_s = -D K \nabla^2 w_b
\]

The equation for equilibrium of vertical forces is

\[
\frac{\partial^2 w_x}{\partial x^2} + 2 \frac{\partial^2 w_x}{\partial x \partial y} + \frac{\partial^2 w_y}{\partial y^2} = -\left( q - P_x \frac{\partial^2 w}{\partial x^2} - P_y \frac{\partial^2 w}{\partial y^2} \right)
\]

where

\[
w = w_b + w_s
\]
By using equations (1), (9), and (11), equation (10) becomes:

$$-DV^2 w_b + P_x \frac{D}{Dx} \frac{\partial^2}{\partial x^2} (\nabla^2 w_b) + P_y \frac{D}{Dy} \frac{\partial^2}{\partial y^2} (\nabla^2 w_b) - P_x \frac{\partial^2 w_b}{\partial x^2} - P_y \frac{\partial^2 w_b}{\partial y^2} = -q$$

(12)

Let

$$\frac{P_x}{D} = p_x, \frac{P_y}{D} = p_y, \frac{D}{R} = R, \frac{q}{D} = q$$

(13)

With this notation and simple transformations, equation (12) becomes:

$$c_1 \frac{\partial^4 w_b}{\partial x^4} + 2c_3 \frac{\partial^4 w_b}{\partial x^2 \partial y^2} + c_2 \frac{\partial^4 w_b}{\partial y^4} + p_x \frac{\partial^2 w_b}{\partial x^2} + p_y \frac{\partial^2 w_b}{\partial y^2} = g$$

(14)

where

$$c_1 = 1 - p_x R, c_2 = 1 - p_y R, 2c_3 = 2 - p_x R - p_y R$$

(15)

Solution of the Differential Equation

The solution of equation (14) is subject to the conditions:

$$w_b = 0, \ x = 0, \ x = a, \ y = 0, \ y = b$$

(16)

$$\frac{\partial^2 w_b}{\partial x^2} = 0, \ x = 0, \ x = a$$

(17)

$$\frac{\partial^2 w_b}{\partial y^2} = 0, \ y = 0, \ y = b$$

(18)

A particular integral, $w_{bl}$, of equation (14) is found by solving the equation:

$$c_1 \frac{\partial^4 w_{bl}}{\partial x^4} + p_x \frac{\partial^2 w_{bl}}{\partial x^2} = g$$

(19)
\[ A_m = \frac{4g}{c_1 a_m^3 (a_m^2 - k^2)} \]

when \( m \) is odd, and

\[ A_m = 0 \] (28)

when \( m \) is even.

It is now necessary to find a solution, \( w_{b2} \), of the homogeneous equation obtained from (14) by setting \( g = 0 \), such that

\[ w_b = w_{b1} + w_{b2} \] (29)

satisfies all of the conditions of equations (16), (17), and (18).

The following series of equations is chosen as a solution of the homogeneous equation:

\[ w_{b2} = \sum_{m=1}^{\infty} Y_m(y) \sin a_m x \] (30)

The functions \( Y_m \) are subject to the conditions (see equations (16) and (18))

\[ Y_m(0) = Y_m(b) = -A_m \] (31)

\[ Y_m^{II}(0) = Y_m^{II}(b) = 0 \] (32)

Substitution of equation (30) in the homogeneous equation corresponding to equation (14) leads to the following differential equation for the function \( Y_m(y) \):

\[ c_2 Y_m^{IV} - \left(2c_3 a_m^2 - p_y\right) Y_m^{II} + \left(c_1 a_m^4 - p_x a_m^2\right) Y_m = 0 \] (33)

Let

\[ Y_m = e^{\lambda_m y} \] (34)

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Equation (47), for \( w_b \) at the center of the panel, can now be written:

\[
\frac{\nu}{\pi S} (a, b) = \frac{LQ}{\pi S} \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{r_m^2} (1 + S_m)
\]

Equation (57)

It may happen that \( r_m = 0 \) for a given \( S \) and a certain pair of values of \( \varepsilon \) and \( m \). In that case, it will be found that \( 1 + S_m = 0 \) also, and the indeterminate term of equation (58) can be evaluated. Computations for the construction of curves for \( w \) were found to be simpler if values of \( \varepsilon \) were chosen to avoid those for which \( r_m = 0 \).

The quantities \( \phi_m \) and \( \psi_m \), which are needed to calculate \( \mu_m \) and \( \nu_m \) in equations (46) and (48), can be expressed in terms of \( S, \varepsilon, \) and \( \rho \) (equations (52) to (54)).

In accordance with equations (15) and (36),

\[
\phi_m = \frac{2 - p_x R - p_y R - \frac{p_y}{\mu_m^2}}{2 (1 - p_y R)}
\]

(59)

By simple transformations (see equations (13) and (51) to (54))

\[
\phi_m = \frac{2 - \varepsilon S - \rho S - \frac{p_y}{m^2}}{2 (1 - \rho S)}
\]

(60)

In like manner, from equation (37),

\[
\psi_m = \sqrt{\frac{1 - \varepsilon S - \frac{p_y}{m^2}}{1 - \rho S}}
\]

(61)
In using formula (48) for \( S_m \), if \( \mu_m \) and \( \nu_m \) are neither real numbers nor pure imaginaries, it is advantageous to express \( S_m \) in terms of \( \gamma_m \) and \( \delta_m \) (equations (40)). Substitution of expressions (42) in equation (48) leads to the formula:

\[
S_m = -\left( \frac{\gamma_m^2 + \delta_m^2}{\gamma_m \delta_m} \right) \sinh \gamma_m \theta_m \sinh \delta_m \theta_m + 2 \gamma_m \delta_m \cosh \gamma_m \theta_m \cosh \delta_m \theta_m \]
\[
\frac{1}{\gamma_m \delta_m} \left( \cosh 2\gamma_m \theta_m + \cosh 2\delta_m \theta_m \right)
\]

(62)

For the calculation of the deflection due to shear, \( w_s \) (equation (9)), and the bending moments, \( M_x \) and \( M_y \) (equation (1)), at the center of the panel, expressions are needed for the values of \( \frac{\partial^2 w_b}{\partial x^2} \) and \( \frac{\partial^2 w_b}{\partial y^2} \) at this point. It is readily found from equation (46) that

\[
\frac{\partial^2 w_b(x, y)}{\partial x^2} = -\sum_{m=1}^{\infty} \left( \frac{m-1}{2} \right)^2 \frac{\alpha_m^2 \lambda_m}{m^3 r_m} \left( 1 + S_m \right)
\]

(63)

\[
= -\frac{\hbar Q}{n^3 \alpha^2} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m^3 r_m} \left( 1 + S_m \right)
\]

where \( S_m \) is defined by equations (48) and (62).

\[
\frac{\partial^2 w_b(x, y)}{\partial y^2} = \frac{\hbar Q}{n^3 \alpha^2} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m^3 r_m} T_m
\]

(64)

where

\[
T_m = \frac{\mu_m^2 \nu_m^2}{\left( \gamma_m^2 - \delta_m^2 \right)^2} \left[ \frac{1}{\cosh \mu_m \theta_m} - \frac{1}{\cosh \nu_m \theta_m} \right]
\]

(65)

\[
= \frac{\gamma_m^2 \delta_m^2 \sinh \gamma_m \theta_m \cosh \delta_m \theta_m \cosh 2\gamma_m \theta_m + \cosh 2\delta_m \theta_m \gamma_m \delta_m \left( \cosh 2\gamma_m \theta_m + \cosh 2\delta_m \theta_m \right)}
\]
For the substitution of equations (63) and (64) in equation (9), note that

\[
\frac{D}{K} \cdot \frac{luQ}{a^2} = \frac{1}{\pi^2} \frac{luQ}{\pi^2} = \frac{Pe}{K} \cdot \frac{luQ}{\pi^5} = \frac{luQS}{\pi^5} \quad (66)
\]

Then

\[
w_s \left( \frac{a}{r}, \frac{b}{r} \right) = \frac{luQS}{\pi^5} \sum_{m=1}^{\infty} \frac{(m-1)}{m^3 \rho_m} \left( 1 + S_m - T_m \right) \quad (67)
\]

In accordance with equation (11), the deflection at the center of the panel, \( w \left( \frac{a}{2}, \frac{b}{2} \right) \), can now be found by adding equations (58) and (67).

Bending Moments

The bending moments \( M_x \) and \( M_y \) at the center of the panel are obtained by substituting equations (63) and (64) in equations (1). Note that

\[
\frac{DluQ}{\pi^3 a^3} = \frac{Dluqa^4}{\pi^3 a^2 D} = \frac{luqa^2}{\pi^3} \quad (68)
\]

Then

\[
M_x \left( \frac{a}{2}, \frac{b}{2} \right) = \frac{luqa^2}{\pi^3} \sum_{m=1}^{\infty} \frac{(m-1)}{m^3 \rho_m} \left( 1 + S_m - T_m \right) \quad (68)
\]

and

\[
M_y \left( \frac{a}{2}, \frac{b}{2} \right) = -\frac{luqa^2}{\pi^3} \sum_{m=1}^{\infty} \frac{(m-1)}{m^3 \rho_m} \left[ T_m - \sigma \left( 1 + S_m \right) \right] \quad (69)
\]

Reactions of the Supports

The reactions of the supports \( R_x \) and \( R_y \) are found from the equations

\[
R_x = Q_x + \frac{\sigma M_{xy}}{\rho_y} \quad (70)
\]
\[ R_y = Q_y + \frac{\partial Q_y}{\partial x} \]  

From equations (1) and (5),

\[
R_x = -D \left[ \frac{\partial^3 w_b}{\partial x^3} + (2 - \sigma) \frac{\partial^3 w_b}{\partial x \partial y^2} \right] \quad (72)
\]

\[
R_y = -D \left[ \frac{\partial^3 w_b}{\partial y^3} + (2 - \sigma) \frac{\partial^3 w_b}{\partial x^2 \partial y} \right] \quad (73)
\]

To determine \( R_x(a, \frac{b}{2}) \) and \( R_y(\frac{a}{2}, b) \), the values of \( R_x \) and \( R_y \) at the midpoints of the respective edges, the following quantities are needed for substitution in equations (72) and (73). They are readily found from equation (46).

\[
D \frac{\partial^3 w_b}{\partial x^3} \left( a, \frac{b}{2} \right) = \frac{4qa}{\pi^2} \sum_{m = 1}^{\infty} \frac{1}{m^2 r_m} \left( 1 + S_m \right) \quad (74)
\]

\[ m = 1, 3, 5, \ldots \]

\[
D \frac{\partial^3 w_b}{\partial x \partial y^2} \left( a, \frac{b}{2} \right) = -\frac{4qa}{\pi^2} \sum_{m = 1}^{\infty} \frac{1}{m^2 r_m} T_m \quad (75)
\]

\[ m = 1, 3, 5, \ldots \]

\[
D \frac{\partial^3 w_b}{\partial x^2 \partial y} \left( \frac{a}{2}, b \right) = -\frac{4qa}{\pi^2} \sum_{m = 1}^{\infty} \frac{1}{m^2 r_m} \frac{(m - 1)}{2} U_m \quad (76)
\]

\[ m = 1, 3, 5, \ldots \]

\[
D \frac{\partial^3 w_b}{\partial y^3} \left( \frac{a}{2}, b \right) = \frac{4qa}{\pi^2} \sum_{m = 1}^{\infty} \frac{1}{m^2 r_m} \frac{(m - 1)}{2} V_m \quad (77)
\]

\[ m = 1, 3, 5, \ldots \]

In these equations, \( S_m \) is defined by equations (48) or (62) and \( T_m \) by equation (65), and
\[ U_m = \frac{\mu_{m} \nu_{m}}{\left( \frac{\mu_{m}}{\nu_{m}} - \frac{\nu_{m}}{\mu_{m}} \right)} \left( \nu_{m} \tanh \mu_{m} \delta_{m} - \mu_{m} \tanh \nu_{m} \delta_{m} \right) \] (78)

\[ V_m = \frac{\mu_{m} \nu_{m}}{\left( \frac{\mu_{m}}{\nu_{m}} - \frac{\nu_{m}}{\mu_{m}} \right)} \left( \mu_{m} \tanh \mu_{m} \delta_{m} - \nu_{m} \tanh \nu_{m} \delta_{m} \right) \] (79)

The substitution of equations (74) to (77) in equations (72) and (73) yields:

\[ P_x \left( a, b \right) = -\frac{Lq_{a}}{\eta_{2}} \sum_{m = 1}^{\infty} \frac{1}{m^2 r_{m}} \left[ 1 + s_{m} - (2 - \sigma) T_{m} \right] \] (80)

and

\[ P_y \left( a, b \right) = -\frac{Lq_{a}}{\eta_{2}} \sum_{m = 1}^{\infty} \frac{(-1)^{m - 1}}{m^2 r_{m}} \left[ v_{m} - (2 - \sigma) U_{m} \right] \] (81)

Solution of the Differential Equation by Means of

a Double Fourier's Series

Although it is not so well adapted to numerical calculation as the solution in a single series, the solution of equation (14) in terms of a double Fourier's series is useful in a number of ways. In particular, it furnishes a ready means of determining the critical combination of loads \( P_x \) and \( P_y \) at which the panel becomes elastically unstable.
The right-hand member of equation (14) is represented by the series:

\[ g = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{a m x}{a} \sin \frac{b n y}{b} \]  

(82)

where

\[ \alpha_m = \frac{m\pi}{a}, \quad \beta_n = \frac{n\pi}{b} \]  

(83)

\[ A_{mn} = \frac{16g}{mn \pi^2} \]  

(84)

when m and n are both odd, and

\[ A_{mn} = 0 \]

when either m or n is even.

The following series, subject to the conditions of equations (16), (17), and (18), is chosen as a solution of equation (14):

\[ w_b = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{a m x}{a} \sin \frac{b n y}{b} \]  

(85)

Substitution of equations (85) and (82) in equation (14) and introduction of the values of \( c_1 \), \( c_2 \), and \( c_3 \) from equations (15) yields:

\[ B_{mn} = \frac{A_{mn}}{\left( 1 - p_x R \right) \alpha_m^4 + \left( 2 - p_x R - p_y R \right) \alpha_m^2 \beta_n^2 \} + \left( 1 - p_y R \right) \beta_n^4 - p_x \alpha_m^2 - p_y \beta_n^2 \]  

(86)

In accordance with equation (84), \( B_{mn} \) is different from zero only when m and n are both odd.

In the notation of equations (52), (53), and (54), the denominator of the fraction in equation (86) can be written in the following form, after introduction of the lengths of the edges of the panel, a and b, by using equations (83):
\[
\frac{n}{a} \left[ (1 - \varepsilon S) m + (2 - \varepsilon S - \rho S) \frac{a^2}{b^2} m^2 n^2 + (1 - \rho S) \frac{a^4}{b^4} n^4 - \varepsilon m^2 - \rho \frac{a^2}{b^2} n^2 \right]
\]

Then

\[
D_{mn} = \frac{A_{mn}}{\frac{n}{a} \left[ \left( \frac{m^2 + \frac{n^2 a^2}{b^2}}{b^2} \right)^2 - \varepsilon m^2 \left( \frac{m^2 + \frac{n^2 a^2}{b^2}}{b^2} \right) - \rho \frac{n^2 a^2}{b^2} \left( m^2 + \frac{n^2 a^2}{b^2} \right) \right]}
\]

The denominator of the fraction in equation (87) will vanish if \( \varepsilon \) and \( \rho \) satisfy the equation obtained by equating to zero the quantity in brackets; that is, if \( \varepsilon \) and \( \rho \) satisfy the equation:

\[
\frac{m^2}{b^2} \left[ 1 + S \left( \frac{m^2 + \frac{n^2 a^2}{b^2}}{b^2} \right) \right] \varepsilon + \frac{n^2 a^2}{b^2} \left[ 1 + S \left( \frac{m^2 + \frac{n^2 a^2}{b^2}}{b^2} \right) \right] \rho = 1
\]

(88)

where \( m \) and \( n \) are both odd numbers. This is the equation of a straight line in the variables \( \varepsilon \) and \( \rho \). It represents a critical combination of \( \varepsilon = \frac{P_x}{P_e} \) and \( \rho = \frac{P_y}{P_e} \) for which the panel buckles into \( m \) half waves in the \( x \) direction and \( n \) half waves in the \( y \) direction. Equation (88) can be used to determine a broken line in the \( \varepsilon \rho \) plane, such that buckling will not occur for values of \( \varepsilon \) and \( \rho \) that are the coordinates of points below this broken line.

Equation (88), for the critical combinations of compressive edge loads, can also be found from equation (46) by requiring that \( \frac{\mu_m a m}{2} \) or \( \frac{\mu_m a m}{2} \) shall be equal to \( i \frac{nr}{2} \), where \( n \) is an odd integer and \( i = \sqrt{-1} \).
From equation (85),
\[
\frac{\partial^2 w_b}{\partial x^2}\left(\frac{a}{2}, \frac{b}{2}\right) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)\left(\frac{m+n}{2} - 1\right) a_m^2 B_{mn}, \text{ m and n odd} \tag{89}
\]
\[
\frac{\partial^2 w_b}{\partial y^2}\left(\frac{a}{2}, \frac{b}{2}\right) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)\left(\frac{m+n}{2} - 1\right) \beta_n^2 B_{mn}, \text{ m and n odd} \tag{90}
\]

It follows from equation (9) that
\[
w_s\left(\frac{a}{2}, \frac{b}{2}\right) = \frac{D}{K} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)\left(\frac{m+n}{2} - 1\right) \left(\alpha_m^2 + \beta_n^2\right) B_{mn}
\]
\[
= \pi^2 D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)\left(\frac{m+n}{2} - 1\right) \left(\frac{m^2}{a^2} + \frac{n^2 a^2}{b^2}\right) B_{mn}
\]
\[
= m \text{ and n odd} \tag{91}
\]

Then, in the notation of equations (51) and (52),
\[
w_s\left(\frac{a}{2}, \frac{b}{2}\right) = S \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)\left(\frac{m+n}{2} - 1\right) \left(\frac{m^2}{a^2} + \frac{n^2 a^2}{b^2}\right) B_{mn}
\]
\[
= m \text{ and n odd} \tag{92}
\]

In accordance with equation (85),
\[
w_b\left(\frac{a}{2}, \frac{b}{2}\right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)\left(\frac{m+n}{2} - 1\right) B_{mn}, \text{ m and n odd} \tag{93}
\]

The central deflection, \(w\left(\frac{a}{2}, \frac{b}{2}\right)\), is the sum of \(w_b\left(\frac{a}{2}, \frac{b}{2}\right)\) and \(w_s\left(\frac{a}{2}, \frac{b}{2}\right)\).

By using equations (1), (89), and (90), the bending moments at the center are found to be:
\[ M_x \left( \frac{a}{2}, \frac{b}{2} \right) = \frac{\pi^2 D}{a^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{m+n}{2} - 1} \left( \frac{m^2}{b^2} + \frac{\sigma n^2 a^2}{b^2} \right) B_{mn} \quad (94) \]

\[ M_y \left( \frac{a}{2}, \frac{b}{2} \right) = \frac{\pi^2 D}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{m+n}{2} - 1} \left( \frac{n^2 a^2}{b^2} + \sigma m^2 \right) B_{mn} \quad (95) \]

In equation (86), \( B_{mn} \) contains the factor \( g = \frac{a}{D} \) in the numerator (see equation (84)), and the factor \( \frac{n}{a} \) in the denominator. Hence, \( M_x \left( \frac{a}{2}, \frac{b}{2} \right) \) and \( M_y \left( \frac{a}{2}, \frac{b}{2} \right) \) are proportional to \( qa^2 \).

The expressions in double series for the reactions of the supports at the midpoints of the edges are found from equations (72) and (73) and the quantities:

\[ \frac{\partial^3 w_b}{\partial x^3} \left( \frac{a}{2}, \frac{b}{2} \right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{n-1}{2}} a_m^3 B_{mn}, \quad m \text{ and } n \text{ odd} \]

\[ \frac{\partial^3 w_b}{\partial x \partial y^2} \left( \frac{a}{2}, \frac{b}{2} \right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{n-1}{2}} a_m^2 a_n^2 B_{mn}, \quad m \text{ and } n \text{ odd} \]

\[ \frac{\partial^3 w_b}{\partial x^2 \partial y} \left( \frac{a}{2}, \frac{b}{2} \right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{m-1}{2}} a_m^2 a_n^2 B_{mn}, \quad m \text{ and } n \text{ odd} \]

\[ \frac{\partial^3 w_b}{\partial y^3} \left( \frac{a}{2}, \frac{b}{2} \right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{m-1}{2}} a_n^3 B_{mn}, \quad m \text{ and } n \text{ odd} \]
Then
\[
R_x(a, b) = -\frac{\pi^3 D}{a^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{m-1}{2}} B_{mn} m \left[ m^2 + (2 - \sigma) \frac{a^2 n^2 b^2}{b^2} \right]
\]
\[m \text{ and } n \text{ odd}\]

\[
R_y(a, b) = -\frac{\pi^3 D}{a^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{\frac{m-1}{2}} B_{mn} n \left[ \frac{a^2 n^2 b^2}{b^2} + (2 - \sigma) m^2 \right]
\]
\[m \text{ and } n \text{ odd}\]

From the discussion of \(B_{mn}\), following equations (94) and (95), it can be shown that \(R_x\) and \(R_y\) are proportional to \(qa\).

The Square Panel

Formulas (58), (67), (68), (69), and (72) were used to compute the deflection and bending moments at the center of a square panel and the reactions of the supports at the midpoint of an edge of the panel. The computations were made for various combinations of compressive edge loads, \(P_x\) and \(P_y\), and for three values, 0, 0.1, and 0.2, of the parameter \(S\). This parameter has the limiting value 0 as the modulus of rigidity of the core becomes infinite. The values 0.1 and 0.2 are associated with cores of relatively low moduli of rigidity, as can be seen from the definition of \(S\) (equation (52)). It is believed that \(S\) will lie between 0 and 0.2 for most panels that are likely to be of practical interest.

The results of the computations are presented in tables 1, 2, and 3. The quantities \(\varepsilon\) and \(\rho\) in the tables are proportional to the compressive edge loads \(P_x\) and \(P_y\), respectively, in accordance with equations (53) and (54).

The method of choosing the values of \(\varepsilon\) and \(\rho\), that is, of \(\frac{P_x}{P_e}\) and \(\frac{P_y}{P_e}\), for which computations were made can be explained in connection with figure 3. The straight line below which a point with coordinates \(\varepsilon\) and \(\rho\) must lie if the panel does not buckle is determined from equation (68). This equation is satisfied by a critical combination of edge loads. For a square panel, equation (68) becomes:
where \( m \) and \( n \) are both odd numbers.

The intercepts of this line on the coordinate axes are:

\[
\varepsilon_0 = \frac{(m^2 + n^2)^2}{m^2 \left[ 1 + S \left( m^2 + n^2 \right) \right]}, \quad \rho_0 = \frac{(m^2 + n^2)^2}{n^2 \left[ 1 + S \left( m^2 + n^2 \right) \right]} \tag{99}
\]

If \( m = n = 1 \),

\[
\varepsilon_0 = \frac{1}{1 + 2S}, \quad \rho_0 = \frac{1}{1 + 2S} \tag{100}
\]

The line \( AB \) with these intercepts is shown in figure 3. No line determined by equation (98) with other integral values of \( m \) and \( n \) will intersect line \( AB \) in the first quadrant of the \( \varepsilon-\rho \) plane, provided that the value of the parameter \( S \) is not greater than 0.3. This implies that, for such values of \( S \), the panel will buckle in a single half wave in each direction. It also implies that the term in which \( m = 1 \) and \( n = 1 \) in the Fourier's double series (equation (85)) is the term that will be most greatly amplified by the application of compressive edge loads.

The combinations of edge loads for which computations were made are shown approximately by the crosses in figure 3. More points were chosen near the line \( AB \) because the quantities to be computed change most rapidly in this region. Because the computations are lengthy, points in the upper triangle \( ACD \) of figure 3 were not chosen.

The maximum deflection, bending moments, and reactions of the supports can be obtained for any combination of edge loads, \( \varepsilon = \frac{P_X}{P_e} \) and \( \rho = \frac{P_Y}{P_e} \), from the computations that were made. The maximum deflection \( w \left( \frac{a}{2}, \frac{a}{2} \right) \) and the maximum bending moments \( M_X \left( \frac{a}{2}, \frac{a}{2} \right) \) and \( M_Y \left( \frac{a}{2}, \frac{a}{2} \right) \) will be denoted by \( w, M_X, \) and \( M_Y \), respectively.

For a combination of edge loads represented by the point \( P_2 \), figure 3, with coordinates \( \varepsilon_2 \) and \( \rho_2 \), the central deflection \( w \) will be, from considerations...
of symmetry, the same as that for the combination represented by the point \( F_1 \), with coordinates \( \frac{-1}{n} \) and \( \frac{-1}{p} \), provided that \( \frac{-1}{2} = \frac{-1}{n} \) and \( \frac{-1}{2} = \frac{-1}{p} \). Similarly, \( M_x \) and \( M_y \) for the combination \( F_2 \) are equal to \( M_{x'} \) and \( M_{y'} \), respectively, for the combination \( F_1 \).

The values of \( w, M_{x'} \), and \( M_{y'} \) for any combination of loads represented by a point \( F_1 \), lying below the line \( DC \) (figure 3), can be found by interpolation from tables 1, 2, and 3. The tables show that, for a given \( S \) and any given combination of \( \varepsilon \) and \( \rho \), the values of \( M_x \) and \( M_y \) are nearly equal. This is to be expected from the preponderance of the first term of the Fourier's series (equation (85)), when \( a \) is equal to \( b \). The letter \( M \) will frequently be used hereafter to denote the mean of the nearly equal values of \( M_x \) and \( M_y \).

The reactions of the supports have their greatest values at the midpoints of the edges. For the square panel, the reaction will be somewhat greater at the midpoints of the pair of edges on which the greater compressive load acts than at the midpoints of the pair of edges with the smaller compressive load. This statement can be verified by considering the expressions for the absolute values of \( R_x \) and \( R_y \) at the points \( \left( a, \frac{a}{2} \right) \) and \( \left( \frac{a}{2}, a \right) \), respectively. These values will be denoted by \( |R_x| \) and \( |R_y| \). It follows from equations (96), (97), and (87) that, with \( \sigma = 0.3\),

\[
|R_x| = \frac{Da}{\pi} \sum_{m = 1}^{\infty} \sum_{n = 1}^{\infty} \frac{(n - 1)}{(m^2 + n^2)^2} \frac{1}{(m^2 + n^2)^2} \left[ \frac{1}{1 + S (m^2 + n^2)} \right] \tag{101}
\]

\[
|R_y| = \frac{Da}{\pi} \sum_{m = 1}^{\infty} \sum_{n = 1}^{\infty} \frac{(m - 1)}{(m^2 + n^2)^2} \frac{1}{(m^2 + n^2)^2} \left[ \frac{1}{1 + S (m^2 + n^2)} \right] \tag{102}
\]

where \( A_{mn} = A_{nm} \) as implied by equation (84). Both \( m \) and \( n \) are odd numbers.

It is readily shown that, for the range of values considered in this report for the parameter \( S \), \( |R_x| > |R_y| \) for a combination of loads represented by any point on the line \( \rho = 0 \), except the point \((0, 0)\), where \( |R_x| = |R_y| \). For a combination of loads represented by a point on the line \( \rho = \varepsilon \) (line \( OC \) in figure 3), \( |R_x| = |R_y| \). For combinations represented by points between the lines \( \rho = 0 \) and \( \rho = \varepsilon \), it is concluded that \( |R_x| > |R_y| \). There is no apparent reason why the difference \( |R_x| - |R_y| \) should vanish and change sign in this region, where \( \varepsilon > \rho \), that is, \( F_x > F_y \). For points in this region, it
is necessary to calculate $R_x$ only, since it is numerically greater than $Rx$. For a combination of loads for which $\varepsilon < \rho$, that is, $P_x < P_y$, it is only necessary to interchange the $x$ and $y$ axes and calculate $Rx$ for the new axes.

Curves for Estimating $w_b$, $M$, $R_x$, $Q_x$

It is known from flat-plate theory that the central deflection of a uniformly loaded square plate associated with bending, $w_b(a, b)$, with $\varepsilon = \rho = 0$ in the present notation, can be obtained quite accurately from the first term of the double Fourier series representing this deflection. The first term of the series is also dominant when either $\varepsilon$ or $\rho$ is near its critical value. These facts suggest that the parameters that enter the first term of the series can be used for predicting the deflection $w_b$ for any combination of edge and uniform loads. From equation (87), this term is written

$$B_{11} = \frac{A_{11}}{\ln \frac{4}{a} \left[ \frac{1}{4} - \frac{(\varepsilon + \rho)(1 + 2S)}{4} \right]}$$

which indicates the possibility of expressing $w_b$ as a function of $(\varepsilon + \rho)(1 + 2S)$. In figure 4, $\frac{Q}{w_b}$ is plotted as a function of this parameter from averages of computed values. It is found that, for any value of $(\varepsilon + \rho)(1 + 2S)$ used in the computations, no individual value of $\frac{Q}{w_b}$ differs from the corresponding average by as much as 1 percent.

With the value of $w_b$ determined from figure 4, the total deflection can be obtained approximately from the formula

$$w = (1 + 2S)w_b$$

which is suggested by the first terms of equations (92) and (93). For $S = 0.1$ and $S = 0.2$, the agreement between the results given in tables 2 and 3 and those obtained from equation (104) is satisfactory. Estimates of $w$ can also be obtained from formula (11):
\[ w = w_b + w_s \]

with \( w_b \) determined from figure 4 and

\[
w_s = \frac{Qs (M_x + M_y)}{2QSM} = \frac{2QSM}{\pi^2 (1 + \sigma) qa^2} \quad (105)\]

where \( M \) is determined from figure 5 and \( Q \) is defined by equation (50). Formula (105) is obtained from formulas (1) and (7).

For practical purposes, formula (104), which is approximate, is sufficiently accurate in the range \( 0 \leq S \leq 0.2 \). On the other hand, formula (11), with \( w_s \) obtained from formula (105), is exact on the basis of the present analysis.

The first term of the double Fourier series for \( M_x \) and \( M_y \) indicates that these two moments are very nearly equal. When these moments are considered as functions of the parameter \( (\epsilon + \rho) (1 + 2S) \) and the average, \( \frac{M}{qa^2} \), is determined for various values of this parameter, it is found that no individual value of \( \frac{M_x}{qa^2} \) or \( \frac{M_y}{qa^2} \) differs from the corresponding \( \frac{M}{qa^2} \) by more than 1-1/2 percent. The average, \( \frac{M}{qa^2} \), can therefore be used with sufficient accuracy for practical purposes in place of individual values of \( \frac{M_x}{qa^2} \) and \( \frac{M_y}{qa^2} \). A plot of \( \frac{qa^2}{M} \) as a function of \( \frac{(\epsilon + \rho) (1 + 2S)}{4} \) is given in figure 5.

As previously discussed, \( |R_x| \) is greater than or less than \( |R_y| \), according as \( \epsilon \) is greater than or less than \( \rho \). The magnitude of these reactions also depends upon \( S \). Consequently, in figure 6, the quantity \( \frac{qa^2}{|R_x|} \) is plotted as a function of \( \frac{(\epsilon + \rho) (1 + 2S)}{4} \) for various values of \( S \). The computed values used in plotting these curves were those for which \( \rho = 0 \). For a given \( (\epsilon + \rho) \) the curves will therefore yield the largest possible estimates of \( \frac{|R_x|}{qa^2} \).

The magnitude of the reaction also increases with \( S \). Conservative results for all values of \( S \) in the range \( 0 \leq S \leq 0.2 \) can therefore be obtained by using the curve for \( S = 0.2 \).
The shear stress resultant at the center of an edge, like $|F_x|$, depends upon the magnitude of the edge load and upon $S$. Plots of $\frac{\alpha a}{Q_x}$ were therefore made for the three values of $S$ used in the computations (figure 7). Again, the plots were constructed from values computed with $\rho = 0$, and, for each value of $S$, the curve yields the largest estimate of $\frac{Q_x}{\alpha a}$ for a given $(\epsilon + \rho)$.

Conclusion

The formulas derived in this report permit calculation of the deflection, bending moments, and reactions of the supports of rectangular panels of sandwich construction under the combined action of lateral and compressive edge loads. Calculations were made for a square panel, and the results are presented in tables and curves. For rectangular panels that are not square, calculations can be made by using the general formulas. Without such calculations, useful estimates of the influence of shear deformation in the core can probably be made for a given panel from the results for square panels.
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Notation

- **a**: length of an edge of the panel
- **A_m**: defined by equation (28)
- **A_mn**: defined by equation (84)
- **b**: length of an edge of the panel
- **B_mn**: defined by equation (86)
- **c**: thickness of the core
- **c_1, c_2, c_3**: defined by equations (15)
- **D**: flexural rigidity of the panel, defined by equation (2)
- **e_1**: defined by equation (24)
- **E_f**: Young's modulus of the facings
- **f_1, f_2**: thicknesses of the facings
- **g**: defined by equation (23)
- **K**: stiffness of the panel in shear, defined by equation (7)
- **m, n**: summation indexes — number of half waves into which the panel buckles in the \( x \) and \( y \) directions, respectively
- **M_x, M_y**: bending moments
- **M_xy**: twisting moment
- **P_x, P_y**: compressive edge loads
- **P_e**: defined by equation (27)
Table 1.—Computed values, for a square panel, of \( \frac{w}{Q}, \frac{M_x}{qa^2}, \frac{M_y}{qa^2}, \) and \( \frac{|R_x|}{qa} \) for \( S = 0 \)

\[
\begin{array}{cccccccc}
\rho & e & \frac{(e + \rho)(1 + 2S)}{4} & w & \frac{M_x}{Q} & \frac{M_y}{qa^2} & \frac{Q_x}{qa} & \frac{|R_x|}{qa} \\
0 & 0 & 0 & 0.0041 & 0.048 & 0.048 & 0.338 & 0.420 \\
0 & 2.00 & 0.500 & 0.0082 & 0.100 & 0.101 & 0.608 & 0.782 \\
0 & 2.60 & 0.650 & 0.0118 & 0.146 & 0.147 & 0.844 & 1.095 \\
0 & 3.20 & 0.800 & 0.0207 & 0.260 & 0.260 & 1.392 & 1.837 \\
0 & 3.60 & 0.900 & 0.0415 & 0.526 & 0.527 & 2.687 & 3.583 \\
1.0 & 0 & 0.250 & 0.0054 & 0.065 & 0.065 & 0.422 & 0.534 \\
1.0 & 1.50 & 0.625 & 0.0109 & 0.136 & 0.136 & 0.775 & 1.008 \\
1.0 & 1.95 & 0.738 & 0.0157 & 0.197 & 0.197 & 1.074 & 1.409 \\
1.0 & 2.40 & 0.850 & 0.0276 & 0.349 & 0.349 & 1.814 & 2.408 \\
1.0 & 2.70 & 0.925 & 0.0554 & 0.704 & 0.705 & 3.537 & 4.733 \\
2.0 & 0 & 0.500 & 0.0082 & 0.101 & 0.100 & 0.593 & 0.764 \\
2.0 & 0.95 & 0.738 & 0.0157 & 0.197 & 0.197 & 1.065 & 1.391 \\
2.0 & 1.30 & 0.825 & 0.0237 & 0.298 & 0.298 & 1.554 & 2.065 \\
2.0 & 1.60 & 0.900 & 0.0415 & 0.527 & 0.527 & 2.666 & 3.560 \\
2.0 & 1.80 & 0.950 & 0.0831 & 1.061 & 1.061 & 5.248 & 7.045 \\
\end{array}
\]

\(^1\)The last digit of the values given in this table may be somewhat uncertain.
Table 2.—Computed values, for a square panel, of $\frac{w}{Q}$, $\frac{M_x}{qa^2}$, $\frac{M_y}{qa^2}$, and $\frac{|R_x|}{qa}$ for $S = 0.1$

| $\rho$  | $\epsilon$ | $\frac{(\epsilon + \rho)(1 + 2S)}{4}$ | $\frac{w}{Q}$ | $\frac{w_b}{Q}$ | $\frac{M_x}{qa^2}$ | $\frac{M_y}{qa^2}$ | $\frac{|R_x|}{qa}$ | $\frac{Q_x}{qa}$ |
|---------|------------|--------------------------------------|---------------|---------------|-------------------|-------------------|-----------------|----------------|
| 0       | 0          | 0                                    | $0.0048$      | $0.0041$      | $0.048$          | $0.048$          | $0.420$        | $0.338$        |
| 0       | 1.667      | $0.500$                              | $0.0098$      | $0.0082$      | $0.099$          | $0.100$          | $0.802$        | $0.628$        |
| 0       | 2.167      | $0.650$                              | $0.0140$      | $0.0118$      | $0.141$          | $0.146$          | $1.116$        | $0.865$        |
| 0       | 2.667      | $0.800$                              | $0.0247$      | $0.0207$      | $0.257$          | $0.260$          | $1.884$        | $1.437$        |
| 0       | 3.000      | $0.900$                              | $0.0493$      | $0.0411$      | $0.520$          | $0.522$          | $3.614$        | $2.720$        |
| 0.833   | 0          | $0.250$                              | $0.0064$      | $0.0054$      | $0.065$          | $0.065$          | $0.533$        | $0.421$        |
| 0.833   | 1.250      | $0.625$                              | $0.0131$      | $0.0110$      | $0.135$          | $0.135$          | $1.018$        | $0.786$        |
| 0.833   | 1.625      | $0.783$                              | $0.0188$      | $0.0158$      | $0.196$          | $0.196$          | $1.421$        | $1.089$        |
| 0.833   | 2.000      | $0.850$                              | $0.0330$      | $0.0276$      | $0.317$          | $0.317$          | $2.433$        | $1.839$        |
| 0.833   | 2.250      | $0.925$                              | $0.0663$      | $0.0553$      | $0.703$          | $0.704$          | $4.765$        | $3.569$        |
| 1.667   | 0          | $0.500$                              | $0.0098$      | $0.0082$      | $0.100$          | $0.100$          | $0.759$        | $0.590$        |
| 1.667   | 0.833      | $0.750$                              | $0.0197$      | $0.0165$      | $0.206$          | $0.206$          | $1.170$        | $1.120$        |
| 1.667   | 1.083      | $0.825$                              | $0.0283$      | $0.0237$      | $0.297$          | $0.297$          | $2.072$        | $1.567$        |
| 1.667   | 1.333      | $0.900$                              | $0.0497$      | $0.0415$      | $0.526$          | $0.526$          | $3.570$        | $2.679$        |
| 1.667   | 1.500      | $0.950$                              | $0.0996$      | $0.0831$      | $1.059$          | $1.059$          | $7.058$        | $5.263$        |

1The last digit of the values given in this table may be somewhat uncertain.
Table 3.—Computed values, for a square panel, of $\frac{w}{Q}$, $\frac{M_X}{qa^2}$, $\frac{M_Y}{qa^2}$, and $\frac{|R_X|}{qa}$ for $S = 0.2$

| $\rho$ | $\epsilon$ | $\frac{(\epsilon + \rho)(1 + 2S)}{4}$ | $\frac{w}{Q}$ | $\frac{w_b}{Q}$ | $\frac{M_X}{qa^2}$ | $\frac{M_Y}{qa^2}$ | $\frac{|R_X|}{qa}$ | $Q_X$ |
|-------|-------------|---------------------------------|----------------|----------------|-----------------|----------------|----------------|-------|
| 0     | 0           | 0                               | 0.0056         | 0.0041         | 0.018           | 0.018          | 0.420          | 0.338 |
| 0     | 1.429       | .500                            | 0.0113         | 0.0082         | 0.099           | 0.100          | 0.823          | 0.649 |
| 0     | 1.857       | .650                            | 0.0162         | 0.0117         | 0.143           | 0.145          | 1.152          | 0.899 |
| 0     | 2.286       | .800                            | 0.0286         | 0.0206         | 0.254           | 0.258          | 1.948          | 1.498 |
| 0     | 2.571       | .900                            | 0.0576         | 0.0414         | 0.518           | 0.525          | 3.737          | 2.836 |
| 0.714| 0           | .250                            | 0.0075         | 0.0054         | 0.065           | 0.065          | 0.531          | 0.421 |
| 0.714| 1.071       | .625                            | 0.0152         | 0.0110         | 0.134           | 0.135          | 1.031          | 0.800 |
| 0.714| 1.393       | .738                            | 0.0218         | 0.0157         | 0.195           | 0.196          | 1.446          | 1.111 |
| 0.714| 1.714       | .850                            | 0.0384         | 0.0276         | 0.346           | 0.348          | 2.463          | 1.869 |
| 0.714| 1.929       | .925                            | 0.0773         | 0.0553         | 0.701           | 0.703          | 4.814          | 3.618 |
| 1.429| 0           | .500                            | 0.0113         | 0.0082         | 0.100           | 0.099          | 0.754          | 0.587 |
| 1.429| .714        | .750                            | 0.0229         | 0.0165         | 0.206           | 0.205          | 1.471          | 1.124 |
| 1.429| .929        | .825                            | 0.0329         | 0.0236         | 0.297           | 0.296          | 2.077          | 1.574 |
| 1.429| 1.143       | .900                            | 0.0578         | 0.0414         | 0.525           | 0.524          | 3.579          | 2.690 |
| 1.429| 1.286       | .950                            | 0.1161         | 0.0830         | 1.060           | 1.058          | 7.070          | 5.277 |

1The last digit of the values given in this table may be somewhat uncertain.

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Figure 4.--The reciprocal of \( \frac{w_0}{Q} \) plotted as a function of \( \frac{(e+\rho)}{(1+2s)} \).
Figure 5.--Reciprocal of $\frac{M}{qa^2}$ plotted as a function of $\frac{(e+\rho)(1+2\rho)}{k}$. 
Figure 6.--Reciprocal of $|F_x|/qa$ plotted as a function of $(e + \rho)(1 + 25)/(1 + 25)$.
Figure 7.--Reciprocal of \( \frac{q_x}{qa} \) plotted as a function of 
\[ \frac{(\varepsilon + \rho)(1 + 2S)}{4} \]