STRESS-STRAIN DISTRIBUTION IN DOUGLAS-FIR BEAMS WITHIN THE PLASTIC RANGE

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WITHIN THE PLASTIC RANGE

By

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Abstract

A method is described to determine the distribution of the bending stresses in a wood beam when some of these stresses exceed the proportional limit. This is used to calculate the position of the neutral axis and the bending strength of some Douglas-fir beams. The calculated values agreed with the results obtained.

Introduction

One of the major problems in the field of timber engineering lies in the evaluation of the strength characteristics of wood beams. The present method of evaluation is usually based on elastic theory. Although this theory is admittedly inadequate to anticipate fully the true structural behavior of wood beams, it still governs present design criteria because of precedence and tradition. In view of the ever-increasing demand for efficient utilization of structural wood, however, the inelastic or plastic concept of design should also be given some thoughtful consideration. This concept envisages loading of the beam beyond the elastic range into the realm of plastic deformation to failure. It therefore provides more rational design criteria in comparison to the pure elastic theory.

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1—This report was prepared from the thesis, "A Preliminary Study of the Feasibility of Plastic Concept of Design in Wood Structures," prepared by the author as a dissertation for the Ph. D. Degree in Civil Engineering at the University of Wisconsin.

2—Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

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This study presents the beam analysis by employing the principles of the plastic theory on the bending behavior of structural materials with rectangular cross sections. Static bending tests using a two-point loading system were conducted on three clear Douglas-fir beams. The longitudinal strain distributions were determined by use of electrical strain gages attached at specific points on the critical sections of the beams. A new method was developed to determine the corresponding longitudinal stress distributions from these strain distributions. This method involves the determination of the relation between the strain levels imposed and the residual strains at the moment the stress is removed. This relation was obtained from tests of minor compression specimens that were cut from the ends of the beam specimens. The internal bending strength of each beam was computed from the resulting longitudinal stress distributions, and compared with the corresponding external bending strength.

To apply the concept of plastic design to wood beams, this study is designed to investigate the following:

(1) The actual stress-strain distribution within the plastic region in Douglas-fir beams.

(2) The correlation between the ultimate strength of Douglas-fir beams and the stress distribution that occurs in them by using Nadai's plastic analysis on beams with rectangular sections.

Review of Theory

Probably the most widely used and accepted theory for design of beams is the elementary elastic theory. It is formulated on the basis that, within certain limits, stresses remain linearly proportional to the accompanying strains. Further, it assumes that plane sections of a beam remain plane during bending. Nadai (10)\(^3\) points out that the validity of this assumption for beams subjected to pure bending has been shown particularly by C. Bach for materials that do not obey Hooke's law and possess no straight-line stress-strain curve when subjected to tension or compression, and by E. Meyer for wrought iron. MacCullough (8) confirms the assumption further for all materials having uniform properties along the length of the beam.

\(^3\) Underlined numbers in parentheses refer to Literature Cited at the end of this report.
When the elastic theory is applied to the mechanics of wood, it predicts within certain limits the flexural behavior of beams. The theory applies to wood beams of any cross section if the tensile strength is reduced by defects or other causes so that tensile failure occurs before the compression stress reaches the proportional limit value. In this case, the stress-strain distribution at the critical section of the beam is linear even up to failure. In clear wood, however, it is found that the proportional limit in bending as determined by standard bending test is much greater than that determined by the compression test parallel to the grain. In fact, it usually exceeds the maximum crushing strength \((9)\). A question remains whether wood is actually stronger in compression when it is bent than when it is merely compressed. Markwardt and Wilson \((9)\) state that the determination of the bending fiber stress at the proportional limit is subject to experimental errors. Therefore, another question arises: Are the experimental errors large enough to account for the greater strength in bending?

The modulus of rupture is usually used for comparing the bending strengths of different species. Since it is based on the maximum load, it is less affected by experimental error than is the proportional limit. The modulus of rupture is the fictitious bending stress at the critical section of the beam at the time of failure. It is computed from the bending moment obtained in a standard bending test \((1)\) by the usual elastic bending formula:

\[
S = \frac{Mc}{I}
\]  

where \(S\) is bending stress, \(M\) is bending moment, \(c\) is distance of extreme fiber from the neutral axis, and \(I\) is moment of inertia of the cross section of the beam.

Usually, however, the linear stress-strain relation on which the elastic theory is based does not hold true within the failing range of the beam. The theory then becomes inadequate to explain why beams of the same material but with different shapes of cross section possess different moduli of rupture. W. G. Youngquist and E. R. Dawley at the Forest Products Laboratory showed in 1947 that, for a rectangular cross section, the modulus of rupture decreases as the height of the beam increases. Newlin and Trayer \((12)\) found that a circular beam yields a higher modulus of rupture than a square-section beam having the same section modulus, and if the square beam is loaded on a corner it exhibits a still higher modulus of rupture.

To account for the difference in shapes of wood beams, a "form factor" is applied to the modulus of rupture. The form factor is the ratio of the modulus of rupture of a wood beam of any cross section to that of a standard 2-by
2-inch cross section. J. B. Kommers and C. B. Norris, in exploratory work at the Forest Products Laboratory in 1920, calculated the form factors for I and box beams by applying Neeley's idealized stress-strain curve (17) in Nadai's theory of bending (10). In a separate study, Newlin and Trayer (12) determined the form factors for a number of beam sections by the "Fiber Support Theory."

The theory of fiber support assumes that the minute fibers of the wood, which are more or less bound together, act as miniature Euler columns when subjected to compression along their length. It contends that when stress is uniformly applied on all the fibers, little support is offered by one fiber to adjacent fibers; when the stress applied is nonuniform, as in a bent beam, the fibers nearer the neutral axis, being less stressed, offer lateral support to the adjacent extreme fibers, and cause them to sustain higher stress. The theory accounts for the increase in compression strength in bending and conforms with the flexural behavior of wood beams in the failing range.

According to Norris in recent work at the Forest Products Laboratory, the fiber support theory can account for the lack of supporting wood under the compression flanges of I and box beams, but the fact that a circular cross-section beam has a form factor greater than unity is not explained. Moreover, Norris stated that the test results for the beams reported by Newlin and Trayer (12) showed excellent agreement with the form factors derived by W. S. Cottingham, who used the method of analysis of J. B. Kommers and Norris.

A further application of the fiber support theory was investigated by Youngquist and Dawley on very deep laminated beams. The investigation was based on the assumption that the lower limit for the modulus of rupture is the maximum crushing strength parallel to the grain. The results showed that the decrease in modulus of rupture with increasing height of beam is greater than that obtained by Newlin and Trayer. The mathematical analysis of this study is reported by Freas and Selbo (6).

An analysis to prove the mathematical validity of the fiber support theory was conducted by Bechtel and Norris (2). The test was conducted by calculating the load-deflection curve of a beam from the stress-strain curves that were obtained from the compression tests on the specimens matched to the beam. It was assumed that the modulus of elasticity in tension is the same as that in compression. Owing to the accurate results of this study, Norris is of the opinion that the fiber support theory should be abandoned.

An analysis of the bonding strength of a beam was developed by Bechtel and Norris (2), who used a simplified longitudinal stress distribution. This
analysis included the effect of the shear stress. Theoretical equations were derived for beams of rectangular cross section, and values computed with these equations checked reasonably well with experimental results. This method of analysis was extended to two-species beams by Ethington (4).

An earlier study conducted by Prager (14) on double-flanged wood beams also used the same longitudinal stress distribution. According to Bechtel and Norris (2), the equations developed by Prager were similar to those obtained by J. B. Kommers and Norris. Kon (7) independently developed the distribution of shear stress associated with the simplified longitudinal stress distribution. His method leads to the same shear-stress distribution as that given by Bechtel and Norris. Smith and Voss (16) derived a more accurate shear distribution in a beam of orthotropic material resulting from a concentrated load. Norris (13) analyzed the effect of combined stresses on the strength of orthotropic materials.

In the development of a mathematical analysis of the flexural behavior, it is necessary to have some knowledge of the mechanical characteristics of wood. When a clear-wood beam is bent, the fibers are stressed as a consequence of the bending. This stress in the fibers varies across the depth of the beam. Figure 1 illustrates the stress variation, as well as the corresponding strain variation, at the critical section of the beam in relation to a progressive bending deformation. Within the elastic limit of the wood beam, the stress and strain variations are linearly distributed (fig. 1A).

The relation between the stress and the strain, as expressed by the elementary elastic theory, is given by:

$$ S = E\delta $$

(2)

where $S$ is bending stress within the elastic limit of the beam, $E$ is modulus of elasticity, and $\delta$ is bending strain deformation within the elastic limit of the beam.

As the stress is gradually increased beyond the proportional limit, the extreme fibers on the compression side of the beam start to yield plastically. This occurs because wood is much stronger in tension than in compression parallel to the grain. Moreover, the stress-strain curve obtained from a standard compression test of wood shows that the proportional limit strength is reached before the maximum crushing strength, whereas in a tension test the stress-strain curve shows substantially a straight line to failure. The compression failures in wood beams are often not prominent, sometimes being almost invisible. This has often led to the erroneous conclusion that tension failures occur before there is a compression failure (12).
As the wood beam is bent, the fibers on the compression side reach their elastic limit and deform plastically before the fibers on the tension side fail. As a result, a stress redistribution occurs. The neutral axis is shifted toward the tension side (fig. 1C). The process of stress redistribution continues with increasing bending moment until tension failure results (fig. 1C). At this point the bending moment usually decreases.

At any stage of beam deformation beyond the proportional limit, the stress and strain across the depth are related as follows:

1. In the elastic portion of the beam, as indicated by the straight-line segment of the stress distribution figures 1B and 1C, the stress-strain relation is expressed by equation (2).

2. In the plastically deformed portion of the beam, as indicated by the curve-line segment of the stress distribution in the compression side, the stress-strain relation is given by:

\[ S_p = E (\delta_p - \delta_r) \]  

where \( S_p \) is bending stress within the plastic region, \( \delta_p \) is strain deformation within the plastic region, and \( \delta_r \) is residual strain.

3. The stress distribution across the depth of the beam is such that the conditions of equilibrium are satisfied. Hence,

\[
\text{Internal compression force} = \text{internal tension force} \quad (4)
\]

\[
\text{Internal moment} = \text{external moment} \quad (5)
\]

For any material of known stress-strain distribution, Nadai (10) derived from equations (4) and (5) a general equation for the bending strength of a beam.

From the above discussion, it is seen that the reliability of a theoretical flexural analysis to predict the characteristics of wood beams at failure depends to a large extent on the accuracy to which the stress-strain distribution can be determined. Bach and Bauman, according to Dietz (3), proposed the first approximation for this distribution for wood beams by superimposing the stress-strain curves obtained from tests of compression and tension specimens (fig. 2). A similar approximation is the simplified longitudinal stress distribution \((2, 7, 14, 17)\) as shown in figure 3. This stress distribution, unlike that proposed by Bach and Bauman, assumes the same modulus of elasticity for both tension and compression. The validity of this assumption is pointed out by Sawada (15). Kuenzi in work at the
Forest Products Laboratory found in 1945 a difference between the modulus of elasticity in compression and tension. The difference, however, was not significant for Douglas-fir.

By applying Nadai's theory (10) to the simplified longitudinal stress distribution, as shown in figure 3, Bechtel and Norris (2) derived the following equations:

\[
\frac{2Scd}{Sc + St} = h
\]

\[
M = Scbd^2 \left[ \frac{2Sc}{3(Sc + St)} \right]^{1/2}
\]

where \( h \) denotes depth in the beam at which stress is proportional to the strain, \( d \) is depth of beam, \( b \) is width of beam, \( Sc \) represents compressive stress in the beam, \( St \) is tensile stress in the beam, and \( M \) is bending moment.

**Investigations**

**Preparation of Specimens**

The wood species used in this investigation was selected on the basis of its general usage for structural purposes. Three air-dried Douglas-fir beams, each having a nominal size of 3 by 8 inches by 14 feet, were selected from the commercially obtained stock at the Forest Products Laboratory. The beams were flat-sawn and free of knots and other visible strength-reducing characteristics. A rough check on the moisture content of the beams, using an electric moisture meter, yielded an average value of about 14 percent. This moisture content was deemed low enough for dressing the beams to the required size of 2 by 5-7/8 inches by 12 feet. The dressed beams were stacked to allow free air circulation in a conditioning room to lower the moisture content to about 12 percent.

Douglas-fir plywood stiffeners, 3/8 by 6 by 45 inches in size, were glued to the outer ends of each beam (fig. 4). The purpose of the stiffeners was to decrease the likelihood of shear failures near the ends of the beam.

At the critical section of each beam, two groups of 20 SR-4 strain gages were attached at cross sections 18 inches from the points of loading. This distance amounted to about three times the depth of the beam, and was considered
enough to avoid the stress concentration effect of the beam loads on the desired performance of the strain gages. For purposes of reference, the relative location of the first group of strain gages is called "Plane I," and that of the second group, "Plane II." The strain gages were of 1-inch gage length with an average gage factor of about 2.05. They were distributed at intervals across the faces of the beam, as shown in figure 4, and oriented to measure the longitudinal strain deformation of the beam. They were glued in place in accordance with standardized procedure. A 6-foot length of copper wire, size No. 22, was soldered to each of the terminals of the strain gage for wiring purposes.

Three graduated steel scales were placed at three selected points on the critical section to obtain the deflection of the beam, as shown in figure 5.

Procedure

The positioning of each beam specimen in the test jig is shown in figure 5. Initial reading at zero load was recorded from each strain gage on the beam specimen using an electronic Baldwin SR-4 strain indicator. A dummy, consisting of a similar strain gage glued to a small piece of Douglas-fir wood, was used in conjunction with the bridge balancing unit. Its purpose was to compensate for any change in the resistance of the strain-measuring gage as a result of a change in temperature and moisture content of the specimen. The strain gages were then connected to an automatic electronic strain recorder in such a way as to record twice the average longitudinal strain resulting in the beam at the various levels.

Static bending was conducted by gradually loading the beam specimen at two selected points, as shown in figures 4 and 5. This two-point loading system subjected the beam specimen to a constant bending moment and to zero shear along its critical section. Therefore, it provided an excellent check of the longitudinal strain readings from the strain gages. The load was applied perpendicularly to the longitudinal axis of the beam specimen at the rate of motion of the movable crosshead of the testing machine of about 0.14 inch per minute. The continuity of the gradual application of load on the beam specimen was momentarily stopped for about 10 to 20 seconds at every predetermined load level to record longitudinal strain readings on the strain recorder, and to take deflection readings from the three graduated scales. The test was carried to failure.

Immediately after the static bending was completed, two 3-inch-long pieces containing the strain gages were cut from the beam. Each piece was further cut into small minor specimens, about 1/2 by 2 by 3 inches in size, as shown in figure 6. This size was considered small enough to relieve the stresses within the wood. Final readings were taken from each of the strain gages on
the minor specimens with the same strain indicator previously used. Values of residual strain at various depths of the beam were then obtained by subtracting the initial readings from the final readings on the strain gages. During the cutting process, special precautions were undertaken not to damage the strain gages and to keep the cutting time to a minimum.

The values of the moduli of elasticity at points where the strain gages were located on the beams were determined from tests of the minor specimens in compression parallel to the grain. The test was conducted by applying the load continuously at the rate of motion of the movable crosshead of the testing machine of about 0.006 inch per minute. This rate of loading was reached by approximating the arithmetical average rate of longitudinal deformation in the beam test.

Strains were recorded from the pair of strain gages on each minor specimen with the bridge balancing unit previously used.

For purposes of comparison, two to four of these minor specimens taken from each beam were tested to failure. These specimens were selected from among those located near the neutral axis in the beam. The other minor specimens were tested only up to the range of their respective proportional limits.

Observations and Results

The vertical distribution of the longitudinal strain at the critical section of each beam at different loads is plotted in figures 7 to 9. The numerical values of the longitudinal strain at failure are given in table 1. Observations of these distributions show substantially a linear variation across the depth of the beam. The deviations of some longitudinal strain values from the linear variation may be accounted for by the experimental limitations in the placement of the individual strain gages. Even with the utmost care, no two strain gages can be glued to the wood exactly in the same manner. Furthermore, the accuracy in the orientation of the individual strain gages with respect to the longitudinal axis of the beam can only be checked by visual alignment.

Table 1 also shows the average values of the residual strain at the different locations of the strain gages in each beam specimen. These are values obtained directly from the strain gages on the beam specimen after it was cut. These recorded residual strain values are compressive on the tension side of the beam as well as the compression side. Those on the tension side may have been caused by two factors: (1) The dynamic resilient effect of the sudden tensile failure in the beam during the static bending test, or (2) the shortening, because of residual strain, of the compression side of the beam, which tends to compress the fibers on the tension side when the wood is removed by failure.

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The moduli of elasticity given in table 1 represent the results of tests conducted on the minor specimens in compression parallel to the grain. Although almost one-half of the minor specimens were stressed in tension in the beams, the application of the compression test to determine the moduli of elasticity was necessary in view of the minute size required of the minor specimens for the determination of the residual strain. The fact that the modulus of elasticity as determined by compression test is the same as that determined in tension on the same wood has been previously stated by Kuenzi and by Sawada (15). A Forest Products Laboratory report (5) on the effect of the stress repetitions on the bending and compressive strengths of wood states that the modulus of elasticity is greater during the first loading than at any subsequent repetition of loading. The values of the modulus of elasticity given in table 1 appear to have more spread than those given by the above report.

Of the three beams tested, accurate load results were obtained only in beam No. 3. The testing machine used was found to have indicated erroneously load readings beyond the 1,800-pound load level during the test for beams Nos. 1 and 2. While this was unfortunate, the strain data obtained from the strain gages were still valuable in the determination of the strength characteristics of the beams. The load-deflection curves at three different sections of beam No. 3 are plotted in figure 10.

Analysis of Results

The results of this study apply to clear Douglas-fir beams where the critical section is subjected only to pure bending moment. Pertinent adjustments should be applied to the results if used under conditions different from those of this study.

The linear variation of the longitudinal strain distributions in figures 7 to 9 satisfactorily shows the validity of the usual assumption that plane sections remain plane during the bending of wood beams. Further, these distributions clearly illustrate the gradual shift of the neutral axis toward the tension side of the beam. This is consistent with the previous explanation of the flexural behavior of wood beams, as illustrated in figure 1.

The longitudinal stress distributions at failure corresponding to the longitudinal and residual strain distributions obtained from each beam were determined by applying equation (3). Graphical comparison shows that the position of the neutral axis as determined correctly from each strain distribution differed considerably from that determined from the corresponding stress distribution. Consequently, the static equilibrium of the beam failed to satisfy equations (4) and (5). To account for the discrepancy in the determination of the stress distributions, the validity of the residual strain
values given in table 1 was re-examined. Because the time that elapsed in cutting the minor specimens from each beam was about 45 minutes, it was decided to investigate the amount of strain recovery that had occurred in the minor specimens before the final residual readings were made, and also to determine the effect, if any, of heat introduced in the strain gages during the cutting process.

The investigation of the effect of heat on the strain gages was conducted by heating a number of selected minor specimens in an open oven. The results showed insignificant changes in the strain gage readings.

The investigation of the effect of strain recovery on the residual strain was conducted on 13 additional minor specimens. These specimens were cut from the end portions of the beam specimens. They were processed and prepared similarly to those shown in figure 6. The test was conducted in compression parallel to the grain. Load was gradually applied along the longitudinal axis of each specimen at the rate of loading equal to that used for the previous minor specimens. Strain values were recorded from a pair of strain gages glued to each specimen, using an electronic strain indicator. During each test, load-strain data were recorded until a predetermined strain level was reached. At this point, the sustained load on the specimen was immediately released. Simultaneously, the time of unloading was recorded. Residual strain readings together with the corresponding time intervals after unloading were then taken from the specimens for the duration of about 1 hour. On the average, the first residual strain reading was recorded about 10 to 15 seconds after the sustained load was completely released. These results are plotted (solid lines) in figure 11. The required residual strain values at the instant the sustained load reached zero were obtained by extrapolation from the residual strain-time lapse curves. However, the evaluations of the static equilibrium of the beams using the extrapolated residual strain values obtained in this way were again found to be erroneous.

A different approach, which later proved to be accurate, was applied on the strain level and load-strain data. Figure 12 illustrates the method of analysis. The method of analysis was based on the following concept pointed out by Nadai and W. J. Kommers (11, 5): A compression specimen subjected to a gradually applied load follows a typical stress-strain curve. If at any time the sustained load is gradually released at the same rate it was applied, the stress-strain curve descends along the straight line substantially parallel to the straight-line portion of the curve within its proportional limit. From figure 12:
Similarly, \[ \Delta \delta = \frac{P}{AE} \] (9)

Substituting equation (9)

\[ \Delta \delta = \frac{P\delta_e}{P_e} \]

Also

\[ \delta_r = \delta_L - \Delta \delta \]

Equation (11) gives an expression for the residual strain at the instant the sustained load on the compression specimen is completely released. For purposes of reference, this strain is called the "instantaneous residual strain."

The values of the instantaneous residual strain for the strain-recovery study were computed from the above analysis for the different strain levels. The values were plotted at the zero-time ordinate in figure 11. They were connected by dotted lines to the corresponding residual strain-time curves to illustrate the relative amount of strain recovery with respect to time. Examinations of the curves show an abrupt decrease in the residual strain values within the first 5 to 15 seconds that accounted for the inaccuracy of the extrapolation method above the 2,000 microinches per inch strain level. Below this strain level, the extrapolation method is more nearly correct. Figure 13 shows the graphical relation of the instantaneous residual strains to that of the different strain levels.

It is significant to note that the slope of the straight-line portion of this curve is very nearly 1 to 1. This indicates that any further deformation in the specimen above a certain strain level in the plastic region becomes a part of the instantaneous residual strain. Further, the curve is quite smooth even though the various specimens were not matched.
The values of the instantaneous residual strain corresponding to that of the compressive longitudinal strain in the beams were determined graphically from figure 13. On the tension side of the beams, the instantaneous residual strains were assumed to be negligible since the stress-strain curve in a standard tension test is essentially a straight line to failure. For purposes of comparison, the instantaneous residual strain distributions are plotted in figures 14 to 19 together with the distributions of the longitudinal and residual strains given in table 1. The longitudinal stress distributions computed from equation (3) are also shown in figures 14 to 19. It is noted that each longitudinal strain distribution shows the same position of the neutral axis as that determined from the corresponding longitudinal stress distribution.

The method of determining the longitudinal stress distribution in a wood beam loaded beyond its proportional limit is one of the major contributions of this study.

The bending strength of each beam specimen was computed from the longitudinal stress distributions at Planes I and II. The computation was simplified by dividing the area under the stress distribution curve into sections, as indicated in figures 14 to 19. The procedure of computation consisted of determining the average internal forces acting at the different sections of the beam specimen and, consequently, the corresponding internal moment from the neutral axis. Accuracy of the test results is shown by the comparison of the total internal forces, $\Sigma F_{iC}$ and $\Sigma F_{it}$, acting at the compression and tension sides of the beam specimen, respectively, and of the total internal moment, $\Sigma M_i$, to the external moment, $M_e$.

A summary of the computed bending strength of the beams is shown in table 2, together with that of the maximum compressive stress values of the matched minor specimens, which were selected for tests up to the range of failure. Comparison of the values in columns 3 and 4 shows that the maximum crushing stresses of the beams and the minor specimens agree satisfactorily. This indicates that the longitudinal compressive strength in a wood beam is substantially the same as its maximum crushing strength parallel to the grain. The significant difference in some of the stress values given in columns 3 and 4 may be accounted for by the fact that the point at which the maximum compressive stress occurred in each beam does not coincide with the point at which the minor specimen was cut.

The equations of static equilibrium apply satisfactorily to the longitudinal stress distributions in the beams. Comparison of the values of internal forces and internal moments given in table 2 shows only a slight difference. For beam No. 3 the internal moments computed at Planes I and II agree closely with the external moments that are computed from the external loads.
on the beam. The close agreement in the values of the computed moments in beam No. 3 gives an indication of the degree of accuracy of the bending strengths of beams Nos. 1 and 2 as computed from the longitudinal stress distributions at Planes I and II.

For purposes of comparison, bending strengths of the beams were computed from equation (7). This equation was derived from the simplified longitudinal stress distribution suggested by Neeley (17) by applying Nadai's theory (10). The values of the computed moments are also given in table 2. Except for the computed bending moment at Plane I in beam No. 3, these values agree reasonably well with the internal moments. In general, the differences in the moment values given in columns 9, 10, and 12 may be attributed primarily to the widespread values of the modulus of elasticity given in table 1.

A check on the accuracy of the longitudinal stress and strain distributions in the beams is illustrated in figures 20 to 22. Comparison of the curves shows that the compressive stress-strain curves obtained from the minor specimens approximate closely the compressive portion of the longitudinal stress-strain curves in the beams. It is noted from the longitudinal stress-strain distributions that considerable plastic deformation is developed on the compression side before the beam specimens finally fail on the tension side. The development of plastic deformation in the failing range is illustrated further from the load-deflection curves of beam No. 3 in figure 10. These curves show clearly that the beam specimen is stressed beyond the proportional limit at its critical section.

Conclusions

1. The usual assumption in beam analysis that plane sections remain plane during bending was satisfactorily proven.

2. The neutral axis of the beam gradually shifted toward the tension side as a result of the stress redistribution across the critical depth.

3. A new method was developed to determine the longitudinal stress distribution in a wood beam loaded beyond its proportional limit.

4. The internal bending strengths of the beam specimens, which were computed from the stress distributions occurring in the beams by using Nadai's plastic analysis, were reasonably accurate.

5. The computed internal bending strengths of the beam specimens agreed reasonably well with those computed using the simplified longitudinal stress distribution.
6. The compressive strength of beams in bending agreed with the maximum crushing strength parallel to the grain.

7. The stress-strain curves of minor specimens tested in compression parallel to the grain closely approximated the compressive portion of the stress-strain curves occurring in the beams.

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(15) Sawada, M.

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The load data for Beams Nos. 1 and 2 were discarded due to erroneous load readings in the testing machine used.
Table 2.--Summary of the results of computation for the strength of the beam specimens

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<th>Beam No.</th>
<th>Section</th>
<th>Maximum Compression</th>
<th>Maximum Longitudinal</th>
<th>Summation of Internal Forces</th>
<th>Summation of Tensile Forces</th>
<th>Difference Between</th>
<th>Summation: External Moment</th>
<th>Difference Between (9) and (10)</th>
<th>Bending Moment Computed from Equation (7)</th>
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<td>(6)</td>
<td>(7)</td>
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1. The values given for Beam No. 3 are the results of two compression tests.
2. The maximum load sustained in Beams Nos. 1 and 2 are discarded due to erroneous load readings in the testing machine used.
Figure 1.--Strain and stress variations across the depth at the critical section of the beam in relation to a progressive bending deformation. A is bending deformation; B, strain variation; and C, stress variation. For Band C, (a) indicates compressive and tensile stresses within the proportional limit; (b) indicates these stresses at yield point; and (c) indicates bending stresses within the plastic region.
Figure 2.--Stress-strain distribution in wood beam as proposed by Bach and Bauman: \( A \), Stress-strain curves from axial tension and compression tests; \( B \), superimposed stress-strain distribution.

Figure 3.--Simplified longitudinal stress distribution in a wood beam.
Figure 4. --Detail sketch of typical beam specimen showing relative placement of strain gages, load points, support, and plywood stiffeners.
Figure 5. -- Beam specimen in testing machine, showing three attached graduated steel scales, and a pair of strain gages that are connected by a cable to an electronic strain indicator.

Figure 6. -- Typical minor specimens cut from each beam specimen for the determinations of residual strains and moduli of elasticity.
Figure 7. --Longitudinal strain distributions at A, Plane I, and B, Plane II, on the critical section of Beam No. 1 under different loads.

Figure 8. --Longitudinal strain distributions at A, Plane I, and B, Plane II, on the critical section of Beam No. 2 under different loads. The dotted lines in Plane II show extrapolated longitudinal strain distribution. Initial tension failure in beam damaged the strain gages before the beam sustained its maximum load.
Figure 9. -- Longitudinal strain distributions at A, Plane I, and B, Plane II, on the critical section of Beam No. 3 under different loads.

Figure 10. -- Load-deflection curves at three different points of Beam No. 3.
Figure 11. -- Curves showing the progressive decrease in residual strain with respect to time at various longitudinal strain levels.

Figure 12. -- Typical stress-strain curve in compression parallel to the grain, illustrating the method of determining the residual strain corresponding to a particular strain level.
Figure 13. -- Curve showing the relation between the residual strain and the corresponding strain level.

Figure 14. -- Distributions of residual strain, longitudinal strain, and stress at failure at the critical section Plane I of Beam No. 1: A, Distributions of residual strains and longitudinal strain; B, stress distribution.
Figure 15. --Distributions of residual strain, longitudinal strain, and stress at failure at the critical section Plane II of Beam No. 1: A, Distributions of residual strains and longitudinal strain; B, stress distribution.

Figure 16. --Distributions of residual strain, longitudinal strain, and stress at failure at the critical section Plane I of Beam No. 2: A, Distributions of residual strains and longitudinal strain; B, stress distribution.
Figure 17.--Distributions of residual strain, longitudinal strain, and stress at failure at the critical section Plane II of Beam No. 2: A, Distributions of residual strains and longitudinal strain; B, stress distribution.

Figure 18.--Distributions of residual strain, longitudinal strain, and stress at failure at the critical section Plane I of Beam No. 3: A, Distributions of residual strains and longitudinal strain; B, stress distribution.
Figure 19. --Distributions of residual strain, longitudinal strain, and stress at failure at the critical section Plane II of Beam No. 3: A, Distributions of residual strains and longitudinal strain; B, stress distribution.

Figure 20. --Graphical comparison between the stress-strain curves of Beam No. 1 and those of the matched minor specimens tested in compression parallel to the grain: A, At Plane I; B, at Plane II.
Figure 21.—Graphical comparison between the stress-strain curves of Beam No. 2 and those of the matched minor specimens tested in compression parallel to the grain: A, At Plane I; B, at Plane II.

Figure 22.—Graphical comparison between the stress-strain curves of Beam No. 3 and those of the matched minor specimens tested in compression parallel to the grain: A, At Plane I; B, at Plane II.