EVALUATION OF THE FACTOR OF SAFETY IN STRUCTURAL TIMBERS

INFORMATION REVIEWED AND REAFFIRMED 1965

No. 2068

February 1957

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Please return to:
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Structural designers give much attention to the factor of safety in determining design stresses for the materials they use. Such attention may often be superficial; the engineer may observe that his material has an average test strength of 10,000 pounds per square inch and, by assuming a factor of safety of 5, arrive at a working stress of 2,000 pounds per square inch.

Determination of a true factor of safety is not so simple. The particular unit of material in the structure under consideration may be stronger or weaker than the test value indicates; its strength may be favorably or unfavorably affected by the conditions of service; or the load coming upon it may be greater or less than the value assumed in design.

With these possibilities in mind, the engineer may conclude that the minimum expected test strength is something less than 10,000 pounds per square inch.

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1—The mathematical analysis used in this report was first developed by Dr. W. S. Ericksen of Wright Air Development Center, Ohio, formerly mathematician at the Forest Products Laboratory.

2—Maintained at Madison, Wis., in cooperation with the University of Wisconsin.
square inch, and that that strength may be reduced by unfavorable condi-
tions of loading to an effective value of 5,000 pounds per square inch. At
the same time, the possibility of a 25 percent overload raises the work-
ing stress to 2,500 pounds per square inch, leaving a value of 2 as the
apparent minimum factor of safety. Recognizing the possibility that
something unforeseen may occur, the engineer may choose to call this
value the "factor of ignorance." Even so, he has little more than intu-
ition to tell him whether his "factor of ignorance" should be 2, or 1-1/4,
or several times that much.

Some attention has been directed in recent years toward more rational
means for evaluating the true factor of safety. Tucker (7) pointed out
how strength can be affected by combinations of probabilities of strength
value in the elementary units that compose a structural member. Later,
Freudenthal (3) recognized that factors of load and of strength affect the
adequacy of structural members, and that each factor may have a differ-
ent value for each individual case. Recently, the Forest Products Lab-
oratory has developed a means of evaluating these factors of adequacy
and combining them in such a way as to indicate the overall performance
to be expected from structural members as a group. This work has been
carried on with particular reference to structural timbers, but the
method is equally applicable to other structural materials.

The work reported here deals with factors of strength and use as they
limit the serviceability of structural timbers. It is recognized that a
structure or structural element may become useless from excessive
deformation under loads that do not cause failure. Plastic deformation
in wood structures is limited by the fact that the ultimate strength of
wood is higher in short-time than in long-time loading. If stresses are
suitable for the duration of load expected, excessive deformation does
does not occur. Deformations in the plastic range, called "irreversible
deformations" by Freudenthal (3), affect the stresses and thus play an
important part in an analysis of the probability and the nature of failure,
but they are beyond the scope of this report.

Basic Concepts

There are a number of definitions for the factor of safety. For the pur-
pose of this work, the true factor of safety is defined as. the ratio of the

3Underlined numbers in parentheses refer to literature cited at the end
of this report.
strength of a structural member to the strength actually required of that member in use. In other terms, this might be called a ratio of adequacy. This ratio is not a fixed constant. It has a different value for each structural member, depending upon the strength of that member and the loading and other conditions under which it is used.

The strength itself may depend upon the loading conditions. Wood has the ability to absorb short-time overloads without damage, but wood under long-time overloading will break at a lower stress than that developed in the usual duration of test load, 5 to 10 minutes. A Forest Products Laboratory report (8) shows the effects of short or long-term loading and how these effects can be recognized in design. Thus, the duration as well as the other conditions of loading affects structural adequacy.

Working stresses for structural materials, including timber, are generally obtained by applying to strength-test values a series of reduction factors based on the nature of the material and the conditions under which it is used. In wood, the principal reduction factors are those for variability of clear wood substance, effect of knots or other growth characteristics, duration of load, and safety. The factor of safety includes many minor factors, notably the "factor of ignorance." For example, an average test value for the modulus of rupture of a given species of wood might be reduced by 1/4 to take account of pieces of less-than-average strength, then by 7/16 to provide for the effects of long-time loading, and then divided by a factor of safety of 1-1/2 to give a basic stress for the clear wood that is 9/32 of the average for the species. In a timber grade permitting knots or the equivalent that reduce the strength by not more than 2/5, the working stress would be 3/5 of the basic stress, or about 1/6 of the average strength of the species.

If an individual beam contains clear wood substance only 3/4 as strong as the average for that species, if it is subjected to full load for such long time as to fail at 9/16 of its short-time strength, if other service conditions are so unfavorable as to reduce even this long-time strength to 2/3 of that under normal conditions, and if it contains a knot of maximum size in the most unfavorable position permitted in its grade, its adequacy may be measured by:

\[
\frac{3}{4} \times \frac{9}{16} \times \frac{2}{3} \times \frac{3}{5} = \frac{27}{160} \text{ or about } 1/6
\]

Under a load resulting in stress equal to the working stress, the ratio of adequacy of the beam is 1. The designer tacitly assumes that this set of
conditions or its equivalent is the worst that will be encountered, since more severe conditions would result in failure.

Another beam may contain clear wood substance less than $3/4$ as strong as the average, while its adequacy in other respects may be more than was assumed. Thus, even with inadequacy in one factor, if the product of all factors exceeds $1/6$, the beam is safe. In other words, its safety is determined by the interplay of all factors and does not require that each individual factor be as good or better than assumed.

Each structural timber has its own strength characteristics and its own conditions of service. Measures of adequacy that are single-valued when applied to any specific timber become multi-valued when applied to timbers as a group. When several reduction factors used as measures of adequacy are considered simultaneously, there is an almost infinite number of possible combinations. Each factor must, therefore, be considered not as a single value but as a distribution of values, and the possible combinations of all factors as a combined distribution. Frequency distributions thus appear as the mathematical expression for each factor and for the combination of factors into an overall measure of adequacy.

The frequency distributions representing the various factors must be chosen and defined in such a way as to (1) operate independently of each other and (2) take account of all the circumstances that affect the adequacy of structural timbers. The first requirement is met in the beam example already cited; strength of clear wood, length of service, conditions of service, and size and position of knots are mutually independent, and each can operate in either direction without affecting any other. To satisfy the second requirement, it is necessary to give full consideration not only to factors affecting strength, but also to those affecting loading and other conditions of service. For example, the timber that is inadequate in strength may never receive full design load, or if it does, may have to sustain it for such a short time that it will be safe.

Once frequency distributions are defined and selected to represent the various factors measuring adequacy, the problem becomes the mathematical one of combining them into an overall frequency distribution that will represent the range of adequacy of timbers in the group being considered. The factors are combined by multiplication. This makes the definition of each factor independent of the order in which it is applied -- a necessary consideration, since all factors are presumed to act simultaneously.
To illustrate the method of combination, reference is made to the beam example mentioned. The strength of its clear wood is \( \frac{3}{4} \) of the average for the species. Its clear-wood strength under long-time loading is \( \frac{9}{16} \) of \( \frac{3}{4} \) of the species average. Its clear-wood strength under its special conditions of service, including the long-time loading, is \( \frac{2}{3} \) of \( \frac{9}{16} \) of \( \frac{3}{4} \) of the species average. Its actual strength as established by the knots in it is \( \frac{3}{5} \) of \( \frac{2}{3} \) of \( \frac{9}{16} \) of \( \frac{3}{4} \) of the species average. Effects of overloading or underloading can be brought into the same combination. If a timber is just adequate under its design load, doubling the load means that its adequacy can be represented by the ratio \( \frac{1}{2} \); likewise, underloading to \( \frac{2}{3} \) the design load can be represented by applying the ratio of \( \frac{3}{2} \) to its other measures of adequacy.

This theory for rational examination of the true factor of safety thus involves first the definition of frequency distributions to represent all the factors of strength and service that affect the adequacy of a group of structural members; and second, the mathematical operation of combining those frequency distributions into a product distribution that represents the overall adequacy of the group.

**Application to Structural Timbers**

How the theory works can be demonstrated by applying it to a hypothetical group of timber beams structurally graded to 60 percent of their clear-wood strength for use in the construction of industrial buildings. Representative values are used, but only to illustrate the theory. Any structural designer may make his own assumptions and combine his own values to arrive at his own estimate of adequacy for any group of structural members of any material. The accuracy of the final result is limited by uncertainties in the choice of frequency distributions and by approximations in the methods of combination.

**Selection of Frequency Distributions**

Frequency distributions are of many types. One of the most common is the "normal frequency distribution" represented by an equation of the type, \( y = e^{-x^2} \), and shown graphically in figure 1, A. The normal distribution presumes a symmetrical arrangement of values, with the majority grouped rather closely about the average and progressively decreasing...
numbers at greater distances from the average. Although statistical examination shows that strength factors in wood are not exactly represented by normal frequency distributions, examination of actual data shows that many approach very closely to the normal distribution. The normal distribution is easily defined, its properties are widely understood, and it is considered to be quite useful in this connection.

Some factors may vary uniformly between definite limits and are represented by the "uniform frequency distribution," shown graphically in figure 1,B. For example, a factor may have values anywhere within the range of 0.60 to 0.74, all values within these limits having equal probability but none beyond.

In this example there are four assumed frequency distributions of the normal type and one of the uniform type. In other examples a smaller number or only one type might be used in the interest of simplicity, or a greater number or more types might yield greater accuracy of results. Calculations have been made with as many as 13 factors, but the purpose of illustration can be served satisfactorily and more briefly with the smaller number. All factors and their products are expressed as ratios to the average strength value for the species as determined by standard laboratory tests. Characteristics of the assumed frequency distributions are tabulated in table 1 and discussed in the following paragraphs.

Variability of Clear Wood

A large amount of data on strength properties of clear wood has been accumulated at the Forest Products Laboratory (6) from tests of carefully selected specimens by standard methods. Examination of the data for dispersion of individual values has shown that the modulus of rupture values for the most common structural species closely approximate a normal frequency distribution with a standard deviation of 16 percent of the average value. Since this average value is taken as unity in the definition of these frequency distributions, the standard deviation for this factor is 0.16. This distribution recognizes the above-average and below-average strengths of individual pieces.

Range of Growth Characteristics

Within Grade

Within any structural grade are knots and other characteristics that affect strength in magnitudes ranging from the greatest permissible to those
just great enough to exclude the material from the next higher grade. Design stress values for such grades are based on the assumption that each timber has the largest permissible knot or equivalent feature. As a matter of fact, there are in any grade some timbers of this kind (low-line pieces) and some near the top of the grade but not quite eligible for the next higher grade (high-line pieces). It is assumed here that individual timbers have a uniform distribution from bottom to top of their grade. The assumed strength ratio, as already noted, is 60 percent of the clear-wood strength. The next higher grade in some species has a ratio of 75 percent. Limits for this assumed uniform frequency distribution are therefore set at 0.60 and 0.74.

Duration of Load

Continued loading reduces the effective strength of wood. It is assumed for this purpose that the most probable duration of full design load during the life of the structure is about 5 years. Durations in some warehouse floors will be greater, and in roof constructions designed for snow loads the duration of full load may be much less. Obviously, the duration of full design load might be quite different in nonindustrial structures. For a duration of about 5 years, the effective strength is about 62 percent of the 5-minute strength obtained in the standard tests upon which species averages are based. A normal frequency distribution with a standard deviation of 0.03 from an average value of 0.62 is assumed to take account of the variations to be expected in this factor.

Other Factors Affecting Strength

The assumed frequency distribution for factors affecting strength, other than those already mentioned, is oversimplified here in the interests of brevity, but will serve for illustrative purposes. It includes factors operating both in manufacture and in use. In other analyses made on structural timbers at the Forest Products Laboratory, the following factors were considered and frequency distributions assumed for each: indeterminacies in design, use of standard sizes in design, form factors, stress concentration factors, efficiency in formulation of grading rules, efficiency of inspection, variations in size, fabrication effects, temperature in service, and other conditions of service. Many of these factors rely heavily upon engineering judgment for their definition, and after all known specific factors are defined there remains some such general factor as "other conditions of service." This suggests the previously mentioned "factor of ignorance," now greatly reduced. A factor such as use
of standard sizes has an effect equivalent to increase of strength and thus tends to balance the factors that usually reduce strength.

A normal frequency distribution with an average value of 1.00 and a standard deviation of 0.20 is considered to represent the overall effects of these factors.

**Loading Effects**

A timber loaded beyond its strength can be represented by a factor proportionate to the degree of overload. A timber that does not receive full design load can likewise be represented. While loading of industrial floors frequently approaches or may even exceed the design load, design loads on roofs may never be realized in service. With both uses in mind, it is arbitrarily assumed that the average and most probable load is about 90 percent of the design load, so that the average timber is approximately 10 percent oversize from this cause. A normal frequency distribution with an average value of 1.10 and a standard deviation of 0.08 recognizes the possibility of overload on 1 in 10 of all such timbers. For more specific purposes, the relation of actual to design loads can be estimated more accurately.

**Methods of Combination**

Development of a suitable method for combining a number of frequency distributions by multiplication has proved to be a difficult task. A procedure that gives exact results has not been found. Three approximate methods, (1) the method of random products, (2) the Gram-Charlier series, and (3) the method of Pearson distributions, can be applied to the illustrative example already outlined. The first may be characterized as an empirical method and the last two as analytical, but all require the use of some approximations. These approximations limit the accuracy of the final result, but this inaccuracy is probably no greater than those arising from uncertainties of judgement in defining the component frequency distributions. Descriptions of the three methods follow. A full discussion of the two analytical methods is given in the Appendix.
Method of Random Products

The method of random products is carried out by taking a random sample value from the frequency distribution representing each factor and multiplying the random values together to get a random product. In this instance, 500 of these random products were computed. The random multipliers can be obtained from a table of random numbers, but in this instance, the electronic computer that performed the multiplication generated its own random numbers. The frequency distribution thus obtained is shown as a polygon in figure 2.

The process of obtaining multipliers for random products by use of a table of random numbers is illustrated in table 2. Random numbers 0000 to 9,999 were divided into 39 classes. Class intervals were equal for dealing with the uniform distribution (column 7 of table 2), because in such a distribution each value has an equal chance to appear. The equal intervals were adjusted to make the midpoints of the first and last intervals fall at the limits of the distribution.

With the normal frequency distributions, class intervals were varied to represent the varying frequencies in the normal distribution throughout its range (column 1 of table 2). The widest intervals were made near the median or mean of the distribution to give the greatest frequency of occurrence of the corresponding multipliers.

A multiplier was calculated to correspond to the midpoint of each of the 39 classes of random numbers, and that multiplier was used in place of any random number drawn from that class. These multipliers appear in columns 3, 4, 5, and 6 of table 2 for the normal distributions, and in column 9 for the uniform distribution. To illustrate, in the fourth class of random numbers with normal intervals, the center of that class interval was at \(-3.2\) times the standard deviation. In the frequency distribution representing variability of clear wood, the mean value was 1.000, and the standard deviation was 0.160. Then \(1.000 - (0.160 \times 3.2) = 0.488\), the value appearing in column 3. In like manner, in the fourth class with uniform intervals, \(0.670 - (16/38 \times 0.14) = 0.611\), the value appearing in column 9.

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4 A table of random numbers consists of numbers in a selected range, often 0000 to 9,999, picked at random so that each number has an equal chance of being used. A table for the selected range may be of any length. Any number in the selected range can appear more than once, depending upon the length of the table.
For calculation by the ordinary desk computer, a number may be drawn from the table of random numbers, the corresponding multiplier from table 2 set down, and the process repeated until each of the 5 multipliers has been drawn 500 times. To illustrate, in one random product the first random number drawn may be 8,522, and its corresponding multiplier from column 3 of table 2 is 1.160; this means that the hypothetical timber represented is 16 percent stronger than the species average. The second random number is 5,265, and the corresponding multiplier from column 4 is 1.000, meaning that other factors affecting strength combine in a way that is just average. The third random number is 2,349, and the corresponding multiplier from column 5 is 0.596, implying that the duration of load on the timber is somewhat more than the designer anticipated. The fourth random number is 2,728, and the corresponding multiplier from column 6 is 1.052; this means that the maximum load sustained is about 95 percent of the assumed design load. The fifth random number is 4,217, and the corresponding multiplier from column 9 is 0.659, meaning that the actual strength ratio of the timber is 66 percent. The product of these 5 multipliers is 0.479, which is in the most probable class of values shown in the random products frequency polygon in figure 2. If the working stress on this hypothetical timber is 1/6 of the average strength-test value for the species, its ratio of adequacy is 0.479/0.167, or approximately 2.9.

The random product distribution shown in figure 2 was calculated on a magnetic data-processing machine. The problem was first "programmed;" that is, the information given in table 2 and the procedure for generating the random numbers were coded on punch cards. After the coded cards, about 100 in this instance, were fed into the machine, the process of computing and classifying the 500 random products took about 15 minutes in the computer and an additional 10 minutes in a tabulator to decode and print the results. This type of operation is well adapted to a digital computer, and 1,000 or more of such products could have been calculated without difficulty.

Gram-Charlier Series

In the Gram-Charlier method, the individual frequency distributions are characterized by their moments around zero, the first 6 moments being used. It is shown in the Appendix that each moment of the product frequency distribution is equal to the product of the corresponding moments from each of the individual frequency distributions. In this way the first 6 moments of the product frequency distribution are obtained, and these are enough to show its character. The product frequency distribution is
in the form of a Gram-Charlier series. In this example, only the first 2 terms of the series were used (see Appendix), because preliminary studies had shown that the addition of more terms had little effect on the results. This frequency distribution is shown in figure 2.

Combinations of normal frequency distributions by this method result in a discontinuity at the point where the variate equals 0. The area under the product frequency curve at and near this discontinuity is indeterminate. Since the possibility of values at or near 0 in the frequency distributions used here is extremely remote, the indeterminate area is believed to be negligible.

Pearson Distributions

The problem of discontinuity where the variate equals 0 in combinations of normal frequency distributions can be avoided by the use of frequency distributions that have limits at or above 0. A number of frequency functions of this character have been developed by Karl Pearson (4). The normal frequency distributions can be approximated by Pearson functions of the type \( y = Kx^m (a - x)^m \), where \( x \) is the variate, \( y \) is the frequency, and \( m \) is a positive integer. Such a function has a lower limit of 0 and an upper limit of \( a \) and is symmetrical about the value \( x = \frac{a}{2} \).

In the Appendix the steps are described by which the Pearson functions are combined with each other and with a function representing the uniform frequency distribution. A significant error in this method arises from the method of combination, which requires that values of the exponent \( m \) in the successive functions be chosen in a series of the form \( m_1 = 2m_2 + 1 \) (see Appendix). While any one or two terms in the series can be chosen to make the Pearson functions fit rather closely to the assumed normal frequency distributions, it is difficult to get a good fit in the case of four or more functions. This is discussed more fully in the Appendix. It is recognized that the assumed normal frequency distributions are not exact and that the Pearson functions, in some instances, may be nearly as accurate; at the same time, it is believed that the necessity of using this series definitely handicaps the Pearson method.

The product frequency distribution by the Pearson method is shown in figure 2.
Product Frequency Distributions

Figure 2 also shows product frequency distributions by the three methods superimposed for comparison. The frequency scale is such that the area under each is unity. Since the width of each bar of the random products polygon is 0.05, the factor for conversion of the polygon from relative to absolute frequency is 0.05 times the number of products in the distribution. In this instance, the factor is 0.05 \times 500 = 25 and the absolute frequency is 25 times the ordinate shown in figure 2.

Frequency distributions by the three methods are quite similar in their general shape as the values for means and standard deviations show. The distributions are visibly skewed to the left, showing elongated tails to the right and a peak or mode at a value slightly less than the mean. Statistical analysis of third moments of the product distributions shows that the skewness is highly significant. Analysis of fourth moments does not show a consistent or significant departure from normality because of excessive sharpening or flattening of the peak.

Figure 2 can be used to estimate the range of the true factor of safety. To illustrate, if a working stress is 1/10 of the average for the species, the range of the product distribution is from about 1 to 9 times that value. This is the range of the true factor of safety. With a working stress at 2/10, the range is from about 1/2 to 4 or 5.

Relative probability of failure of an individual piece at a specific working stress value can be estimated by the relative area under the frequency curve to the left of the line that represents that value (fig. 2). Cumulative frequency distributions by the three methods, shown in part in figure 3, are convenient for this purpose. The cumulative distribution is the continuous integral of area under the frequency curve or polygon. If, for example, a working stress for the grade assumed is 1/6 of the average value for the species, the probability of failure is indicated in figure 3 as a small fraction of 1 percent. Figure 3 also shows how rapidly the probability of failure may increase with increased working stresses.

While it is difficult to make a choice among the product frequencies obtained by the three methods used here, it is believed that greater errors are possible by the Pearson than by the other two methods. Further, the Pearson method involves the greatest burden of computational work. Since each of the methods is approximate, all three are shown, partly for confirmation, but also to indicate the limits of accuracy of the results.
Conclusion

This report calls attention to the many factors that affect the adequacy of structural timbers and indicates a means for evaluating their effects. While the factors chosen for illustration are representative, they are based largely on judgment and may be over-simplified. The results indicate general trends, but are not to be taken as precise and accurate answers to the problem. This is emphasized by the differences between the distributions obtained by three mathematical methods, all of which are based on the same assumed factors of strength and use.

The structural engineer may make his own assumptions as to factors of strength and conditions of use, and compute his own estimates of the adequacy of the material, using any of the three methods discussed. This report presents information helpful to engineers in considering the factor of safety as multi-valued instead of single-valued. It shows that safety is affected by use as well as strength, and indicates the relation of the factor of safety to the level of working stresses.
APPENDIX

The Distribution of a Product

The problem of finding the frequency function of a product of 2 normally distributed variates, \( x_1 \) and \( x_2 \), has been studied by Craig (1). He obtained an expansion in the form of an infinite series of Bessel functions and found that the series converged quite rapidly for small values of the ratios \( \frac{m_{x_1}}{\sigma_{x_1}} \), \( \frac{m_{x_2}}{\sigma_{x_2}} \), where \( m \) and \( \sigma \) denote respectively the mean and standard deviation, and the subscripts associate these quantities with the respective distributions. The frequency distribution function \( f(x_1 x_2) \) has a logarithmic discontinuity at the origin. For large values of the ratios, the convergence is slow, and the series does not adequately define the frequency function. The problem of finding the distribution under this condition apparently has not been solved. Unfortunately, the ratio is large for each distribution considered in this report.

Because no indication was found in the literature of how to proceed in determining the frequency function of the product of a number of normal variates together with uniform variates, three approximate methods were attempted. The first, suggested by J. G. Osborne, is described in this report as the "random products method." The second is a method of approximating the product distribution by means of a Gram-Charlier series. The third method is the so-called Pearson distribution. This last method is exact but uses functions that approximate the given distributions.

Gram-Charlier Series

The work of Craig indicates that the distribution of a product has a logarithmic discontinuity at the origin (zero in this instance). It is not expected therefore, that an expansion can be obtained in the form of a Gram-Charlier series that defines the frequency function in the vicinity of this point. In all distributions used in this report, however, the means are far removed from the origin, and the standard deviations are small. The work of Craig is therefore not applicable, and, while it is certain that the discontinuity does occur at the origin, its effect on this product
distribution is believed negligible. On the basis of this assumption, this product distribution function is assumed to be defined by its moments.

The moments of the product distribution are obtained from the moments of the component distributions in the following manner. Suppose $x_1$ and $x_2$ are two variates with frequency functions of $f_1(x_1)$ and $f_2(x_2)$, respectively, and with ranges $a_1 \leq x_1 \leq b_1$ and $a_2 \leq x_2 \leq b_2$. Then, by definition, the $r^{th}$ moment, $\alpha_1^{(r)}$ of the frequency of $x_1$ about the origin ($x_1 = 0$) is

$$\alpha_1^{(r)} = \int_{a_1}^{b_1} x_1^r f_1(x_1) \, dx_1$$  \hspace{2cm} (1)

The $r^{th}$ moment, $\alpha_2^{(r)}$, of the second distribution is

$$\alpha_2^{(r)} = \int_{a_2}^{b_2} x_2^r f_2(x_2) \, dx_2$$  \hspace{2cm} (2)

The $r^{th}$ moment $\beta_r$ of the product distribution about the origin is defined as

$$\beta_r = \int_{a_1}^{b_1} \int_{a_2}^{b_2} (x_1 x_2)^r f_1(x_1) f_2(x_2) \, dx_1 \, dx_2$$  \hspace{2cm} (3)

Since $x_1$ and $x_2$ vary independently, this may be written as

$$\beta_r = \int_{a_1}^{b_1} x_1^r f_1(x_1) \, dx_1 \int_{a_2}^{b_2} x_2^r f_2(x_2) \, dx_2$$

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or, by equations (1) and (2),

\[ \beta_r = \alpha_1^{(r)} \alpha_2^{(r)} \]  

(4)

This may be carried out for \( n \) variates, and the \( r^{th} \) moment of their product is

\[ \beta_r = \alpha_1^{(r)} \alpha_2^{(r)} \alpha_3^{(r)} \ldots \alpha_n^{(r)} \]  

(5)

Thus, the \( r^{th} \) moment of the product distribution about the origin is the product of the \( r^{th} \) moment of the component distributions about their respective origins.

A normal frequency distribution function with mean \( m_1 \) and standard deviation \( \sigma_1 \) is of the form

\[ f_1(x_1) = \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(x_1 - m_1)^2}{2\sigma_1^2}} \]

For such a function,

\[ \alpha_1^{(1)} = m_1 \]

\[ \alpha_1^{(2)} = \sigma_1^2 + m_1^2 \]  

(6)

\[ \alpha_1^{(3)} = m_1 (3\sigma_1^2 + m_1^2) \]

\[ \alpha_1^{(4)} = 3\sigma_1^4 + 6\sigma_1^2 m_1^2 + m_1^4 \]
\[ \alpha_1^{(5)} = m_1 (15 \sigma_1^4 + 10 \sigma_1^2 m_1^2 + m_1^4) \]

(6) continued

\[ \alpha_1^{(6)} = 15 \sigma_1^6 + 45 \sigma_1^4 m_1^2 + 15 \sigma_1^2 m_1^4 + m_1^6 \]

A uniform frequency distribution function with limits \( a_2 \) and \( b_2 \) is of the form

\[ f_2(x_2) = \frac{1}{b_2 - a_2} \]

Therefore, in this case,

\[ \alpha_2^{(r)} = \frac{1}{b_2 - a_2} \int_{a_2}^{b_2} x_2^r \, dx_2 = \frac{b_2^{r+1} - a_2^{r+1}}{(r+1)(b_2 - a_2)} \]  

(7)

Using formula (5) together with (6) and (7), the moments with respect to the origin of the frequency distribution of the product of any number of normal and uniform variates can be computed.

In order to carry out the Gram-Charlier expansion, the moments \( \mu_r \) of the product distribution with respect to its mean are needed. These are obtained in the following manner.

\[ \mu_1 = m = \beta_1 \]

\[ \mu_2 = \sigma^2 = \beta_2 - m^2 \]

\[ \mu_3 = \beta_3 - 3m\beta_2 + 2m^3 \]

\[ \mu_4 = \beta_4 - 4m\beta_3 + 6m^2\beta_2 - 3m^4 \]

(8)
\[ \mu_5 = \beta_5 - 5m\beta_4 + 10m^2\beta_3 - 10m^3\beta_2 + 4m^4 \]  

(8) continued

\[ \mu_6 = \beta_6 - 6m\beta_5 + 15m^2\beta_4 - 20m^3\beta_3 + 15m^4\beta_2 - 5m^6 \]

Let \( u \) denote the product variate, and then let \( v = \frac{u - m}{\sigma} \). The frequency function of \( v \), \( g(v) \) may be expressed in terms of the normal function

\[
\phi(v) = \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}}
\]

and its derivatives

\[
\phi^{(r)}(v) = \frac{d^r \phi(v)}{dv^r}
\]

\[
g(v) = \phi(v) - \frac{1}{3!} \lambda_3 \phi^{(3)}(v) + \frac{1}{4!} \lambda_4 \phi^{(4)}(v) - \frac{1}{5!} \lambda_5 \phi^{(5)}(v)
\]

\[
+ \frac{10\lambda_3^2 + \lambda_6}{6!} \phi^{(6)}(v) - \frac{35\lambda_3\lambda_4}{7!} \phi^{(7)}(v) + \left( \frac{\lambda_4^2}{2(4!)^2} \right)
\]

\[
+ \frac{\lambda_3\lambda_5}{3!5!} \phi^{(8)}(v) - \frac{280}{9!} \lambda_3^3 \phi^{(9)}(v) + \ldots
\]

(9)
where
\[
\begin{align*}
\lambda_3 &= \frac{\mu_3}{\sigma^3}, \\
\lambda_4 &= \frac{\mu_4}{\sigma^4} - 3, \\
\lambda_5 &= \frac{\mu_5}{\sigma^5} - \frac{10\mu_3}{\sigma^3}, \\
\lambda_6 &= \frac{\mu_6}{\sigma^6} - \frac{15\mu_4}{\sigma^4} - 10 \left( \frac{\mu_3}{\sigma^3} \right)^2 + 30
\end{align*}
\]

The frequency function of \( u \) is defined as
\[
f(u) = \frac{1}{\sigma} g \left( \frac{u - m}{\sigma} \right)
\]

Formula (9) is a form of the Gram-Charlier series (2). In this case only the first two terms of the expansion were needed to give a good approximation. If more definition in the tails of the frequency distribution is desired, more terms may be used.

The cumulative function \( h(v) \) is obtained by substituting
\[
\int_{\phi(v)}^{\infty} \phi(v) \, dv
\]

for \( \phi(v) \) throughout formula (9). The cumulative function of \( u \) is then defined as
\[
F(u) = \frac{1}{\sigma} h \left( \frac{u - m}{\sigma} \right)
\]

Pearson Distributions

The method of expansion by Gram-Charlier series may be characterized as one that leads to an approximate solution using component distributions.
exactly as given. Another method approximates given data by using the Pearson type II functions (4) and combining them in an exact manner.

The Pearson functions are polynomials that define a distribution over a finite range. For the purpose of simplifying the analysis, the range \(0 \leq x \leq \alpha\), in which \(\alpha\) is twice the mean of the given normal distribution, has been chosen in each case. The polynomials are then of the form

\[ f(x) = k_n x^n (\alpha - x)^n, \]

where \(n\) is an integer and the factor \(k_n\) is so adjusted that the area under the curve is unity. Specifically,

\[
k_n = \frac{(2n + 1)!}{(n!)^2 \alpha^{2n+1}}
\]  

(13)

The standard deviation of this distribution is

\[
\sigma = \frac{\alpha}{2 \sqrt{2n + 3}}
\]  

(14)

**Normally Distributed Variates**

The following theorem, stated without proof, is a generalization of one stated by Huntington (5).

**Theorem.** --Suppose a variable \(x_1\) is distributed in accordance with a probability law

\[
\int_0^{\alpha_1} f_1(x_1) \, dx_1 = 1
\]

and a variable \(x_2\) in accordance with a probability law

\[
\int_0^{\alpha_2} f_2(x_2) \, dx_2 = 1
\]  

\[\tag{14}
\]
\( x_1 \) and \( x_2 \) being independently distributed. Then, the product \( y_2 = x_1 x_2 \) is distributed according to the law

\[
\int_0^{\alpha_1 \alpha_2} g(y_2) \, dy_2 = 1
\]

where

\[
g(y_2) = \int_{\frac{y_2}{\alpha_2}}^{\alpha_1} f_1(x_1) f_2 \left( \frac{y_2}{x_1} \right) \frac{dx_1}{x_1}
\]

(15)

The proof of this theorem follows without difficulty from the laws of probability of independent events and the fundamentals of the calculus.

In the process of combining the Pearson distributions according to formula (15), it is convenient to have on hand the evaluation of the following integral:

\[
I = \int_a^b (x - a)^m (b - x)^n \, dx
\]

where \( m \) and \( n \) are integers.

Let \( x - a = y(ba) \)

so that \( dx = dy(b - a) \)

and \( b - x = (b - a)(1 - y) \)

Then,

\[
I = (b - a)^{m+n+1} \int_0^1 y^m (1 - y)^n \, dy
\]
The integral on the right is a Beta function, the evaluation of which gives

\[ I = (b - a)^{m+n+1} \frac{m!n!}{(m + n + 1)!} \]  \hspace{1cm} (16)

Let

\[ f_1(x_1) = K m_1 x_1^{m_1} (\alpha_1 - x_1)^{m_1} \]

\[ f_2(x_2) = K m_2 x_2^{m_2} (\alpha_2 - x_2)^{m_2} \]

\[ f_3(x_3) = K m_3 x_3^{m_3} (\alpha_3 - x_3)^{m_3} \]  \hspace{1cm} (17)

\[ \ldots \]

\[ f_i(x_i) = K m_i x_i^{m_i} (\alpha_i - x_i)^{m_i} \]

be the frequency functions of a number of variates \( x_1, x_2, \ldots, x_i \).

And let

\[ y_2 = x_1 x_2 \]

\[ y_3 = x_1 x_2 x_3 \]

\[ \ldots \]

\[ y_i = x_1 x_2 \ldots x_i \]  \hspace{1cm} (18)
The problem is to determine the distribution of $y_i$, that is, the distribution of the product of variates $x_1, x_2, \ldots, x_i$. In general, the frequency function $g_i(y_i)$ is a polynomial in $y_i$, which is too complicated for use in computation. However, if the exponents $m_i$ are chosen in the sequence

\[
\begin{align*}
m_1 &= 2m_2 + 1 \\
m_2 &= 2m_3 + 1 \\
&\cdots \\
m_{i-1} &= 2m_i + 1
\end{align*}
\]  

(19)

the polynomial is of the form

\[
g_i(y_i) = A_i y_i^{m_i} (\beta_i - y_i)^{n_i}
\]  

(20)

where

\[
\begin{align*}
\beta_i &= \alpha_1 \alpha_2 \cdots \alpha_i \\
n_i &= m_1 + m_2 + \cdots + m_i + i - 1
\end{align*}
\]  

(21)

The factor $A_i$ is so determined that the area under the curve from 0 to $\beta_i$ is unity. That is

\[
\int_0^{\beta_i} g_i(y_i) \, dy_i = 1
\]
\[ l = A_i \int_0^{\beta_i} y_1^{m_i} (\beta_i - y_i)^{n_i} \, dy_i \]

By formula (16)

\[ l = A_i \beta_i \frac{m_i!n_i!}{(m_i + n_i + 1)!} \]

so that

\[ A_i = \frac{(m_i + n_i + 1)}{m_i!n_i!\beta_i} \]

To establish the form (20), substitute \( f_1(x_1) \) and \( f_2(x_2) \) as given by (17) into (15) to obtain

\[ g_2(y_2) = K_{m_1} K_{m_2} \int_{\frac{y_2}{\alpha_2}}^{\alpha_1} x_1 \frac{m_1}{x_1} \left( \frac{y_2}{x_1} \right)^{m_2} \left( \frac{\alpha_2 - y_2}{x_1} \right)^{m_2} \, dx_1 \]

\[ = K_{m_1} K_{m_2} \frac{\alpha_1}{\alpha_2} y_2 \frac{m_2}{x_1} \int_{\frac{y_2}{\alpha_2}}^{\alpha_1} x_1 \left( \frac{m_1 - 2m_2 - 1}{x_1} \right)^{m_1} \left( \frac{x_1 - y_2}{\alpha_2} \right)^{m_2} \, dx_1 \]

Now since \( m_1 = 2m_2 + 1 \), the integral can be evaluated by (16). Thus,
The method of obtaining formulas (27) and (28) based on analysis of the plane bounded by the limits in (26) has been developed by the U. S. Forest Products Laboratory.

To carry out the integration, let

\[
\frac{u}{a} \int_{\frac{u}{b}}^{\frac{u}{a}} g(y) h\left(\frac{u}{y}\right) \frac{dy}{y} = \int_{\frac{u}{b}}^{\frac{u}{a}} g(y) h\left(\frac{u}{y}\right) \frac{dy}{y} - \int_{\frac{u}{a}}^{\frac{\beta}{a}} g(y) h\left(\frac{u}{y}\right) \frac{dy}{y} \tag{29}
\]

Then, formula (16) can be applied to obtain the following function.
\[ f(u) = \frac{A}{L} \left\{ (\beta - \frac{u}{a})^{190} \frac{10! 179!}{190!} + 10 \left( \frac{u}{a} \right) (\beta - \frac{u}{b})^{189} \frac{9! 179!}{189!} + 45 \left( \frac{u}{b} \right)^2 (\beta - \frac{u}{b})^{188} \frac{8! 179!}{188!} \right. \\
+ 120 \left( \frac{u}{b} \right)^3 (\beta - \frac{u}{b})^{187} \frac{7! 179!}{187!} + 210 \left( \frac{u}{b} \right)^4 (\beta - \frac{u}{b})^{186} \frac{6! 179!}{186!} + 252 \left( \frac{u}{b} \right)^5 (\beta - \frac{u}{b})^{185} \frac{5! 179!}{185!} \\
+ 210 \left( \frac{u}{b} \right)^6 (\beta - \frac{u}{b})^{184} \frac{4! 179!}{184!} + 120 \left( \frac{u}{b} \right)^7 (\beta - \frac{u}{b})^{183} \frac{3! 179!}{183!} + 45 \left( \frac{u}{b} \right)^8 (\beta - \frac{u}{b})^{182} \frac{2! 179!}{182!} \right. \\
+ 10 \left( \frac{u}{b} \right)^9 (\beta - \frac{u}{b})^{181} \frac{179!}{181!} + \left( \frac{u}{b} \right)^{10} (\beta - \frac{u}{b})^{180} \frac{1}{180} - (\beta - \frac{u}{a})^{190} \frac{10! 179!}{190!} \right. \\
- 10 \left( \frac{u}{a} \right) (\beta - \frac{u}{a})^{189} \frac{9! 179!}{189!} - 45 \left( \frac{u}{a} \right)^2 (\beta - \frac{u}{a})^{188} \frac{8! 179!}{188!} - 120 \left( \frac{u}{a} \right)^3 (\beta - \frac{u}{a})^{187} \frac{7! 179!}{187!} \right. \\
- 210 \left( \frac{u}{a} \right)^4 (\beta - \frac{u}{a})^{186} \frac{6! 179!}{186!} - 252 \left( \frac{u}{a} \right)^5 (\beta - \frac{u}{a})^{185} \frac{5! 179!}{185!} - 210 \left( \frac{u}{a} \right)^6 (\beta - \frac{u}{a})^{184} \frac{4! 179!}{184!} \right. \\
- 120 \left( \frac{u}{a} \right)^7 (\beta - \frac{u}{a})^{183} \frac{3! 179!}{183!} - 45 \left( \frac{u}{a} \right)^8 (\beta - \frac{u}{a})^{182} \frac{2! 179!}{182!} \right. \\
- 10 \left( \frac{u}{a} \right)^9 (\beta - \frac{u}{a})^{181} \frac{179!}{181!} - \left( \frac{u}{a} \right)^{10} (\beta - \frac{u}{a})^{180} \frac{1}{180} \right\} \\
\]

A comparison of the frequency function obtained by the three methods is given in figure 2.

Report No. 2068
(1) Craig, Cecil C.

(2) Cramer, Harold

(3) Freudenthal, Alfred M.
    1956. Safety and the Probability of Structural Failure. Transactions American Society of Civil Engineers 121.

(4) Fry, Thornton C.

(5) Huntington, E. V.

(6) Markwardt, L. J., and Wilson, T. R. C.

(7) Tucker, John, Jr.

(8) Wood, L. W.
<table>
<thead>
<tr>
<th>Factor</th>
<th>Type of distribution</th>
<th>Most probable value</th>
<th>Limiting values</th>
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<td>Variability of clear wood</td>
<td>Normal</td>
<td>1.00</td>
<td>0.16</td>
</tr>
<tr>
<td>Range of defects within grade</td>
<td>Uniform</td>
<td></td>
<td>0.60 0.74</td>
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<tr>
<td>Other factors affecting strength</td>
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<td>0.20</td>
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<tr>
<td>Duration of load</td>
<td>Normal</td>
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<td>0.03</td>
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<tr>
<td>Expected versus actual load</td>
<td>Normal</td>
<td>1.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Intervals in: Multiples of table of standard deviation of random numbers</td>
<td>Multipliers corresponding to centers of intervals: Variability: Other factors: Duration: Expected clear wood: strength: number of intervals: of centers: of load: versus: actual load:</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
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<td>0000</td>
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<td>0.240</td>
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<td>0.424</td>
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<td>0.542</td>
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<td>0.572</td>
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<td>0.800</td>
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<td>0.920</td>
<td>0.608</td>
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<tr>
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<td>0.960</td>
<td>0.614</td>
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<td>4,602</td>
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<td>1.000</td>
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<td>1.040</td>
<td>0.626</td>
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<td>1.064</td>
<td>1.080</td>
<td>0.632</td>
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<td>1.096</td>
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<td>0.644</td>
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<td>1.200</td>
<td>0.650</td>
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<td>9,032</td>
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<td>9,332</td>
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<td>1.320</td>
<td>0.668</td>
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<td>9,594</td>
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<td>1.360</td>
<td>0.674</td>
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<td>9,713</td>
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<td>1.400</td>
<td>0.680</td>
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<td>9,821</td>
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<td>1.440</td>
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<td>1.600</td>
<td>0.710</td>
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<td>9,999</td>
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<td>0.734</td>
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Table 2.--Use of random numbers 0000 to 9,999 to find multipliers for random products.
<table>
<thead>
<tr>
<th>Variate</th>
<th>Factor</th>
<th>Exponent used in Pearson function</th>
<th>Standard deviation in Pearson</th>
<th>Standard deviation in Normal</th>
<th>Average value of both functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>Duration of load</td>
<td>95</td>
<td>0.045</td>
<td>0.03</td>
<td>0.62</td>
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<td>$X_2$</td>
<td>Expected versus actual load</td>
<td>47</td>
<td>0.111</td>
<td>0.08</td>
<td>1.10</td>
</tr>
<tr>
<td>$X_3$</td>
<td>Variability of clear wood</td>
<td>23</td>
<td>0.143</td>
<td>0.16</td>
<td>1.00</td>
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<tr>
<td>$X_4$</td>
<td>Other factors affecting strength</td>
<td>11</td>
<td>0.200</td>
<td>0.20</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Figure 1.--Types of frequency distribution; A, normal; B, uniform.
Figure 2. -- Product frequency distribution by three methods. (Frequency scale is adjusted so that the areas under the two curves and the random products polygon are each equal to unity.)
Figure 3. -- Portion of cumulative frequency distribution by three methods.
Figure 4. - Comparison of four normal frequency functions with their Pearson approximations (see Table 3).
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FOREST PRODUCTS LABORATORY

Boxing and Crating -- Strength and serviceability of shipping containers, methods of packing.

Building Construction Subjects -- Partial list of Government publications of interest to architects, engineers, builders, and retail lumbermen.

Chemistry of Wood and Derived Products -- Chemical properties and uses of wood and chemical wood products, such as turpentine, alcohol, and acetic acid.

Fire Protection -- Fire test methods, fire retarding chemicals and treatments, and fire behavior of treated and untreated wood, wood products, and wood structures.

Fungus Defects in Forest Products -- Decay, stains, and molds in timber, buildings, and various wood products; antiseptic properties of protective materials.

Furniture Manufacturers, Woodworkers and Teachers of Wood Shop Practice -- Partial list of Government publications on growth, structure, and identification of wood; moisture content, physical properties, air seasoning, and kiln drying; grading, manufacturing, and waste utilization; strength and related properties and joints and fastenings; glues and gluing, veneer and plywood fabrication; box and crate construction.

Glue and Plywood -- Development of waterproof glues, preparation and application of various glues, plywood manufacturing problems.

Report No. 2068 -38-
Growth, Structure, and Identification of Wood -- Structure and identification of wood; the effect of cellular structure of wood on its strength, shrinkage, permeability, and other properties; the influence of environmental factors such as light, soil, moisture, and fire, on the quality of wood produced; and secretions of economic value produced by trees and their exploitation.

Logging, Milling, and Utilization of Timber Products -- Methods and practices in the lumber-producing and wood-consuming industries; standard lumber grades, sizes, and nomenclature; production and use of small dimension stock; specifications for small wooden products; uses for little-used species and commercial woods; and low-grade and wood waste surveys.

Mechanical Properties of Timber -- Strength of timber and factors affecting strength; design of wooden articles or parts where strength or resistance to external forces is of importance.

Pulp and Paper -- Suitability of various woods for pulp and paper; fundamental principles underlying the pulping and bleaching processes; methods of technical control of these processes; relation of the chemical and physical properties of pulps and the relation of these properties to the paper making qualities of the pulps; waste in the industry, for example, decay in wood and pulp, utilization of bark, white water losses, etc.

Seasoning of Wood -- Experimental and applied kiln drying, physical properties, air drying, steam bending.

Structural Sandwich, Plastic Laminates, and Wood-Base Aircraft Components -- Strength, selection, and character of aircraft wood, plywood, and wood and composite laminated and sandwich materials; fabrication and assembly problems; methods of calculating the strength.

Report No. 2068 -39-
Wood Finishing Subjects -- Effect of coatings in preventing moisture absorption; painting characteristics of different woods and weathering of wood.

Wood Preservation -- Preservative materials and methods of application; durability and service records of treated and untreated wood in various forms.

Note: Since Forest Products Laboratory publications are so varied in subject matter no single big list is issued. Instead a list is made up for each Laboratory division. Twice a year, December 31 and June 30, a list is made up showing new reports for the previous 6 months. This is the only item sent regularly to the Laboratory's mailing list. Any one who has asked for and received the proper subject lists and who has had his name placed on the mailing list can keep up to date on Forest Products Laboratory publications. Each subject list carries descriptions of all other subject lists.