EVALUATION OF THE STIFFNESS OF A
ROOF SYSTEM MADE OF GLUED-
LAMINATED BEAMS AND HEAVY
TIMBER DECKING

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EVALUATION OF THE STIFFNESS OF A ROOF SYSTEM MADE OF
GLUED-LAMINATED BEAMS AND HEAVY TIMBER DECKING

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Summary

Glued-laminated beams with heavy timber decking are widely used as roof systems. Few data are available, however, concerning the deflection of such constructions under load and the change of deflection with service.

An addition was built to the fire-test building at the U.S. Forest Products Laboratory early in 1960 that was about 28 feet wide and 40 feet long. The roof system was made of glued-laminated beams in conjunction with 3- by 6-inch western redcedar decking.

The stiffness of the beams and of a sample lot of decking was determined before erection. After the roof was finished, it was loaded to a uniform live load of 40.5 pounds per square foot. The deflection of the beams and of representative runs of decking was measured during loading and at various times after loading.

From the results of this investigation, it was concluded that the decking did not contribute appreciably to the stiffness of the beams and that the roof decking did not deflect as a continuous beam of uniform stiffness.

A mathematical development of elastic curves for tapered beams is presented in the Appendix.

Introduction

Structural members of glued-laminated wood are well suited for many types of buildings, including schools, churches, auditoriums, gymnasiums, and warehouses. Frequently, timber decking of nominal 3- or 4-inch thickness and random length is used as the structural roof material in conjunction with

1Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

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laminated beams or arches. The decking is usually of a double tongue-and-
groove pattern and spans the distance between the heavy laminated structural
beams or arches. The combination of laminated members and decking provides a
"finished" ceiling that has particular architectural appeal for many applica-
tions. The need for purlins and false ceilings can also be eliminated and in
most cases additional roof insulation is not required.

The timber decking is supplied in random lengths and applied in a random order
but there are limitations on factors, such as the spacing of end joints, dis-
tribution of lengths, and spans for various thicknesses and species. Individ-
ual runs of decking are spiked to the adjacent runs through predrilled holes,
and each run is also spiked to the laminated beam or arch. Hence, the decking
is essentially a large diaphragm intended to resist adequately direct dead and
live loads and to provide shear rigidity of the roof system. A built-up roof
or other covering protects the outside of the decking from the weather.

It has been suggested that the decking may serve another purpose—that is, to
add stiffness to the roof assembly when the decking is on the compressive
face of the laminated member subjected to bending. With simply supported
laminated beams, for example, adjacent runs of decking in juxtaposition might
result in an effective moment of inertia of the beam-decking combination that
is somewhat higher than that of the moment of inertia of the beam alone. If
individual runs of decking are not in contact, such as might occur after
shrinkage of decking in service, then the decking could not be expected to
contribute to stiffness. Since deflection rather than strength governs in
many designs, it is desirable to know whether or not the decking does contrib-
ute to stiffness. If it does, somewhat smaller beams could be used where
deflection is the factor governing design.

The deflection of the timber decking is also of interest, since deflection
rather than strength usually governs the allowable spans that are permitted.
Limited tests have been made of panels of double tongue-and-groove decking,
including tests of continuous panels over two spans. The general conclusion
from these earlier tests was that the usual engineering formulas for deflec-
tion of continuous beams of uniform stiffness would apply, except that the
effective width should be reduced because of the presence of end joints.

In 1959, the U.S. Forest Products Laboratory started erection of an addition
to the existing fire-test house in order to provide further facilities for
research on the fire resistance of wood. The building is about 28 feet wide
and 40 feet long, the walls are of concrete block, and each of four laminated
beams span the width. Nominal 3- by 6-inch western redcedar decking consti-
tutes the roof, supported on one end by a beam adjacent to the existing build-
ing and on the other by a concrete block wall. With the construction of this
building, it was possible to determine the stiffness of each laminated beam
before erection and the stiffness of each beam-decking combination. Opportu-
nity also was provided to make measurements of the deflection characteristics
of a large panel of heavy timber decking supported at five reactions. Furth-
more, erection of the building provides the opportunity of (1) observing the
long-time performance of the roof system, (2) making stiffness determinations
of the beams and decking at any time deemed desirable, and (3) observing the
creep of the beams and decking after various periods of service.

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This report presents the results of tests of four laminated beams before and after erection, showing the effect of the roof decking on the stiffness, and data on the deflection characteristics of the timber decking. The work was done at the Forest Products Laboratory in cooperation with Unit Structures, Inc., of Peshtigo, Wis.  

Description of Material

Beams

Four glued-laminated beams of southern pine were laminated at and purchased from Unit Structures, Inc. Specifications called for the beams to be 5-1/4 inches wide, 16-1/4 to 18-1/4 inches deep with a single taper, 28 feet long, Industrial grade southern pine, laminated with a waterproof adhesive, and surfaced four sides but without sealer or other surface finish. The grade combination was not specified, but beams were to be capable of withstanding a roof load of 50 pounds per square foot in service.

The beams were delivered to the Forest Products Laboratory on January 15, 1960. Spot checks with a portable electric moisture meter showed that the moisture content was about 8 percent at 1/4-inch depth and about 10 percent at 1-1/8-inch depth. The beams were then wrapped in a polyethylene film that was retained during storage, except for the time required to attach necessary fittings, apply a sealer, and make tests. The wrapping was also retained on the beams during erection of the building and for about 2 months thereafter.

Each beam was cut to a length of 26 feet 11-1/2 inches to correspond to the exact length required in the building. Anchor plates (shoes) that were provided were 5-1/2 inches wide and 8 inches long. The effective span of the beams, with anchor plates in place, was therefore 26 feet 3-1/2 inches, if simple support conditions are assumed.

The beams, unfinished when they were received, were coated with two applications of conventional clear wood sealer before erection in the building. In the late spring of 1960, one coat of satin-finish interior varnish was also applied.

Decking

Nominal 3- by 6-inch Standard grade western redcedar roof decking was purchased and spikes were supplied with the decking. Standard grade is equal to or better than the following when graded in accordance with the Standard Grading and Dressing Rules of the West Coast Lumber Inspection Bureau:

Acknowledgement is made to Maurice J. Rhude, Vice President and Chief Engineer of Unit Structures, Inc., for assistance and suggestions in developing and conducting this evaluation.
20 percent Select Merchantable
60 percent No. 1 Common or Better
20 percent No. 2

In addition, there are requirements for minimum length, in that 40 percent or more of the board footage shall be 14 feet or longer and 90 percent shall be 10 feet or longer. Following is a tally of the material received at the Laboratory:

<table>
<thead>
<tr>
<th>Length (Ft.)</th>
<th>Lineal Feet</th>
<th>Percentage of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 to 9</td>
<td>104</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>209</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>348</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>247</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>914</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>280</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>960</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>1,690</td>
<td></td>
</tr>
</tbody>
</table>

When received at the Laboratory, the decking had a moisture content of about 15 percent, so it was placed in a kiln and dried to the desired level of about 10 percent. A moisture content of 10 percent was considered to be between the minimum and maximum that the decking would attain in service. Kiln drying was done in accordance with a mild schedule. After drying, the material was removed from the kiln and bulk piled, except that every sixth piece 11 feet in length or longer was set aside for stiffness tests. The decking was stored indoors until ready for installation, which was started within a week after kiln drying.

**Tests Before Erection**

**Beams**

The cross section of each beam was measured at each end reaction and at the center and quarter points. Slight variations from an exactly uniform taper were observed, as expected, but these variations were reasonable. For purposes of calculation, it was assumed that the taper was at a constant rate, and depth adjustments were made accordingly. Each beam also was weighed, the moisture content determined with an electric moisture meter, and the specific gravity calculated. Information on each of the beams is presented in table 1.

Permanent reference marks were placed on each beam so that center deflection could be determined in the static bending test and also after the beams were
erected. Brass disks, 1 inch in diameter and 1/2 inch long, were prepared with a shallow circumferential groove near one edge. A hole was drilled in the center of each disk so that the disk could be screwed to the side of the beam. Disks were fastened to the side of a beam at midheight and on a vertical line directly above the inner edge of the anchor plates (307-1/2 inches apart). The shallow groove was on the outer edge. A steel scale with 0.01-inch graduations was securely fastened to the center of the beam, and a strip of mirror was placed adjacent to it. A 6-pound-test nylon filament line was supported in the top grooves of the disks and was kept taut with a 3/4-pound weight hung from one end. The scale reading was determined with the aid of the mirror to eliminate parallax at the eye-level position.

Reference marks and a scale were placed on only the east face of beam 1 because the other face would be adjacent to the wall of the existing building. On the other three beams, reference marks and a scale were placed on both sides.

Beams were tested in static bending in a million-pound-capacity screw-type testing machine. Each beam was simply supported over a span of 26 feet 3-1/2 inches, and loaded at the quarter points. Load was applied through wood blocks with a radius of 36 inches. Each block was separated from the beam with a steel plate of about 3/32-inch thickness. The load at one reaction was measured with a hydraulic capsule and weighing mechanism (fig. 1), and thus represented one-half of the load on the beam. Each end reaction was mounted on roller beds to prevent horizontal components of force. Figure 2 is an end view of a beam in the testing machine.

Load-deflection data were taken at 1,000-pound increments of load. Center deflection was measured on both sides using a taut line and scale. Load was applied continuously at a head speed of 0.13 inch per minute to a maximum of 10,000 pounds. The outer fiber stress at maximum load was about 1,600 pounds per square inch. The load was then reduced to 1,000 pounds and deflection was again read on both sides. The deflection was measured over a distance of 307.5 inches instead of the total span of 315.5 inches.

As soon as each test was completed, the beams were removed from the testing machine and rewrapped in polyethylene film. They were then stored indoors until erected in the building.

Decking

Thirty pieces of decking were used to determine the modulus of elasticity of a representative sample of the decking. As mentioned earlier, every sixth piece 11 feet long or longer was set aside for these tests when removed from the kiln. These pieces were considered representative of the lot.

The dimensions, moisture content, and weight of each piece were determined at the time of test. Moisture content was measured with a portable electric moisture meter at 1-1/8-inch needle penetration. Each piece was then placed in a testing machine and center loaded as a beam over a span of 10 feet.
Load was applied at a head speed of 0.3 inch per minute. Center deflection was measured by means of a taut wire and scale; load-deflection data were observed at each 20-pound increment of load up to a maximum of 160 pounds. The outer fiber stress at maximum load was less than 900 pounds per square inch. The specimen was then unloaded and removed from the machine.

The modulus of elasticity of each piece was determined from the slope of the load-deflection curve and the dimensions of the specimen. Minimum, maximum, and average values of modulus of elasticity and other properties are presented in table 2.

**Erection of Roof Assembly**

The beams were placed in position in the building during February 1960 and individual beams were spaced 9 feet 8-3/4 inches apart. The 3- by 6-inch roof decking was installed within a few days. A built-up roofing was then applied over the decking so that the decking and beams were fully protected from the weather. The polyethylene film around the beams was kept in place, however, to minimize any moisture changes associated with further building construction.

The heavy timber decking was applied in the manner commonly used by the building industry with the exception that lengths were laid down according to a predetermined pattern. The pattern of lengths was selected in an attempt to set up severe concentrations of end joints and lengths within certain pattern requirements so as to obtain the minimum stiffness. In setting up patterns, the requirements that were followed included:

(a) All end joints in the same general line had to be separated by at least two intervening runs.

(b) No floaters were permitted; each length of decking extended over at least one support.

(c) A minimum distance of 4 feet between joints in adjacent runs was required.

(d) Available lengths as furnished in an order had to be utilized with reasonable efficiency. The supplier reported that the lengths furnished with this order conformed with a typical specification.

Deflections were to be measured on various runs of decking. It was evident, however, that the location of joints in adjacent runs would affect the deflection in any selected run so, arbitrarily, it was decided that seven runs would constitute a pattern, and deflections would be measured over the center run of each pattern. Each pattern was to comply approximately with the requirements stated above. In patterns 1, 2, and 3, an effort was made to concentrate end joints and short lengths in the bay nearest the existing fire-test house, using locations reasonably near the area of maximum deflection. A broad general pattern was also selected and used in two different seven-run
patterns. A top view of the entire decking pattern that was finally used is shown in figure 4.

A tally of the different patterns showed the following distribution of lengths, expressed in terms of surface measure:

<table>
<thead>
<tr>
<th>Pattern</th>
<th>14 feet longer</th>
<th>10 feet longer</th>
<th>Less than 10 feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38</td>
<td>88</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>88</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>38</td>
<td>82</td>
<td>18</td>
</tr>
<tr>
<td>General</td>
<td>49</td>
<td>90</td>
<td>10</td>
</tr>
</tbody>
</table>

Hence, all of the patterns contained a somewhat higher proportion of short lengths than would be expected from the proportional distribution in a shipment. The patterns, however, seem reasonable as to the distribution that might occur.

The first run was nailed in place starting on the north side, and successive runs were then added in accordance with the pattern. Each run of decking was spiked to the previous run with 8-inch-long spikes that had a diameter of about 0.207 inch. The spikes had annular grooves on the point end for a distance of about 3-3/4 inches from the point. The spikes were inserted in pre-drilled holes spaced about 30 inches apart and driven tight. Each piece of decking was also spiked to the previous run at about 6 inches from each end. The decking was fastened to each support with a single fortypenny spike, face nailing through the decking to the beam. End splines were not used.

**Tests After Erection**

On March 1, 1960, the entire roof assembly was tested by live loading. The built-up roofing was covered with a protective layer of 55-pound asphalt roofing paper and the top surface was marked into 1-foot squares. This condition was considered to be that representing a live load of 0.5 pound per square foot. At that time, the deflections of the beams and decking were read; details of each method of measurement will be described later.

When the initial readings were completed, a 20-pound concrete block was placed in alternate squares of the roof, corresponding to a live load of 10.5 pounds per square foot. Deflection readings were observed and recorded. The alternate squares of the roof were then filled with 20-pound blocks, and deflection readings were made at a live load of 20.5 pounds per square foot. The loading and reading process was repeated for loadings of 30.5 and 40.5 pounds per square foot. Figure 5 shows the workmen adding the last few blocks at the maximum load condition. When all readings were completed, the live load was removed and deflections at a live load of 0.5 pound per square foot were again measured.
It was impractical to load the roof assembly simultaneously so that a uniform live load was on the structure at all times during loading. An attempt was made, however, to apply and remove blocks in such a manner that the loading sequence would not significantly affect the deflection at any condition of uniform load. For example, the first set of blocks were put in a row at the top of the south wall and loading proceeded in rows towards the north until the uniform live load of 10.5 pounds per square foot was in place. The next increment of loading started from the north side and loading proceeded to the south. In unloading, a similar procedure was followed. After these tests were completed, the protective layer of asphalt roofing was removed and the live load was thus reduced to zero. Two sets of deflection readings at zero live load were taken on March 2, one set in the morning and another set in the afternoon. Readings were also taken on March 7 and September 21, 1960, and on May 5 and September 14, 1961. The deflections observed during the test on March 1 and at zero live load at later dates are presented in tables 3 and 4.

Beams

Methods of determining the beam deflections are identical to those described for the condition of quarter-point loading. Readings were made on the east side of beam 1 and on both sides of each of the other three beams. The scale, mirror, and taut nylon line at the center of a beam are shown in figure 6.

Decking

Measurements of decking deflection must be made to a high degree of accuracy, since the decking deflections are relatively small for this specific structure. Furthermore, such measurements must be based on permanent and positive reference points so that readings can be made periodically with the desired accuracy. The measurement of deflections on the underside of a large roof panel poses many problems, but a practical and acceptable technique was developed for this work.

It was impossible to measure deflection of any run from the center of a reaction, so the accepted alternative was to establish base points as close to the edge of each beam as possible. From these base points, the deflection of a run of decking could then be measured at some point between the beams. For a four-span beam of length $L$, of uniform stiffness and uniformly loaded, the point of maximum deflection is at a point $0.440L$ from the outer support for the two outer spans and $0.460L$ from the inner support for the two inner spans. The span was 9 feet 8-3/4 inches, so the point of theoretical maximum deflection for the outer span was at 7 inches and for the inner span at 4-3/8 inches from the center of the span. It was decided that the deflection at these points would be measured with respect to the established base points, and that such deflection would be measured for the center run of each decking pattern in each of the four bays. In addition, deflections would be measured in bay No. 1 adjacent to the roof opening. Hence, deflection was measured in 21 positions. These positions are shown as closed circles in figure 4.
A deflection-measuring yoke was designed to meet the requirements needed. It consisted of 5/8-inch-outside-diameter aluminum tubing welded to short aluminum bars to form a stiff but lightweight assembly (fig. 7). Adjustable stainless steel hanger pins with spherical heads were fitted into each end block (fig. 8), and these pins in turn were fit into special hangers made of brass (fig. 9). The longitudinal hole in each hanger was drilled and reamed so its radius was identical to the radius on the spherical head of the hanger pin. Also, positive positioning stops were placed on each hanger so the deflection-measuring yoke would be in an identical position at any time that repetitive measurements might be made at a specific location. Each hanger was positioned so that the yoke would hang at the center of the run of decking, and was securely fastened to the underside of the decking with two wood screws.

Two 1-inch micrometers were mounted in short lengths of an aluminum bar that was welded between the two top pieces of tubing of the yoke (fig. 10). One micrometer was 7 inches off center and the other 4-3/4 inches off center. Vertical adjustment of the micrometers was possible by use of set screws that held them in place. Small brass plates, 1/8 by 1-1/2 by 1-1/2 inches, were screwed to the underside of the decking on each run where deflection was to be measured. These plates provided a positive point from which deflection could be measured with respect to the hangers.

Techniques were developed so that deflection measurements could be read and duplicated to the nearest 0.0005 inch. The yoke was suspended between hangers, and a taut nylon line was set into shallow grooves machined into the top of each hanger pin (fig. 8). The suspended yoke with this line in place is shown in figure 7. Each micrometer was set to read 0.500 and was adjusted vertically so that the tip of the micrometer would just contact the line. The micrometer was then locked in position with the set screw. In figure 10, the left micrometer is in this position. This technique, in effect, fixed the top of the micrometer at a reading of 0.500 inch with respect to the hanger pins which in turn were positively positioned in relation to the bottom of the decking by the hangers.

In reading the deflection of any run of decking, the yoke was placed in position and the micrometer was turned until it just made contact with the brass plate. It was difficult to observe this contact visually because of lack of headroom, so an electrical signal was improvised (fig. 11). As soon as the micrometer tip made contact with the brass plate, an electrical circuit was completed and the flashlight was turned on. The micrometer was then read to the nearest 0.001 inch, and the deflection of the decking was determined relative to the arbitrary condition equivalent to zero live load.

The distance between beams (supports) of the decking was 9 feet 3-1/2 inches and, for deflection limitations of decking in structures, deflection might be measured by considering the inner edge of the support as zero deflection. The hangers were 1 inch inside the supports, so deflection was actually measured over a span of 9 feet 1-1/2 inches. The difference in deflection due to the 2-inch difference between these two spans is not of practical significance and will be disregarded in further discussions in this report.
Discussion of Data Relating to Beams

Load-deflection data from the tests under quarter-point loading showed a linear relationship throughout the range of loading. At the center, the observed deflection was for a distance of 307.5 inches, but the actual deflection should have been measured over the full 315.5-inch span. It can be shown that the actual deflection was about 4 percent greater than the observed values, and this correction was made when calculating the modulus of elasticity of each beam.

The modulus of elasticity of each beam was calculated from the slope of the load-deflection curve, using the formula

\[ E = \frac{K}{768} \frac{W L^3}{\delta I_1} \]

where:
- \( K \) = the taper factor, depending upon beam taper; it is described in Appendix I.
- \( W \) = the total applied load, pounds
- \( L \) = the beam span, inches
- \( \delta \) = the center deflection due to load \( W \), inches. The observed deflection was multiplied by 1.04 to obtain deflection over a span of 315.5 inches.
- \( I_1 \) = the moment of inertia of the smaller end of the beam at a support, inches\(^4\).

This is similar to the conventional engineering formula used for nontapered beams, except for \( K \), and that \( I_1 \) is based on the smaller dimensions of the beam. The calculated modulus of elasticity for each beam is shown in table 1.

After the dimensions and modulus of elasticity of each beam were known, it was possible to consider each beam as a weighing platform when installed in the building. Roof decking and built-up roofing were applied as described earlier. If each beam was simply supported and if a uniform load was placed along its length, then the deflection of the beam alone would be in accordance with the formula for the deflection of a tapered beam under uniform load. On the other hand, if the decking contributed to stiffness, the deflection would be less than that predicted by assuming the decking to be ineffective.

Table 3 is a tabulation of the center deflection of the beams at various times after erection in the building. The first line represents the deflection due to the dead load of the roof system. This deflection is the difference between the readings noted when the beam was in the testing machine on February 8 and when it was in place ready for load tests of the roof system on March 1. The initial readings taken on March 1 were at a live load of 0.5 pound per square foot, just prior to loading the roof with concrete blocks. This initial reading was then corrected to a condition of zero live load, based on the data obtained during the live load tests, and the corrected reading considered to be the zero base. Deflections are considered as zero at
zero live load and they increase with increasing increments of live load. A minus deflection designates downward deflection from the zero base, while a plus deflection designates upward deflection.

**Deflection Due to Dead Load**

The dead load was approximately 7.5 pounds per square foot. The deflections noted, however, were as much as double those expected from the dead load. The reason for this is not explained, since (1) there appeared to be no significant change in moisture content between February 8 and March 1, (2) the beams would not be expected to creep appreciably during the short interval of time under low stress, and (3) at no time during construction were the beams subjected to loads significantly higher than the equivalent of the dead load.

**Deflection Due to Live Load**

Each beam, as positioned in the building, is not in the simply supported condition that prevailed during loading in the testing machine. The bearing plates rest flatwise on a load-bearing wall. Although the center of the reaction is theoretically through the center of the anchor plate, it appears more logical that rotation of the beam under initial loading will take place at the inner edge of the bearing plate. Hence, the effective span would be 307.5 instead of 315.5 inches, and the deflection measured with the permanent reference marks would represent the deflection for this span. With increased loading, however, the point of rotation would shift outward from the inner edge of the support.

In the following discussion, it was assumed that:

(a) The effective span was 307.5 inches.

(b) The area of the roof subjected to uniform loading was $116.75 \times 4$ feet wide and $\frac{12}{12}$ feet long, or about 1,000 square feet. The concrete blocks used during test increased the live load by 40 pounds per square foot, and therefore they added 40,000 pounds to the test area.

(c) Modulus of elasticity was as determined from tests under quarter-point loading.

(d) Each beam was loaded uniformly along its length.

(e) The load supported by the east wall was the same as that supported by beam 1.

Appendix I presents the derivation for the elastic deformation of a tapered beam subjected to uniform loading. The formula is

$$\delta = K \frac{5}{384} \frac{WL^3}{EI}$$

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where $K$ is the taper factor for the condition of uniform loading. All values of the equation are known except $W$, which is the total uniform load on each beam. A summation of loads on each beam would then be

$$W = W_1 + W_2 + W_3 + W_4 + W_5$$

where $W_1$ is the load carried by beam 1, $W_2$ the load carried by beam 2, and so on. Since $W_5$ is assumed to be the same as $W_1$, then

$$W = 2W_1 + W_2 + W_3 + W_4$$

Individual values of $W$ were calculated for each beam based on the slope of the load-deflection curve obtained during the full-scale roof loading (fig. 12), the predetermined modulus of elasticity, and the physical dimensions. The physical dimensions also influence the value of $K$. The indicated values of load on each beam due to the live load of 40 pounds per square foot are shown in table 5.

Table 5 reveals two significant facts. First of all, the total load as measured by beam deflections was 30,400 pounds compared with an actual known value of 40,000 pounds. From this, it can be judged that the roof decking contributed about 4 percent to the stiffness. The second factor concerns load distribution. The actual load distribution between beams obviously does not conform with the percentages indicated by theory. In other words, the roof decking does not act as a continuous beam of uniform stiffness. This fact is even more evident from measurements of roof deflection and will be discussed later in this report.

**Change of Deflection With Time**

About 1-1/2 years have passed since the building was erected and as yet no definite trends of beam deflection in service are evident. Periodic measurements of beam deflection will be made, however, and such data will be used to evaluate the performance of a roof system of this type.

It is of interest, however, to note that changes in deflection were appreciable. On September 21, 1960 (table 3), the downward deflection of beams Nos. 2 and 4 was about 1/8 inch; this is equivalent to the deflection caused by a uniform live load of about 5 pounds per square foot. On May 4, 1961, this deflection was no longer evident and all beams were near zero deflection. The snow loads during the winter were minor until March 1961. On September 13, 1961, however, the deflection of beams Nos. 2 and 4 was about 1/4 inch. These observations were made about a day after heavy rainfall deposited approximately 5 inches of rain in 2 days.
Discussion of Data on Deflection of Decking

The usual engineering analysis of deflections in continuous beams would assume the length of span to be the center-to-center distance, or 116.75 inches. Actually, however, the deflection that should be considered from the standpoint of use is that relative to the inner edge of supports, or a span of 111.5 inches. It was impractical to measure deflection exactly at the inner edge of the support, so the deflection was measured over a span of 109.5 inches. This resulted in the measured deflection being slightly smaller than the true deflection, but the difference will be considered negligible in further discussion. Deflections measured over a span of 109.5 inches will be compared with theoretical deflections for a span of 116.75 inches. This is a practical comparison because actual and theoretical deflections would be considered from such bases.

The underside of the decking was coated with two applications of satin-finish interior varnish late in the spring of 1960. Care was taken to avoid applying varnish over any of the fittings used in measuring deflections.

Decking

A wide variation occurred in the modulus of elasticity of the pieces of sample decking (table 2). The average specific gravity and modulus of elasticity of the material were, however, about equal to the average of the species.

Theoretical and Observed Deflection

The maximum deflection of a four-span beam of uniform stiffness and with a uniformly distributed load can be calculated by ordinary engineering equations. For the outer spans, maximum deflection occurs at a distance of 0.440L from the center of the outer reaction and is computed from

\[ \delta_o = 0.00645 \frac{wL^4}{EI} \]

where \( w = \) load per inch of length, pounds per inch
\( L = \) span, inches
\( E = \) modulus of elasticity, pounds per square inch, and
\( I = \) moment of inertia, inches^4.

For the inner spans, the maximum deflection occurs at a distance 0.460L from the center of the middle reaction and is computed from

\[ \delta_i = 0.00188 \frac{wL^4}{EI} \]

where values are as before. The largest deflection for such a beam, therefore, occurs in the outer span and is more than three times the maximum deflection that occurs in the inner spans.
Theoretical values of maximum deflection in each span were calculated for the roof decking, using various values of \( w \) as live load, \( L = 116.75 \), \( E = 1,270,000 \), and \( I = 1/12\) bd\(^3\), where \( b \) was taken as unit width and \( d \) was the average thickness of the sample of decking. The values are presented in Table 4. At a live load of 40.5 pounds per square foot, the following comparison of average deflections is of interest.

<table>
<thead>
<tr>
<th>Bay No.</th>
<th>Observed deflection (0.001 inch)</th>
<th>Theoretical deflection (0.001 inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTER BAYS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-155</td>
<td>-209</td>
</tr>
<tr>
<td>4</td>
<td>-159</td>
<td>-213</td>
</tr>
<tr>
<td>INNER BAYS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-94</td>
<td>-120</td>
</tr>
<tr>
<td>3</td>
<td>-114</td>
<td>-163</td>
</tr>
</tbody>
</table>

In the outer bays, the average deflection was about equal to that predicted by assuming the decking to act as a continuous beam of uniform stiffness. The effective width of a panel may be expressed as some percentage of the actual width that is effective in resisting deflection due to a bending moment, and would be about 100 percent in the outer bays. In bays 2 and 3, however, the average effective widths indicated are only 50 and 40 percent of the actual widths, respectively.

The presence of butt joints in a roof deck of this type is expected to affect both strength and stiffness. Certainly, the section modulus at any cross section is reduced in proportion to the number of end joints at that cross section. Stiffness may not be reduced in as large a proportion as strength, but some reduction is to be expected. It is unreasonable, therefore, to expect that the effective width should be equal to the actual width as indicated by the deflection in the outer bays.

A comparison of theoretical and actual deflections points out clearly that the roof deck did not deflect as a continuous beam of uniform stiffness, and therefore, the deflection in each bay was not measured at the point of maximum deflection. It is likely, however, that the measurements were taken close to that point and are near the maximum for each run.

Theoretically, the maximum deflection of a continuous beam of uniform stiffness and under uniform loading will occur in the outer spans. This is to be expected, since the beam is simply supported at the outer reaction. Ratios of theoretical maximum deflection in inner spans to outer spans are as follows for 3- to 5-span beams:

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<table>
<thead>
<tr>
<th>Number of spans</th>
<th>Bay</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>2 and 3</td>
<td>0.29</td>
</tr>
<tr>
<td>5</td>
<td>2 and 4</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Although it appears reasonable to expect that the maximum deflection of panels of heavy timber decking will occur in the outer spans under uniform loading, this may not always be so.

The maximum deflection of a uniformly loaded continuous beam of uniform stiffness and equal spans may be calculated from

$$\delta = K \frac{wL^4}{EI}$$

where \(K\) is 0.00688 for a 3-span beam, 0.00645 for a 4-span beam, and 0.00656 for a 5-span beam. It is the usual industry practice\(^3\) to use the largest of these constants (0.00688) when calculating deflections of uniformly loaded members composed of controlled random layups of decking that are continuous over three or more equal spans. The constant is commonly expressed as 1/145. In addition, the effective width of the decking panel is assumed to be 0.8. On the basis of such practice, the maximum deflection of the decking in the addition to the fire-test house due to a live load of 40.5 pounds per square foot would be

$$\delta = \frac{wL^4}{145 (0.8) EI}$$

or 0.242 inch for the outer bays. It may be noted from table 4 that none of the observed values during the live load test were as large as this calculated one.

In analyzing the data from these tests, attempts were made to develop theoretical expressions that would explain the behavior of the decking under various increments of load, and thereby provide a method of predicting the deflection of such decking under various conditions of loading. None of the methods used were successful. Methods of analysis considered such factors as (1) using smaller effective widths in inner bays than those in outer bays, and developing moment and deflection curves; (2) developing moment and deflection curves, taking into account the settlement of the supports; and (3) the loading sequence as related to effective width in various runs of decking.

Additional tests are recommended on panels of heavy timber decking with spans approaching the maximum used in current industry practice. For a condition of uniform loading, panels of five spans might be evaluated, since the theoretical deflection in the center bay is about one-half that of the outer bays.

Change of Deflection With Time

Deflection of the decking varied substantially with time even though there was not a change in load conditions. This is evident from values in table 4, and from the plotted data in figure 13. The average deflection in each bay is plotted as a dot and the bracket represents the range in values at that bay.

The observed variation of deflection with time indicates that precise theoretical methods of predicting deflection under various conditions of load may be of little practical value. The greatest change occurred in bay No. 4, where the average deflection between September 21, 1960, and May 3, 1961, decreased 0.206 inch and between May 3 and September 14, 1961, increased 0.240 inch. This change is more than that resulting from a uniform live load of 40.5 pounds per square foot. No explanation is suggested as to the reason for such large variations with time.

The deflection readings taken on March 2, 1960, represent readings taken about midmorning and about midafternoon. Continual small variations of deflection are evident even though no changes in the load conditions take place. On this particular date, the change was probably due primarily to temperature differences.

The moisture content of the decking at 1-1/8-inch depth was substantially the same at each time the deflections were read. Undoubtedly, there is a variable moisture gradient in the decking that changes in service and these moisture changes, combined with temperature effects, are probably major factors contributing to deflection changes.

Periodic measurements of deflection will be made to observe the performance of the decking in service. Some piping, ducts, lights, and heaters have been installed since the building was erected, but these are suspended primarily from the decking in the south part of the building. In general, this equipment will not have an appreciable effect on the deflection in the test area.

Effect of Butt Joints on Stiffness

Butt joints affect the strength of a deck panel, but it is sometimes stated that such joints will have no appreciable effect on stiffness. The reasoning behind the latter statement is that the tongues and grooves of the decking, combined with spiking to the adjacent run of decking, will provide side fixity to transmit bending moment.

An exploratory investigation was conducted from remnants of decking. The first panel was made by clamping together eight runs of decking that were 120 inches long. Three nailing strips were then attached to the panel, one near each end and one at the center, and the clamps removed. The panel then underwent quarter-point loading in a mechanical testing machine; the span was 112 inches. The center deflection of the panel with respect to the supports was determined at each edge. Load-deflection data were taken at 100-pound load increments to a maximum load of 1,600 pounds.
As soon as this test was completed, the panel was disassembled and each run of decking was cut in two, crosswise at a point 1 foot from the center. Alternate runs were cut on alternate sides of the center. The panel was reassembled with each run in the identical position as before but now containing a butt joint. Each run was spiked to the adjacent one, in the conventional way, through the predrilled holes with 8-inch spikes, except that extra holes were drilled and spikes were used near the end joints. A top view of the panel is shown in figure 14. The butt joints were 2 feet apart and there was a continuous member between each joint. The dotted lines represent the position of the 8-inch spikes in the panel.

The assembled panel, spiked together but containing end joints, was tested on the same day as and by the same methods used for the panel with full-length runs, except that loading was stopped at 1,400 pounds. Figure 15 shows the panel in the machine and ready to test. The stiffness of the panel with end joints was only 61 percent of that for the panel with full-length runs.

These two panels were identical except for the presence of end joints. The joints were placed closer than those permitted in industry practice. They show conclusively, however, that such joints can cause an appreciable reduction in stiffness.

Conclusions

A typical roof system made of glued-laminated beams and heavy timber decking was evaluated with respect to stiffness. Based on the observations made thus far, the following conclusions are presented.

(1) The heavy timber decking does not contribute appreciably to the stiffness of the glued-laminated beams. Deflection requirements, therefore, must be based on stiffness of the beams.

(2) Panels of random-length heavy timber decking do not deflect according to the theory for continuous beams of uniform stiffness. Other methods were tried for predicting the stiffness but none were successful.

(3) Some change in deflection of the laminated beams at zero live load was observed during the first 18 months of service, but this was not large. Deflection of the decking was erratic, however, and changed appreciably during this period. The behavior of the decking indicates that precise methods of calculating deflection may be of little practical value.

(4) Concentrations of butt joints in decking will result in an appreciable reduction in stiffness.

The Appendix to this report presents the mathematical development of elastic curves for a tapered beam subjected to two-point and uniform loading.
APPENDIX I

Development of Elastic Curves for a Tapered Beam Subjected To Two-Point Loading and To Uniform Loading

Equations are developed for the elastic curve for a uniformly tapered beam, simply supported and subjected to bending. The derivations apply to (1) symmetrical two-point loading and (2) uniform loading. Curves of taper factor versus degree of taper are presented.

Symmetrical Two-Point Loading

Consider the beam shown in figure 16A. Let the origin be at the left end. The moment of inertia of the cross section of the beam with respect to the neutral surface is a function of \( x \). Any conventional approach to finding the equation of the elastic curve will be confounded with this variable second moment. In this derivation, the double-integration technique will be used.

The depth of the beam at any point is

\[
dx = d_1 + \left( \frac{d_2 - d_1}{L} \right) x = d_1 (1 + \beta x)
\]

where

\[
\beta = \frac{d_2 - d_1}{Ld_1}
\]

The corresponding moment of inertia is

\[
I_x = I_1 (1 + \beta x)^3
\]

assuming a rectangular cross section, where \( I_1 \) is the moment of inertia of the small end about its centroidal axis.

This appendix was prepared by R. L. Ethington, technologist, and E. W. Kuenzi, engineer.

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It is known, from elementary mechanics, that

$$EI_x \frac{d^2 y}{dx^2} = M_x$$

or, in this case

$$EI_1 (1 + \beta_x)^3 \frac{d^2 y}{dx^2} = M_x$$

The classical double-integration approach is to write three separate expressions for $M$ over the three spaces between loads on the beam, and to integrate each expression twice, yielding six constants of integration. There are two boundary conditions and four continuity conditions for evaluating these constants. A somewhat simpler technique, which entails evaluating only two constants, is to define the following unit step function:

Let

$$U(x - k) = \begin{cases} 
0, & x < k \\
1, & x > k 
\end{cases}$$

Then, over the entire span,

$$M_x = Px - U(x - a) \cdot P(x - a) - U(x - b) \cdot P(x - b)$$

where

$$b = L - a.$$ 

The equation to be integrated is

$$EI_1 \frac{d^2 y}{dx^2} = P \left[ \frac{x}{(1 + \beta x)^3} - U(x - a) \cdot \frac{(x - a)}{(1 + \beta x)^3} - U(x - b) \cdot \frac{(x - b)}{(1 + \beta x)^3} \right]$$
It is helpful at this point to make a transformation of variables.

Let \( z = 1 + \beta x \). Then

\[
\frac{d}{dx} \frac{EI_1}{z} = P \int_1^z \frac{z - 1}{\beta^2 z^3} \, dz - U(x - a) \int_1^z \frac{z - (1 + \beta a)}{\beta^2 z^3} \, dz
\]

\[
- U(x - b) \int_1^z \frac{z - (1 + \beta b)}{\beta^2 z^3} \, dz + C_1
\]

The integration yields

\[
\frac{d}{dx} \frac{EI_1}{z} = P \frac{z - 1}{\beta^2 z^3} \left[ \frac{1}{2} \frac{1}{z} + \frac{1}{2z^2} - U(x - a) \cdot \left[ \frac{1}{2(1 + \beta a)} \frac{1}{z} + \frac{1 + \beta a}{z^2} \right] \right]
\]

\[- U(x - b) \cdot \left[ \frac{1}{2(1 + \beta b)} \frac{1}{z} + \frac{1 + \beta b}{z^2} \right] + C_1
\]

A second similar integration yields

\[
EI_1 y = \frac{P}{\beta^3} \left\{ \frac{z - 1}{2} - \frac{1}{2z} + \frac{1}{2} \ln z - U(x - a) \cdot \left[ \frac{z}{2(1 + \beta a)} - \frac{1 + \beta a}{2z} \right] \right. \\
+ \frac{1}{2} + \ln \left( \frac{1 + \beta a}{z} \right) - U(x - b) \cdot \left[ \frac{z}{2(1 + \beta b)} - \frac{1 + \beta b}{2z} \right] \\
+ \frac{1}{2} + \ln \left( \frac{1 + \beta b}{z} \right) \right\} + C_1 \left( \frac{z - 1}{\beta} \right) + C_2
\]
The boundary conditions are:

(a) When $x = 0$, $y = 0$

(b) When $x = L$, $y = 0$

Condition (a) implies that $C_2 = 0$. Condition (b) permits evaluation of $C_1$. Details of the algebra will be omitted here. The deflection equation is

$$EI_y = \frac{PL^3}{y^3} \left\{ \frac{z^2}{2} - \frac{1}{2z} + \frac{1}{2} - \ln z + \left[ \frac{\ln \zeta \epsilon}{\eta} \right] \right. $$

$$- \frac{(y + 1)}{2} \left( \frac{1}{\zeta} - \frac{1}{\epsilon} + \frac{1}{\eta} \right) \frac{z - 1}{\gamma} - U(x - a) \left[ \ln \frac{\zeta}{z} + \frac{z}{2\epsilon} - \frac{\zeta}{2z} \right] $$

$$- U(x - b) \left[ \ln \frac{\epsilon}{z} + \frac{z}{2\epsilon} - \frac{\epsilon}{2z} \right] \right\} $$

Where

$$\zeta = 1 + \beta a$$
$$\epsilon = 1 + \beta b$$
$$\eta = 1 + \beta L$$
$$\gamma = \beta L$$

The particular case of center deflection in a quarter-point loaded beam was studied. If the center deflection is expressed as a constant multiple of the center deflection for a nontapered beam, then

$$\delta_x = \frac{L}{2} = K \frac{11}{768} \frac{WL^3}{EI_1}$$
where \( W = 2P \) and \( K \) is the taper factor. By substitution of these conditions into the deflection equation, the taper factor is found to be

\[
K = \frac{192}{11 \gamma^3} \left[ \ln \frac{\epsilon}{\xi \eta} + \frac{\gamma(7 \gamma^2 + 32 \gamma + 32)}{2(\gamma + 2)(3 \gamma + 4)(\gamma + 4)} \right]
\]

A curve of the taper factor versus the magnitude of the taper is presented in figure 18.

**Uniform Loading**

Consider the beam shown in figure 16B. The approach here is essentially the same as before. The moment anywhere in the span is given by

\[
M_x = \frac{W}{2} \left( x - \frac{x^2}{L} \right)
\]

Then

\[
EI_1 \frac{d^2 y}{dx^2} = \frac{W}{2} \left[ \frac{x - \frac{x^2}{L}}{(1 + \beta x)^3} \right]
\]

For ease of integration, the righthand side may be expressed in two fractions.

\[
EI_1 \frac{dy}{dx} = \frac{W}{2} \int \frac{x}{(1 + \beta x)^3} \, dx - \frac{W}{2} \int \frac{x^2}{L(1 + \beta x)} \, dx + \frac{W}{2} C_1
\]

Let \( z = 1 + \beta x \). Then

\[
EI_1 \frac{dy}{dx} = \frac{W}{2 \beta^2} \int \frac{z - 1}{z^3} \, dz - \frac{W}{2 \beta^3} \int \frac{z^2 - 2z + 1}{z^3} \, dz + \frac{W}{2} C_1
\]
The integration yields

\[ EI_1 \frac{dy}{dx} = \frac{W}{2\beta^2} \left\{ \frac{1}{2z^2} - \frac{1}{z} - \frac{\ln z}{\beta L} - \frac{2}{\beta L z^2} + \frac{1}{2\beta L z^2} + \beta^2 C_1 \right\} \]

A second integration gives

\[ EI_1 y = \frac{W}{2\beta^3} \left\{ -\left(1 + \frac{2}{\beta L}\right) \ln z - \frac{1}{2} \left(1 + \frac{1}{\beta L}\right) \frac{1}{z} - \frac{1}{\beta L} \ln z \right\} \]

\[ + \frac{z}{\beta L} + \beta^2 C_1 (z - 1) + \beta^3 C_2 \]

The boundary conditions are:
(a) When \( x = 0 \), \( y = 0 \)
(b) When \( x = L \), \( y = 0 \)

These conditions permit evaluation of the two constants of integration.

The deflection equation is

\[ EI_1 y = \frac{WL^3}{2\gamma^3} \left\{ -\frac{1}{\gamma} (2 + \gamma + z) \ln z - \frac{\eta}{2\gamma z} + \frac{z}{\gamma} + \frac{\eta - 1}{2\gamma} \right\} \]

\[ + \frac{z - 1}{\gamma^2} (3 + 2\gamma) \ln \eta - \frac{3}{2} \frac{z - 1}{\gamma} \]

The particular case of center deflection was studied. If the center deflection is expressed as a constant multiple of the center deflection for a nontapered beam, then

\[ \delta x = \frac{L}{2} = K \frac{5}{384} \frac{WL^3}{EI_1} \]
where $K$ is the taper factor. The taper factor may be found as

$$K = \frac{96}{5y^4} \left\{ (3 + 2\gamma) \ln (1 + \gamma) - 3 (2 + \gamma) \ln \left(1 + \frac{\gamma}{2}\right) - \frac{2(1 + \gamma)}{2 + \gamma} + \frac{\gamma}{2} + 1 \right\}$$

The curve of taper factor versus magnitude of taper turns out to be essentially the same as the curve for quarter-point loading presented in figure 17.
Table 1.—Summary of information on each of the four laminated beams at the time of the preliminary stiffness tests

<table>
<thead>
<tr>
<th>Beam No. and dimension</th>
<th>Dimensions</th>
<th>Weight:Moisture content</th>
<th>Specific:gravity</th>
<th>Modulus of elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam No. 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measured width</td>
<td>5.25 :</td>
<td>10.0</td>
<td>0.49</td>
<td>1,620</td>
</tr>
<tr>
<td>assumed depth</td>
<td>18.05 :</td>
<td>17.80 : 17.36 :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.26 : 17.78 : 17.30 :</td>
<td>16.34 :</td>
<td>17.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam No. 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measured width</td>
<td>5.25 :</td>
<td>10.5</td>
<td>0.50</td>
<td>1,700</td>
</tr>
<tr>
<td>assumed depth</td>
<td>18.21 :</td>
<td>17.80 : 17.16 :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam No. 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measured width</td>
<td>5.25 :</td>
<td>10.2</td>
<td>0.50</td>
<td>1,590</td>
</tr>
<tr>
<td>assumed depth</td>
<td>18.25 :</td>
<td>17.60 : 17.17 :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam No. 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measured width</td>
<td>5.25 :</td>
<td>11.2</td>
<td>0.49</td>
<td>1,630</td>
</tr>
<tr>
<td>assumed depth</td>
<td>18.13 :</td>
<td>17.74 : 17.21 :</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1Measurements at centerline of end reaction. Distance between North and South measuring points was 26 feet 3-1/2 inches. Beams were 26 feet 11-1/2 inches long.
2Average based on weighted dimensions.
3Average of 10 values determined with portable electric moisture meter, using insulated pins that were driven to 1-1/8-inch depth.
4Based on volume from measured dimensions and estimated ovenlyre weight.
5For purposes of calculations, depth was assumed to taper at a constant rate. The assumed value is an approximation that reasonably represents the measured values of depth.
Table 2.—Summary of data from 30 samples of 3- by 6-inch western redcedar decking tested in static bending before erection in addition to fire-test house

<table>
<thead>
<tr>
<th>Property</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moisture content</td>
<td>8.3</td>
<td>11.2</td>
<td>10.0</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>0.276</td>
<td>0.387</td>
<td>0.333</td>
</tr>
<tr>
<td>Depth or thickness</td>
<td>2.48</td>
<td>2.66</td>
<td>2.593</td>
</tr>
<tr>
<td>Modulus of elasticity...1,000 p.s.i.</td>
<td>855</td>
<td>1,680</td>
<td>1,270</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>6.68</td>
<td>8.28</td>
<td>7.67</td>
</tr>
</tbody>
</table>

1 Determined with an electric moisture meter, using insulated pins that were driven to 1-1/8-inch depth.
2 Based on volume at test and estimated ovendry weight.
Table 3.--Moisture content and deflection data of four glued-laminated southern pine beams

<table>
<thead>
<tr>
<th>Date</th>
<th>Live load</th>
<th>East Moisture (%)</th>
<th>East Deflection (in)</th>
<th>West Moisture (%)</th>
<th>West Deflection (in)</th>
<th>East Moisture (%)</th>
<th>East Deflection (in)</th>
<th>West Moisture (%)</th>
<th>West Deflection (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/1/60</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
</tr>
<tr>
<td>3/2/60</td>
<td>0 : 10</td>
<td>0 : .06</td>
<td>0 : .03</td>
<td>0 : 10</td>
<td>0 : .06</td>
<td>0 : .03</td>
<td>0 : .06</td>
<td>0 : .06</td>
<td>0 : .06</td>
</tr>
<tr>
<td>3/7/60</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
</tr>
<tr>
<td>3/14/60</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
</tr>
<tr>
<td>3/21/60</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
</tr>
<tr>
<td>3/28/60</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : 10</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
<td>0 : .01</td>
</tr>
</tbody>
</table>

DEFLECTION DUE TO DEAD LOAD OF ROOF SYSTEM

<table>
<thead>
<tr>
<th>Date</th>
<th>Deflection (in)</th>
<th>Date</th>
<th>Deflection (in)</th>
<th>Date</th>
<th>Deflection (in)</th>
<th>Date</th>
<th>Deflection (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/1/60</td>
<td>-.01</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
<td>-.26</td>
<td>3/28/60</td>
<td>-.25</td>
</tr>
<tr>
<td>3/2/60</td>
<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
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<td>-.26</td>
<td>3/28/60</td>
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<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
<td>-.26</td>
<td>3/28/60</td>
<td>-.25</td>
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<tr>
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<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
<td>-.26</td>
<td>3/28/60</td>
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</tr>
<tr>
<td>3/28/60</td>
<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
<td>-.26</td>
<td>3/28/60</td>
<td>-.25</td>
</tr>
</tbody>
</table>

OTHER DATA OBSERVED AFTER ERECTION OF ROOF SYSTEM

<table>
<thead>
<tr>
<th>Date</th>
<th>Deflection (in)</th>
<th>Date</th>
<th>Deflection (in)</th>
<th>Date</th>
<th>Deflection (in)</th>
<th>Date</th>
<th>Deflection (in)</th>
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</tr>
<tr>
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<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
<td>-.26</td>
<td>3/28/60</td>
<td>-.25</td>
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<tr>
<td>3/7/60</td>
<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
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</tr>
<tr>
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<td>-.02</td>
<td>3/14/60</td>
<td>-.23</td>
<td>3/21/60</td>
<td>-.26</td>
<td>3/28/60</td>
<td>-.25</td>
</tr>
</tbody>
</table>

1Negative values denote downward deflection; positive values upward.
2The deflection of each beam was measured when set in place in the testing machine without external load. Deflection was measured again after the decking, built-up roofing, and protective asphalt paper were in place. The protective asphalt paper resulted in a live load of 0.5 pound per square foot. The dead load of the decking and built-up roof was about 7.5 pounds per square foot.
3On March 1, deflection readings were taken when the asphalt protective layer was in place and at subsequent increments of live load; the readings at 0 live load were then estimated. Zero deflection was assumed as that occurring on March 1, just prior to the addition of the live load to the roof.
Table 4.--Deflection data for various runs of decking in the addition to the fire-test house

<table>
<thead>
<tr>
<th>Date</th>
<th>Live load</th>
<th>Deflection at run</th>
<th>Outer bays</th>
<th>Theo-Moist</th>
<th>Inner bays</th>
<th>Theo-Moist</th>
<th>Deflection at run</th>
<th>Theo-Moist</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
<tr>
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<td>0.001</td>
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<tr>
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<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
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</tr>
<tr>
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<tr>
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<td>0.001</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

1Negative values denote downward deflection; positive values upward.
2Run adjacent to opening for ventilation duct. Value not used in calculation of average.
3Calculated by usual engineering equations, assuming length of span as 9 feet 8-3/4 inches and assuming the assembled deck to act as a continuous beam of uniform stiffness.
4In bay 3, deflection read on run 28.
Table 5.—Indicated loads on each laminated beam when roof system was subjected to an increment of 40 pounds per square foot uniform live load. Total increment of live load on roof was 40,000 pounds

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>East wall</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : 2 : 3 : 4</td>
<td>10.85 : 27.1 : 26.5 : 24.7 : 10.85 : 100.0</td>
<td></td>
</tr>
</tbody>
</table>

Load carried, based on deflection...lb.: 4,180 : 10,420 : 10,200 : 9,500 : 4,180 : 38,480

Load distribution\(^2\)..........percent: 10.85 : 27.1 : 26.5 : 24.7 : 10.85 : 100.0

Theoretical distribution\(^3\)......percent: 9.8 : 28.6 : 23.2 : 28.6 : 9.8 : 100.0

\(^1\)Assumed equal to beam 1.

\(^2\)Based on values in line 1 divided by total indicated load, 38,480 pounds.

\(^3\)Distribution of load at each reaction assuming a 4-span continuous beam of uniform stiffness.
Figure 1. -- Hydraulic capsule and weighing mechanism used in static bending tests of laminated beams. Note the brass disk (reference mark) at midheight and directly above inner edge of support plate.
Figure 2. --End view of a laminated beam in the testing machine.
Figure 3. --Static bending test of a piece of decking to determine the modulus of elasticity.
Figure 4. Top view of overall layout of the addition to fire-test house showing joint spacing of heavy timber decking. (Closed circles indicate points of measurement.)
Figure 5. -- Workmen placing last few blocks during loading of roof system.
Figure 6. -- Scale, mirror, and taut nylon line used to measure center deflection of the laminated beams.
Figure 7. --Overall view of the deflection-measuring yoke.
Figure 8. --Adjustable hanger pin at end of the yoke. Underside of head is spherical. Groove in top of the head, with line, is used for calibration.
Figure 9. -- Hanger used to support end of the yoke.
Figure 10. - Micrometers near center of the yoke measure deflection of decking.
Figure 11. --Measuring deflection of a run of decking with the aid of an electrical signal incorporated with the micrometer.
Figure 12. --Deflection of four glued-laminated southern pine beams when roof system was subjected to uniform live loads.
Figure 13. --Deflection of decking on various dates, showing average and range of values.
Figure 14. -- Sketch of test panel of heavy timber decking with a concentration of butt joints near the center. Panel was about 3-1/2 feet wide and 10 feet long.

NOTE: SPIKES DRIVEN FROM OTHER SIDE OF DECKING EXCEPT FOR ONE AT THIS LOCATION.
Figure 15. --Panel of heavy timber decking ready to be tested for stiffness in bending.
Figure 16. -- A tapered beam subjected to (A) two-point symmetrical loading and (B) uniformly distributed loading.
Figure 17. -- Curve of taper factor, $K$, versus amount of taper, $d_1/d_2$, for a beam loaded at the quarter points.