DEFLECTION CHARACTERISTICS OF TWO 20-FOOT-DIAMETER LAMINATED WOOD RINGS SUBJECTED TO COMPRESSIVE LOADING ALONG A DIAMETER

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Tests were made to determine if the existing theory for curved members applied to a large glued-laminated wood ring loaded along a diameter. A trial southern pine ring and a white oak ring were fabricated. The rings were tested by compressive loading along a diameter, and the vertical deflection, the horizontal deflection, and the strain in the outer fiber at various locations were measured. The compressive loads imposed bending stresses to approximately the proportional limit.

The rings were cut into quadrants and tested as curved beams. The modulus of elasticity and modulus of rupture were determined, and outer fiber strain data were obtained.

Horizontal and vertical ring deflections were computed from theory and compared with those from test. Strains recorded at the various strain gage locations were compared for the rings and beams at equivalent values of bending moment. It was concluded that the theory was applicable to a laminated wooden ring of the type tested.
The integrity of glue bonds was studied by block shear and delamination tests of the oak ring. With one exception, the quality of the glue bond met the requirements of MIL-W-0015154B.

Introduction

Glued laminated wood has been increasing in importance as a structural material in the United States during the past quarter of a century. Many factors have had a play in the increased use, such as (1) the excellent architectural effects and unusual structural elements that can be produced by varied shapes and forms to permit flexibility and imagination in design, (2) the proven serviceability and satisfactory performance of laminated structural members, and (3) the development of specifications that aid the designer. Other factors, such as the nonmagnetic nature of wood, have been of major importance to the Navy in building of nonmagnetic minesweepers.

Most structural materials are essentially isotropic, having approximately equal strength properties in all directions. Wood, however, is an orthotropic material that, by definition, "has three mutually perpendicular axes of elastic symmetry." Glued laminated wood, plywood, and many of the reinforced plastics are also orthotropic materials. The usual engineering equations are derived for isotropic materials. They apply to the design of orthotropic structural elements if the principal stresses act in the directions of the orthotropic axes throughout the major portion of the elements. For unusual or new types of elements made of an orthotropic material, however, it is desirable to make check tests to determine if the usual engineering theory is applicable.

Fabrication of a closed wooden ring is possible through the use of glued laminated construction. Large thin rings of this type have possibilities for special structural and architectural applications, but data on laminated rings of this type have not been available. At the request of, and in cooperation with, the Bureau of Ships, a white oak laminated ring of 20-foot diameter was fabricated at Unit Structures, Inc., Peshtigo, Wis., and tested at the Forest Products Laboratory. This report presents the results of the tests.

3 Military Specification: Wood Laminates, White Oak (For Ship and Boat Use).

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Prior to laminating the white oak ring, Unit Structures laminated a similar ring of southern yellow pine. This ring was fabricated as a preliminary check of the laminating procedures but was reasonably well made. Tests of this ring were made by the Laboratory, and results are also presented in this report.

The primary purpose of these tests was to determine the deflection characteristics of a large laminated ring when subjected to a load along its diameter, and the strength properties of beams cut from the ring. A further objective was to determine the integrity of the adhesive joints in such a structure. The tests were made in July and August, 1959.

Development of Laminating Procedure

Various complicating factors are involved in the laminating of a closed structure such as a ring. One of the principal problems occurs in the closure of the ring, since the end joints in the zone of closure must be spaced in some predetermined and acceptable pattern. The requirements for the laminated ring were provided to Unit Structures by the Bureau of Ships. One butt joint was permitted in each lamination, and these butt joints were required to have certain minimum spacing requirements. Furthermore, not more than 3/8 inch of opening was permitted between the butted ends of each joint in the completed ring.

According to information received from Unit Structures, many methods of laminating the ring were considered. Two or three small rings of one-fifth scale were made to try out proposed techniques before selecting the procedure for laminating the large ring. Unit Structures personnel decided, however, that a full-size experimental ring should first be made of southern yellow pine. From this experiment, it would be possible to try out the selected procedure and gain experience in laminating a large oak ring.

Laminations were prepared from nominal 12-inch-wide No. 2 southern yellow pine. Each lamination was 5/8 inch thick and was scarfed to the desired length. Since the ring was experimental, Unit Structures personnel were not concerned with precise spacing and closure of the butt joints, relation of butt joints to scarf joints, or the exact alinement of the laminations. The laminations were spread with a phenol-resorcinol

Acknowledgement is made to Unit Structures, Inc. who laminated the rings, donated the full-size southern pine ring for test, and otherwise provided assistance and offered suggestions relating to this work.

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adhesive, placed on the form, and clamping was started. At the zone of closure, the laminations were forced into position by hand—an awkward and unsatisfactory procedure. In the process of closure, one of the laminations broke about 3 feet from one end while a workman was trying to force an overlapping lamination into position. It was not possible to correct this deficiency, so the broken end was discarded and closure was continued until the entire assembly was clamped. The laminating procedure was reanalyzed and Unit Structure personnel then designed and built a clockarm-type apparatus to facilitate bringing the laminations into position. This apparatus was reported to be a very helpful tool in laminating the oak ring.

Details of laminating the oak ring are presented in the following section. The southern pine ring had been laminated by essentially the same procedures, except for the use of the clockarm apparatus for positioning the laminations.

**Fabrication of Oak Ring**

White oak was procured as 1-inch-thick boards of random width and length from Hamer Lumber Sales, Kenova, W. Va. The lumber was furnished in accordance with the requirements of Military Specification MIL-W-0015154, Grade A, Class 2 or better. A phenol-resorcinol adhesive conforming with MIL-A-397 was used wherever glue was required.

The circumferential lengths of laminations for this 20-foot-diameter ring varied from about 58 to 63 feet, so it was necessary to end-scarf boards to attain these lengths. In addition, edge-gluing was required to obtain a 12-1/2 inch width for many of the boards. The depth of ring, approximately 9 inches, required 15 laminations of 5/8-inch thickness.

Edge joints were prepared, spread with adhesive, clamped, and cured. When all full-width boards were ready, they were surfaced on two sides and scarfed. Plain scarfs with a 1-in-12 slope were cut on both ends of all boards except those forming the ends of the lamination; these boards were scarfed only on one end. Care was taken to make end boards of the proper length so that no scarf would come within 18 inches of any butt joint in the assembly. Scarfs in what would be the three inside and three outside laminations of the ring were spaced in accordance with arrangement 1 of MIL-W-0015154B; the scarfs in the remaining laminations were generously dispersed. The scarfs were spread with

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5Discussion of fabrication was condensed from information supplied by Maurice J. Rhude, Chief Engineer, Unit Structures, Inc.

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adhesive, clamped, and cured. Laminations were then properly marked with a form line near the center of their lengths, taking into consideration the spacing requirements of the butt joints that would be created at their ends in the ring. Both ends were accurately trimmed to the exact length required.

Just prior to laminating, the boards were surfaced and the adhesive was spread in a room at a temperature of about 70° F. and a relative humidity of about 40 percent. The spreading required 9 minutes. The first and last laminations were spread on one side only, but all others were spread on both sides simultaneously. The adhesive was spread at a rate of 35 pounds per 1,000 square feet of single surface. The laminations were laid flat on top of one another with form lines corresponding to a mark on the gluing form. All laminations were then tipped simultaneously onto their edges and clamping to the form was started.

The gluing form consisted of a steel framework, with bucks set in a circular shape inside a continuous steel caul 3/8 inch by 12-1/2 inches, to provide a form with an outside diameter of 18 feet and 6 inches. Taylor clamps with front and rear rocker heads were used to clamp the laminations against the bucks and the steel caul, and also to hold the laminations in line along their width.

One group of workmen proceeded clockwise, another counterclockwise around the form until 180 degrees of the circle had been clamped in place. It was then necessary to bring the ends of successive laminations against the form, one at a time. Additional open assembly time resulted, especially at the staggered ends of laminations already in open rather than closed assembly. To assist in bringing the laminations against the form, a clockarm-type apparatus was used. It was provided with a roller attachment at the outside end to catch the laminations and pull them against the form. Holes drilled in the inner end of this arm permitted adjustment of the arm in increments of 5/8 inch; hence, each lamination could be rolled down against the previous inner lamination. This innovation was, to a major degree, responsible for the speed and success with which the ends of the laminations were brought into position in accordance with the specified arrangement for butt joints.

When all laminations were reasonably tight against the form and the ends of each were but a few inches from meeting, clamping was continued around the form. It was necessary to work back and forth across the butt-joint area during the clamping operation in order to bring all laminations into their final position. The total assembly time from
Start of spreading to completion of clamping was 58 minutes. A torque wrench was used to check the pressure of all screws, and workmen proceeded from the form line in both directions around the ring.

A test block package containing representative edge joints was assembled and placed on the form. An insulated and waterproofed cover was then put over the entire assembly. The temperature under the cover was raised to 200° F. by steam pipes beneath the form at a rate not exceeding 50° F. per hour. The curing was carried out until the innermost glue line had been at 150° F. or more for at least 6 hours. A perforated steam pipe provided humidification so as to maintain a wood equilibrium moisture content of approximately 12 percent during the heating, curing, and cooling process.

Each lamination in the cured ring had one butt joint, with such joints spaced essentially 30 inches or more in adjacent laminations. For a given cross section of the completed assembly, there were no more than two butt joints and these two joints were separated by at least five laminations. Cross sections containing butt joints were separated by at least 12 inches.

Preparation and Delivery of Rings

Both the white oak and southern pine rings were surfaced on the edges, and the inside and outside surfaces were sanded smooth. The outside diameter of each finished ring was about 20 feet and 3/4 inch. The rings were wrapped in waterproof paper and laid flatwise on a framework mounted on a large truck-trailer. The top of the framework was about 10 feet off the ground, so the rings would clear most vehicles and other objects along the highway.

Delivery was made to the U.S. Forest Products Laboratory on June 23, 1959. A local transfer company removed the rings from the truck and brought them into the building through large double doors where they could be handled by available Laboratory cranes.

Inspection of the rings at the Laboratory showed that the oak ring was well made. There was good closure at the butt joints, and there appeared to be good gluing throughout. Spacing of the butt joints was measured, and the results are shown in table 1. The southern pine ring had one defective area of about 3-foot length that contained only 14 laminations. This was a result of breakage of one lamination at the final closure, attributable to the unsatisfactory preliminary procedure. In addition,
there were a few laminations near the point of closure that were not quite full width, and there was also evidence of some slight openings in or adjacent to the glue lines. With these exceptions, however, the southern pine ring appeared to be well made. Mr. Rhude of Unit Structures emphasized that the southern pine ring had been made expressly to work out laminating procedures. He believes that a ring of southern pine could be made equally as well as an oak ring if the clockarm apparatus is used during laminating.

Tests

Ring Tests

The primary purpose of testing an entire ring was to determine whether it deflected under load according to theory. Keeping this and subsequent beam tests in mind, each ring was inspected so as to establish a suitable testing procedure. It was decided that the area containing butt joints should not be placed in zones of maximum bending moment when tested as a ring so that higher ring loads could be applied without danger of causing failure.

For the oak ring, a horizontal diameter was so oriented that the zone containing butt joints extended from 27 inches below to 78-1/2 inches above it, as measured along the arc of the ring. The horizontal reference line was placed midway between the butt joints in the two inner laminations. Since the pine ring had a reduced cross section in a portion of the butt joint zone, it was oriented so that the center of this reduced section fell about 50° above the horizontal, and thus at the point of approximately zero bending moment.

An overall sketch of a ring is presented in figure 1. The testing procedure first involved loading of the entire ring in compression, by load \( P \), with the ring oriented as shown. After this test, the ring was cut into quadrants. Sections 1a, 2a, 3a, and 4a were about 1 inch in length and were used in the determination of average moisture content. The four \( b \) sections were about 15 inches long and were used for block shear and cyclic delamination tests. The major portion of each quadrant was used for bending tests.

Before testing, 12 metaelectric strain gages of 1-inch gage length were placed on each ring. The gages were mounted on both the inside and outside rims of the ring, with their lengths parallel to the circumferential
direction. In three quadrants the gages were placed at midwidth with one gage on the inner rim and one along the outer rim in the same radial plane. In quadrant 2, however, two pairs of gages were used, each pair being 2 inches from the edge of the ring. The radial position of all gages is shown in figure 1. Figure 2 is a photograph of the gages on the inner rim of quadrant 2. Gages 7, 8, 9, and 10 were along the horizontal diameter, 0°, corresponding to the point of maximum bending moment in quadrants 1 and 3. Gages 11 and 12 were at 45° with the horizontal at a point of low bending moment, while the other six gages were at 85° with the horizontal diameter. The maximum bending moment in the ring occurs where the load is applied. Gages 1, 3, and 5 could obviously not be placed along this axis in the ring test, so they were offset 5° in order to avoid contact with the heads of the testing machine. Gages 2, 4, and 6 were offset 5° to be on the same radial line as those mentioned above.

The rings were placed vertically in a million-pound testing machine. Guide blocks had previously been attached to the movable and stationary heads of the machine to insure absolute vertical orientation of the ring. Small irregularities in the line of contact between the ring and heads were shimmed with brass shim stock and paper to obtain uniform bearing.

A wire was dropped from a screw centered in the middle lamination at the top of the ring and a steel scale and weight were attached to the lower end of the wire near the base of the ring. Vertical deflections were obtained by observing the movement of the scale past a stationary hairline attached to a column that was fixed to the bottom of the ring.

One end of a 1- by 4-inch board about 21 feet long was attached to the ring with a single nail at the center of quadrant 1 and at the center of the middle lamination. The opposite end of the board was permitted to slide over a metal support located at the center of quadrant 3. A scale was attached to the sliding arm at the movable end, and its motion relative to the ring (horizontal deflection) was measured as the scale passed an index point fixed to the middle lamination.

Figure 3 is a photograph of the white oak ring in place and ready for test. An initial load of 1,000 pounds was applied to the ring, and readings were taken of the horizontal and vertical gages and all metalelectric gages. Load was applied at a convenient head speed and stopped at successive 1,000-pound increments of load to permit recording of the deflections and strains. Horizontal and vertical deflections were determined to
strains at each metalectric gage were read to the nearest 10 micro-inches, using a Baldwin balancing bridge and indicator. The pine ring was loaded to 18,000 pounds and the oak ring to 20,000 pounds. These loads were equivalent to maximum outer fiber stresses of about 4,420 and 4,230 pounds per square inch, respectively, at the cross section of maximum bending moment. It was desired that the load applied to each ring would not exceed the proportional limit of the material. The load-deflection curves, however, showed a slight deviation of horizontal and vertical deflection from linearity at about 13,000 to 15,000 pounds load for both rings (figure 4).

The No. 1 metalectric gage on the pine ring did not function after the ring was in place, so data are not available.

After each ring test was completed, the ring was removed from the testing machine and cut into quadrants and sections as indicated in figure 1.

Curved Beam Tests

The deflection of each ring is dependent upon several factors, one of which is the modulus of elasticity of the material in the ring. Tests of a curved beam from each quadrant, therefore, permit obtaining four separate values of modulus of elasticity from four sections of the ring. In addition to providing data on modulus of elasticity, beam tests provide limited comparative strength values and also load conditions such that bending moments within the beam are quite accurately known. Hence, outer fiber strain measurements as determined from metalectric gage readings under known bending moments can be correlated with readings determined from the ring test under theoretical bending moments.

A curved beam was prepared from each quadrant. The beams were 150 inches long and the ends were prepared to the form shown in the sketch of figure 5. The bearing ends were 6 inches long and were carefully machined with a portable router so that they were flat, in the same plane, and in a plane perpendicular to the sides of the curved beam.

The beams were tested over a 144-inch span by two-point loading. The load points were 36 inches apart. Details of the test method are shown in figure 5. Rollers were used under both reactions and one load point to insure that loads would be in the vertical direction. The support assembly at each reaction was free to rotate in the plane of the beam.
The knife edges at the load points prevented restraint parallel to the plane of the beam and allowed adjustment perpendicular to the plane. The birch load blocks were fitted to the curvature of the beam but, because of local irregularities, were not in intimate contact throughout the entire surface. This was not considered to be a significant factor, however, because the bending moment between the load blocks would not be affected by such local irregularities. A set of tie rods was placed around the blocks as a safety precaution. These tie rods were kept loose throughout the test and would not have been necessary since there was no slippage of the load blocks. A photograph of the test method is shown in figure 6.

Total deflection, deflection over the center 24 inches, and readings of the two or four metalectric strain gages located between the load points were determined at each 2,000-pound increment of load throughout the test. Total deflection was measured to 0.01 inch by means of a scale suspended from the middle lamination at the center of the beam and a taut wire stretched across the base. The deflection over the center 24 inches, an area subjected to the maximum bending moment with no shear, was measured to the nearest 0.0005 inch with a yoke and dial arrangement. Metalectric strain gage readings were recorded as in the ring tests. In addition to these readings, the elongation of the beam during test was measured. This elongation was small and not of practical significance, so it will not be discussed further in this report.

The beams were loaded in a testing machine at a head speed of 0.3 inch per minute. The machine was stopped at each 2,000-pound increment of load, and deflections and strains were determined. All beams were tested to failure. Photographs of typical failures of oak beams are shown in figures 7 and 8. Figure 7 shows how the bottom lamination failed in tension adjacent to the butt joint in the second lamination. Figure 8 shows a front view of the failure of beam No. WO-3.

Further information concerning the failure of each of the beams, with WO indicating white oak and SP indicating southern pine, includes:

WO-1.--Tensile failure started in bottom lamination, adjacent to the butt joint in the second lamination, at a load of about 30,000 pounds. Shear failures adjacent to butt joints progressed as loading was continued. The bottom lamination failed completely with a brash tension failure at the maximum load, 41,040 pounds, and failure progressed longitudinally in the beam.
WO-2. --The bottom lamination failed in tension at a load of 64,180 pounds. Local cross grain in the second lamination probably contributed to the failure.

WO-3. --The bottom lamination failed partly in tension at 64,000 pounds load. Continued loading resulted in progressive failure along the grain, although cross grain was not severe. The maximum load attained was 68,750 pounds. There was some steep local cross grain in the third lamination that may have contributed to failure.

WO-4. --The specimen failed suddenly at a maximum load of 78,000 pounds. The bottom lamination failed in tension; its general slope of grain was about 1 in 10. Failure also occurred along local cross grain in the second lamination.

SP-1. --There was slight evidence of open glue lines and checking in the beam before it was tested. The beam started to "creak" beginning at a load of about 20,000 pounds, but no failures were observed until the maximum load was reached. The beam failed completely at a load of 38,000 pounds. Local cross grain at a knot in the bottom lamination may have contributed substantially to the failure. Stresses in shear and tension perpendicular to the grain also possibly contributed to failure.

SP-2. --The beam failed suddenly at a load of 59,530 pounds. The major part of the three bottom laminations failed in tension.

SP-3. --The beam failed suddenly at a load of 46,000 pounds, accompanied by splitting and tension failures of the four bottom laminations. Local cross grain at a knot in the bottom lamination contributed to tensile failures. There was some evidence of a weak glue bond between portions of the fourth and fifth laminations.

SP-4. --Failure started by a "peeling away" of the bottom lamination at a butt joint below one of the load points at a load of 24,000 pounds. Small local failures progressed as distinguished by "creaking" of the beam with increased load. The beam failed completely at 36,780 pounds, by splintering and shattering along the laminations. This beam had a nonuniform cross section near one end but this was outside of the zone where maximum bending moment was applied. This nonuniformity was due to the breakage of one lamination at the time the ring was being laminated, as was mentioned previously. The values of modulus of rupture and modulus of elasticity obtained by measurements between load points were, however, probably not affected by this nonuniformity near the end.
After testing, a section about 1 inch long was cut from each beam about 2 feet from one end. The overall moisture content and specific gravity of the section were determined.

**Block-Shear and Delamination Tests**

Block-shear and delamination tests were made on the portions of the oak ring marked with a lower case \(b\), as indicated in figure 1. One block-shear test was made on each glue joint at each of these locations in the ring. In addition, a section cut from the test block package of six randomly selected and edge-glued 18-inch-long boards (provided by the fabricator) were subjected to a cyclic exposure test. These tests were made in accordance with Specification MIL-W-0015154B.

**Presentation of Data**

Data on the opening and spacing of each of the 15 butt joints in the laminated oak ring are shown in table 1. All butt joints were located within a circumferential length of about 9 feet. The opening between the butted ends of each lamination is shown in the second column. Spacing of butt joints between adjacent laminations, as determined from an arbitrary reference line, is readily determined from the data of the third column.

The vertical and horizontal deflections of the two rings at a load of 12,000 pounds are presented in table 2. Both the observed and theoretical values are shown.

Table 3 presents data obtained from the eight beam tests. The moisture content and specific gravity were obtained from a section cut about 2 feet from the end of the beam.

Comparisons of strain observed in the ring and beam tests at theoretical or assumed bending moments are presented in table 4.

Results of the block shear and delamination tests are shown in table 5.
Deflection of Rings

A thin ring is one for which the thickness is sufficiently small with respect to the mean radius that the effects of longitudinal and shearing forces are negligible. Perhaps the simplest theory relating to such a ring is one based on Castigliano's theorem. It states that

\[ M = \frac{PR}{2} \left( \cos \phi - \frac{2}{\pi} \right) \]  

(1)

where

- \( M \) = moment at any cross section
- \( P \) = load applied along a vertical diameter
- \( R \) = mean radius
- \( \phi \) = angle between the radial cross section considered and the horizontal

Positive moments tend to decrease the radius of curvature. Furthermore, application of theorem shows that

\[ \delta_v = 0.149 \frac{PR^3}{EI} \]  

(2)

\[ \delta_h = 0.137 \frac{PR^3}{EI} \]  

(3)

where

- \( \delta_v \) = vertical deflection
- \( \delta_h \) = horizontal deflection
- \( E \) = modulus of elasticity
- \( I \) = moment of inertia of cross section with respect to its horizontal centroidal axis

The comparison of theoretical deflections and actual deflections is presented in table 2. For the oak ring, the difference between computed and actual deflections is 8 percent in the vertical direction and 5 percent in the horizontal. The corresponding values for the


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southern pine ring are 14 percent and 10 percent. The only parameter in the deflection equations (2) and (3) which could not be estimated independent of the theory being studied in this report is the modulus of elasticity. This property was determined from load-deflection data over the center 24 inches from the curved beam tests. The standard straight-beam theory for a two-point loaded beam was used. It can be shown that, for the ratio of thickness to mean radius used, the error induced by assuming a straight beam is less than 1 percent. Although the moduli of elasticity used are, at best, only estimates, any deviation from true value of modulus of elasticity cannot explain the difference in percent error for the horizontal and vertical deflections in a single ring. This may, in part, be explained by the assumption of a thin ring, for it may be seen that if longitudinal and shear forces are accounted for, a greater deflection would be predicted. It is felt, however, that the thin ring theory fits quite well, considering the nonhomogeneity of the rings. Even the reduced section and poor butt joints in the pine ring appear to have had little effect on the elastic behavior of the ring. It is significant, of course, that the ring was intentionally oriented so that the weakest section was subjected to small bending moments during the ring test.

Beam Tests

The method used for the curved-beam tests was considered to be satisfactory. Results of the tests (table 3) show that beams containing butt joints had about the same stiffness as beams without butt joints. In the oak beams, only beam No. WO-1 contained butt joints. In the southern pine beams, however, both SP-1 and SP-4 had butt joints. As expected, the beams with butt joints were markedly weaker than those without butt joints.

By chance, none of the beams had a scarf joint in the bottom tensile lamination in the zone of maximum bending moment. A scarf joint in such a position may or may not have had an effect on strength. There was no evidence that a weak scarf joint was the primary cause of failure in any of the beams.

Moisture Content and Specific Gravity

The overall moisture content of the oak rings and beams was about 10 percent and for pine rings and beams about 12 percent. Surface moisture content at about 1/8-inch depth was checked at several points with an
electric moisture meter; measurements were made at the time of the
ring test and then again at the time of the beam tests. The moisture
content near the surface was about 1 percent less than that for the
average cross section. Both the surface and average moisture content
obtained at the time of the ring test and subsequent beam tests of
each species were about the same. When correlating data, therefore,
it was not necessary to adjust for changes in moisture content that
occurred in the interval of time between ring and beam tests.

The specific gravity of the two rings, as determined from sections cut
from each beam (table 3), was about average for the species.

Strains Measured by Metalectric Gages

The metalectric strain gage readings were not entirely consistent when
readings were taken from the ring and beam tests. Theoretically, the
strains on the concave side are expected to be slightly higher than on
the convex side when the curved element is subjected to a constant
bending moment. For a homogeneous material, the difference in
strain would be about 5 percent for a curved member having the
curvature and dimensions of these rings. For a nonhomogeneous material
composed of layers of different stiffnesses, other relationships would
be expected because of the varying position of the neutral axis. In
both the ring and beam tests, however, the strain reading on the
concave side was generally larger than the corresponding strain on the
convex side.

Gages that were at midheight in the ring test (gages 7, 8, 9, and 10)
were subjected to strains due to a direct compressive force as well as
bending moment. Consequently, compressive strains were slightly
higher and tensile strains slightly lower than strains that would be due
to bending moment alone. The strain due to the compressive load was
relatively small, being in the order of about 5 percent of the outer fiber
strain.

Metalectric strain gages measure strain over a very small area.
Readings can be influenced by local irregularities or by irregularities
through the cross section. It is not to be expected, therefore, that
strains measured over a very short length will agree exactly with strains
or deformations measured over greater lengths.

In view of the curvature of the sections, the nonhomogeneous nature of
the material, and the variation in the position of the neutral axis, it
seems most proper to make comparisons of the algebraic difference in strain at each cross section. Moreover, this comparison automatically accounts for the effect of direct strain at gages 7, 8, 9, and 10. Although the strains on the outer fibers may not be equal, it can be assumed with but minor error that the strains are linear through the depth of the cross section. Algebraic differences in strains observed between gages that were opposite each other at various cross sections are presented in table 4. The strains are those (1) observed when the ring was subjected to 12,000 pounds load, (2) observed from beam tests at the same theoretical bending moment imposed by a 12,000-pound load on the ring, and (3) calculated on the basis of the same theoretical bending moment, the moment of inertia of the beam, and the modulus of elasticity of the beam measured over the 24-inch center section.

An example of the procedure used in determining the values of outer fiber strain (table 4) is presented for WO-1. In this section, strains measured by gages 9 and 10 were $+820 \times 10^{-6}$ and $-900 \times 10^{-6}$ inches per inch, respectively, when the ring was subjected to a load of 12,000 pounds (figure 9). The algebraic difference of these strains is $1,720 \times 10^{-6}$. The theoretical bending moment in the ring at the load was $0.182 PR$, where $P$ is the load and $R$ the mean radius, or

$$M = 0.182 (115.6)(12,000) = 252,600 \text{ inch-pounds}$$

When WO-1 was tested as a beam, the total load, $P$, required to produce a bending moment of 252,600 inch-pounds is twice the bending moment divided by the moment arm or

$$P = \frac{2(252,600)}{54} = 9,350 \text{ pounds}.$$ 

At 9,350 pounds load (figure 9), the strains measured by gages 9 and 10 were $-780 \times 10^{-6}$ and $+880 \times 10^{-6}$ inches per inch, respectively, or an algebraic difference of $1,660 \times 10^{-6}$ inches per inch. Based on the test of WO-1 as a beam, the modulus of elasticity in the center 24 inches was 1,810,000 pounds per square inch. The moment of inertia was $810 \text{ inches}^4$ and the depth 9.32 inches. Assuming a straight homogeneous beam, the usual engineering equations, with $e$ as outer fiber strain, result in:

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\[ f = \frac{Mc}{I} = \frac{27PC}{I} = Ee \]  
\[ e = \frac{27(9,350)(9.32)}{2(810)(1,810,000)} = 803 \times 10^{-6} \text{ inches per inch.} \]

Thus the difference in outer fiber strain would be \(1,606 \times 10^{-6}\) inches per inch, assuming the neutral axis to be at the center.

Presumably if the predicted moment distribution is correct at the quarter points, it should be correct throughout the ring. Gages 11 and 12 were used to check at an intermediate point. No data are available for these gages from the beam test because they fell in a region between two beams. The calculated values of strain deviated 37 percent and 4 percent respectively from the observed values for the oak ring and the pine ring. No explanation is suggested for the large difference in the oak values other than that it may possibly be attributed to wood with a low modulus of elasticity in the zone of the strain gages.

Based on the data summarized in table 4, the strains obtained from the ring test were reasonably similar to the strains obtained at the known bending moment in the beam. The moment distribution in each ring must have been nearly that predicted by curved beam theory. Furthermore, the observed strains were nearly the same as those expected, based on the known bending moment, moment of inertia, and modulus of elasticity of the beam measured over the center 24-inch section.

Block-Shear and Delamination

Block-shear and delamination tests were made from sections of the oak ring and delamination tests were made on the edge joints of the special test block. All average values, except for the wood failure for section R-WO-2b, met the requirements of MIL-W-0015154B. It was not possible to tell from the samples what might have been the reason for some of the low percentages of wood failure. A long assembly time is sometimes a factor but this would be expected to have had the most pronounced effect on sections R-WO-1b and R-WO-4b, since these were nearest to the closure end. Instead, the low values of wood failure may have been caused by too short an assembly time since clamping was started in the region opposite the closure end.
In general, the delamination values were very low, the maximum average being 0.4 percent between laminations. Figure 10 shows the section from specimen No. R-WO-1b after the delamination test. For edge joints of sections, the delamination for section R-WO-2b was 9.5 percent but was zero for the other three sections. The test block package for edge joint quality tests showed zero delamination in edge joints and joints between laminations.

Conclusions

The moment distribution predicted for a thin isotropic ring by Castigliano's theorem may be applied to glued laminated wood rings of the type tested. Deflections may also be predicted with reasonable accuracy.

The limitation on application of the theory probably lies in the nonhomogeneity of the laminated structure. This limitation does not appear to be of practical significance.

In general, the procedures used in making the laminated oak ring resulted in good glue bonds even though somewhat complex laminating techniques were necessary.
Table 1.—Opening and spacing of the 15 butt joints in the laminated oak ring

<table>
<thead>
<tr>
<th>Lamination ¹</th>
<th>Opening between butted ends</th>
<th>Distance from reference line to butt joint ², ³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16 inches</td>
<td>Inches</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>-15.4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>+15.8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>+46.4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>+76.4</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>-27.0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>+3.0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>+32.3</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>+63.1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>-11.0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>+18.9</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>+48.6</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>+78.5</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>-25.7</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>+4.5</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>+34.3</td>
</tr>
</tbody>
</table>

¹ Numbered outward from concave side.
² Measured along curvature of lamination.
³ Positive distances were measured upward from the horizontal diameter when the ring was in the test position.

Report No. 1877
Table 2.—Comparison of observed and theoretical deflections of laminated rings, and the moisture content of the rings at time of test

<table>
<thead>
<tr>
<th></th>
<th>White oak</th>
<th>Southern pine</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deflection at 12,000 Pounds Load</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Type of deflection</strong></td>
<td>Inches</td>
<td>Inches</td>
</tr>
<tr>
<td>Vertical Observed</td>
<td>2.00</td>
<td>2.52</td>
</tr>
<tr>
<td>Theoretical</td>
<td>1.84</td>
<td>2.17</td>
</tr>
<tr>
<td>Horizontal Observed</td>
<td>1.78</td>
<td>2.22</td>
</tr>
<tr>
<td>Theoretical</td>
<td>1.69</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Moisture Content of Sample

<table>
<thead>
<tr>
<th></th>
<th>Percent</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>9.7</td>
<td>12.3</td>
</tr>
<tr>
<td>2a</td>
<td>9.9</td>
<td>14.0</td>
</tr>
<tr>
<td>3a</td>
<td>10.3</td>
<td>11.8</td>
</tr>
<tr>
<td>4a</td>
<td>10.1</td>
<td>12.3</td>
</tr>
</tbody>
</table>

Theoretical values were calculated from the applicable average properties obtained from beam tests (table 3). The modulus of elasticity used was that obtained in the zone of constant bending moment, which was the center 24 inches of each beam.

Report No. 1877
Table 3.—Results of tests of quadrants from laminated wood rings tested as curved beams

<table>
<thead>
<tr>
<th>Specimen: Depth: Width: Moisture: Specific gravity: Modulus of elasticity: Modulus of rupture: Deflection at maximum load:</th>
<th>Specimen No.</th>
<th>Width</th>
<th>Moisture content</th>
<th>Specific gravity</th>
<th>Modulus over 1</th>
<th>Modulus of rupture</th>
<th>Deflection at maximum load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In.</td>
<td>In.</td>
<td>Percent</td>
<td>1,000 p.s.i.</td>
<td>1,000 p.s.i.</td>
<td>P.s.i.</td>
<td>In.</td>
</tr>
<tr>
<td><strong>WHITE OAK</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WO-1 : 9.32 : 12.00 : 10.2 : 0.63 : 1,810 : 1,620 : 6,370 : 1.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WO-2 : 9.32 : 12.00 : 9.8 : 0.66 : 1,770 : 1,600 : 6,960 : 3.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WO-3 : 9.33 : 12.01 : 9.8 : 0.67 : 1,920 : 1,840 : 10,650 : 3.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WO-4 : 9.32 : 12.01 : 10.8 : 0.62 : 1,930 : 1,830 : 12,110 : 3.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average : 9.32 : 12.00 : 10.2 : 0.64 : 1,860 : 1,720 : 9,770 : 3.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SOUTHERN PINE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP-1 : 9.15 : 10.85 : 12.2 : 0.54 : 1,840 : 1,670 : 6,770 : 1.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP-2 : 9.18 : 10.76 : 12.1 : 0.52 : 1,690 : 1,610 : 10,620 : 4.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP-3 : 9.14 : 10.79 : 13.0 : 0.55 : 1,900 : 1,610 : 8,260 : 2.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP-4 : 9.15 : 10.54 : 11.5 : 0.54 : 2,020 : 1,700 : 6,770 : 2.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average : 9.16 : 10.74 : 12.2 : 0.54 : 1,860 : 1,650 : 8,100 : 2.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Based on volume at test and oven dry moisture content.
2 Conventional straight-beam theory was used to determine modulus of elasticity and modulus of rupture.
3 Determined from deflections measured over 24 or 144 inches.
4 Measured over 144-inch span.
Table 4.--Difference in outer fiber strain at four radial axes of laminated rings. Observed strain measured with pair of metalectric strain gages on faces at inner and outer radii.

<table>
<thead>
<tr>
<th>Quadrant:Strain gage: Bending Nos.:</th>
<th>Observed strain</th>
<th>Calculated strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In ring</td>
<td>In beam</td>
</tr>
<tr>
<td><strong>WHITE OAK</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 : 9,10 : +252,600 : 1,720 : 1,660</td>
<td>1,605</td>
<td></td>
</tr>
<tr>
<td>2 : 1,2 : -381,600 : 2,460 : 2,435</td>
<td>2,485</td>
<td></td>
</tr>
<tr>
<td>3,4 : -381,600 : 2,395 : 2,375</td>
<td>2,485</td>
<td></td>
</tr>
<tr>
<td>11,12 : + 48,600 : 475 : .................</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>3 : 7,8 : +252,600 : 1,615 : 1,450 : 1,510</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 : 5,6 : -381,600 : 2,330 : 2,205 : 2,275</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SOUTHERN PINE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 : 9,10 : +252,600 : 1,840 : 1,665</td>
<td>1,810</td>
<td></td>
</tr>
<tr>
<td>2 : 1,2 : -381,600 : 2,690 : 2,740</td>
<td>2,985</td>
<td></td>
</tr>
<tr>
<td>3,4 : -381,600 : 335 : .................</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>11,12 : + 48,600 : 335 : .................</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>3 : 7,8 : +252,600 : 1,845 : 1,720 : 1,770</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 : 5,6 : -381,600 : 2,730 : 2,585 : 2,570</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Bending moment is a theoretical moment in ring test but is known in beam test.
2 Positive moments tend to decrease the radius of curvature.
3 Calculated on basis of moment of inertia and modulus of elasticity of the 24-inch center of each beam.

Report No. 1877
Table 5.--Results of block shear and delamination tests\(^1,2\) on four sections from a laminated white oak ring\(^3\)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Specimen No.</th>
<th>Specimen No.</th>
<th>Specimen No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-WO-1b</td>
<td>R-WO-2b</td>
<td>R-WO-3b</td>
<td>R-WO-4b</td>
</tr>
<tr>
<td>Shear</td>
<td>Wood Shear</td>
<td>Shear</td>
<td>Wood Shear</td>
</tr>
<tr>
<td>P.s.i.</td>
<td>Percent</td>
<td>P.s.i.</td>
<td>Percent</td>
</tr>
<tr>
<td>1,970</td>
<td>25</td>
<td>2,130</td>
<td>95</td>
</tr>
<tr>
<td>2,290</td>
<td>100</td>
<td>2,240</td>
<td>25</td>
</tr>
<tr>
<td>2,170</td>
<td>85</td>
<td>1,930</td>
<td>80</td>
</tr>
<tr>
<td>1,750</td>
<td>100</td>
<td>2,730</td>
<td>100</td>
</tr>
<tr>
<td>1,210</td>
<td>95</td>
<td>1,990</td>
<td>100</td>
</tr>
<tr>
<td>2,310</td>
<td>60</td>
<td>1,610</td>
<td>55</td>
</tr>
<tr>
<td>2,070</td>
<td>55</td>
<td>2,410</td>
<td>90</td>
</tr>
<tr>
<td>2,480</td>
<td>90</td>
<td>2,630</td>
<td>60</td>
</tr>
<tr>
<td>2,550</td>
<td>95</td>
<td>2,170</td>
<td>100</td>
</tr>
<tr>
<td>2,980</td>
<td>95</td>
<td>2,950</td>
<td>50</td>
</tr>
<tr>
<td>2,230</td>
<td>60</td>
<td>1,970</td>
<td>45</td>
</tr>
<tr>
<td>1,830</td>
<td>95</td>
<td>2,570</td>
<td>55</td>
</tr>
<tr>
<td>2,040</td>
<td>75</td>
<td>2,250</td>
<td>35</td>
</tr>
<tr>
<td>1,890</td>
<td>95</td>
<td>2,180</td>
<td>80</td>
</tr>
<tr>
<td>Av. 2,130</td>
<td>80</td>
<td>2,270</td>
<td>69</td>
</tr>
</tbody>
</table>

\(^1\) Tests made in accordance with MIL-W-0015154B.

\(^2\) Percentages of delamination in joints between laminations were 0.3 for specimen 1b, 0.4 for 2b, and 0.1 for 3b and 4b. Percentages of delamination in edge joints were 9.5 for 2b and 0.0 for the others.

\(^3\) The ring had 15 laminations and one block shear test was made on each glue joint for each of the four sections.

Report No. 1877
Figure 1. -- Sketch of 20-foot-diameter ring subjected to load, P. Angle notations designate where metallectric strain gages were applied, the angle being measured from the horizontal diameter. Small numbers at inner and outer surface identify gages. Markings on ring show how ring was cut after the ring test. Circled numbers identify quadrant and beam, a sections are for determination of moisture content, and b sections are for block shear and cyclic delamination tests.
Figure 2. -- Two metalectric gages, Nos. 2 and 4, on the inner rim of the oak ring in quadrant 2.
Figure 3. -- Test of 20-foot diameter laminated white oak ring.

ZM 115 903
Figure 4.--Load-deflection curves for laminated rings. Deflection increased in horizontal direction and decreased in vertical direction.
Figure 5. --Details of method used in testing curved laminated beams.
Figure 6.--Method used in testing curved beams cut from laminated rings.
Figure 7. — Failure adjacent to butt joint of white oak beam R-W0-1.

ZM 115 924
Figure 8. --Front view of failure of white oak beam R-WO-3.

ZM 115 923
Figure 9.--Load-strain relationship observed at metalastic gages 9 and 10 during ring test and beam test.
Figure 10. Appearance of section from specimen No. R-WO-1b after completion of delamination test. Section is almost entirely free of delamination.
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- List of publications on Chemistry of Wood and Derived Products
- List of publications on Fungus Defects in Forest Products and Decay in Trees
- List of publications on Glue, Glued Products and Veneer
- List of publications on Growth, Structure, and Identification of Wood
- List of publications on Mechanical Properties and Structural Uses of Wood and Wood Products
- Partial list of publications for Architects, Builders, Engineers, and Retail Lumbermen
- List of publications on Fire Protection
- List of publications on Logging, Milling, and Utilization of Timber Products
- List of publications on Pulp and Paper
- List of publications on Seasoning of Wood
- List of publications on Structural Sandwich, Plastic Laminates, and Wood-Base Aircraft Components
- List of publications on Wood Finishing
- List of publications on Wood Preservation
- Partial list of publications for Furniture Manufacturers, Woodworkers and Teachers of Woodshop Practice

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